Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.2.3-g-sec-^p-a+b-sec-^m-c+d-sec-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [286]. This is test number [122].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (286)	% 0. (0)
Mathematica	% 95.45 (273)	% 4.55 (13)
Maple	% 91.61 (262)	% 8.39 (24)
Maxima	% 58.04 (166)	% 41.96 (120)
Fricas	% 81.12 (232)	% 18.88 (54)
Sympy	% 0.35 (1)	% 99.65 (285)
Giac	% 67.13 (192)	% 32.87 (94)

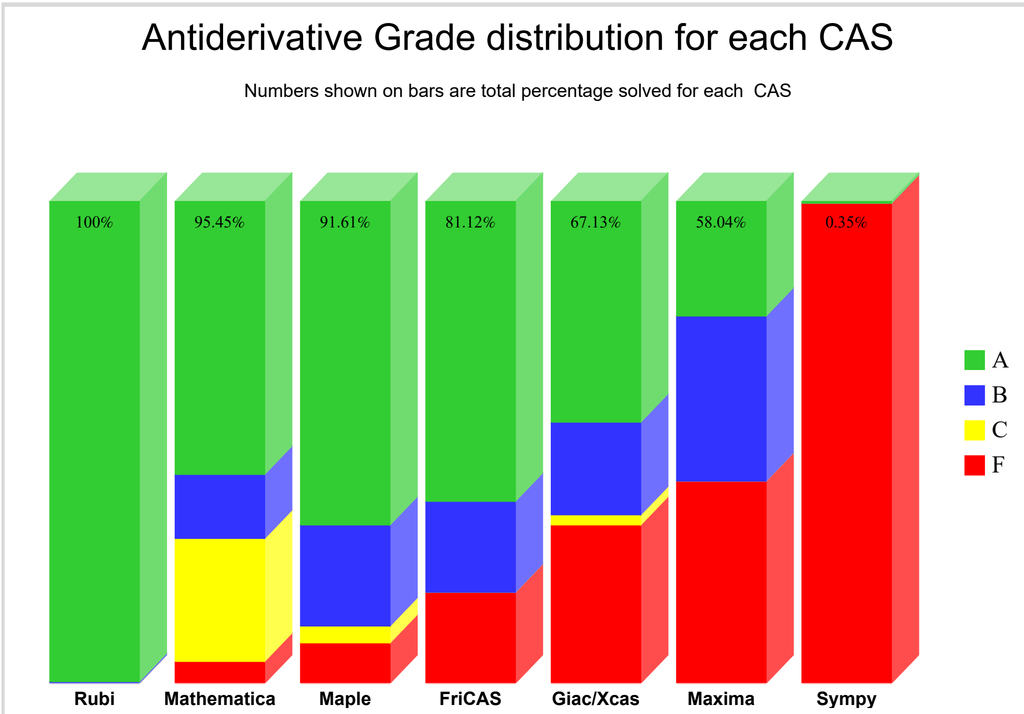
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

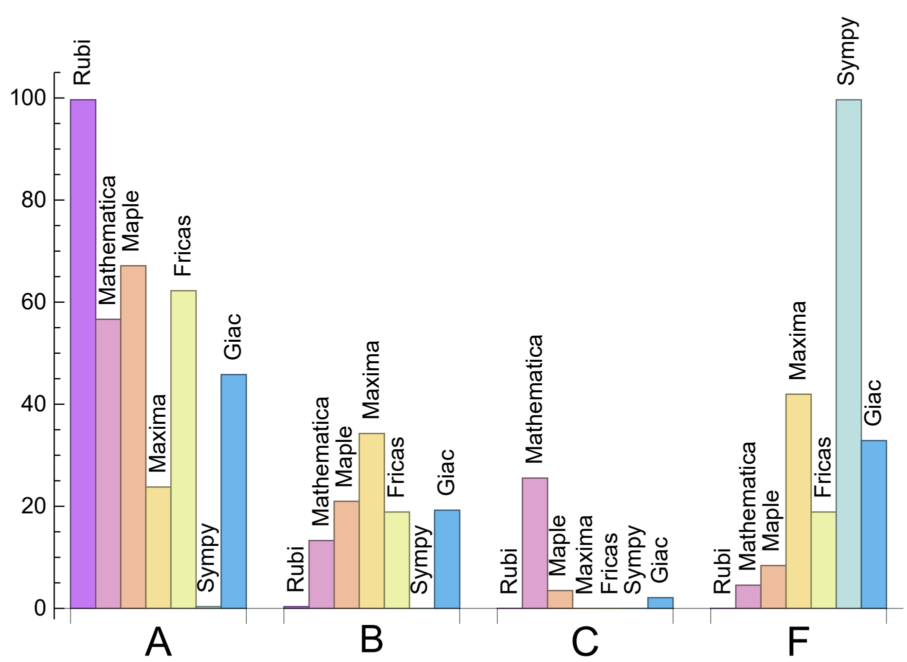
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.65	0.35	0.	0.
Mathematica	56.64	13.29	25.52	4.55
Maple	67.13	20.98	3.5	8.39
Maxima	23.78	34.27	0.	41.96
Fricas	62.24	18.88	0.	18.88
Sympy	0.35	0.	0.	99.65
Giac	45.8	19.23	2.1	32.87

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.26	135.98	1.05	121.	1.
Mathematica	2.46	1430.54	10.18	154.	1.35
Maple	0.17	214.4	1.58	147.5	1.33
Maxima	2.84	612.2	5.99	295.5	2.98
Fricas	1.88	756.91	5.81	411.5	4.17
Sympy	6.85	53.	3.12	53.	3.12
Giac	1.67	284.18	1.98	183.	1.83

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {42, 53, 77, 162, 163, 164, 174, 175, 176, 178, 180, 182, 216, 224, 232, 240, 265, 269, 275, 277}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

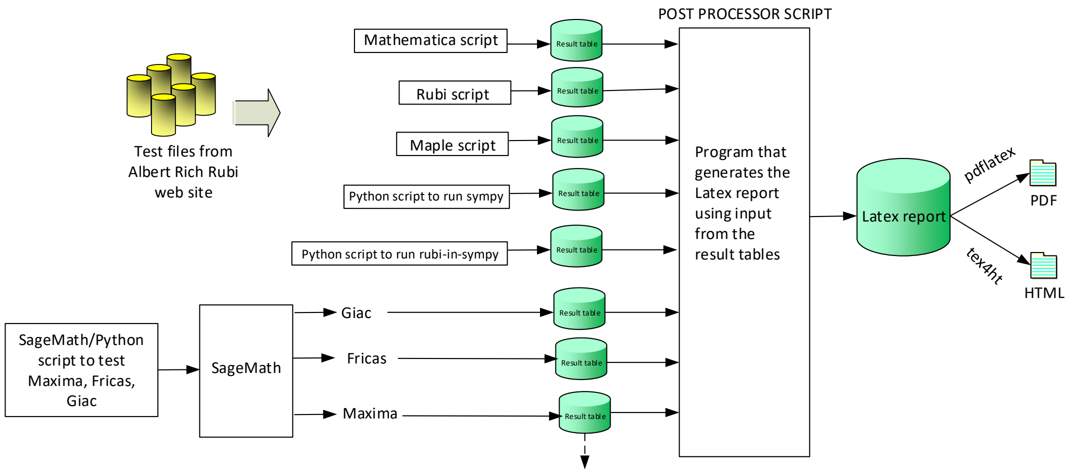
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286 }

B grade: { 197 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 37, 38, 39, 40, 41, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 71, 72, 73, 74, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 102, 103, 108, 109, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 130, 131, 132, 141, 146, 147, 152, 153, 168, 169, 170, 173, 175, 178, 179, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 200, 201, 202, 203, 209, 217, 218, 221, 226, 227, 229, 230, 234, 235, 236, 237, 238, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 259, 260, 261, 262, 263, 264, 266, 267, 268, 271, 276, 279, 281, 282, 283, 286 }

B grade: { 1, 2, 3, 15, 27, 28, 34, 35, 36, 42, 43, 44, 54, 61, 62, 107, 126, 171, 172, 180, 181, 195, 196, 204, 210, 211, 212, 213, 219, 220, 225, 228, 252, 253, 258, 273, 275, 285 }

C grade: { 68, 69, 70, 75, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 104, 105, 106, 110, 117, 118, 127, 128, 129, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 148, 149, 150, 158, 162, 163, 164, 165, 174, 176, 183, 197, 198, 199, 205, 206, 207, 208, 214, 215, 216, 222, 223, 224, 231, 232, 233, 239, 240, 265, 269, 270, 272, 277, 280, 284 }

F grade: { 151, 154, 155, 156, 157, 159, 160, 161, 166, 167, 177, 274, 278}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 108, 109, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 131, 132, 133, 134, 136, 137, 138, 139, 141, 142, 143, 144, 148, 149, 150, 168, 169, 171, 172, 173, 181, 182, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 208, 209, 213, 214, 215, 216, 220, 221, 222, 223, 224, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 249, 250, 251, 255, 256, 257, 262, 263, 264, 265, 266, 268, 269, 273, 275, 276, 279, 282, 285, 286 }

B grade: { 69, 70, 77, 78, 91, 92, 98, 99, 106, 107, 110, 118, 128, 129, 130, 135, 140, 145, 146, 147, 170, 178, 179, 180, 183, 184, 189, 197, 198, 205, 206, 207, 210, 211, 212, 217, 218, 219, 225, 226, 227, 228, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 252, 253, 254, 258, 259, 260, 261, 281 }

C grade: { 267, 270, 271, 272, 274, 277, 278, 280, 283, 284 }

F grade: { 38, 48, 60, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 174, 175, 176, 177 }

2.1.4 Maxima

A grade: { 2, 3, 4, 7, 8, 9, 14, 26, 38, 39, 40, 41, 47, 48, 49, 50, 51, 57, 58, 59, 60, 61, 62, 63, 86, 87, 88, 93, 94, 95, 100, 101, 109, 110, 116, 118, 126, 129, 135, 136, 140, 141, 145, 147, 156, 157, 162, 163, 164, 168, 173, 183, 185, 186, 187, 188, 195, 196, 203, 221, 229, 230, 244, 245, 246, 247, 285, 286

B grade: { 1, 5, 6, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 52, 53, 54, 55, 56, 89, 96, 102, 103, 107, 108, 111, 112, 113, 114, 115, 117, 119, 120, 121, 122, 123, 124, 125, 127, 128, 130, 131, 132, 133, 134, 137, 138, 139, 142, 143, 146, 148, 150, 158, 169, 170, 171, 172, 178, 180, 181, 182, 193, 194, 202, 204, 210, 211, 212, 213, 217, 218, 219, 220, 225, 226, 227, 228 }

C grade: { }

F grade: { 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 90, 91, 92, 97, 98, 99, 104, 105, 106, 144, 149, 151, 152, 153, 154, 155, 159, 160, 161, 165, 166, 167, 174, 175, 176, 177, 179, 184, 189, 190, 191, 192, 197, 198, 199, 200, 201, 205, 206, 207, 208, 209, 214, 215, 216, 222, 223, 224, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 113, 114, 115, 120, 121, 123, 124, 125, 132, 136, 138, 143, 144, 148, 149, 150, 156, 157, 158, 162, 163, 164, 168, 169, 171, 172, 173, 178, 179, 180, 181, 182, 185, 186, 187, 188, 189, 193, 194, 195, 196, 197, 202, 203, 204, 205, 210, 211, 213, 214, 217, 218, 220, 221, 225, 226, 227, 228, 229, 230, 235, 236, 237, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 255, 256, 262, 286 }

B grade: { 17, 30, 56, 69, 74, 82, 107, 108, 111, 112, 116, 119, 122, 126, 130, 131, 137, 141, 142, 146, 147, 170, 183, 184, 190, 191, 192, 198, 199, 200, 201, 206, 207, 208, 209, 212, 215, 216, 219, 222, 223, 224, 231, 232, 233, 234, 238, 250, 251, 253, 254, 260, 261, 285 }

C grade: { }

F grade: { 110, 117, 118, 127, 128, 129, 133, 134, 135, 139, 140, 145, 151, 152, 153, 154, 155, 159, 160, 161, 165, 166, 167, 174, 175, 176, 177, 243, 252, 257, 258, 259, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

2.1.6 Sympy

A grade: { 170 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286 }

2.1.7 Giac

A grade: { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 105, 106, 133, 134, 135, 139, 140, 145, 168, 169, 170, 171, 172, 173, 196, 199, 203, 204, 208, 213, 214, 215, 216, 220, 221, 226, 227, 229, 230, 248, 249, 255, 261, 262, 285, 286 }

B grade: { 3, 14, 103, 141, 146, 147, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 198, 200, 201, 202, 205, 206, 207, 209, 210, 211, 212, 217, 218, 219, 222, 223, 224, 225, 228, 231, 232, 233, 244, 245, 246, 247, 250, 251, 252, 253, 254, 256, 257, 258, 259, 260, 263 }

C grade: { 68, 75, 83, 90, 97, 104 }

F grade: { 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 136, 137, 138, 142, 143, 144, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	499	130	290	347	0	207
normalized size	1	1.	4.75	1.24	2.76	3.3	0.	1.97
time (sec)	N/A	0.195	1.671	0.027	0.977	0.494	0.	1.504

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	887	107	180	302	0	182
normalized size	1	1.	10.31	1.24	2.09	3.51	0.	2.12
time (sec)	N/A	0.157	6.46	0.022	0.956	0.486	0.	1.482

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	313	84	146	263	0	158
normalized size	1	1.	5.13	1.38	2.39	4.31	0.	2.59
time (sec)	N/A	0.104	0.718	0.022	0.967	0.483	0.	1.55

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	42	92	178	0	80
normalized size	1	1.	1.	1.11	2.42	4.68	0.	2.11
time (sec)	N/A	0.05	0.029	0.015	0.951	0.476	0.	1.824

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	77	63	136	174	0	85
normalized size	1	1.	1.83	1.5	3.24	4.14	0.	2.02
time (sec)	N/A	0.054	0.067	0.063	0.996	0.472	0.	1.386

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	50	21	131	123	0	28
normalized size	1	1.	1.39	0.58	3.64	3.42	0.	0.78
time (sec)	N/A	0.05	0.263	0.07	0.975	0.435	0.	1.322

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	87	37	158	189	0	53
normalized size	1	1.	1.14	0.49	2.08	2.49	0.	0.7
time (sec)	N/A	0.098	0.333	0.079	0.991	0.442	0.	1.291

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	113	50	239	258	0	73
normalized size	1	1.	0.97	0.43	2.06	2.22	0.	0.63
time (sec)	N/A	0.15	0.393	0.084	1.028	0.446	0.	1.315

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	139	63	266	321	0	93
normalized size	1	1.	0.88	0.4	1.68	2.03	0.	0.59
time (sec)	N/A	0.21	0.356	0.093	1.028	0.454	0.	1.227

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	102	192	497	448	0	279
normalized size	1	1.	0.6	1.12	2.91	2.62	0.	1.63
time (sec)	N/A	0.274	1.857	0.033	1.021	0.512	0.	1.312

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	91	167	433	404	0	252
normalized size	1	1.	0.61	1.11	2.89	2.69	0.	1.68
time (sec)	N/A	0.241	1.333	0.03	0.993	0.507	0.	1.35

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	82	142	306	356	0	225
normalized size	1	1.	0.87	1.51	3.26	3.79	0.	2.39
time (sec)	N/A	0.149	0.742	0.026	0.977	0.497	0.	1.268

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	51	75	203	243	0	124
normalized size	1	1.	0.7	1.03	2.78	3.33	0.	1.7
time (sec)	N/A	0.108	0.149	0.02	0.967	0.483	0.	1.249

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	45	84	146	263	0	158
normalized size	1	1.	0.74	1.38	2.39	4.31	0.	2.59
time (sec)	N/A	0.103	0.102	0.02	1.008	0.481	0.	1.331

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	220	116	304	275	0	142
normalized size	1	1.	2.97	1.57	4.11	3.72	0.	1.92
time (sec)	N/A	0.099	1.717	0.063	1.032	0.476	0.	1.311

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	109	91	271	313	0	119
normalized size	1	1.	1.22	1.02	3.04	3.52	0.	1.34
time (sec)	N/A	0.128	0.085	0.077	0.984	0.479	0.	1.262

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	255	196	0	31
normalized size	1	1.	0.66	0.61	6.71	5.16	0.	0.82
time (sec)	N/A	0.075	0.116	0.083	0.987	0.44	0.	1.341

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	115	39	365	267	0	58
normalized size	1	1.	1.44	0.49	4.56	3.34	0.	0.72
time (sec)	N/A	0.15	0.43	0.097	1.058	0.444	0.	1.338

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	141	52	363	342	0	81
normalized size	1	1.	1.17	0.43	3.	2.83	0.	0.67
time (sec)	N/A	0.23	0.421	0.102	1.057	0.456	0.	1.264

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	167	65	525	414	0	104
normalized size	1	1.	1.02	0.4	3.22	2.54	0.	0.64
time (sec)	N/A	0.315	0.704	0.115	1.051	0.462	0.	1.324

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	122	242	598	544	0	333
normalized size	1	1.	0.54	1.07	2.63	2.4	0.	1.47
time (sec)	N/A	0.335	3.399	0.143	1.008	0.532	0.	1.399

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	111	217	551	489	0	306
normalized size	1	1.	0.54	1.05	2.67	2.37	0.	1.49
time (sec)	N/A	0.302	2.345	0.035	1.011	0.522	0.	1.427

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	102	192	497	444	0	279
normalized size	1	1.	0.84	1.59	4.11	3.67	0.	2.31
time (sec)	N/A	0.178	1.715	0.029	1.029	0.511	0.	1.376

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	60	100	329	285	0	147
normalized size	1	1.	0.6	1.	3.29	2.85	0.	1.47
time (sec)	N/A	0.131	0.248	0.024	0.985	0.496	0.	1.361

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	81	142	306	356	0	225
normalized size	1	1.	0.86	1.51	3.26	3.79	0.	2.39
time (sec)	N/A	0.148	0.848	0.027	0.988	0.494	0.	1.357

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	70	107	180	302	0	182
normalized size	1	1.	0.81	1.24	2.09	3.51	0.	2.12
time (sec)	N/A	0.156	0.616	0.024	1.012	0.488	0.	1.317

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	287	166	522	320	0	167
normalized size	1	1.	2.87	1.66	5.22	3.2	0.	1.67
time (sec)	N/A	0.129	3.04	0.076	1.014	0.484	0.	1.273

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	402	140	471	410	0	165
normalized size	1	1.	3.38	1.18	3.96	3.45	0.	1.39
time (sec)	N/A	0.183	3.247	0.078	1.035	0.487	0.	1.324

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	139	113	417	446	0	144
normalized size	1	1.	1.05	0.86	3.16	3.38	0.	1.09
time (sec)	N/A	0.214	0.108	0.09	1.029	0.485	0.	1.288

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	481	261	0	31
normalized size	1	1.	0.66	0.61	12.66	6.87	0.	0.82
time (sec)	N/A	0.072	0.152	0.095	1.058	0.449	0.	1.342

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	141	39	482	333	0	58
normalized size	1	1.	1.76	0.49	6.02	4.16	0.	0.72
time (sec)	N/A	0.152	0.385	0.113	1.069	0.457	0.	1.252

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	167	52	699	413	0	81
normalized size	1	1.	1.38	0.43	5.78	3.41	0.	0.67
time (sec)	N/A	0.23	0.677	0.125	1.093	0.467	0.	1.297

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	193	65	698	486	0	104
normalized size	1	1.	1.19	0.4	4.31	3.	0.	0.64
time (sec)	N/A	0.317	0.575	0.135	1.135	0.475	0.	1.353

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	1036	212	798	386	0	188
normalized size	1	1.	8.56	1.75	6.6	3.19	0.	1.55
time (sec)	N/A	0.156	6.409	0.086	1.051	0.492	0.	1.322

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	287	164	521	344	0	165
normalized size	1	1.	2.87	1.64	5.21	3.44	0.	1.65
time (sec)	N/A	0.13	3.171	0.078	1.016	0.482	0.	1.393

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	220	116	302	301	0	138
normalized size	1	1.	2.97	1.57	4.08	4.07	0.	1.86
time (sec)	N/A	0.103	1.58	0.059	0.961	0.479	0.	1.328

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	77	61	136	190	0	82
normalized size	1	1.	1.88	1.49	3.32	4.63	0.	2.
time (sec)	N/A	0.053	0.069	0.055	0.954	0.47	0.	1.305

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	24	32	0	26
normalized size	1	1.	1.	0.	1.5	2.	0.	1.62
time (sec)	N/A	0.089	0.028	180.	0.962	0.434	0.	1.238

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	81	48	104	123	0	80
normalized size	1	1.	1.37	0.81	1.76	2.08	0.	1.36
time (sec)	N/A	0.137	0.459	0.049	0.992	0.435	0.	1.241

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	107	61	131	185	0	99
normalized size	1	1.	1.37	0.78	1.68	2.37	0.	1.27
time (sec)	N/A	0.179	0.866	0.055	0.984	0.445	0.	1.324

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	145	74	158	255	0	117
normalized size	1	1.	1.21	0.62	1.32	2.12	0.	0.98
time (sec)	N/A	0.229	0.878	0.059	1.012	0.452	0.	1.219

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	380	234	1033	521	0	221
normalized size	1	1.	2.32	1.43	6.3	3.18	0.	1.35
time (sec)	N/A	0.247	1.274	0.102	1.049	0.5	0.	1.368

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	349	186	717	487	0	198
normalized size	1	1.	2.33	1.24	4.78	3.25	0.	1.32
time (sec)	N/A	0.215	1.947	0.092	1.023	0.494	0.	1.273

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	485	136	460	440	0	171
normalized size	1	1.	4.08	1.14	3.87	3.7	0.	1.44
time (sec)	N/A	0.186	4.256	0.074	1.011	0.485	0.	1.383

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	109	89	265	340	0	126
normalized size	1	1.	1.24	1.01	3.01	3.86	0.	1.43
time (sec)	N/A	0.131	0.094	0.07	0.995	0.477	0.	1.313

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	23	21	127	126	0	28
normalized size	1	1.	0.64	0.58	3.53	3.5	0.	0.78
time (sec)	N/A	0.048	0.088	0.066	0.978	0.437	0.	1.357

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	83	48	103	124	0	97
normalized size	1	1.	1.41	0.81	1.75	2.1	0.	1.64
time (sec)	N/A	0.136	0.571	0.048	0.989	0.439	0.	1.229

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	0	42	111	0	45
normalized size	1	1.	0.87	0.	1.11	2.92	0.	1.18
time (sec)	N/A	0.098	0.05	180.	0.96	0.439	0.	1.231

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	147	76	163	259	0	136
normalized size	1	1.	1.84	0.95	2.04	3.24	0.	1.7
time (sec)	N/A	0.146	0.984	0.055	0.984	0.452	0.	1.284

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	179	87	189	290	0	155
normalized size	1	1.	1.83	0.89	1.93	2.96	0.	1.58
time (sec)	N/A	0.189	0.916	0.058	1.019	0.458	0.	1.23

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	211	102	217	400	0	174
normalized size	1	1.	1.5	0.72	1.54	2.84	0.	1.23
time (sec)	N/A	0.246	1.279	0.066	1.001	0.468	0.	1.34

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	406	256	1262	663	0	250
normalized size	1	1.	1.89	1.19	5.87	3.08	0.	1.16
time (sec)	N/A	0.336	2.2	0.123	1.114	0.516	0.	1.331

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	193	193	380	208	918	620	0	225
normalized size	1	1.	1.97	1.08	4.76	3.21	0.	1.17
time (sec)	N/A	0.309	1.377	0.103	1.062	0.504	0.	1.354

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	826	160	635	579	0	200
normalized size	1	1.	5.04	0.98	3.87	3.53	0.	1.22
time (sec)	N/A	0.275	6.318	0.085	1.041	0.49	0.	1.373

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	139	111	410	479	0	154
normalized size	1	1.	1.06	0.85	3.13	3.66	0.	1.18
time (sec)	N/A	0.215	0.119	0.084	1.024	0.486	0.	1.263

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	250	196	0	31
normalized size	1	1.	0.66	0.61	6.58	5.16	0.	0.82
time (sec)	N/A	0.074	0.114	0.082	1.007	0.441	0.	1.36

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	87	37	155	192	0	53
normalized size	1	1.	1.14	0.49	2.04	2.53	0.	0.7
time (sec)	N/A	0.099	0.329	0.069	1.007	0.438	0.	1.278

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	109	61	128	186	0	123
normalized size	1	1.	1.4	0.78	1.64	2.38	0.	1.58
time (sec)	N/A	0.181	0.773	0.053	0.971	0.444	0.	1.281

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	147	76	162	261	0	146
normalized size	1	1.	1.84	0.95	2.02	3.26	0.	1.82
time (sec)	N/A	0.142	0.767	0.055	1.011	0.448	0.	1.286

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	50	0	55	180	0	59
normalized size	1	1.	0.85	0.	0.93	3.05	0.	1.
time (sec)	N/A	0.109	0.068	180.	0.96	0.456	0.	1.236

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	211	102	215	394	0	182
normalized size	1	1.	2.13	1.03	2.17	3.98	0.	1.84
time (sec)	N/A	0.149	1.259	0.063	0.989	0.478	0.	1.364

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	257	115	244	462	0	203
normalized size	1	1.	2.14	0.96	2.03	3.85	0.	1.69
time (sec)	N/A	0.204	1.562	0.065	1.03	0.476	0.	1.407

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	289	128	270	540	0	221
normalized size	1	1.	1.78	0.79	1.67	3.33	0.	1.36
time (sec)	N/A	0.26	2.157	0.068	1.017	0.491	0.	1.262

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	86	83	0	298	0	150
normalized size	1	1.	0.53	0.51	0.	1.83	0.	0.92
time (sec)	N/A	0.278	0.89	0.186	0.	0.483	0.	2.535

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	76	73	0	259	0	113
normalized size	1	1.	0.62	0.6	0.	2.12	0.	0.93
time (sec)	N/A	0.2	0.479	0.18	0.	0.469	0.	1.832

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	64	63	0	205	0	82
normalized size	1	1.	0.79	0.78	0.	2.53	0.	1.01
time (sec)	N/A	0.128	0.272	0.171	0.	0.466	0.	1.493

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	51	53	0	158	0	43
normalized size	1	1.	1.31	1.36	0.	4.05	0.	1.1
time (sec)	N/A	0.059	0.175	0.221	0.	0.454	0.	1.361

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	132	85	0	686	0	228
normalized size	1	1.	1.71	1.1	0.	8.91	0.	2.96
time (sec)	N/A	0.107	0.616	0.239	0.	0.583	0.	1.825

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	246	164	0	864	0	142
normalized size	1	1.	3.24	2.16	0.	11.37	0.	1.87
time (sec)	N/A	0.114	1.1	0.199	0.	0.606	0.	1.598

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	309	308	0	1042	0	178
normalized size	1	1.	2.73	2.73	0.	9.22	0.	1.58
time (sec)	N/A	0.155	1.209	0.214	0.	0.619	0.	1.886

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	88	85	0	359	0	153
normalized size	1	1.	0.51	0.5	0.	2.1	0.	0.89
time (sec)	N/A	0.448	1.595	0.2	0.	0.489	0.	4.106

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	78	75	0	313	0	116
normalized size	1	1.	0.61	0.59	0.	2.45	0.	0.91
time (sec)	N/A	0.327	1.192	0.19	0.	0.482	0.	2.652

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	66	65	0	254	0	85
normalized size	1	1.	0.78	0.76	0.	2.99	0.	1.
time (sec)	N/A	0.208	0.734	0.187	0.	0.472	0.	1.949

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	55	0	200	0	46
normalized size	1	1.	1.34	1.34	0.	4.88	0.	1.12
time (sec)	N/A	0.097	0.426	0.253	0.	0.465	0.	1.426

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	123	173	145	0	851	0	267
normalized size	1	1.05	1.48	1.24	0.	7.27	0.	2.28
time (sec)	N/A	0.213	0.889	0.25	0.	0.597	0.	1.95

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	124	184	144	0	907	0	234
normalized size	1	1.1	1.63	1.27	0.	8.03	0.	2.07
time (sec)	N/A	0.228	1.431	0.218	0.	0.612	0.	1.708

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	130	359	230	0	1076	0	181
normalized size	1	1.11	3.07	1.97	0.	9.2	0.	1.55
time (sec)	N/A	0.238	2.411	0.227	0.	0.625	0.	2.196

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	398	402	0	1281	0	219
normalized size	1	1.	2.43	2.45	0.	7.81	0.	1.34
time (sec)	N/A	0.282	5.692	0.236	0.	0.665	0.	2.728

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	88	85	0	400	0	153
normalized size	1	1.	0.51	0.5	0.	2.34	0.	0.89
time (sec)	N/A	0.444	2.59	0.218	0.	0.505	0.	5.747

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	78	75	0	355	0	116
normalized size	1	1.	0.61	0.59	0.	2.77	0.	0.91
time (sec)	N/A	0.323	1.542	0.208	0.	0.489	0.	3.544

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	66	65	0	290	0	85
normalized size	1	1.	0.78	0.76	0.	3.41	0.	1.
time (sec)	N/A	0.209	1.03	0.199	0.	0.479	0.	2.44

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	55	0	231	0	46
normalized size	1	1.	1.34	1.34	0.	5.63	0.	1.12
time (sec)	N/A	0.096	0.708	0.248	0.	0.469	0.	1.551

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	185	206	0	942	0	308
normalized size	1	1.	1.13	1.26	0.	5.74	0.	1.88
time (sec)	N/A	0.325	1.533	0.273	0.	0.617	0.	2.169

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	324	157	0	1067	0	300
normalized size	1	1.	1.93	0.93	0.	6.35	0.	1.79
time (sec)	N/A	0.336	2.961	0.232	0.	0.656	0.	1.822

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	263	206	0	1107	0	305
normalized size	1	1.	1.51	1.18	0.	6.36	0.	1.75
time (sec)	N/A	0.352	4.587	0.233	0.	0.668	0.	2.47

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	86	83	220	209	0	151
normalized size	1	1.	0.61	0.58	1.55	1.47	0.	1.06
time (sec)	N/A	0.234	0.756	0.187	1.568	0.481	0.	2.066

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	74	73	185	176	0	115
normalized size	1	1.	0.69	0.68	1.71	1.63	0.	1.06
time (sec)	N/A	0.191	0.43	0.177	1.502	0.469	0.	1.637

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	54	63	149	116	0	86
normalized size	1	1.	0.75	0.88	2.07	1.61	0.	1.19
time (sec)	N/A	0.153	0.241	0.174	1.522	0.463	0.	1.49

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	43	113	103	0	85
normalized size	1	1.	0.74	1.1	2.9	2.64	0.	2.18
time (sec)	N/A	0.107	0.131	0.235	1.525	0.456	0.	1.42

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	155	107	0	675	0	184
normalized size	1	1.	1.74	1.2	0.	7.58	0.	2.07
time (sec)	N/A	0.157	0.496	0.194	0.	0.588	0.	1.405

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	183	266	0	857	0	177
normalized size	1	1.	1.5	2.18	0.	7.02	0.	1.45
time (sec)	N/A	0.211	1.431	0.21	0.	0.617	0.	1.602

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	306	471	0	1049	0	220
normalized size	1	1.	1.96	3.02	0.	6.72	0.	1.41
time (sec)	N/A	0.255	3.019	0.22	0.	0.664	0.	1.585

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	84	85	254	243	0	144
normalized size	1	1.	0.54	0.55	1.64	1.57	0.	0.93
time (sec)	N/A	0.317	0.941	0.193	1.726	0.487	0.	4.218

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	68	75	220	193	0	109
normalized size	1	1.	0.55	0.61	1.79	1.57	0.	0.89
time (sec)	N/A	0.274	0.397	0.208	1.589	0.471	0.	2.926

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	60	53	149	171	0	84
normalized size	1	1.	0.67	0.6	1.67	1.92	0.	0.94
time (sec)	N/A	0.219	0.249	0.207	1.536	0.465	0.	2.229

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	53	147	142	0	89
normalized size	1	1.	1.34	1.29	3.59	3.46	0.	2.17
time (sec)	N/A	0.1	0.15	0.256	1.526	0.46	0.	1.394

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	259	131	0	864	0	224
normalized size	1	1.	1.88	0.95	0.	6.26	0.	1.62
time (sec)	N/A	0.271	2.143	0.219	0.	0.617	0.	1.58

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	365	320	0	948	0	221
normalized size	1	1.	2.16	1.89	0.	5.61	0.	1.31
time (sec)	N/A	0.343	1.399	0.233	0.	0.661	0.	1.512

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	434	551	0	1239	0	265
normalized size	1	1.	2.14	2.71	0.	6.1	0.	1.31
time (sec)	N/A	0.389	2.381	0.242	0.	0.76	0.	1.644

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	78	85	289	261	0	143
normalized size	1	1.	0.46	0.5	1.71	1.54	0.	0.85
time (sec)	N/A	0.402	0.626	0.241	1.497	0.484	0.	6.36

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	74	65	255	250	0	111
normalized size	1	1.	0.55	0.48	1.89	1.85	0.	0.82
time (sec)	N/A	0.348	0.344	0.247	1.508	0.472	0.	4.376

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	60	63	220	211	0	81
normalized size	1	1.	0.68	0.72	2.5	2.4	0.	0.92
time (sec)	N/A	0.222	0.317	0.237	1.517	0.469	0.	2.858

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	55	184	176	0	194
normalized size	1	1.	1.34	1.34	4.49	4.29	0.	4.73
time (sec)	N/A	0.102	0.153	0.29	1.492	0.467	0.	1.645

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	225	155	0	1053	0	261
normalized size	1	1.	1.24	0.86	0.	5.82	0.	1.44
time (sec)	N/A	0.394	1.58	0.244	0.	0.66	0.	1.616

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	398	370	0	1242	0	257
normalized size	1	1.	1.88	1.75	0.	5.86	0.	1.21
time (sec)	N/A	0.477	6.446	0.27	0.	0.697	0.	1.752

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	468	631	0	1207	0	298
normalized size	1	1.	1.9	2.57	0.	4.91	0.	1.21
time (sec)	N/A	0.532	6.639	0.278	0.	0.835	0.	1.89

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	87	82	861	225	0	0
normalized size	1	1.	2.02	1.91	20.02	5.23	0.	0.
time (sec)	N/A	0.133	0.464	0.3	1.877	0.473	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	73	72	402	186	0	0
normalized size	1	1.	1.7	1.67	9.35	4.33	0.	0.
time (sec)	N/A	0.133	0.301	0.313	1.865	0.465	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	56	62	74	132	0	0
normalized size	1	1.	1.37	1.51	1.8	3.22	0.	0.
time (sec)	N/A	0.12	0.163	0.304	1.557	0.466	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	99	136	124	0	0	0
normalized size	1	1.	1.94	2.67	2.43	0.	0.	0.
time (sec)	N/A	0.125	0.853	0.309	1.523	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	62	60	694	186	0	0
normalized size	1	1.	1.48	1.43	16.52	4.43	0.	0.
time (sec)	N/A	0.139	0.252	0.303	1.834	0.461	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	69	70	1023	254	0	0
normalized size	1	1.	1.6	1.63	23.79	5.91	0.	0.
time (sec)	N/A	0.137	0.384	0.293	2.094	0.472	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	108	103	2268	274	0	0
normalized size	1	1.	1.21	1.16	25.48	3.08	0.	0.
time (sec)	N/A	0.275	1.071	0.275	1.891	0.49	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	97	93	1492	273	0	0
normalized size	1	1.	1.09	1.04	16.76	3.07	0.	0.
time (sec)	N/A	0.276	0.592	0.263	1.859	0.481	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	78	83	743	197	0	0
normalized size	1	1.	0.88	0.93	8.35	2.21	0.	0.
time (sec)	N/A	0.274	0.431	0.27	1.848	0.471	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	73	73	76	186	0	0
normalized size	1	1.	1.7	1.7	1.77	4.33	0.	0.
time (sec)	N/A	0.13	0.275	0.322	1.531	0.467	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	174	162	371	0	0	0
normalized size	1	1.	1.83	1.71	3.91	0.	0.	0.
time (sec)	N/A	0.264	1.448	0.338	1.903	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	134	251	165	0	0	0
normalized size	1	1.	1.35	2.54	1.67	0.	0.	0.
time (sec)	N/A	0.275	0.656	0.308	1.595	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	63	73	720	225	0	0
normalized size	1	1.	1.5	1.74	17.14	5.36	0.	0.
time (sec)	N/A	0.149	0.492	0.267	1.82	0.471	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	80	83	2105	321	0	0
normalized size	1	1.	0.91	0.94	23.92	3.65	0.	0.
time (sec)	N/A	0.301	0.553	0.283	4.423	0.483	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	90	93	3521	383	0	0
normalized size	1	1.	0.98	1.01	38.27	4.16	0.	0.
time (sec)	N/A	0.285	0.847	0.283	19.998	0.498	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	100	103	5273	458	0	0
normalized size	1	1.	1.09	1.12	57.32	4.98	0.	0.
time (sec)	N/A	0.287	1.238	0.277	109.892	0.505	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	113	105	3313	360	0	0
normalized size	1	1.	0.84	0.78	24.72	2.69	0.	0.
time (sec)	N/A	0.421	1.343	0.265	1.96	0.503	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	92	95	2060	251	0	0
normalized size	1	1.	0.69	0.71	15.37	1.87	0.	0.
time (sec)	N/A	0.424	0.779	0.26	1.91	0.483	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	96	85	1493	273	0	0
normalized size	1	1.	1.08	0.96	16.78	3.07	0.	0.
time (sec)	N/A	0.275	0.599	0.261	1.885	0.48	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	88	75	78	225	0	0
normalized size	1	1.	2.05	1.74	1.81	5.23	0.	0.
time (sec)	N/A	0.128	0.463	0.28	1.551	0.469	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	328	189	995	0	0	0
normalized size	1	1.	2.33	1.34	7.06	0.	0.	0.
time (sec)	N/A	0.405	6.621	0.319	1.993	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	188	288	2747	0	0	0
normalized size	1	1.	1.3	1.99	18.94	0.	0.	0.
time (sec)	N/A	0.42	1.431	0.277	2.554	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	182	366	228	0	0	0
normalized size	1	1.	1.26	2.52	1.57	0.	0.	0.
time (sec)	N/A	0.431	1.33	0.276	1.546	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	76	75	2450	298	0	0
normalized size	1	1.	1.81	1.79	58.33	7.1	0.	0.
time (sec)	N/A	0.146	0.551	0.248	1.923	0.481	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	92	85	3671	394	0	0
normalized size	1	1.	1.05	0.97	41.72	4.48	0.	0.
time (sec)	N/A	0.298	0.827	0.257	19.682	0.493	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	102	95	5546	471	0	0
normalized size	1	1.	0.77	0.71	41.7	3.54	0.	0.
time (sec)	N/A	0.456	1.158	0.264	109.035	0.506	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	141	165	995	0	0	188
normalized size	1	1.	1.01	1.19	7.16	0.	0.	1.35
time (sec)	N/A	0.409	1.494	0.315	1.924	0.	0.	3.665

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	173	149	373	0	0	154
normalized size	1	1.	1.84	1.59	3.97	0.	0.	1.64
time (sec)	N/A	0.266	1.847	0.303	1.881	0.	0.	2.174

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	140	116	86	0	0	116
normalized size	1	1.	2.8	2.32	1.72	0.	0.	2.32
time (sec)	N/A	0.122	0.371	0.303	1.524	0.	0.	1.632

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	94	85	59	487	0	0
normalized size	1	1.	2.	1.81	1.26	10.36	0.	0.
time (sec)	N/A	0.135	0.793	0.267	1.835	0.607	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	79	131	548	952	0	0
normalized size	1	1.	0.83	1.38	5.77	10.02	0.	0.
time (sec)	N/A	0.28	0.881	0.299	1.929	0.668	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	91	170	1621	1138	0	0
normalized size	1	1.	0.65	1.21	11.58	8.13	0.	0.
time (sec)	N/A	0.43	0.84	0.301	2.163	0.692	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	183	235	2747	0	0	188
normalized size	1	1.	1.29	1.65	19.35	0.	0.	1.32
time (sec)	N/A	0.424	1.044	0.263	2.515	0.	0.	4.752

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	132	193	134	0	0	123
normalized size	1	1.	1.39	2.03	1.41	0.	0.	1.29
time (sec)	N/A	0.278	1.053	0.253	1.532	0.	0.	3.57

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	73	73	186	0	116
normalized size	1	1.	1.	1.74	1.74	4.43	0.	2.76
time (sec)	N/A	0.134	0.184	0.284	1.522	0.467	0.	2.672

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	157	123	536	952	0	0
normalized size	1	1.	1.65	1.29	5.64	10.02	0.	0.
time (sec)	N/A	0.279	1.403	0.3	1.867	0.665	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	69	133	765	979	0	0
normalized size	1	1.	0.66	1.28	7.36	9.41	0.	0.
time (sec)	N/A	0.182	0.772	0.261	1.973	0.695	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	122	211	0	1315	0	0
normalized size	1	1.	0.84	1.45	0.	9.01	0.	0.
time (sec)	N/A	0.341	1.505	0.269	0.	0.745	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	178	281	180	0	0	153
normalized size	1	1.	1.23	1.94	1.24	0.	0.	1.06
time (sec)	N/A	0.436	1.423	0.261	1.507	0.	0.	7.379

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	68	75	132	225	0	115
normalized size	1	1.	1.62	1.79	3.14	5.36	0.	2.74
time (sec)	N/A	0.147	0.284	0.251	1.544	0.475	0.	4.044

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	71	83	78	254	0	117
normalized size	1	1.	1.65	1.93	1.81	5.91	0.	2.72
time (sec)	N/A	0.134	0.246	0.29	1.536	0.472	0.	1.919

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	91	164	1608	1138	0	0
normalized size	1	1.	0.65	1.17	11.49	8.13	0.	0.
time (sec)	N/A	0.424	1.053	0.3	2.149	0.693	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	130	204	0	1315	0	0
normalized size	1	1.	0.89	1.4	0.	9.01	0.	0.
time (sec)	N/A	0.344	1.393	0.265	0.	0.74	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	84	173	2240	1177	0	0
normalized size	1	1.	0.52	1.08	14.	7.36	0.	0.
time (sec)	N/A	0.202	1.391	0.277	2.932	0.772	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	1.27	0.523	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	89	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.248	0.433	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	85	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.163	0.366	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.555	0.305	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.995	0.389	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	0	0	308	435	0	0
normalized size	1	1.	0.	0.	1.92	2.72	0.	0.
time (sec)	N/A	0.38	15.778	0.265	1.563	0.52	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	A	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	0	0	231	263	0	0
normalized size	1	1.	0.	0.	2.31	2.63	0.	0.
time (sec)	N/A	0.222	38.373	0.261	1.561	0.498	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	163	0	154	171	0	0
normalized size	1	1.	3.54	0.	3.35	3.72	0.	0.
time (sec)	N/A	0.102	18.772	0.313	1.56	0.474	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	0.643	0.305	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	0.693	0.257	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	2.031	0.255	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	321	0	211	296	0	0
normalized size	1	1.	1.9	0.	1.25	1.75	0.	0.
time (sec)	N/A	0.374	9.026	0.658	1.539	0.513	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	250	0	144	227	0	0
normalized size	1	1.	2.4	0.	1.38	2.18	0.	0.
time (sec)	N/A	0.225	3.142	0.668	1.517	0.503	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	47	47	208	0	84	169	0	0
normalized size	1	1.	4.43	0.	1.79	3.6	0.	0.
time (sec)	N/A	0.101	1.146	0.689	1.511	0.495	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	257	0	0	0	0	0
normalized size	1	1.	2.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	1.45	0.52	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	2.287	0.647	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	2.777	0.665	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	68	130	232	342	0	207
normalized size	1	1.	0.65	1.24	2.21	3.26	0.	1.97
time (sec)	N/A	0.193	0.309	0.027	1.037	0.494	0.	1.225

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	57	107	216	297	0	182
normalized size	1	1.	0.66	1.24	2.51	3.45	0.	2.12
time (sec)	N/A	0.156	0.183	0.023	0.991	0.487	0.	1.297

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	36	49	86	53	22
normalized size	1	1.	1.	2.12	2.88	5.06	3.12	1.29
time (sec)	N/A	0.071	0.019	0.02	0.977	0.439	6.854	1.268

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	154	104	262	270	0	124
normalized size	1	1.	2.75	1.86	4.68	4.82	0.	2.21
time (sec)	N/A	0.108	0.607	0.058	1.024	0.476	0.	1.208

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	335	81	194	323	0	115
normalized size	1	1.	4.79	1.16	2.77	4.61	0.	1.64
time (sec)	N/A	0.161	0.451	0.062	0.978	0.477	0.	1.198

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	43	37	155	192	0	53
normalized size	1	1.	0.5	0.43	1.8	2.23	0.	0.62
time (sec)	N/A	0.163	0.178	0.071	1.008	0.44	0.	1.254

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	13374	0	0	0	0	0
normalized size	1	1.	95.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	55.689	0.695	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	72	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.34	0.62	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	3396	0	0	0	0	0
normalized size	1	1.	18.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.232	17.365	0.897	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	226	226	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.413	3.118	0.504	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	143	162	234	1322	855	0	0
normalized size	1	1.38	1.56	2.25	12.71	8.22	0.	0.
time (sec)	N/A	0.278	1.533	0.342	2.068	0.804	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	116	73	204	0	676	0	0
normalized size	1	1.43	0.9	2.52	0.	8.35	0.	0.
time (sec)	N/A	0.232	0.474	0.238	0.	0.55	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	213	724	317	1769	1229	0	0
normalized size	1	1.52	5.17	2.26	12.64	8.78	0.	0.
time (sec)	N/A	0.281	6.455	0.346	2.072	0.647	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	150	236	152	724	818	0	0
normalized size	1	1.29	2.03	1.31	6.24	7.05	0.	0.
time (sec)	N/A	0.303	2.944	0.305	1.954	0.51	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	242	328	294	1890	1401	0	0
normalized size	1	1.35	1.83	1.64	10.56	7.83	0.	0.
time (sec)	N/A	0.358	2.301	0.334	2.139	0.913	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	94	138	76	613	0	0
normalized size	1	1.	2.04	3.	1.65	13.33	0.	0.
time (sec)	N/A	0.255	0.789	0.276	1.887	0.735	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	98	411	0	802	0	0
normalized size	1	1.	1.51	6.32	0.	12.34	0.	0.
time (sec)	N/A	0.159	0.26	0.272	0.	1.049	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	153	431	512	687	0	802
normalized size	1	1.	0.65	1.83	2.17	2.91	0.	3.4
time (sec)	N/A	0.437	1.8	0.057	0.991	0.541	0.	1.518

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	103	290	359	510	0	539
normalized size	1	1.	0.6	1.7	2.1	2.98	0.	3.15
time (sec)	N/A	0.295	0.677	0.049	0.981	0.514	0.	1.629

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	75	174	223	371	0	329
normalized size	1	1.	0.69	1.61	2.06	3.44	0.	3.05
time (sec)	N/A	0.165	0.413	0.042	0.965	0.494	0.	1.627

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	75	86	119	247	0	177
normalized size	1	1.	1.34	1.54	2.12	4.41	0.	3.16
time (sec)	N/A	0.072	0.03	0.035	0.977	0.481	0.	1.51

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	107	135	0	622	0	178
normalized size	1	1.	1.55	1.96	0.	9.01	0.	2.58
time (sec)	N/A	0.144	0.185	0.078	0.	0.642	0.	2.036

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	75	105	0	780	0	193
normalized size	1	1.	0.95	1.33	0.	9.87	0.	2.44
time (sec)	N/A	0.139	0.226	0.089	0.	0.509	0.	1.708

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	167	178	0	1604	0	370
normalized size	1	1.	1.27	1.36	0.	12.24	0.	2.82
time (sec)	N/A	0.27	1.175	0.102	0.	0.579	0.	1.39

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	247	271	0	2731	0	632
normalized size	1	1.	1.31	1.43	0.	14.45	0.	3.34
time (sec)	N/A	0.454	3.189	0.117	0.	0.692	0.	1.504

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	371	460	602	922	884	0	1038
normalized size	1	1.13	1.41	1.84	2.82	2.7	0.	3.17
time (sec)	N/A	0.424	2.018	0.075	1.028	0.567	0.	1.454

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	277	326	420	633	667	0	714
normalized size	1	1.14	1.35	1.74	2.62	2.76	0.	2.95
time (sec)	N/A	0.343	1.455	0.063	0.988	0.531	0.	1.472

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	234	479	268	437	490	0	452
normalized size	1	1.33	2.72	1.52	2.48	2.78	0.	2.57
time (sec)	N/A	0.262	1.001	0.052	1.02	0.511	0.	1.403

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	481	141	225	336	0	252
normalized size	1	1.	4.67	1.37	2.18	3.26	0.	2.45
time (sec)	N/A	0.117	6.242	0.041	0.984	0.49	0.	1.433

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	C	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	208	329	291	0	953	0	274
normalized size	1	2.19	3.46	3.06	0.	10.03	0.	2.88
time (sec)	N/A	0.245	1.989	0.081	0.	0.953	0.	1.285

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	231	312	330	0	1289	0	323
normalized size	1	1.97	2.67	2.82	0.	11.02	0.	2.76
time (sec)	N/A	0.257	1.475	0.11	0.	0.988	0.	1.383

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	184	249	167	0	1339	0	297
normalized size	1	1.42	1.92	1.28	0.	10.3	0.	2.28
time (sec)	N/A	0.197	1.161	0.107	0.	0.562	0.	1.367

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	268	211	228	0	2585	0	567
normalized size	1	1.26	0.99	1.07	0.	12.14	0.	2.66
time (sec)	N/A	0.283	4.735	0.128	0.	0.678	0.	1.448

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	330	322	352	0	4093	0	998
normalized size	1	1.2	1.17	1.28	0.	14.83	0.	3.62
time (sec)	N/A	0.557	9.697	0.146	0.	0.836	0.	1.601

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	333	380	523	946	791	0	825
normalized size	1	1.16	1.32	1.82	3.28	2.75	0.	2.86
time (sec)	N/A	0.427	2.778	0.075	1.015	0.554	0.	1.337

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	273	433	342	620	587	0	532
normalized size	1	1.06	1.68	1.33	2.41	2.28	0.	2.07
time (sec)	N/A	0.3	2.554	0.06	1.014	0.517	0.	1.284

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	273	188	354	393	0	301
normalized size	1	1.	2.18	1.5	2.83	3.14	0.	2.41
time (sec)	N/A	0.153	1.304	0.047	0.995	0.5	0.	1.205

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	257	419	491	0	1235	0	398
normalized size	1	1.68	2.74	3.21	0.	8.07	0.	2.6
time (sec)	N/A	0.325	2.288	0.097	0.	1.522	0.	1.305

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	274	455	548	0	1891	0	444
normalized size	1	1.7	2.83	3.4	0.	11.75	0.	2.76
time (sec)	N/A	0.355	4.051	0.107	0.	1.573	0.	1.302

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	301	393	768	0	2534	0	528
normalized size	1	1.6	2.09	4.09	0.	13.48	0.	2.81
time (sec)	N/A	0.394	3.378	0.127	0.	1.685	0.	1.36

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	227	398	227	0	2178	0	432
normalized size	1	1.28	2.24	1.28	0.	12.24	0.	2.43
time (sec)	N/A	0.223	3.52	0.128	0.	0.643	0.	1.46

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	327	274	303	0	3699	0	845
normalized size	1	1.23	1.03	1.14	0.	13.91	0.	3.18
time (sec)	N/A	0.337	9.662	0.156	0.	0.804	0.	1.535

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	236	1243	596	805	693	0	487
normalized size	1	1.29	6.79	3.26	4.4	3.79	0.	2.66
time (sec)	N/A	0.335	6.453	0.079	1.029	0.517	0.	1.355

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	171	275	371	524	512	0	311
normalized size	1	1.46	2.35	3.17	4.48	4.38	0.	2.66
time (sec)	N/A	0.254	2.522	0.069	1.023	0.5	0.	1.245

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	125	237	196	301	370	0	193
normalized size	1	1.84	3.49	2.88	4.43	5.44	0.	2.84
time (sec)	N/A	0.146	1.566	0.051	0.968	0.481	0.	1.296

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	109	78	134	197	0	100
normalized size	1	1.	2.53	1.81	3.12	4.58	0.	2.33
time (sec)	N/A	0.085	0.263	0.043	0.987	0.477	0.	1.283

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	134	160	74	0	783	0	154
normalized size	1	1.61	1.93	0.89	0.	9.43	0.	1.86
time (sec)	N/A	0.155	0.664	0.065	0.	0.506	0.	1.239

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	196	286	146	0	1497	0	308
normalized size	1	1.35	1.97	1.01	0.	10.32	0.	2.12
time (sec)	N/A	0.253	3.35	0.082	0.	0.563	0.	1.301

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	207	268	1422	221	0	2865	0	505
normalized size	1	1.29	6.87	1.07	0.	13.84	0.	2.44
time (sec)	N/A	0.4	6.781	0.1	0.	0.692	0.	1.305

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	315	446	766	1042	1037	0	716
normalized size	1	1.22	1.73	2.97	4.04	4.02	0.	2.78
time (sec)	N/A	0.438	4.154	0.099	1.042	0.539	0.	1.295

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	249	310	514	724	841	0	508
normalized size	1	1.29	1.61	2.66	3.75	4.36	0.	2.63
time (sec)	N/A	0.32	2.832	0.083	1.04	0.519	0.	1.306

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	193	294	316	462	632	0	354
normalized size	1	1.45	2.21	2.38	3.47	4.75	0.	2.66
time (sec)	N/A	0.233	1.523	0.064	1.002	0.5	0.	1.232

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	149	181	170	263	383	0	224
normalized size	1	1.67	2.03	1.91	2.96	4.3	0.	2.52
time (sec)	N/A	0.154	0.737	0.074	1.006	0.482	0.	1.354

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	76	60	126	144	0	86
normalized size	1	1.	1.17	0.92	1.94	2.22	0.	1.32
time (sec)	N/A	0.087	0.21	0.051	0.973	0.436	0.	1.149

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	183	209	122	0	1307	0	348
normalized size	1	1.42	1.62	0.95	0.	10.13	0.	2.7
time (sec)	N/A	0.241	1.629	0.07	0.	0.531	0.	1.159

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	260	376	203	0	2631	0	662
normalized size	1	1.23	1.78	0.96	0.	12.47	0.	3.14
time (sec)	N/A	0.373	3.911	0.09	0.	0.645	0.	1.26

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	284	346	2220	280	0	4329	0	1048
normalized size	1	1.22	7.82	0.99	0.	15.24	0.	3.69
time (sec)	N/A	0.558	7.295	0.105	0.	0.783	0.	1.31

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	405	1338	956	1277	1470	0	950
normalized size	1	1.12	3.69	2.63	3.52	4.05	0.	2.62
time (sec)	N/A	0.539	6.704	0.119	1.121	0.629	0.	1.478

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	329	439	679	930	1200	0	713
normalized size	1	1.15	1.53	2.37	3.24	4.18	0.	2.48
time (sec)	N/A	0.416	2.148	0.104	1.082	0.571	0.	1.304

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	265	292	454	641	910	0	529
normalized size	1	1.29	1.42	2.21	3.13	4.44	0.	2.58
time (sec)	N/A	0.28	2.374	0.079	1.053	0.533	0.	1.227

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	193	295	286	414	601	0	367
normalized size	1	1.45	2.22	2.15	3.11	4.52	0.	2.76
time (sec)	N/A	0.201	1.484	0.071	1.025	0.508	0.	1.349

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	180	128	248	270	0	185
normalized size	1	1.	1.57	1.11	2.16	2.35	0.	1.61
time (sec)	N/A	0.163	0.47	0.064	1.021	0.46	0.	1.245

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	135	64	155	227	0	108
normalized size	1	1.	1.32	0.63	1.52	2.23	0.	1.06
time (sec)	N/A	0.115	0.337	0.053	1.015	0.451	0.	1.261

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	235	345	203	0	2187	0	660
normalized size	1	1.3	1.91	1.12	0.	12.08	0.	3.65
time (sec)	N/A	0.323	3.146	0.084	0.	0.572	0.	1.189

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	288	325	1772	284	0	3638	0	1284
normalized size	1	1.13	6.15	0.99	0.	12.63	0.	4.46
time (sec)	N/A	0.486	7.013	0.104	0.	0.706	0.	1.217

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	414	1096	365	0	5948	0	1916
normalized size	1	1.12	2.98	0.99	0.	16.16	0.	5.21
time (sec)	N/A	0.706	7.693	0.116	0.	0.895	0.	1.538

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	102	302	0	738	0	0
normalized size	1	1.	1.67	4.95	0.	12.1	0.	0.
time (sec)	N/A	0.155	0.227	0.364	0.	0.966	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	187	504	0	2603	0	0
normalized size	1	1.	1.34	3.6	0.	18.59	0.	0.
time (sec)	N/A	0.462	16.948	0.366	0.	1.143	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	107	170	0	625	0	0
normalized size	1	1.	1.37	2.18	0.	8.01	0.	0.
time (sec)	N/A	0.157	0.218	0.331	0.	0.601	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	171	403	0	2743	0	0
normalized size	1	1.	1.21	2.86	0.	19.45	0.	0.
time (sec)	N/A	0.529	0.274	0.345	0.	1.333	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	94	431	0	801	0	0
normalized size	1	1.	1.54	7.07	0.	13.13	0.	0.
time (sec)	N/A	0.136	0.226	0.255	0.	1.041	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	427	568	0	2753	0	0
normalized size	1	1.	2.87	3.81	0.	18.48	0.	0.
time (sec)	N/A	0.57	1.322	0.379	0.	21.063	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	228531	518	0	2403	0	0
normalized size	1	1.	1873.2	4.25	0.	19.7	0.	0.
time (sec)	N/A	0.301	33.619	0.238	0.	1.21	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	141	518	0	2543	0	0
normalized size	1	1.	1.14	4.18	0.	20.51	0.	0.
time (sec)	N/A	0.368	0.411	0.249	0.	1.51	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	198	473	0	2654	0	0
normalized size	1	1.	1.19	2.83	0.	15.89	0.	0.
time (sec)	N/A	0.583	0.394	0.338	0.	2.566	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	155	726	0	0	0	0
normalized size	1	1.	0.67	3.14	0.	0.	0.	0.
time (sec)	N/A	0.816	0.391	0.346	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	201	431	512	687	0	1207
normalized size	1	1.	0.8	1.72	2.05	2.75	0.	4.83
time (sec)	N/A	0.501	4.429	0.048	1.156	0.616	0.	1.452

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	143	290	359	510	0	833
normalized size	1	1.	0.79	1.61	1.99	2.83	0.	4.63
time (sec)	N/A	0.356	1.085	0.04	1.168	0.578	0.	1.257

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	88	174	223	371	0	419
normalized size	1	1.	0.77	1.51	1.94	3.23	0.	3.64
time (sec)	N/A	0.185	0.594	0.035	1.026	0.56	0.	1.342

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	75	86	119	247	0	219
normalized size	1	1.	1.23	1.41	1.95	4.05	0.	3.59
time (sec)	N/A	0.075	0.027	0.026	1.015	0.535	0.	1.345

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	112	135	0	707	0	178
normalized size	1	1.	1.47	1.78	0.	9.3	0.	2.34
time (sec)	N/A	0.129	0.208	0.058	0.	1.959	0.	1.291

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	97	132	0	861	0	242
normalized size	1	1.	0.98	1.33	0.	8.7	0.	2.44
time (sec)	N/A	0.144	0.36	0.067	0.	0.53	0.	1.235

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	172	236	0	1631	0	564
normalized size	1	1.	1.04	1.42	0.	9.83	0.	3.4
time (sec)	N/A	0.301	0.823	0.079	0.	0.629	0.	1.481

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	405	376	0	2707	0	980
normalized size	1	1.	1.71	1.59	0.	11.42	0.	4.14
time (sec)	N/A	0.512	1.036	0.092	0.	0.751	0.	1.431

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	580	1066	0	0	0	848
normalized size	1	1.	2.35	4.32	0.	0.	0.	3.43
time (sec)	N/A	0.445	4.346	0.098	0.	0.	0.	1.461

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	389	593	0	1621	0	474
normalized size	1	1.	2.29	3.49	0.	9.54	0.	2.79
time (sec)	N/A	0.35	1.406	0.081	0.	85.835	0.	1.342

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	135	288	0	1139	0	273
normalized size	1	1.	1.31	2.8	0.	11.06	0.	2.65
time (sec)	N/A	0.297	0.804	0.065	0.	12.732	0.	1.415

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	112	135	0	694	0	178
normalized size	1	1.	1.47	1.78	0.	9.13	0.	2.34
time (sec)	N/A	0.127	0.184	0.063	0.	1.532	0.	1.284

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	119	108	0	2214	0	710
normalized size	1	1.	0.98	0.89	0.	18.3	0.	5.87
time (sec)	N/A	0.275	0.232	0.083	0.	6.496	0.	1.342

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	229	208	0	0	0	459
normalized size	1	1.	1.22	1.11	0.	0.	0.	2.45
time (sec)	N/A	0.612	0.714	0.1	0.	0.	0.	1.47

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	379	1137	1870	0	0	0	1197
normalized size	1	1.	3.	4.93	0.	0.	0.	3.16
time (sec)	N/A	0.668	6.545	0.145	0.	0.	0.	1.442

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	511	1249	0	0	0	770
normalized size	1	1.	1.72	4.21	0.	0.	0.	2.59
time (sec)	N/A	0.527	3.469	0.118	0.	0.	0.	1.33

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	362	790	0	2800	0	757
normalized size	1	1.	1.59	3.46	0.	12.28	0.	3.32
time (sec)	N/A	0.467	1.579	0.092	0.	154.86	0.	1.437

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	180	486	0	1719	0	374
normalized size	1	1.	0.91	2.45	0.	8.68	0.	1.89
time (sec)	N/A	0.367	0.669	0.088	0.	21.652	0.	1.441

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	97	132	0	861	0	243
normalized size	1	1.	0.97	1.32	0.	8.61	0.	2.43
time (sec)	N/A	0.136	0.362	0.068	0.	0.527	0.	1.297

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	176	210	0	0	0	458
normalized size	1	1.	0.95	1.13	0.	0.	0.	2.46
time (sec)	N/A	0.605	1.016	0.105	0.	0.	0.	1.321

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	185	355	0	0	0	0
normalized size	1	1.	0.87	1.67	0.	0.	0.	0.
time (sec)	N/A	0.293	3.653	0.33	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	44216	351	0	0	0	0
normalized size	1	1.	225.59	1.79	0.	0.	0.	0.
time (sec)	N/A	0.193	32.473	0.404	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	233	219	0	0	0	0
normalized size	1	1.	1.21	1.14	0.	0.	0.	0.
time (sec)	N/A	0.168	3.486	0.389	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	176	177	0	0	0	0
normalized size	1	1.	1.6	1.61	0.	0.	0.	0.
time (sec)	N/A	0.124	1.751	0.339	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	176	170	0	0	0	0
normalized size	1	1.	1.41	1.36	0.	0.	0.	0.
time (sec)	N/A	0.12	0.445	0.309	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	396	396	39039	291	0	0	0	0
normalized size	1	1.	98.58	0.73	0.	0.	0.	0.
time (sec)	N/A	0.611	32.378	0.411	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	223	465	0	0	0	0
normalized size	1	1.	1.31	2.74	0.	0.	0.	0.
time (sec)	N/A	0.849	4.028	0.393	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	236	0	0	0	0
normalized size	1	1.	1.	2.84	0.	0.	0.	0.
time (sec)	N/A	0.433	0.243	0.355	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	222	479	0	0	0	0
normalized size	1	1.	1.32	2.85	0.	0.	0.	0.
time (sec)	N/A	1.07	3.942	0.397	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	264	153	0	0	0	0
normalized size	1	1.	2.78	1.61	0.	0.	0.	0.
time (sec)	N/A	0.119	6.329	0.306	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	295	295	0	292	0	0	0	0
normalized size	1	1.	0.	0.99	0.	0.	0.	0.
time (sec)	N/A	0.922	18.917	0.355	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	2173	225	0	0	0	0
normalized size	1	1.	10.4	1.08	0.	0.	0.	0.
time (sec)	N/A	0.288	17.722	0.293	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	156	224	0	0	0	0
normalized size	1	1.	0.73	1.05	0.	0.	0.	0.
time (sec)	N/A	0.36	5.112	0.312	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	1019	222	0	0	0	0
normalized size	1	1.	4.45	0.97	0.	0.	0.	0.
time (sec)	N/A	0.44	8.382	0.332	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	312	312	0	355	0	0	0	0
normalized size	1	1.	0.	1.14	0.	0.	0.	0.
time (sec)	N/A	0.889	14.601	0.336	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	185	355	0	0	0	0
normalized size	1	1.	0.87	1.67	0.	0.	0.	0.
time (sec)	N/A	0.277	0.391	0.343	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	223	465	0	0	0	0
normalized size	1	1.	1.31	2.74	0.	0.	0.	0.
time (sec)	N/A	0.843	3.828	0.391	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	189	236	0	0	0	0
normalized size	1	1.	1.85	2.31	0.	0.	0.	0.
time (sec)	N/A	0.122	4.318	0.342	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	167	236	0	0	0	0
normalized size	1	1.	0.8	1.13	0.	0.	0.	0.
time (sec)	N/A	0.361	3.009	0.339	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	236	0	0	0	0
normalized size	1	1.	1.	2.84	0.	0.	0.	0.
time (sec)	N/A	0.424	0.24	0.352	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	246	344	0	0	0	0
normalized size	1	1.	1.48	2.07	0.	0.	0.	0.
time (sec)	N/A	0.856	4.047	0.354	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	151	49	92	309	0	68
normalized size	1	1.	2.25	0.73	1.37	4.61	0.	1.01
time (sec)	N/A	0.298	0.915	0.12	1.028	0.665	0.	2.18

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	175	62	119	377	0	86
normalized size	1	1.	1.97	0.7	1.34	4.24	0.	0.97
time (sec)	N/A	0.336	1.072	0.135	0.986	0.523	0.	2.266

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [258] had the largest ratio of [0.3226]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	7	1.	30	0.233
2	A	9	6	1.	30	0.2
3	A	6	5	1.	30	0.167
4	A	3	3	1.	28	0.107
5	A	2	2	1.	30	0.067
6	A	1	1	1.	30	0.033
7	A	2	2	1.	30	0.067
8	A	3	2	1.	30	0.067
9	A	4	2	1.	30	0.067
10	A	14	7	1.	32	0.219
11	A	11	6	1.	32	0.188
12	A	7	5	1.	32	0.156
13	A	4	3	1.	32	0.094
14	A	6	5	1.	30	0.167
15	A	5	5	1.	32	0.156
16	A	3	2	1.	32	0.062
17	A	1	1	1.	32	0.031
18	A	2	2	1.	32	0.062
19	A	3	2	1.	32	0.062
20	A	4	2	1.	32	0.062
21	A	16	7	1.	32	0.219
22	A	13	6	1.	32	0.188
23	A	8	5	1.	32	0.156
24	A	5	3	1.	32	0.094
25	A	7	5	1.	32	0.156
26	A	9	6	1.	30	0.2
27	A	6	6	1.	32	0.188
28	A	6	5	1.	32	0.156
29	A	4	2	1.	32	0.062
30	A	1	1	1.	32	0.031
31	A	2	2	1.	32	0.062
32	A	3	2	1.	32	0.062
33	A	4	2	1.	32	0.062
34	A	10	6	1.	32	0.188
35	A	6	6	1.	32	0.188
36	A	5	5	1.	32	0.156
37	A	2	2	1.	30	0.067
38	A	3	3	1.	32	0.094
39	A	6	4	1.	32	0.125
40	A	10	6	1.	32	0.188
41	A	13	7	1.	32	0.219

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
42	A	11	6	1.	32	0.188
43	A	7	6	1.	32	0.188
44	A	6	5	1.	32	0.156
45	A	3	2	1.	32	0.062
46	A	1	1	1.	30	0.033
47	A	6	4	1.	32	0.125
48	A	3	2	1.	32	0.062
49	A	7	5	1.	32	0.156
50	A	10	6	1.	32	0.188
51	A	13	7	1.	32	0.219
52	A	12	6	1.	32	0.188
53	A	8	6	1.	32	0.188
54	A	7	5	1.	32	0.156
55	A	4	2	1.	32	0.062
56	A	1	1	1.	32	0.031
57	A	2	2	1.	30	0.067
58	A	10	6	1.	32	0.188
59	A	7	5	1.	32	0.156
60	A	4	3	1.	32	0.094
61	A	7	5	1.	32	0.156
62	A	10	6	1.	32	0.188
63	A	13	7	1.	32	0.219
64	A	4	2	1.	32	0.062
65	A	3	2	1.	32	0.062
66	A	2	2	1.	32	0.062
67	A	1	1	1.	32	0.031
68	A	3	3	1.	32	0.094
69	A	3	3	1.	32	0.094
70	A	4	4	1.	32	0.125
71	A	4	2	1.	34	0.059
72	A	3	2	1.	34	0.059
73	A	2	2	1.	34	0.059
74	A	1	1	1.	34	0.029
75	A	4	3	1.05	34	0.088
76	A	4	4	1.1	34	0.118
77	A	4	3	1.11	34	0.088
78	A	5	4	1.	34	0.118
79	A	4	2	1.	34	0.059
80	A	3	2	1.	34	0.059
81	A	2	2	1.	34	0.059
82	A	1	1	1.	34	0.029
83	A	5	3	1.	34	0.088
84	A	5	4	1.	34	0.118
85	A	5	4	1.	34	0.118

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	3	1.	34	0.088
87	A	3	3	1.	34	0.088
88	A	2	2	1.	34	0.059
89	A	1	1	1.	34	0.029
90	A	3	3	1.	34	0.088
91	A	4	4	1.	34	0.118
92	A	5	4	1.	34	0.118
93	A	4	3	1.	34	0.088
94	A	3	2	1.	34	0.059
95	A	2	2	1.	34	0.059
96	A	1	1	1.	34	0.029
97	A	4	3	1.	34	0.088
98	A	5	4	1.	34	0.118
99	A	6	4	1.	34	0.118
100	A	4	2	1.	34	0.059
101	A	3	2	1.	34	0.059
102	A	2	2	1.	34	0.059
103	A	1	1	1.	34	0.029
104	A	5	3	1.	34	0.088
105	A	6	4	1.	34	0.118
106	A	7	4	1.	34	0.118
107	A	1	1	1.	36	0.028
108	A	1	1	1.	36	0.028
109	A	1	1	1.	36	0.028
110	A	1	1	1.	36	0.028
111	A	1	1	1.	36	0.028
112	A	1	1	1.	36	0.028
113	A	2	2	1.	36	0.056
114	A	2	2	1.	36	0.056
115	A	2	2	1.	36	0.056
116	A	1	1	1.	36	0.028
117	A	2	2	1.	36	0.056
118	A	2	2	1.	36	0.056
119	A	1	1	1.	36	0.028
120	A	2	2	1.	36	0.056
121	A	2	2	1.	36	0.056
122	A	2	2	1.	36	0.056
123	A	3	2	1.	36	0.056
124	A	3	2	1.	36	0.056
125	A	2	2	1.	36	0.056
126	A	1	1	1.	36	0.028
127	A	3	2	1.	36	0.056
128	A	3	3	1.	36	0.083
129	A	3	2	1.	36	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	1	1	1.	36	0.028
131	A	2	2	1.	36	0.056
132	A	3	2	1.	36	0.056
133	A	3	2	1.	36	0.056
134	A	2	2	1.	36	0.056
135	A	1	1	1.	36	0.028
136	A	2	2	1.	36	0.056
137	A	3	3	1.	36	0.083
138	A	4	3	1.	36	0.083
139	A	3	3	1.	36	0.083
140	A	2	2	1.	36	0.056
141	A	1	1	1.	36	0.028
142	A	3	3	1.	36	0.083
143	A	3	3	1.	36	0.083
144	A	4	4	1.	36	0.111
145	A	3	2	1.	36	0.056
146	A	1	1	1.	36	0.028
147	A	1	1	1.	36	0.028
148	A	4	3	1.	36	0.083
149	A	4	4	1.	36	0.111
150	A	4	3	1.	36	0.083
151	A	3	3	1.	32	0.094
152	A	3	3	1.	32	0.094
153	A	3	3	1.	30	0.1
154	A	3	3	1.	32	0.094
155	A	3	3	1.	32	0.094
156	A	3	2	1.	34	0.059
157	A	2	2	1.	34	0.059
158	A	1	1	1.	34	0.029
159	A	2	2	1.	34	0.059
160	A	2	2	1.	34	0.059
161	A	2	2	1.	34	0.059
162	A	3	2	1.	36	0.056
163	A	2	2	1.	36	0.056
164	A	1	1	1.	36	0.028
165	A	3	3	1.	34	0.088
166	A	3	3	1.	36	0.083
167	A	3	3	1.	36	0.083
168	A	10	7	1.	32	0.219
169	A	7	6	1.	32	0.188
170	A	3	3	1.	30	0.1
171	A	5	5	1.	32	0.156
172	A	4	4	1.	32	0.125
173	A	3	3	1.	32	0.094

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
174	A	5	3	1.	34	0.088
175	A	2	2	1.	32	0.062
176	A	6	4	1.	34	0.118
177	A	7	4	1.	34	0.118
178	A	5	5	1.38	40	0.125
179	A	4	4	1.43	36	0.111
180	A	8	8	1.52	38	0.21
181	A	4	4	1.29	40	0.1
182	A	8	8	1.35	40	0.2
183	A	3	3	1.	38	0.079
184	A	2	2	1.	34	0.059
185	A	8	6	1.	29	0.207
186	A	7	6	1.	29	0.207
187	A	6	6	1.	29	0.207
188	A	5	5	1.	27	0.185
189	A	5	5	1.	29	0.172
190	A	5	5	1.	29	0.172
191	A	6	5	1.	29	0.172
192	A	7	5	1.	29	0.172
193	A	9	8	1.13	31	0.258
194	A	8	7	1.14	31	0.226
195	A	8	7	1.33	31	0.226
196	A	6	6	1.	29	0.207
197	B	8	8	2.19	31	0.258
198	A	8	8	1.97	31	0.258
199	A	5	4	1.42	31	0.129
200	A	6	5	1.26	31	0.161
201	A	8	6	1.2	31	0.194
202	A	9	7	1.16	31	0.226
203	A	9	7	1.06	31	0.226
204	A	10	6	1.	29	0.207
205	A	9	9	1.68	31	0.29
206	A	9	9	1.7	31	0.29
207	A	9	9	1.6	31	0.29
208	A	6	4	1.28	31	0.129
209	A	7	5	1.23	31	0.161
210	A	7	7	1.29	31	0.226
211	A	6	6	1.46	31	0.194
212	A	6	6	1.84	31	0.194
213	A	3	3	1.	29	0.103
214	A	4	4	1.61	31	0.129
215	A	6	6	1.35	31	0.194
216	A	7	7	1.29	31	0.226
217	A	8	8	1.22	31	0.258

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
218	A	7	7	1.29	31	0.226
219	A	6	6	1.45	31	0.194
220	A	6	6	1.67	31	0.194
221	A	2	2	1.	29	0.069
222	A	6	6	1.42	31	0.194
223	A	7	6	1.23	31	0.194
224	A	8	7	1.22	31	0.226
225	A	9	8	1.12	31	0.258
226	A	8	7	1.15	31	0.226
227	A	7	7	1.29	31	0.226
228	A	6	6	1.45	31	0.194
229	A	4	4	1.	31	0.129
230	A	3	3	1.	29	0.103
231	A	7	6	1.3	31	0.194
232	A	8	6	1.13	31	0.194
233	A	9	7	1.12	31	0.226
234	A	2	2	1.	35	0.057
235	A	5	5	1.	35	0.143
236	A	2	2	1.	35	0.057
237	A	5	5	1.	37	0.135
238	A	2	2	1.	33	0.061
239	A	5	4	1.	39	0.103
240	A	5	5	1.	33	0.152
241	A	5	5	1.	35	0.143
242	A	5	4	1.	39	0.103
243	A	8	6	1.	39	0.154
244	A	8	6	1.	29	0.207
245	A	7	6	1.	29	0.207
246	A	6	6	1.	29	0.207
247	A	5	5	1.	27	0.185
248	A	5	5	1.	29	0.172
249	A	5	5	1.	29	0.172
250	A	6	5	1.	29	0.172
251	A	7	5	1.	29	0.172
252	A	12	8	1.	31	0.258
253	A	10	8	1.	31	0.258
254	A	8	7	1.	31	0.226
255	A	5	5	1.	29	0.172
256	A	6	4	1.	31	0.129
257	A	7	5	1.	31	0.161
258	A	16	10	1.	31	0.323
259	A	14	10	1.	31	0.323
260	A	12	9	1.	31	0.29
261	A	10	7	1.	31	0.226

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
262	A	5	5	1.	29	0.172
263	A	7	5	1.	31	0.161
264	A	3	3	1.	33	0.091
265	A	1	1	1.	35	0.029
266	A	1	1	1.	35	0.029
267	A	1	1	1.	35	0.029
268	A	1	1	1.	35	0.029
269	A	3	3	1.	37	0.081
270	A	7	5	1.	39	0.128
271	A	3	3	1.	39	0.077
272	A	8	6	1.	39	0.154
273	A	1	1	1.	33	0.03
274	A	11	11	1.	39	0.282
275	A	3	3	1.	33	0.091
276	A	3	3	1.	35	0.086
277	A	7	7	1.	39	0.18
278	A	11	11	1.	39	0.282
279	A	3	3	1.	33	0.091
280	A	7	5	1.	39	0.128
281	A	1	1	1.	33	0.03
282	A	3	3	1.	35	0.086
283	A	3	3	1.	39	0.077
284	A	7	5	1.	39	0.128
285	A	4	2	1.	28	0.071
286	A	4	2	1.	28	0.071

Chapter 3

Listing of integrals

3.1 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx$

Optimal. Leaf size=105

$$\frac{ac^4\tan^5(e+fx)}{5f}+\frac{4ac^4\tan^3(e+fx)}{3f}+\frac{7ac^4\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3ac^4\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{ac^4\tan(e+fx)}{8f}$$

```
[Out] (7*a*c^4*ArcTanh[Sin[e + f*x]])/(8*f) - (a*c^4*Sec[e + f*x]*Tan[e + f*x])/(
8*f) - (3*a*c^4*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (4*a*c^4*Tan[e + f*x]^
3)/(3*f) + (a*c^4*Tan[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.195434, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3958, 2611, 3770, 2607, 30, 3768, 14}

$$\frac{ac^4\tan^5(e+fx)}{5f}+\frac{4ac^4\tan^3(e+fx)}{3f}+\frac{7ac^4\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3ac^4\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{ac^4\tan(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (7*a*c^4*ArcTanh[Sin[e + f*x]])/(8*f) - (a*c^4*Sec[e + f*x]*Tan[e + f*x])/(
8*f) - (3*a*c^4*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (4*a*c^4*Tan[e + f*x]^
3)/(3*f) + (a*c^4*Tan[e + f*x]^5)/(5*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx = -\left((ac)\int \left(c^3\sec(e+fx)\tan^2(e+fx)-3c^3\sec^2(e+fx)\tan^2(\right.\right.$$
$$= -\left(\left(ac^4\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)+\left(ac^4\right)\int \sec^4(e+fx$$
$$= -\frac{ac^4\sec(e+fx)\tan(e+fx)}{2f}-\frac{3ac^4\sec^3(e+fx)\tan(e+fx)}{4f}$$
$$= \frac{ac^4\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac^4\sec(e+fx)\tan(e+fx)}{8f}-\frac{3ac^4}{}$$
$$= \frac{7ac^4\tanh^{-1}(\sin(e+fx))}{8f}-\frac{ac^4\sec(e+fx)\tan(e+fx)}{8f}-\frac{3ac}{}$$

Mathematica [B] time = 1.67133, size = 499, normalized size = 4.75

$$-\frac{ac^4\sec(e)\sec^5(e+fx)\left(-1920\sin(2e+fx)+780\sin(e+2fx)+780\sin(3e+2fx)+640\sin(2e+3fx)-720\sin(4e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -(a*c^4*Sec[e]*Sec[e + f*x]^5*(525*Cos[2*e + 3*f*x]*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]] + 525*Cos[4*e + 3*f*x]*Log[Cos[(e + f*x)/2] - Sin[(e + f*
x)/2]] + 105*Cos[4*e + 5*f*x]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 10
```

```
5*Cos[6*e + 5*f*x]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 1050*Cos[f*x]
*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2]]) + 1050*Cos[2*e + f*x]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
 - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 525*Cos[2*e + 3*f*x]*Log[Cos
[(e + f*x)/2] + Sin[(e + f*x)/2]] - 525*Cos[4*e + 3*f*x]*Log[Cos[(e + f*x)/
2] + Sin[(e + f*x)/2]] - 105*Cos[4*e + 5*f*x]*Log[Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2]] - 105*Cos[6*e + 5*f*x]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
 + 800*Sin[f*x] - 1920*Sin[2*e + f*x] + 780*Sin[e + 2*f*x] + 780*Sin[3*e +
2*f*x] + 640*Sin[2*e + 3*f*x] - 720*Sin[4*e + 3*f*x] + 30*Sin[3*e + 4*f*x]
+ 30*Sin[5*e + 4*f*x] + 272*Sin[4*e + 5*f*x]))/(3840*f)
```

Maple [A] time = 0.027, size = 130, normalized size = 1.2

$$-\frac{3\,ac^4\left(\sec\left(fx+e\right)\right)^3\tan\left(fx+e\right)}{4f}-\frac{ac^4\sec\left(fx+e\right)\tan\left(fx+e\right)}{8f}+\frac{7\,ac^4\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{8f}-\frac{17\,ac^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x)
```

```
[Out] -3/4*a*c^4*sec(f*x+e)^3*tan(f*x+e)/f-1/8*a*c^4*sec(f*x+e)*tan(f*x+e)/f+7/8/
f*a*c^4*ln(sec(f*x+e)+tan(f*x+e))-17/15/f*a*c^4*tan(f*x+e)+14/15/f*a*c^4*ta
n(f*x+e)*sec(f*x+e)^2+1/5/f*a*c^4*tan(f*x+e)*sec(f*x+e)^4
```

Maxima [B] time = 0.977239, size = 290, normalized size = 2.76

$$16\left(3\,\tan\left(fx+e\right)^5+10\,\tan\left(fx+e\right)^3+15\,\tan\left(fx+e\right)\right)ac^4+160\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)ac^4+45\,ac^4\left(\frac{2(}{\mathrm{si}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/240*(16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a*c^4 +
160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^4 + 45*a*c^4*(2*(3*sin(f*x + e)^3
 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x
+ e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*a*c^4*(2*sin(f*x + e)/(sin(f*x +
 e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a*c^4*log
(sec(f*x + e) + tan(f*x + e)) - 720*a*c^4*tan(f*x + e))/f
```

Fricas [A] time = 0.493873, size = 347, normalized size = 3.3

$$\frac{105\,ac^4\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-105\,ac^4\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)-2\left(136\,ac^4\cos\left(fx+e\right)^4\right.}{240\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/240*(105*a*c^4*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 105*a*c^4*cos(f*x +
 e)^5*log(-sin(f*x + e) + 1) - 2*(136*a*c^4*cos(f*x + e)^4 + 15*a*c^4*cos(f
*x + e)^3 - 112*a*c^4*cos(f*x + e)^2 + 90*a*c^4*cos(f*x + e) - 24*a*c^4)*si
n(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$ac^4\left(\int \sec\left(e+fx\right)dx + \int -3\sec^2\left(e+fx\right)dx + \int 2\sec^3\left(e+fx\right)dx + \int 2\sec^4\left(e+fx\right)dx + \int -3\sec^5\left(e+fx\right)dx -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a*c**4*(Integral(sec(e + f*x), x) + Integral(-3*sec(e + f*x)**2, x) + Integ
ral(2*sec(e + f*x)**3, x) + Integral(2*sec(e + f*x)**4, x) + Integral(-3*se
c(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Giac [A] time = 1.50418, size = 207, normalized size = 1.97

$$\frac{105\,ac^4\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right) - 105\,ac^4\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right) - \frac{2\left(105\,ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9+790\,ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-896\,a\right.}{\left(\tan\left(\right.\right.}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] 1/120*(105*a*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 105*a*c^4*log(abs(tan
(1/2*f*x + 1/2*e) - 1)) - 2*(105*a*c^4*tan(1/2*f*x + 1/2*e)^9 + 790*a*c^4*t
an(1/2*f*x + 1/2*e)^7 - 896*a*c^4*tan(1/2*f*x + 1/2*e)^5 + 490*a*c^4*tan(1/
2*f*x + 1/2*e)^3 - 105*a*c^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2
 - 1)^5)/f
```

3.2 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=86

$$\frac{2ac^3\tan^3(e+fx)}{3f}+\frac{5ac^3\tanh^{-1}(\sin(e+fx))}{8f}-\frac{ac^3\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{3ac^3\tan(e+fx)\sec(e+fx)}{8f}$$

```
[Out] (5*a*c^3*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a*c^3*Sec[e + f*x]*Tan[e + f*x])
/(8*f) - (a*c^3*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (2*a*c^3*Tan[e + f*x]^
3)/(3*f)
```

Rubi [A] time = 0.157387, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3958, 2611, 3770, 2607, 30, 3768}

$$\frac{2ac^3\tan^3(e+fx)}{3f}+\frac{5ac^3\tanh^{-1}(\sin(e+fx))}{8f}-\frac{ac^3\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{3ac^3\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (5*a*c^3*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a*c^3*Sec[e + f*x]*Tan[e + f*x])
/(8*f) - (a*c^3*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (2*a*c^3*Tan[e + f*x]^
3)/(3*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx = -\left((ac)\int \left(c^2\sec(e+fx)\tan^2(e+fx)-2c^2\sec^2(e+fx)\tan^2(\right.\right.$$
$$= -\left(\left(ac^3\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)-\left(ac^3\right)\int \sec^3(e+fx$$
$$= -\frac{ac^3\sec(e+fx)\tan(e+fx)}{2f}-\frac{ac^3\sec^3(e+fx)\tan(e+fx)}{4f}+$$
$$= \frac{ac^3\tanh^{-1}(\sin(e+fx))}{2f}-\frac{3ac^3\sec(e+fx)\tan(e+fx)}{8f}-\frac{ac^3}{}$$
$$= \frac{5ac^3\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3ac^3\sec(e+fx)\tan(e+fx)}{8f}-\frac{ac}{}$$

Mathematica [B] time = 6.4605, size = 887, normalized size = 10.31

$$a\left(\frac{5\cos^3(e+fx)\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)(c-c\sec(e+fx))^3\csc^6\left(\frac{e}{2}+\frac{fx}{2}\right)}{64f}-\frac{5\cos^3(e+fx)\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] a*((5*Cos[e + f*x]^3*Csc[e/2 + (f*x)/2]^6*Log[Cos[e/2 + (f*x)/2] - Sin[e/2
+ (f*x)/2]]*(c - c*Sec[e + f*x])^3)/(64*f) - (5*Cos[e + f*x]^3*Csc[e/2 + (f
*x)/2]^6*Log[Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^
3)/(64*f) + (Cos[e + f*x]^3*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^3)/(1
28*f*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^4) - (Cos[e + f*x]^3*Csc[e/2
 + (f*x)/2]^6*(c - c*Sec[e + f*x])^3*Sin[(f*x)/2])/(24*f*(Cos[e/2] - Sin[e/
2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^3*Csc[e/2
+ (f*x)/2]^6*(c - c*Sec[e + f*x])^3*(Cos[e/2] - 17*Sin[e/2]))/(384*f*(Cos[e
/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^2) + (Cos[e + f*x
]^3*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^3*Sin[(f*x)/2])/(12*f*(Cos[e/
2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])) - (Cos[e + f*x]^3
*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^3)/(128*f*(Cos[e/2 + (f*x)/2] +
Sin[e/2 + (f*x)/2])^4) - (Cos[e + f*x]^3*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e
+ f*x])^3*Sin[(f*x)/2])/(24*f*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/2] + S
in[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^3*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e +
 f*x])^3*(-Cos[e/2] - 17*Sin[e/2]))/(384*f*(Cos[e/2] + Sin[e/2])*(Cos[e/2 +
 (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (Cos[e + f*x]^3*Csc[e/2 + (f*x)/2]^6*(
c - c*Sec[e + f*x])^3*Sin[(f*x)/2])/(12*f*(Cos[e/2] + Sin[e/2])*(Cos[e/2 +
(f*x)/2] + Sin[e/2 + (f*x)/2])))
```

Maple [A] time = 0.022, size = 107, normalized size = 1.2

$$-\frac{2\,ac^{3}\tan\left(fx+e\right)}{3\,f}+\frac{2\,ac^{3}\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^{2}}{3\,f}+\frac{5\,ac^{3}\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{8\,f}-\frac{ac^{3}\left(\sec\left(fx+e\right)\right.}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x)
```

```
[Out] -2/3/f*a*c^3*tan(f*x+e)+2/3/f*a*c^3*tan(f*x+e)*sec(f*x+e)^2+5/8/f*a*c^3*ln(
sec(f*x+e)+tan(f*x+e))-1/4*a*c^3*sec(f*x+e)^3*tan(f*x+e)/f-3/8*a*c^3*sec(f*
x+e)*tan(f*x+e)/f
```

Maxima [A] time = 0.956496, size = 180, normalized size = 2.09

$$\frac{32\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)ac^{3}+3\,ac^{3}\left(\frac{2\left(3\,\sin\left(fx+e\right)^{3}-5\,\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^{4}-2\,\sin\left(fx+e\right)^{2}+1}-3\,\log\left(\sin\left(fx+e\right)+1\right)+3\,\log\left(\sin\left(fx+\right.\right.\right.}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(32*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^3 + 3*a*c^3*(2*(3*sin(f*x +
e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(
f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 48*a*c^3*log(sec(f*x + e) + tan(
f*x + e)) - 96*a*c^3*tan(f*x + e))/f
```

Fricas [A] time = 0.486274, size = 302, normalized size = 3.51

$$\frac{15\,ac^{3}\cos\left(fx+e\right)^{4}\log\left(\sin\left(fx+e\right)+1\right)-15\,ac^{3}\cos\left(fx+e\right)^{4}\log\left(-\sin\left(fx+e\right)+1\right)-2\left(16\,ac^{3}\cos\left(fx+e\right)^{3}+9\right.}{48\,f\cos\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/48*(15*a*c^3*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 15*a*c^3*cos(f*x + e)
^4*log(-sin(f*x + e) + 1) - 2*(16*a*c^3*cos(f*x + e)^3 + 9*a*c^3*cos(f*x +
e)^2 - 16*a*c^3*cos(f*x + e) + 6*a*c^3)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-ac^{3}\left(\int -\sec\left(e+fx\right)dx+\int 2\sec^{2}\left(e+fx\right)dx+\int -2\sec^{4}\left(e+fx\right)dx+\int \sec^{5}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a*c**3*(Integral(-sec(e + f*x), x) + Integral(2*sec(e + f*x)**2, x) + Inte
gral(-2*sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))
```

Giac [A] time = 1.48159, size = 182, normalized size = 2.12

$$\frac{15\,ac^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,ac^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(15\,ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+73\,ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-55\,ac^3\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/24*(15*a*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a*c^3*log(abs(tan(1/
2*f*x + 1/2*e) - 1)) - 2*(15*a*c^3*tan(1/2*f*x + 1/2*e)^7 + 73*a*c^3*tan(1/
2*f*x + 1/2*e)^5 - 55*a*c^3*tan(1/2*f*x + 1/2*e)^3 + 15*a*c^3*tan(1/2*f*x +
 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

3.3 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=61

$$\frac{ac^2\tan^3(e+fx)}{3f}+\frac{ac^2\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac^2\tan(e+fx)\sec(e+fx)}{2f}$$

```
[Out] (a*c^2*ArcTanh[Sin[e + f*x]])/(2*f) - (a*c^2*Sec[e + f*x]*Tan[e + f*x])/(2*
f) + (a*c^2*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.104285, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3958, 2611, 3770, 2607, 30}

$$\frac{ac^2\tan^3(e+fx)}{3f}+\frac{ac^2\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac^2\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a*c^2*ArcTanh[Sin[e + f*x]])/(2*f) - (a*c^2*Sec[e + f*x]*Tan[e + f*x])/(2*
f) + (a*c^2*Tan[e + f*x]^3)/(3*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx = -\left((ac)\int \left(c\sec(e+fx)\tan^2(e+fx)-c\sec^2(e+fx)\tan^2(e+\right.\right.$$
$$= -\left(\left(ac^2\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)+\left(ac^2\right)\int \sec^2(e+fx$$
$$= -\frac{ac^2\sec(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}\left(ac^2\right)\int \sec(e+fx)\,dx+\frac{(a}{}$$
$$= \frac{ac^2\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac^2\sec(e+fx)\tan(e+fx)}{2f}+\frac{ac^2\,\mathrm{t}}{}$$

Mathematica [B] time = 0.717691, size = 313, normalized size = 5.13

$$-\frac{ac^2\sec(e)\sec^3(e+fx)\left(-12\sin(2e+fx)+6\sin(e+2fx)+6\sin(3e+2fx)+4\sin(2e+3fx)+3\cos(2e+3fx)\log\left(\mathrm{c}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] -(a*c^2*Sec[e]*Sec[e + f*x]^3*(3*Cos[2*e + 3*f*x]*Log[Cos[(e + f*x)/2] - Si
n[(e + f*x)/2]] + 3*Cos[4*e + 3*f*x]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2
]] + 9*Cos[f*x]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]]) + 9*Cos[2*e + f*x]*(Log[Cos[(e + f*x)/2] - Sin[(
e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 3*Cos[2*e + 3*f*
x]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 3*Cos[4*e + 3*f*x]*Log[Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]] - 12*Sin[2*e + f*x] + 6*Sin[e + 2*f*x] + 6*S
in[3*e + 2*f*x] + 4*Sin[2*e + 3*f*x]))/(48*f)
```

Maple [A] time = 0.022, size = 84, normalized size = 1.4

$$-\frac{c^2a\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}+\frac{c^2a\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}-\frac{c^2a\tan\left(fx+e\right)}{3f}+\frac{c^2a\tan\left(fx+e\right)\left(\sec\left(fx+e\right.\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x)
```

```
[Out] -1/2*a*c^2*sec(f*x+e)*tan(f*x+e)/f+1/2/f*c^2*a*ln(sec(f*x+e)+tan(f*x+e))-1/
3/f*c^2*a*tan(f*x+e)+1/3/f*c^2*a*tan(f*x+e)*sec(f*x+e)^2
```

Maxima [A] time = 0.966504, size = 146, normalized size = 2.39

$$\frac{4\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)ac^2+3ac^2\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+12ac^2\log}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^2 + 3*a*c^2*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 12*a
*c^2*log(sec(f*x + e) + tan(f*x + e)) - 12*a*c^2*tan(f*x + e))/f
```

Fricas [A] time = 0.482892, size = 263, normalized size = 4.31

$$\frac{3\,ac^2\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\,ac^2\cos\left(fx+e\right)^3\log\left(-\sin\left(fx+e\right)+1\right)-2\left(2\,ac^2\cos\left(fx+e\right)^2+3\,ac^2\right.}{12\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*a*c^2*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*a*c^2*cos(f*x + e)^3
*log(-sin(f*x + e) + 1) - 2*(2*a*c^2*cos(f*x + e)^2 + 3*a*c^2*cos(f*x + e)
- 2*a*c^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$ac^2\left(\int\sec\left(e+fx\right)dx+\int-\sec^2\left(e+fx\right)dx+\int-\sec^3\left(e+fx\right)dx+\int\sec^4\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a*c**2*(Integral(sec(e + f*x), x) + Integral(-sec(e + f*x)**2, x) + Integra
l(-sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4, x))
```

Giac [B] time = 1.54994, size = 158, normalized size = 2.59

$$\frac{3\,ac^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\,ac^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(3\,ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+8\,ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,ac^2\tan\left(\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] 1/6*(3*a*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a*c^2*log(abs(tan(1/2*f
*x + 1/2*e) - 1)) - 2*(3*a*c^2*tan(1/2*f*x + 1/2*e)^5 + 8*a*c^2*tan(1/2*f*x
 + 1/2*e)^3 - 3*a*c^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)
/f
```

3.4 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=38

$$\frac{ac\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac\tan(e+fx)\sec(e+fx)}{2f}$$

```
[Out] (a*c*ArcTanh[Sin[e + f*x]])/(2*f) - (a*c*Sec[e + f*x]*Tan[e + f*x])/(2*f)
```

Rubi [A] time = 0.0497687, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3958, 2611, 3770}

$$\frac{ac\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a*c*ArcTanh[Sin[e + f*x]])/(2*f) - (a*c*Sec[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx &= -\left((ac)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)\\ &= -\frac{ac\sec(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}(ac)\int \sec(e+fx)\,dx\\ &= \frac{ac\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac\sec(e+fx)\tan(e+fx)}{2f}\end{aligned}$$

Mathematica [A] time = 0.0288633, size = 38, normalized size = 1.

$$-ac\left(\frac{\tan(e+fx)\sec(e+fx)}{2f}-\frac{\tanh^{-1}(\sin(e+fx))}{2f}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] -(a*c*(-ArcTanh[Sin[e + f*x]]/(2*f) + (Sec[e + f*x]*Tan[e + f*x])/(2*f)))
```

Maple [A] time = 0.015, size = 42, normalized size = 1.1

$$\frac{ac\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}-\frac{ac\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] 1/2/f*a*c*ln(sec(f*x+e)+tan(f*x+e))-1/2*a*c*sec(f*x+e)*tan(f*x+e)/f
```

Maxima [A] time = 0.951144, size = 92, normalized size = 2.42

$$\frac{ac\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+4\,ac\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] 1/4*(a*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log
(sin(f*x + e) - 1)) + 4*a*c*log(sec(f*x + e) + tan(f*x + e)))/f
```

Fricas [A] time = 0.476123, size = 178, normalized size = 4.68

$$\frac{ac\cos\left(fx+e\right)^{2}\log\left(\sin\left(fx+e\right)+1\right)-ac\cos\left(fx+e\right)^{2}\log\left(-\sin\left(fx+e\right)+1\right)-2\,ac\sin\left(fx+e\right)}{4\,f\cos\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/4*(a*c*cos(f*x + e)^2*log(sin(f*x + e) + 1) - a*c*cos(f*x + e)^2*log(-sin
(f*x + e) + 1) - 2*a*c*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-ac\left(\int -\sec\left(e+fx\right)dx+\int \sec^3\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] -a*c*(Integral(-sec(e + f*x), x) + Integral(sec(e + f*x)**3, x))
```

Giac [A] time = 1.82412, size = 80, normalized size = 2.11

$$\frac{ac\log\left(\sin\left(fx+e\right)+1\right)-ac\log\left(-\sin\left(fx+e\right)+1\right)+\frac{2\,ac\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*(a*c*log(sin(f*x + e) + 1) - a*c*log(-sin(f*x + e) + 1) + 2*a*c*sin(f*x
 + e)/(sin(f*x + e)^2 - 1))/f
```

3.5 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=42

$$-\frac{a\tanh^{-1}(\sin(e+fx))}{cf}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}$$

```
[Out] -((a*ArcTanh[Sin[e + f*x]])/(c*f)) - (2*a*Tan[e + f*x])/(f*(c - c*Sec[e + f
*x]))
```

Rubi [A] time = 0.0536762, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3957, 3770}

$$-\frac{a\tanh^{-1}(\sin(e+fx))}{cf}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x]),x]
```

```
[Out] -((a*ArcTanh[Sin[e + f*x]])/(c*f)) - (2*a*Tan[e + f*x])/(f*(c - c*Sec[e + f
*x]))
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx &= -\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}-\frac{a\int \sec(e+fx)\,dx}{c}\\ &= -\frac{a\tanh^{-1}(\sin(e+fx))}{cf}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}\end{aligned}$$

Mathematica [A] time = 0.0672002, size = 77, normalized size = 1.83

$$-\frac{a\left(-\frac{2\cot\left(\frac{1}{2}(e+fx)\right)}{f}-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x]),x]
```

```
[Out] -((a*((-2*Cot[(e + f*x)/2])/f - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f
+ Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]/f))/c)
```

Maple [A] time = 0.063, size = 63, normalized size = 1.5

$$-\frac{a}{fc}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{a}{fc}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+2\,\frac{a}{fc\tan\left(1/2\,fx+e/2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

```
[Out] -1/f*a/c*ln(tan(1/2*f*x+1/2*e)+1)+1/f*a/c*ln(tan(1/2*f*x+1/2*e)-1)+2/f*a/c/
tan(1/2*f*x+1/2*e)
```

Maxima [B] time = 0.995552, size = 136, normalized size = 3.24

$$-\frac{a\left(\frac{\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{c}-\frac{\cos\left(fx+e\right)+1}{c\sin\left(fx+e\right)}\right)-\frac{a\left(\cos\left(fx+e\right)+1\right)}{c\sin\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] -(a*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - a*(cos(f*x + e)
+ 1)/(c*sin(f*x + e)))/f
```

Fricas [A] time = 0.471912, size = 174, normalized size = 4.14

$$-\frac{a\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-a\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-4\,a\cos\left(fx+e\right)-4\,a}{2\,cf\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] -1/2*(a*log(sin(f*x + e) + 1)*sin(f*x + e) - a*log(-sin(f*x + e) + 1)*sin(f
*x + e) - 4*a*cos(f*x + e) - 4*a)/(c*f*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a\left(\int \frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int \frac{\sec^{2}\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

```
[Out] -a*(Integral(sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**2
/(sec(e + f*x) - 1), x))/c
```

Giac [A] time = 1.38624, size = 85, normalized size = 2.02

$$-\frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{c}-\frac{a\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{c}-\frac{2a}{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] -(a*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c - a*log(abs(tan(1/2*f*x + 1/2*e) -
 1))/c - 2*a/(c*tan(1/2*f*x + 1/2*e)))/f
```

3.6 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=36

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{3f(c-c\sec(e+fx))^2}$$

```
[Out] -((a + a*Sec[e + f*x])*Tan[e + f*x])/(3*f*(c - c*Sec[e + f*x])^2)
```

Rubi [A] time = 0.0502279, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{3f(c-c\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^2,x]

[Out] -((a + a*Sec[e + f*x])*Tan[e + f*x])/(3*f*(c - c*Sec[e + f*x])^2)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2}$$

Mathematica [A] time = 0.263198, size = 50, normalized size = 1.39

$$\frac{a\csc\left(\frac{e}{2}\right)\left(\sin\left(e+\frac{3fx}{2}\right)-3\sin\left(e+\frac{fx}{2}\right)\right)\csc^3\left(\frac{1}{2}(e+fx)\right)}{12c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^2,x]

[Out] (a*Csc[e/2]*Csc[(e + f*x)/2]^3*(-3*Sin[e + (f*x)/2] + Sin[e + (3*f*x)/2]))/
(12*c^2*f)
```

Maple [A] time = 0.07, size = 21, normalized size = 0.6

$$-\frac{a}{3fc^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x)
```

```
[Out] -1/3/f*a/c^2/tan(1/2*f*x+1/2*e)^3
```

Maxima [B] time = 0.974534, size = 131, normalized size = 3.64

$$-\frac{\frac{a\left(\frac{3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)\left(\cos\left(fx+e\right)+1\right)^3}{c^2\sin\left(fx+e\right)^3}-\frac{a\left(\frac{3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-1\right)\left(\cos\left(fx+e\right)+1\right)^3}{c^2\sin\left(fx+e\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/6*(a*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c
^2*sin(f*x + e)^3) - a*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x
 + e) + 1)^3/(c^2*sin(f*x + e)^3))/f
```

Fricas [A] time = 0.434553, size = 123, normalized size = 3.42

$$\frac{a\cos\left(fx+e\right)^2+2\,a\cos\left(fx+e\right)+a}{3\left(c^2f\cos\left(fx+e\right)-c^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*(a*cos(f*x + e)^2 + 2*a*cos(f*x + e) + a)/((c^2*f*cos(f*x + e) - c^2*f)
*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\left(\int\frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int\frac{\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Giac [A] time = 1.32167, size = 28, normalized size = 0.78

$$-\frac{a}{3\,c^2 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -1/3*a/(c^2*f*tan(1/2*f*x + 1/2*e)^3)
```

3.7 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=76

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{15cf(c-c\sec(e+fx))^2}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{5f(c-c\sec(e+fx))^3}$$

```
[Out] -((a + a*Sec[e + f*x])*Tan[e + f*x])/(5*f*(c - c*Sec[e + f*x])^3) - ((a + a
*Sec[e + f*x])*Tan[e + f*x])/(15*c*f*(c - c*Sec[e + f*x])^2)
```

Rubi [A] time = 0.0980012, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3951, 3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{15cf(c-c\sec(e+fx))^2}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{5f(c-c\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -((a + a*Sec[e + f*x])*Tan[e + f*x])/(5*f*(c - c*Sec[e + f*x])^3) - ((a + a
*Sec[e + f*x])*Tan[e + f*x])/(15*c*f*(c - c*Sec[e + f*x])^2)
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx &= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{5f(c-c\sec(e+fx))^3}+\frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx}{5c}\\ &= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{5f(c-c\sec(e+fx))^3}-\frac{(a+a\sec(e+fx))\tan(e+fx)}{15cf(c-c\sec(e+fx))^2}\end{aligned}$$

Mathematica [A] time = 0.332773, size = 87, normalized size = 1.14

$$-\frac{a\csc\left(\frac{e}{2}\right)\left(15\sin\left(e+\frac{fx}{2}\right)-5\sin\left(e+\frac{3fx}{2}\right)-15\sin\left(2e+\frac{3fx}{2}\right)+4\sin\left(2e+\frac{5fx}{2}\right)+25\sin\left(\frac{fx}{2}\right)\right)\csc^5\left(\frac{1}{2}(e+fx)\right)}{240c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -(a*Csc[e/2]*Csc[(e + f*x)/2]^5*(25*Sin[(f*x)/2] + 15*Sin[e + (f*x)/2] - 5*
Sin[e + (3*f*x)/2] - 15*Sin[2*e + (3*f*x)/2] + 4*Sin[2*e + (5*f*x)/2]))/(24
0*c^3*f)
```

Maple [A] time = 0.079, size = 37, normalized size = 0.5

$$\frac{a}{2fc^3}\left(-\frac{1}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}+\frac{1}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x)
```

```
[Out] 1/2/f*a/c^3*(-1/3/tan(1/2*f*x+1/2*e)^3+1/5/tan(1/2*f*x+1/2*e)^5)
```

Maxima [A] time = 0.9913, size = 158, normalized size = 2.08

$$-\frac{\frac{a\left(\frac{10\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{15\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-3\right)\left(\cos\left(fx+e\right)+1\right)^5}{c^3\sin\left(fx+e\right)^5}+\frac{3a\left(\frac{5\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-1\right)\left(\cos\left(fx+e\right)+1\right)^5}{c^3\sin\left(fx+e\right)^5}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] -1/60*(a*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) + 3*a*(5*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)
^5))/f
```

Fricas [A] time = 0.441625, size = 189, normalized size = 2.49

$$\frac{4a\cos\left(fx+e\right)^3+7a\cos\left(fx+e\right)^2+2a\cos\left(fx+e\right)-a}{15\left(c^3f\cos\left(fx+e\right)^2-2c^3f\cos\left(fx+e\right)+c^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*(4*a*cos(f*x + e)^3 + 7*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/((c^3
*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a\left(\int \frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a*(Integral(sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e +
f*x) - 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e + f*x)*
*2 + 3*sec(e + f*x) - 1), x))/c**3
```

Giac [A] time = 1.29068, size = 53, normalized size = 0.7

$$-\frac{5\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,a}{30\,c^3 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] -1/30*(5*a*tan(1/2*f*x + 1/2*e)^2 - 3*a)/(c^3*f*tan(1/2*f*x + 1/2*e)^5)
```

3.8 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=116

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{105f\left(c^2-c^2\sec(e+fx)\right)^2}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{35cf(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}$$

```
[Out] -((a + a*Sec[e + f*x])*Tan[e + f*x])/(7*f*(c - c*Sec[e + f*x])^4) - (2*(a +
 a*Sec[e + f*x])*Tan[e + f*x])/(35*c*f*(c - c*Sec[e + f*x])^3) - (2*(a + a*
Sec[e + f*x])*Tan[e + f*x])/(105*f*(c^2 - c^2*Sec[e + f*x])^2)
```

Rubi [A] time = 0.150324, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3951, 3950}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{105f\left(c^2-c^2\sec(e+fx)\right)^2}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{35cf(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -((a + a*Sec[e + f*x])*Tan[e + f*x])/(7*f*(c - c*Sec[e + f*x])^4) - (2*(a +
 a*Sec[e + f*x])*Tan[e + f*x])/(35*c*f*(c - c*Sec[e + f*x])^3) - (2*(a + a*
Sec[e + f*x])*Tan[e + f*x])/(105*f*(c^2 - c^2*Sec[e + f*x])^2)
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{7f(c-c\sec(e+fx))^4}+\frac{2\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx}{7c}$$

$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{7f(c-c\sec(e+fx))^4}-\frac{2(a+a\sec(e+fx))\tan(e+fx)}{35cf(c-c\sec(e+fx))^3}+\frac{2\int \frac{\sec(e+f}{(c-}}{}$$

$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{7f(c-c\sec(e+fx))^4}-\frac{2(a+a\sec(e+fx))\tan(e+fx)}{35cf(c-c\sec(e+fx))^3}-\frac{2(a+a\sec}{105f(c}$$

Mathematica [A] time = 0.39332, size = 113, normalized size = 0.97

$$-\frac{a\csc\left(\frac{e}{2}\right)\left(455\sin\left(e+\frac{fx}{2}\right)-273\sin\left(e+\frac{3fx}{2}\right)-210\sin\left(2e+\frac{3fx}{2}\right)+56\sin\left(2e+\frac{5fx}{2}\right)+105\sin\left(3e+\frac{5fx}{2}\right)-23\sin\right.}{6720c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -(a*Csc[e/2]*Csc[(e + f*x)/2]^7*(350*Sin[(f*x)/2] + 455*Sin[e + (f*x)/2] -
273*Sin[e + (3*f*x)/2] - 210*Sin[2*e + (3*f*x)/2] + 56*Sin[2*e + (5*f*x)/2]
 + 105*Sin[3*e + (5*f*x)/2] - 23*Sin[3*e + (7*f*x)/2]))/(6720*c^4*f)
```

Maple [A] time = 0.084, size = 50, normalized size = 0.4

$$\frac{a}{4fc^4}\left(-\frac{1}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}+\frac{2}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}-\frac{1}{7}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x)
```

```
[Out] 1/4/f*a/c^4*(-1/3/tan(1/2*f*x+1/2*e)^3+2/5/tan(1/2*f*x+1/2*e)^5-1/7/tan(1/2
*f*x+1/2*e)^7)
```

Maxima [A] time = 1.02754, size = 239, normalized size = 2.06

$$\frac{\frac{a\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}+\frac{3a\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{35\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-5\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}}{840f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/840*(a*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x + e
) + 1)^7/(c^4*sin(f*x + e)^7) + 3*a*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f
```

Fricas [A] time = 0.446291, size = 258, normalized size = 2.22

$$\frac{23a\cos(fx+e)^4+36a\cos(fx+e)^3+5a\cos(fx+e)^2-6a\cos(fx+e)+2a}{105\left(c^4f\cos(fx+e)^3-3c^4f\cos(fx+e)^2+3c^4f\cos(fx+e)-c^4f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/105*(23*a*cos(f*x + e)^4 + 36*a*cos(f*x + e)^3 + 5*a*cos(f*x + e)^2 - 6*a
*cos(f*x + e) + 2*a)/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^
4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\left(\int\frac{\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int\frac{\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f
*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**
4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

Giac [A] time = 1.31515, size = 73, normalized size = 0.63

$$-\frac{35\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-42\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+15\,a}{420\,c^4f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] -1/420*(35*a*tan(1/2*f*x + 1/2*e)^4 - 42*a*tan(1/2*f*x + 1/2*e)^2 + 15*a)/(
c^4*f*tan(1/2*f*x + 1/2*e)^7)
```

3.9 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=158

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{315cf\left(c^2-c^2\sec(e+fx)\right)^2}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{105c^2f(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{21cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx}{9f(c-}$$

```
[Out] -((a + a*Sec[e + f*x])*Tan[e + f*x])/(9*f*(c - c*Sec[e + f*x])^5) - ((a + a
*Sec[e + f*x])*Tan[e + f*x])/(21*c*f*(c - c*Sec[e + f*x])^4) - (2*(a + a*Se
c[e + f*x])*Tan[e + f*x])/(105*c^2*f*(c - c*Sec[e + f*x])^3) - (2*(a + a*Se
c[e + f*x])*Tan[e + f*x])/(315*c*f*(c^2 - c^2*Sec[e + f*x])^2)
```

Rubi [A] time = 0.209773, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3951, 3950}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{315cf\left(c^2-c^2\sec(e+fx)\right)^2}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{105c^2f(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{21cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx}{9f(c-}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -((a + a*Sec[e + f*x])*Tan[e + f*x])/(9*f*(c - c*Sec[e + f*x])^5) - ((a + a
*Sec[e + f*x])*Tan[e + f*x])/(21*c*f*(c - c*Sec[e + f*x])^4) - (2*(a + a*Se
c[e + f*x])*Tan[e + f*x])/(105*c^2*f*(c - c*Sec[e + f*x])^3) - (2*(a + a*Se
c[e + f*x])*Tan[e + f*x])/(315*c*f*(c^2 - c^2*Sec[e + f*x])^2)
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx}{3c}$$

$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))\tan(e+fx)}{21cf(c-c\sec(e+fx))^4} + \frac{2\int \frac{\sec(e+fx}{(c-c}}{}$$

$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))\tan(e+fx)}{21cf(c-c\sec(e+fx))^4} - \frac{2(a+a\sec}{105c^2f(c}$$

$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))\tan(e+fx)}{21cf(c-c\sec(e+fx))^4} - \frac{2(a+a\sec}{105c^2f(c}$$

Mathematica [A] time = 0.356075, size = 139, normalized size = 0.88

$$-\frac{a\csc\left(\frac{e}{2}\right)\left(3465\sin\left(e+\frac{fx}{2}\right)-2247\sin\left(e+\frac{3fx}{2}\right)-2625\sin\left(2e+\frac{3fx}{2}\right)+1143\sin\left(2e+\frac{5fx}{2}\right)+945\sin\left(3e+\frac{5fx}{2}\right)-20}{80640c^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -(a*Csc[e/2]*Csc[(e + f*x)/2]^9*(3843*Sin[(f*x)/2] + 3465*Sin[e + (f*x)/2]
- 2247*Sin[e + (3*f*x)/2] - 2625*Sin[2*e + (3*f*x)/2] + 1143*Sin[2*e + (5*f
*x)/2] + 945*Sin[3*e + (5*f*x)/2] - 207*Sin[3*e + (7*f*x)/2] - 315*Sin[4*e
+ (7*f*x)/2] + 58*Sin[4*e + (9*f*x)/2]))/(80640*c^5*f)
```

Maple [A] time = 0.093, size = 63, normalized size = 0.4

$$\frac{a}{8fc^5}\left(-\frac{1}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}+\frac{3}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}-\frac{3}{7}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}+\frac{1}{9}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x)
```

```
[Out] 1/8/f*a/c^5*(-1/3/tan(1/2*f*x+1/2*e)^3+3/5/tan(1/2*f*x+1/2*e)^5-3/7/tan(1/2
*f*x+1/2*e)^7+1/9/tan(1/2*f*x+1/2*e)^9)
```

Maxima [A] time = 1.02831, size = 266, normalized size = 1.68

$$-\frac{\frac{a\left(\frac{180\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{315\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}+\frac{5a\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{63\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-7\right)(\cos(fx+e}{c^5\sin(fx+e)^9}}{5040f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x, algorithm="maxi
ma")
```

```
[Out] -1/5040*(a*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)
 + 5*a*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) +
1)^9/(c^5*sin(f*x + e)^9))/f
```

Fricas [A] time = 0.453771, size = 321, normalized size = 2.03

$$\frac{58\,a\cos\left(fx+e\right)^{5}+83\,a\cos\left(fx+e\right)^{4}+4\,a\cos\left(fx+e\right)^{3}-11\,a\cos\left(fx+e\right)^{2}+8\,a\cos\left(fx+e\right)-2\,a}{315\left(c^{5}f\cos\left(fx+e\right)^{4}-4\,c^{5}f\cos\left(fx+e\right)^{3}+6\,c^{5}f\cos\left(fx+e\right)^{2}-4\,c^{5}f\cos\left(fx+e\right)+c^{5}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x, algorithm="fric
as")
```

```
[Out] 1/315*(58*a*cos(f*x + e)^5 + 83*a*cos(f*x + e)^4 + 4*a*cos(f*x + e)^3 - 11*
a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - 2*a)/((c^5*f*cos(f*x + e)^4 - 4*c^5*f
*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*si
n(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a\left(\int\frac{\sec\left(e+fx\right)}{\sec^{5}\left(e+fx\right)-5\sec^{4}\left(e+fx\right)+10\sec^{3}\left(e+fx\right)-10\sec^{2}\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int\frac{\sec^{2}\left(e+fx\right)}{\sec^{5}\left(e+fx\right)-5\sec^{4}\left(e+fx\right)+10\sec^{3}\left(e+fx\right)-10\sec^{2}\left(e+fx\right)}\right.}{c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a*(Integral(sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e +
 f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e +
f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(
e + f*x)**2 + 5*sec(e + f*x) - 1), x))/c**5
```

Giac [A] time = 1.22664, size = 93, normalized size = 0.59

$$-\frac{105\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-189\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+135\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-35\,a}{2520\,c^{5}f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x, algorithm="giac
")
```

```
[Out] -1/2520*(105*a*tan(1/2*f*x + 1/2*e)^6 - 189*a*tan(1/2*f*x + 1/2*e)^4 + 135*
a*tan(1/2*f*x + 1/2*e)^2 - 35*a)/(c^5*f*tan(1/2*f*x + 1/2*e)^9)
```

3.10 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$

Optimal. Leaf size=171

$$-\frac{a^2c^5\tan^7(e+fx)}{7f}-\frac{4a^2c^5\tan^5(e+fx)}{5f}+\frac{9a^2c^5\tanh^{-1}(\sin(e+fx))}{16f}+\frac{a^2c^5\tan^3(e+fx)\sec^3(e+fx)}{2f}-\frac{3a^2c^5\tan(e+$$

```
[Out] (9*a^2*c^5*ArcTanh[Sin[e + f*x]])/(16*f) - (3*a^2*c^5*Sec[e + f*x]*Tan[e +
f*x])/(16*f) - (3*a^2*c^5*Sec[e + f*x]^3*Tan[e + f*x])/(8*f) + (a^2*c^5*Sec
[e + f*x]*Tan[e + f*x]^3)/(4*f) + (a^2*c^5*Sec[e + f*x]^3*Tan[e + f*x]^3)/(
2*f) - (4*a^2*c^5*Tan[e + f*x]^5)/(5*f) - (a^2*c^5*Tan[e + f*x]^7)/(7*f)
```

Rubi [A] time = 0.273989, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3958, 2611, 3770, 2607, 30, 3768, 14}

$$-\frac{a^2c^5\tan^7(e+fx)}{7f}-\frac{4a^2c^5\tan^5(e+fx)}{5f}+\frac{9a^2c^5\tanh^{-1}(\sin(e+fx))}{16f}+\frac{a^2c^5\tan^3(e+fx)\sec^3(e+fx)}{2f}-\frac{3a^2c^5\tan(e+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```


```
[Out] (9*a^2*c^5*ArcTanh[Sin[e + f*x]])/(16*f) - (3*a^2*c^5*Sec[e + f*x]*Tan[e +
f*x])/(16*f) - (3*a^2*c^5*Sec[e + f*x]^3*Tan[e + f*x])/(8*f) + (a^2*c^5*Sec
[e + f*x]*Tan[e + f*x]^3)/(4*f) + (a^2*c^5*Sec[e + f*x]^3*Tan[e + f*x]^3)/(
2*f) - (4*a^2*c^5*Tan[e + f*x]^5)/(5*f) - (a^2*c^5*Tan[e + f*x]^7)/(7*f)
```


Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = \left(a^2c^2\right)\int \left(c^3\sec(e+fx)\tan^4(e+fx)-3c^3\sec^2(e+fx)\,\mathrm{ta}\right.$$

$$= \left(a^2c^5\right)\int \sec(e+fx)\tan^4(e+fx)\,dx-\left(a^2c^5\right)\int \sec^4(e+$$

$$= \frac{a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f}+\frac{a^2c^5\sec^3(e+fx)\tan^3(e+}{2f}$$

$$= -\frac{3a^2c^5\sec(e+fx)\tan(e+fx)}{8f}-\frac{3a^2c^5\sec^3(e+fx)\tan(e}{8f}$$

$$= \frac{3a^2c^5\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3a^2c^5\sec(e+fx)\tan(e+fx}{16f}$$

$$= \frac{9a^2c^5\tanh^{-1}(\sin(e+fx))}{16f}-\frac{3a^2c^5\sec(e+fx)\tan(e+fx}{16f}$$

Mathematica [A] time = 1.85716, size = 102, normalized size = 0.6

$$\frac{a^2c^5\left(10080\tanh^{-1}(\sin(e+fx))-(2520\sin(e+fx)-455\sin(2(e+fx))-616\sin(3(e+fx))+2380\sin(4(e+fx))\right.}{17920f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*c^5*(10080*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^7*(2520*Sin[e + f*x] -
 455*Sin[2*(e + f*x)] - 616*Sin[3*(e + f*x)] + 2380*Sin[4*(e + f*x)] - 392*
Sin[5*(e + f*x)] + 245*Sin[6*(e + f*x)] + 184*Sin[7*(e + f*x)]))))/(17920*f)
```

Maple [A] time = 0.033, size = 192, normalized size = 1.1

$$-\frac{23\,a^2c^5\tan\left(fx+e\right)}{35\,f}-\frac{13\,a^2c^5\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^4}{35\,f}+\frac{41\,a^2c^5\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^2}{35\,f}-\frac{5\,a^2c^5\left(\sec\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x)
```

```
[Out] -23/35/f*a^2*c^5*tan(f*x+e)-13/35/f*a^2*c^5*tan(f*x+e)*sec(f*x+e)^4+41/35/f
*a^2*c^5*tan(f*x+e)*sec(f*x+e)^2-5/8*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)/f-7/16
*a^2*c^5*sec(f*x+e)*tan(f*x+e)/f+9/16/f*a^2*c^5*ln(sec(f*x+e)+tan(f*x+e))+1
/2/f*a^2*c^5*tan(f*x+e)*sec(f*x+e)^5-1/7/f*a^2*c^5*tan(f*x+e)*sec(f*x+e)^6
```

Maxima [B] time = 1.02112, size = 497, normalized size = 2.91

$$-\frac{96\left(5\tan\left(fx+e\right)^{7}+21\tan\left(fx+e\right)^{5}+35\tan\left(fx+e\right)^{3}+35\tan\left(fx+e\right)\right)a^{2}c^{5}+224\left(3\tan\left(fx+e\right)^{5}+10\tan\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] -1/3360*(96*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 35*
tan(f*x + e))*a^2*c^5 + 224*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(
f*x + e))*a^2*c^5 - 5600*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^5 + 105*a^
2*c^5*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x
 + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) +
1) + 15*log(sin(f*x + e) - 1)) - 1050*a^2*c^5*(2*(3*sin(f*x + e)^3 - 5*sin(
f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1)
 + 3*log(sin(f*x + e) - 1)) + 840*a^2*c^5*(2*sin(f*x + e)/(sin(f*x + e)^2 -
 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 3360*a^2*c^5*log(sec
(f*x + e) + tan(f*x + e)) + 10080*a^2*c^5*tan(f*x + e))/f
```

Fricas [A] time = 0.512487, size = 448, normalized size = 2.62

$$\frac{315\,a^{2}c^{5}\cos\left(fx+e\right)^{7}\log\left(\sin\left(fx+e\right)+1\right)-315\,a^{2}c^{5}\cos\left(fx+e\right)^{7}\log\left(-\sin\left(fx+e\right)+1\right)-2\left(368\,a^{2}c^{5}\cos\left(fx+e\right)^{6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/1120*(315*a^2*c^5*cos(f*x + e)^7*log(sin(f*x + e) + 1) - 315*a^2*c^5*cos(
f*x + e)^7*log(-sin(f*x + e) + 1) - 2*(368*a^2*c^5*cos(f*x + e)^6 + 245*a^2
*c^5*cos(f*x + e)^5 - 656*a^2*c^5*cos(f*x + e)^4 + 350*a^2*c^5*cos(f*x + e)
^3 + 208*a^2*c^5*cos(f*x + e)^2 - 280*a^2*c^5*cos(f*x + e) + 80*a^2*c^5)*si
n(f*x + e))/(f*cos(f*x + e)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^{2}c^{5}\left(\int -\sec\left(e+fx\right)dx+\int 3\sec^{2}\left(e+fx\right)dx+\int -\sec^{3}\left(e+fx\right)dx+\int -5\sec^{4}\left(e+fx\right)dx+\int 5\sec^{5}\left(e+fx\right)d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**2*c**5*(Integral(-sec(e + f*x), x) + Integral(3*sec(e + f*x)**2, x) + I
ntegral(-sec(e + f*x)**3, x) + Integral(-5*sec(e + f*x)**4, x) + Integral(5
*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x) + Integral(-3*sec(e + f
*x)**7, x) + Integral(sec(e + f*x)**8, x))
```

Giac [A] time = 1.3115, size = 279, normalized size = 1.63

$$\frac{315\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-315\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(315\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}-2100\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.}{}}{560\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/560*(315*a^2*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 315*a^2*c^5*log(abs
(tan(1/2*f*x + 1/2*e) - 1)) - 2*(315*a^2*c^5*tan(1/2*f*x + 1/2*e)^13 - 2100
*a^2*c^5*tan(1/2*f*x + 1/2*e)^11 - 8393*a^2*c^5*tan(1/2*f*x + 1/2*e)^9 + 92
16*a^2*c^5*tan(1/2*f*x + 1/2*e)^7 - 5943*a^2*c^5*tan(1/2*f*x + 1/2*e)^5 + 2
100*a^2*c^5*tan(1/2*f*x + 1/2*e)^3 - 315*a^2*c^5*tan(1/2*f*x + 1/2*e))/(tan
(1/2*f*x + 1/2*e)^2 - 1)^7)/f
```

3.11 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$

Optimal. Leaf size=150

$$-\frac{2a^2c^4\tan^5(e+fx)}{5f}+\frac{7a^2c^4\tanh^{-1}(\sin(e+fx))}{16f}+\frac{a^2c^4\tan^3(e+fx)\sec^3(e+fx)}{6f}-\frac{a^2c^4\tan(e+fx)\sec^3(e+fx)}{8f}+\frac{a}{}$$

```
[Out] (7*a^2*c^4*ArcTanh[Sin[e + f*x]])/(16*f) - (5*a^2*c^4*Sec[e + f*x]*Tan[e +
f*x])/(16*f) - (a^2*c^4*Sec[e + f*x]^3*Tan[e + f*x])/(8*f) + (a^2*c^4*Sec[e
 + f*x]*Tan[e + f*x]^3)/(4*f) + (a^2*c^4*Sec[e + f*x]^3*Tan[e + f*x]^3)/(6*
f) - (2*a^2*c^4*Tan[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.24069, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3958, 2611, 3770, 2607, 30, 3768}

$$-\frac{2a^2c^4\tan^5(e+fx)}{5f}+\frac{7a^2c^4\tanh^{-1}(\sin(e+fx))}{16f}+\frac{a^2c^4\tan^3(e+fx)\sec^3(e+fx)}{6f}-\frac{a^2c^4\tan(e+fx)\sec^3(e+fx)}{8f}+\frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (7*a^2*c^4*ArcTanh[Sin[e + f*x]])/(16*f) - (5*a^2*c^4*Sec[e + f*x]*Tan[e +
f*x])/(16*f) - (a^2*c^4*Sec[e + f*x]^3*Tan[e + f*x])/(8*f) + (a^2*c^4*Sec[e
 + f*x]*Tan[e + f*x]^3)/(4*f) + (a^2*c^4*Sec[e + f*x]^3*Tan[e + f*x]^3)/(6*
f) - (2*a^2*c^4*Tan[e + f*x]^5)/(5*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx = \left(a^2c^2\right)\int \left(c^2\sec(e+fx)\tan^4(e+fx)-2c^2\sec^2(e+fx)\,\mathrm{ta}\right.$$
$$= \left(a^2c^4\right)\int \sec(e+fx)\tan^4(e+fx)\,dx+\left(a^2c^4\right)\int \sec^3(e+$$
$$= \frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{4f}+\frac{a^2c^4\sec^3(e+fx)\tan^3(e+}{6f}$$
$$= -\frac{3a^2c^4\sec(e+fx)\tan(e+fx)}{8f}-\frac{a^2c^4\sec^3(e+fx)\tan(e}{8f}$$
$$= \frac{3a^2c^4\tanh^{-1}(\sin(e+fx))}{8f}-\frac{5a^2c^4\sec(e+fx)\tan(e+fx}{16f}$$
$$= \frac{7a^2c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{5a^2c^4\sec(e+fx)\tan(e+fx}{16f}$$

Mathematica [A] time = 1.33315, size = 91, normalized size = 0.61

$$\frac{a^2c^4\left(1680\tanh^{-1}(\sin(e+fx))+(330\sin(e+fx)-240\sin(2(e+fx))-445\sin(3(e+fx))+192\sin(4(e+fx))-13\right.}{3840f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*c^4*(1680*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]^6*(330*Sin[e + f*x] - 2
40*Sin[2*(e + f*x)] - 445*Sin[3*(e + f*x)] + 192*Sin[4*(e + f*x)] - 135*Sin
[5*(e + f*x)] - 48*Sin[6*(e + f*x)])))/(3840*f)
```

Maple [A] time = 0.03, size = 167, normalized size = 1.1

$$-\frac{a^2c^4\left(\sec\left(fx+e\right)\right)^3\tan\left(fx+e\right)}{24\,f}-\frac{9\,a^2c^4\sec\left(fx+e\right)\tan\left(fx+e\right)}{16\,f}+\frac{7\,a^2c^4\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{16\,f}-\frac{2\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x)
```

```
[Out] -1/24*a^2*c^4*sec(f*x+e)^3*tan(f*x+e)/f-9/16*a^2*c^4*sec(f*x+e)*tan(f*x+e)/
f+7/16/f*a^2*c^4*ln(sec(f*x+e)+tan(f*x+e))-2/5/f*a^2*c^4*tan(f*x+e)+4/5/f*a
^2*c^4*tan(f*x+e)*sec(f*x+e)^2-2/5/f*a^2*c^4*tan(f*x+e)*sec(f*x+e)^4+1/6/f*
a^2*c^4*tan(f*x+e)*sec(f*x+e)^5
```

Maxima [B] time = 0.993228, size = 433, normalized size = 2.89

$$-\frac{64\left(3\tan\left(fx+e\right)^{5}+10\tan\left(fx+e\right)^{3}+15\tan\left(fx+e\right)\right)a^{2}c^{4}-640\left(\tan\left(fx+e\right)^{3}+3\tan\left(fx+e\right)\right)a^{2}c^{4}+5a^{2}c^{4}\left(\frac{2(\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] -1/480*(64*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^4
 - 640*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^4 + 5*a^2*c^4*(2*(15*sin(f*x
 + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x
+ e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x
+ e) - 1)) - 30*a^2*c^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e
)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e)
- 1)) - 120*a^2*c^4*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e)
 + 1) + log(sin(f*x + e) - 1)) - 480*a^2*c^4*log(sec(f*x + e) + tan(f*x + e
)) + 960*a^2*c^4*tan(f*x + e))/f
```

Fricas [A] time = 0.507252, size = 404, normalized size = 2.69

$$\frac{105a^{2}c^{4}\cos\left(fx+e\right)^{6}\log\left(\sin\left(fx+e\right)+1\right)-105a^{2}c^{4}\cos\left(fx+e\right)^{6}\log\left(-\sin\left(fx+e\right)+1\right)-2\left(96a^{2}c^{4}\cos\left(fx+e\right)^{5}+\cdots\right.}{480f\cos\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/480*(105*a^2*c^4*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 105*a^2*c^4*cos(f
*x + e)^6*log(-sin(f*x + e) + 1) - 2*(96*a^2*c^4*cos(f*x + e)^5 + 135*a^2*c
^4*cos(f*x + e)^4 - 192*a^2*c^4*cos(f*x + e)^3 + 10*a^2*c^4*cos(f*x + e)^2
+ 96*a^2*c^4*cos(f*x + e) - 40*a^2*c^4)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}c^{4}\left(\int\sec\left(e+fx\right)dx+\int-2\sec^{2}\left(e+fx\right)dx+\int-\sec^{3}\left(e+fx\right)dx+\int4\sec^{4}\left(e+fx\right)dx+\int-\sec^{5}\left(e+fx\right)dx\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**2*c**4*(Integral(sec(e + f*x), x) + Integral(-2*sec(e + f*x)**2, x) + In
tegral(-sec(e + f*x)**3, x) + Integral(4*sec(e + f*x)**4, x) + Integral(-se
c(e + f*x)**5, x) + Integral(-2*sec(e + f*x)**6, x) + Integral(sec(e + f*x)
**7, x))
```

Giac [A] time = 1.35006, size = 252, normalized size = 1.68

$$\frac{105\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-105\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(105\,a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}-595\,a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/240*(105*a^2*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 105*a^2*c^4*log(abs
(tan(1/2*f*x + 1/2*e) - 1)) - 2*(105*a^2*c^4*tan(1/2*f*x + 1/2*e)^11 - 595*
a^2*c^4*tan(1/2*f*x + 1/2*e)^9 - 1686*a^2*c^4*tan(1/2*f*x + 1/2*e)^7 + 1386
*a^2*c^4*tan(1/2*f*x + 1/2*e)^5 - 595*a^2*c^4*tan(1/2*f*x + 1/2*e)^3 + 105*
a^2*c^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^6)/f
```

3.12 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=94

$$-\frac{a^2c^3\tan^5(e+fx)}{5f}+\frac{3a^2c^3\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2c^3\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^2c^3\tan(e+fx)\sec(e+fx)}{8f}$$

```
[Out] (3*a^2*c^3*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^2*c^3*Sec[e + f*x]*Tan[e + f
*x])/(8*f) + (a^2*c^3*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f) - (a^2*c^3*Tan[e +
 f*x]^5)/(5*f)
```

Rubi [A] time = 0.148851, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3958, 2611, 3770, 2607, 30}

$$-\frac{a^2c^3\tan^5(e+fx)}{5f}+\frac{3a^2c^3\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2c^3\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^2c^3\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (3*a^2*c^3*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^2*c^3*Sec[e + f*x]*Tan[e + f
*x])/(8*f) + (a^2*c^3*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f) - (a^2*c^3*Tan[e +
 f*x]^5)/(5*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx = \left(a^2c^2\right)\int \left(c\sec(e+fx)\tan^4(e+fx)-c\sec^2(e+fx)\tan^4\right.$$
$$= \left(a^2c^3\right)\int \sec(e+fx)\tan^4(e+fx)\,dx-\left(a^2c^3\right)\int \sec^2(e+$$
$$= \frac{a^2c^3\sec(e+fx)\tan^3(e+fx)}{4f}-\frac{1}{4}\left(3a^2c^3\right)\int \sec(e+fx)\,\mathrm{t}$$
$$= -\frac{3a^2c^3\sec(e+fx)\tan(e+fx)}{8f}+\frac{a^2c^3\sec(e+fx)\tan^3(e}{4f}$$
$$= \frac{3a^2c^3\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3a^2c^3\sec(e+fx)\tan(e+fx}{8f}$$

Mathematica [A] time = 0.741826, size = 82, normalized size = 0.87

$$\frac{a^2c^3\left(120\tanh^{-1}(\sin(e+fx))-(40\sin(e+fx)+10\sin(2(e+fx))-20\sin(3(e+fx))+25\sin(4(e+fx))+4\sin(5(\right.}{320f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*c^3*(120*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^5*(40*Sin[e + f*x] + 10*
Sin[2*(e + f*x)] - 20*Sin[3*(e + f*x)] + 25*Sin[4*(e + f*x)] + 4*Sin[5*(e +
 f*x)])))/(320*f)
```

Maple [A] time = 0.026, size = 142, normalized size = 1.5

$$-\frac{a^2c^3\tan\left(fx+e\right)}{5f}+\frac{2\,a^2c^3\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^2}{5f}-\frac{5\,a^2c^3\sec\left(fx+e\right)\tan\left(fx+e\right)}{8f}+\frac{3\,a^2c^3\ln\left(\sec\left(fx+e\right)\right.}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x)
```

```
[Out] -1/5/f*a^2*c^3*tan(f*x+e)+2/5/f*a^2*c^3*tan(f*x+e)*sec(f*x+e)^2-5/8*a^2*c^3
*sec(f*x+e)*tan(f*x+e)/f+3/8/f*a^2*c^3*ln(sec(f*x+e)+tan(f*x+e))+1/4/f*a^2*
c^3*tan(f*x+e)*sec(f*x+e)^3-1/5/f*a^2*c^3*tan(f*x+e)*sec(f*x+e)^4
```

Maxima [B] time = 0.977232, size = 306, normalized size = 3.26

$$-\frac{16\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)a^2c^3-160\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^2c^3+15\,a^2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/240*(16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^3
 - 160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3 + 15*a^2*c^3*(2*(3*sin(f*x
 + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(s
in(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*a^2*c^3*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 240*
a^2*c^3*log(sec(f*x + e) + tan(f*x + e)) + 240*a^2*c^3*tan(f*x + e))/f
```

Fricas [A] time = 0.49673, size = 356, normalized size = 3.79

$$\frac{15\,a^2c^3\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\,a^2c^3\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)-2\left(8\,a^2c^3\cos\left(fx+e\right)^4+25\right.}{80\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/80*(15*a^2*c^3*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 15*a^2*c^3*cos(f*x
+ e)^5*log(-sin(f*x + e) + 1) - 2*(8*a^2*c^3*cos(f*x + e)^4 + 25*a^2*c^3*co
s(f*x + e)^3 - 16*a^2*c^3*cos(f*x + e)^2 - 10*a^2*c^3*cos(f*x + e) + 8*a^2*
c^3)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^2c^3\left(\int -\sec\left(e+fx\right)dx+\int \sec^2\left(e+fx\right)dx+\int 2\sec^3\left(e+fx\right)dx+\int -2\sec^4\left(e+fx\right)dx+\int -\sec^5\left(e+fx\right)dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**2*c**3*(Integral(-sec(e + f*x), x) + Integral(sec(e + f*x)**2, x) + Int
egral(2*sec(e + f*x)**3, x) + Integral(-2*sec(e + f*x)**4, x) + Integral(-s
ec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Giac [A] time = 1.26778, size = 225, normalized size = 2.39

$$\frac{15\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(15\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9-70\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-128\,a^2\right.}{\left(\tan\right.}}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/40*(15*a^2*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^2*c^3*log(abs(ta
n(1/2*f*x + 1/2*e) - 1)) - 2*(15*a^2*c^3*tan(1/2*f*x + 1/2*e)^9 - 70*a^2*c^
3*tan(1/2*f*x + 1/2*e)^7 - 128*a^2*c^3*tan(1/2*f*x + 1/2*e)^5 + 70*a^2*c^3*
tan(1/2*f*x + 1/2*e)^3 - 15*a^2*c^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/
2*e)^2 - 1)^5)/f
```

3.13 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=73

$$\frac{3a^2c^2\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2c^2\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^2c^2\tan(e+fx)\sec(e+fx)}{8f}$$

```
[Out] (3*a^2*c^2*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^2*c^2*Sec[e + f*x]*Tan[e + f
*x])/(8*f) + (a^2*c^2*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f)
```

Rubi [A] time = 0.108332, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3958, 2611, 3770}

$$\frac{3a^2c^2\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2c^2\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^2c^2\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (3*a^2*c^2*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^2*c^2*Sec[e + f*x]*Tan[e + f
*x])/(8*f) + (a^2*c^2*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \left(a^2c^2\right)\int \sec(e+fx)\tan^4(e+fx)\,dx$$
$$= \frac{a^2c^2\sec(e+fx)\tan^3(e+fx)}{4f} - \frac{1}{4}\left(3a^2c^2\right)\int \sec(e+fx)\,\text{t}$$
$$= -\frac{3a^2c^2\sec(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^2\sec(e+fx)\tan^3(e}{4f}$$
$$= \frac{3a^2c^2\tanh^{-1}(\sin(e+fx))}{8f} - \frac{3a^2c^2\sec(e+fx)\tan(e+fx}{8f}$$

Mathematica [A] time = 0.149303, size = 51, normalized size = 0.7

$$\frac{a^2c^2\left(6\tanh^{-1}(\sin(e+fx)) - (5\cos(2(e+fx))+1)\tan(e+fx)\sec^3(e+fx)\right)}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*c^2*(6*ArcTanh[Sin[e + f*x]] - (1 + 5*Cos[2*(e + f*x)])*Sec[e + f*x]^3
*Tan[e + f*x]))/(16*f)
```

Maple [A] time = 0.02, size = 75, normalized size = 1.

$$-\frac{5\,a^2c^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{8\,f} + \frac{3\,a^2c^2\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{8\,f} + \frac{a^2c^2\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^3}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x)
```

```
[Out] -5/8*a^2*c^2*sec(f*x+e)*tan(f*x+e)/f+3/8/f*a^2*c^2*ln(sec(f*x+e)+tan(f*x+e)
)+1/4/f*a^2*c^2*tan(f*x+e)*sec(f*x+e)^3
```

Maxima [B] time = 0.967191, size = 203, normalized size = 2.78

$$-\frac{a^2c^2\left(\frac{2\left(3\,\sin\left(fx+e\right)^3-5\,\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^4-2\,\sin\left(fx+e\right)^2+1} - 3\,\log\left(\sin\left(fx+e\right)+1\right) + 3\,\log\left(\sin\left(fx+e\right)-1\right)\right) - 8\,a^2c^2\left(\frac{2\,\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1} - \log\left(\sin\left(\right.\right.\right.}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/16*(a^2*c^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*s
in(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 8
*a^2*c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log
(sin(f*x + e) - 1)) - 16*a^2*c^2*log(sec(f*x + e) + tan(f*x + e)))/f
```

Fricas [A] time = 0.48342, size = 243, normalized size = 3.33

$$\frac{3\,a^2c^2\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-3\,a^2c^2\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)-2\left(5\,a^2c^2\cos\left(fx+e\right)^2-2\,a^2c}{16\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/16*(3*a^2*c^2*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 3*a^2*c^2*cos(f*x +
e)^4*log(-sin(f*x + e) + 1) - 2*(5*a^2*c^2*cos(f*x + e)^2 - 2*a^2*c^2)*sin(
f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2c^2\left(\int\sec\left(e+fx\right)dx+\int-2\sec^3\left(e+fx\right)dx+\int\sec^5\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**2*c**2*(Integral(sec(e + f*x), x) + Integral(-2*sec(e + f*x)**3, x) + In
tegral(sec(e + f*x)**5, x))
```

Giac [A] time = 1.24941, size = 124, normalized size = 1.7

$$\frac{3\,a^2c^2\log\left(\sin\left(fx+e\right)+1\right)-3\,a^2c^2\log\left(-\sin\left(fx+e\right)+1\right)+\frac{2\left(5\,a^2c^2\sin\left(fx+e\right)^3-3\,a^2c^2\sin\left(fx+e\right)\right)}{\left(\sin\left(fx+e\right)^2-1\right)^2}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/16*(3*a^2*c^2*log(sin(f*x + e) + 1) - 3*a^2*c^2*log(-sin(f*x + e) + 1) +
2*(5*a^2*c^2*sin(f*x + e)^3 - 3*a^2*c^2*sin(f*x + e))/(sin(f*x + e)^2 - 1)^
2)/f
```

3.14 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=61

$$-\frac{a^2c\tan^3(e+fx)}{3f}+\frac{a^2c\tanh^{-1}(\sin(e+fx))}{2f}-\frac{a^2c\tan(e+fx)\sec(e+fx)}{2f}$$

```
[Out] (a^2*c*ArcTanh[Sin[e + f*x]])/(2*f) - (a^2*c*Sec[e + f*x]*Tan[e + f*x])/(2*
f) - (a^2*c*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.102527, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3958, 2611, 3770, 2607, 30}

$$-\frac{a^2c\tan^3(e+fx)}{3f}+\frac{a^2c\tanh^{-1}(\sin(e+fx))}{2f}-\frac{a^2c\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*c*ArcTanh[Sin[e + f*x]])/(2*f) - (a^2*c*Sec[e + f*x]*Tan[e + f*x])/(2*
f) - (a^2*c*Tan[e + f*x]^3)/(3*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = -\left((ac)\int \left(a\sec(e+fx)\tan^2(e+fx)+a\sec^2(e+fx)\tan^2(e+\right.\right.$$
$$= -\left(\left(a^2c\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)-\left(a^2c\right)\int \sec^2(e+fx$$
$$= -\frac{a^2c\sec(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}\left(a^2c\right)\int \sec(e+fx)\,dx-\frac{(a^2}{}$$
$$= \frac{a^2c\tanh^{-1}(\sin(e+fx))}{2f}-\frac{a^2c\sec(e+fx)\tan(e+fx)}{2f}-\frac{a^2c\,\mathrm{t}}{}$$

Mathematica [A] time = 0.102027, size = 45, normalized size = 0.74

$$\frac{a^2c\left(-2\tan^3(e+fx)+3\tanh^{-1}(\sin(e+fx))-3\tan(e+fx)\sec(e+fx)\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*c*(3*ArcTanh[Sin[e + f*x]] - 3*Sec[e + f*x]*Tan[e + f*x] - 2*Tan[e + f
*x]^3))/(6*f)
```

Maple [A] time = 0.02, size = 84, normalized size = 1.4

$$\frac{a^2c\tan\left(fx+e\right)}{3f}+\frac{a^2c\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}-\frac{a^2c\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}-\frac{a^2c\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

```
[Out] 1/3/f*a^2*c*tan(f*x+e)+1/2/f*a^2*c*ln(sec(f*x+e)+tan(f*x+e))-1/2*a^2*c*sec(
f*x+e)*tan(f*x+e)/f-1/3/f*a^2*c*tan(f*x+e)*sec(f*x+e)^2
```

Maxima [A] time = 1.00824, size = 146, normalized size = 2.39

$$-\frac{4\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^2c-3a^2c\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-12a^2c\log}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c - 3*a^2*c*(2*sin(f*x + e)/
(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 12*
a^2*c*log(sec(f*x + e) + tan(f*x + e)) - 12*a^2*c*tan(f*x + e))/f
```

Fricas [A] time = 0.481272, size = 263, normalized size = 4.31

$$\frac{3\,a^2c\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\,a^2c\cos\left(fx+e\right)^3\log\left(-\sin\left(fx+e\right)+1\right)+2\left(2\,a^2c\cos\left(fx+e\right)^2-3\,a^2c\right.}{12\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*a^2*c*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*a^2*c*cos(f*x + e)^3
*log(-sin(f*x + e) + 1) + 2*(2*a^2*c*cos(f*x + e)^2 - 3*a^2*c*cos(f*x + e)
- 2*a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^2c\left(\int-\sec\left(e+fx\right)dx+\int-\sec^2\left(e+fx\right)dx+\int\sec^3\left(e+fx\right)dx+\int\sec^4\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*c*(Integral(-sec(e + f*x), x) + Integral(-sec(e + f*x)**2, x) + Integ
ral(sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4, x))
```

Giac [B] time = 1.33076, size = 158, normalized size = 2.59

$$\frac{3\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(3\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-8\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a^2c\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*a^2*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a^2*c*log(abs(tan(1/2*f
*x + 1/2*e) - 1)) - 2*(3*a^2*c*tan(1/2*f*x + 1/2*e)^5 - 8*a^2*c*tan(1/2*f*x
 + 1/2*e)^3 - 3*a^2*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)
/f
```

3.15 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=74

$$-\frac{3a^2\tan(e+fx)}{cf}-\frac{3a^2\tanh^{-1}(\sin(e+fx))}{cf}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}$$

```
[Out] (-3*a^2*ArcTanh[Sin[e + f*x]])/(c*f) - (3*a^2*Tan[e + f*x])/(c*f) - (2*(a^2
 + a^2*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x]))
```

Rubi [A] time = 0.0993604, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3957, 3787, 3770, 3767, 8}

$$-\frac{3a^2\tan(e+fx)}{cf}-\frac{3a^2\tanh^{-1}(\sin(e+fx))}{cf}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (-3*a^2*ArcTanh[Sin[e + f*x]])/(c*f) - (3*a^2*Tan[e + f*x])/(c*f) - (2*(a^2
 + a^2*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x]))
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = -\frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{(3a)\int \sec(e+fx)(a+a\sec(e+fx))}{c}$$
$$= -\frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{\left(3a^2\right)\int \sec(e+fx)\,dx}{c} - \frac{\left(3a^2\right)\int \sec}{c}$$
$$= -\frac{3a^2\tanh^{-1}(\sin(e+fx))}{cf} - \frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{f(c-c\sec(e+fx))} + \frac{\left(3a^2\right)\text{Sub}}{}$$
$$= -\frac{3a^2\tanh^{-1}(\sin(e+fx))}{cf} - \frac{3a^2\tan(e+fx)}{cf} - \frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e}{f(c-c\sec(e+fx))}$$

Mathematica [B] time = 1.71673, size = 220, normalized size = 2.97

$$\frac{2a^2\sin\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(4\csc\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sec\left(\frac{1}{2}(e+fx)\right)+\tan\left(\frac{1}{2}(e+fx)\right)\left(\frac{}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)(\sin}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^2*Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]*(4*Csc[e/2]*Sec[(e +
f*x)/2]*Sin[(f*x)/2] + (-3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 3*Log
[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + Sin[f*x]/((Cos[e/2] - Sin[e/2])*(Co
s[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])))*Tan[(e + f*x)/2]))/(f*(c - c*Sec[e + f*x]))
```

Maple [A] time = 0.063, size = 116, normalized size = 1.6

$$\frac{a^2}{fc}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1} - 3\,\frac{a^2\ln\left(\tan\left(1/2\,fx+e/2\right)+1\right)}{fc} + \frac{a^2}{fc}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{-1} + 3\,\frac{a^2\ln\left(\tan\left(1/2\,fx+e/2\right)\right.}{fc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x)
```

```
[Out] 1/f*a^2/c/(tan(1/2*f*x+1/2*e)+1)-3/f*a^2/c*ln(tan(1/2*f*x+1/2*e)+1)+1/f*a^2
/c/(tan(1/2*f*x+1/2*e)-1)+3/f*a^2/c*ln(tan(1/2*f*x+1/2*e)-1)+4/f*a^2/c/tan(
1/2*f*x+1/2*e)
```

Maxima [B] time = 1.03162, size = 304, normalized size = 4.11

$$-\frac{a^2\left(\frac{\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}\right) + 2\,a^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c} - \frac{\cos(fx+e}{c\sin(fx+}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -(a^2*((3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c*sin(f*x + e)/(cos(f*x
 + e) + 1) - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + log(sin(f*x + e)/(cos
(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c) + 2*a^2
*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x +
e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - a^2*(cos(f*x + e) +
 1)/(c*sin(f*x + e)))/f
```

Fricas [A] time = 0.476103, size = 275, normalized size = 3.72

$$-\frac{3\,a^{2}\cos\left(fx+e\right)\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-3\,a^{2}\cos\left(fx+e\right)\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-10\,a^{2}\cos}{2\,cf\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/2*(3*a^2*cos(f*x + e)*log(sin(f*x + e) + 1)*sin(f*x + e) - 3*a^2*cos(f*x
 + e)*log(-sin(f*x + e) + 1)*sin(f*x + e) - 10*a^2*cos(f*x + e)^2 - 8*a^2*c
os(f*x + e) + 2*a^2)/(c*f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^{2}\left(\int\frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{2\sec^{2}\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{\sec^{3}\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*(Integral(sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(2*sec(e + f*
x)**2/(sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) - 1),
 x))/c
```

Giac [A] time = 1.3107, size = 142, normalized size = 1.92

$$-\frac{\frac{3\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c}-\frac{3\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c}-\frac{2\left(3\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-2\,a^{2}\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -(3*a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c - 3*a^2*log(abs(tan(1/2*f*x +
1/2*e) - 1))/c - 2*(3*a^2*tan(1/2*f*x + 1/2*e)^2 - 2*a^2)/((tan(1/2*f*x + 1
/2*e)^3 - tan(1/2*f*x + 1/2*e))*c))/f
```

3.16 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=89

$$\frac{a^2\tanh^{-1}(\sin(e+fx))}{c^2f}+\frac{2a^2\tan(e+fx)}{f\left(c^2-c^2\sec(e+fx)\right)}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^2}$$

```
[Out] (a^2*ArcTanh[Sin[e + f*x]])/(c^2*f) - (2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f
*x])/(3*f*(c - c*Sec[e + f*x])^2) + (2*a^2*Tan[e + f*x])/(f*(c^2 - c^2*Sec[
e + f*x]))
```

Rubi [A] time = 0.127974, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3957, 3770}

$$\frac{a^2\tanh^{-1}(\sin(e+fx))}{c^2f}+\frac{2a^2\tan(e+fx)}{f\left(c^2-c^2\sec(e+fx)\right)}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[e + f*x]])/(c^2*f) - (2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f
*x])/(3*f*(c - c*Sec[e + f*x])^2) + (2*a^2*Tan[e + f*x])/(f*(c^2 - c^2*Sec[
e + f*x]))
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = -\frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^2}-\frac{a\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx}{c}$$

$$= -\frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^2}+\frac{2a^2\tan(e+fx)}{f\left(c^2-c^2\sec(e+fx)\right)}+\frac{a^2\int\sec(e+f$$

$$= \frac{a^2\tanh^{-1}(\sin(e+fx))}{c^2f}-\frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^2}+\frac{2a^2\tan(e+}{f\left(c^2-c^2\sec(e\right.}$$

Mathematica [A] time = 0.0846671, size = 109, normalized size = 1.22

$$\frac{a^2\left(-\frac{4\cot\left(\frac{1}{2}(e+fx)\right)}{3f}-\frac{2\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{3f}-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*((-4*Cot[(e + f*x)/2])/(3*f) - (2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2)
/(3*f) - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f + Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]/f))/c^2
```

Maple [A] time = 0.077, size = 91, normalized size = 1.

$$\frac{a^2}{fc^2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-\frac{a^2}{fc^2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\frac{2a^2}{3fc^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}-2\frac{a^2}{fc^2\tan\left(1/2\,fx+e/2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)
```

```
[Out] 1/f*a^2/c^2*ln(tan(1/2*f*x+1/2*e)+1)-1/f*a^2/c^2*ln(tan(1/2*f*x+1/2*e)-1)-2
/3/f*a^2/c^2/tan(1/2*f*x+1/2*e)^3-2/f*a^2/c^2/tan(1/2*f*x+1/2*e)
```

Maxima [B] time = 0.983811, size = 271, normalized size = 3.04

$$\frac{a^2\left(\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2}-\frac{\left(\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}\right)-\frac{2a^2\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}+\frac{a^2\left(\frac{3\sin(f}{(\cos(fx+}\right.}{}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(a^2*(6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^2 - 6*log(sin(f*x +
e)/(cos(f*x + e) + 1) - 1)/c^2 - (9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) - 2*a^2*(3*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3) + a^2*(3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x +
e)^3))/f
```

Fricas [A] time = 0.478872, size = 313, normalized size = 3.52

$$-\frac{8a^2\cos\left(fx+e\right)^2-8a^2\cos\left(fx+e\right)-3\left(a^2\cos\left(fx+e\right)-a^2\right)\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)+3\left(a^2\cos\left(fx+\right.\right.}{6\left(c^2f\cos\left(fx+e\right)-c^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(8*a^2*cos(f*x + e)^2 - 8*a^2*cos(f*x + e) - 3*(a^2*cos(f*x + e) - a^2
)*log(sin(f*x + e) + 1)*sin(f*x + e) + 3*(a^2*cos(f*x + e) - a^2)*log(-sin(
f*x + e) + 1)*sin(f*x + e) - 16*a^2)/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x
+ e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\left(\int \frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{2\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + In
tegral(2*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Giac [A] time = 1.26181, size = 119, normalized size = 1.34

$$\frac{\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c^2}-\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c^2}-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^2\right)}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^2 - 3*a^2*log(abs(tan(1/2*f
*x + 1/2*e) - 1))/c^2 - 2*(3*a^2*tan(1/2*f*x + 1/2*e)^2 + a^2)/(c^2*tan(1/2
*f*x + 1/2*e)^3))/f
```

3.17 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=38

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$

```
[Out] -((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(c - c*Sec[e + f*x])^3)
```

Rubi [A] time = 0.0751821, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(c - c*Sec[e + f*x])^3)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3}$$

Mathematica [A] time = 0.115587, size = 25, normalized size = 0.66

$$\frac{a^2\cot^5\left(\frac{1}{2}(e+fx)\right)}{5c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*Cot[(e + f*x)/2]^5)/(5*c^3*f)
```

Maple [A] time = 0.083, size = 23, normalized size = 0.6

$$\frac{a^2}{5fc^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x)
```

```
[Out] 1/5/f*a^2/c^3/tan(1/2*f*x+1/2*e)^5
```

Maxima [B] time = 0.986597, size = 255, normalized size = 6.71

$$\frac{\frac{a^2\left(\frac{10 \sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{15 \sin(fx+e)^4}{(\cos(fx+e)+1)^4}+3\right)(\cos(fx+e)+1)^5}{c^3 \sin(fx+e)^5}-\frac{a^2\left(\frac{10 \sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{15 \sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{c^3 \sin(fx+e)^5}-\frac{6\,a^2\left(\frac{5 \sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)(\cos(fx+e)+1)^5}{c^3 \sin(fx+e)^5}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(a^2*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) - a^2*(10*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) - 6*a^2*(5*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5))/f
```

Fricas [B] time = 0.439728, size = 196, normalized size = 5.16

$$\frac{a^2\cos\left(fx+e\right)^3+3\,a^2\cos\left(fx+e\right)^2+3\,a^2\cos\left(fx+e\right)+a^2}{5\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)+c^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) + a^2)/
((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2\left(\int\frac{\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int\frac{2\sec^2(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int\frac{\sec^3(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\right.}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) - 1), x) + Integral(2*sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e +
f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)*
```

```
*3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Giac [A] time = 1.34113, size = 31, normalized size = 0.82

$$\frac{a^2}{5\,c^3 f \tan\left(\frac{1}{2}\,f x + \frac{1}{2}\,e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/5*a^2/(c^3*f*tan(1/2*f*x + 1/2*e)^5)
```

3.18 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=80

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{35cf(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{7f(c-c\sec(e+fx))^4}$$

```
[Out] -((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(7*f*(c - c*Sec[e + f*x])^4) - ((a +
 a*Sec[e + f*x])^2*Tan[e + f*x])/(35*c*f*(c - c*Sec[e + f*x])^3)
```

Rubi [A] time = 0.150209, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3951, 3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{35cf(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{7f(c-c\sec(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(7*f*(c - c*Sec[e + f*x])^4) - ((a +
 a*Sec[e + f*x])^2*Tan[e + f*x])/(35*c*f*(c - c*Sec[e + f*x])^3)
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx &= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{7f(c-c\sec(e+fx))^4}+\frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx}{7c}\\ &= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{7f(c-c\sec(e+fx))^4}-\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{35cf(c-c\sec(e+fx))^3}\end{aligned}$$

Mathematica [A] time = 0.43033, size = 115, normalized size = 1.44

$$-\frac{a^2\csc\left(\frac{e}{2}\right)\left(140\sin\left(e+\frac{fx}{2}\right)-91\sin\left(e+\frac{3fx}{2}\right)-35\sin\left(2e+\frac{3fx}{2}\right)+7\sin\left(2e+\frac{5fx}{2}\right)+35\sin\left(3e+\frac{5fx}{2}\right)-6\sin\left(3e+\cdots\right.\right.}{2240c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -(a^2*Csc[e/2]*Csc[(e + f*x)/2]^7*(70*Sin[(f*x)/2] + 140*Sin[e + (f*x)/2] -
 91*Sin[e + (3*f*x)/2] - 35*Sin[2*e + (3*f*x)/2] + 7*Sin[2*e + (5*f*x)/2] +
 35*Sin[3*e + (5*f*x)/2] - 6*Sin[3*e + (7*f*x)/2]))/(2240*c^4*f)
```

Maple [A] time = 0.097, size = 39, normalized size = 0.5

$$\frac{a^{2}}{2\,fc^{4}}\left(\frac{1}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}-\frac{1}{7}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x)
```

```
[Out] 1/2/f*a^2/c^4*(1/5/tan(1/2*f*x+1/2*e)^5-1/7/tan(1/2*f*x+1/2*e)^7)
```

Maxima [B] time = 1.05842, size = 365, normalized size = 4.56

$$\frac{\frac{2\,a^{2}\left(\frac{21\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{35\,\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{105\,\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-15\right)\left(\cos\left(fx+e\right)+1\right)^{7}}{c^{4}\sin\left(fx+e\right)^{7}}+\frac{3\,a^{2}\left(\frac{21\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{35\,\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{35\,\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-5\right)\left(\cos\left(fx+e\right)+1\right)^{7}}{c^{4}\sin\left(fx+e\right)^{7}}-\cdots}{840\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(2*a^2*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x
 + e) + 1)^7/(c^4*sin(f*x + e)^7) + 3*a^2*(21*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) - a^2*(21*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15)*(cos(f*x + e) + 1)^7/(c^4*sin
(f*x + e)^7))/f
```

Fricas [A] time = 0.443729, size = 267, normalized size = 3.34

$$\frac{6\,a^{2}\cos\left(fx+e\right)^{4}+17\,a^{2}\cos\left(fx+e\right)^{3}+15\,a^{2}\cos\left(fx+e\right)^{2}+3\,a^{2}\cos\left(fx+e\right)-a^{2}}{35\left(c^{4}f\cos\left(fx+e\right)^{3}-3\,c^{4}f\cos\left(fx+e\right)^{2}+3\,c^{4}f\cos\left(fx+e\right)-c^{4}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/35*(6*a^2*cos(f*x + e)^4 + 17*a^2*cos(f*x + e)^3 + 15*a^2*cos(f*x + e)^2
+ 3*a^2*cos(f*x + e) - a^2)/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2
 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\left(\int \frac{\sec(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx + \int \frac{2\sec^2(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx + \int \frac{}{\sec^4(e+fx)}\right.}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e
+ f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(2*sec(e + f*x)**2/(sec(e + f
*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) +
Integral(sec(e + f*x)**3/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f
*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

Giac [A] time = 1.33813, size = 58, normalized size = 0.72

$$\frac{7\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-5\,a^2}{70\,c^4f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/70*(7*a^2*tan(1/2*f*x + 1/2*e)^2 - 5*a^2)/(c^4*f*tan(1/2*f*x + 1/2*e)^7)
```

3.19 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=121

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{315c^2f(c-c\sec(e+fx))^3}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{63cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}$$

```
[Out] -((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(9*f*(c - c*Sec[e + f*x])^5) - (2*(a
 + a*Sec[e + f*x])^2*Tan[e + f*x])/(63*c*f*(c - c*Sec[e + f*x])^4) - (2*(a
+ a*Sec[e + f*x])^2*Tan[e + f*x])/(315*c^2*f*(c - c*Sec[e + f*x])^3)
```

Rubi [A] time = 0.230276, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3951, 3950}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{315c^2f(c-c\sec(e+fx))^3}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{63cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(9*f*(c - c*Sec[e + f*x])^5) - (2*(a
 + a*Sec[e + f*x])^2*Tan[e + f*x])/(63*c*f*(c - c*Sec[e + f*x])^4) - (2*(a
+ a*Sec[e + f*x])^2*Tan[e + f*x])/(315*c^2*f*(c - c*Sec[e + f*x])^3)
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx &= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{9f(c-c\sec(e+fx))^5}+\frac{2\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx}{9c}\\
&= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{9f(c-c\sec(e+fx))^5}-\frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{63cf(c-c\sec(e+fx))^4}+\frac{2\int
\\
&= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{9f(c-c\sec(e+fx))^5}-\frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{63cf(c-c\sec(e+fx))^4}-\frac{2(a
\end{aligned}$$

Mathematica [A] time = 0.420944, size = 141, normalized size = 1.17

$$-\frac{a^2 \csc\left(\frac{e}{2}\right)\left(2520 \sin\left(e+\frac{fx}{2}\right)-1638 \sin\left(e+\frac{3fx}{2}\right)-2310 \sin\left(2e+\frac{3fx}{2}\right)+1062 \sin\left(2e+\frac{5fx}{2}\right)+630 \sin\left(3e+\frac{5fx}{2}\right)-10\right.}{80640 c^5 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -(a^2*Csc[e/2]*Csc[(e + f*x)/2]^9*(3402*Sin[(f*x)/2] + 2520*Sin[e + (f*x)/2
] - 1638*Sin[e + (3*f*x)/2] - 2310*Sin[2*e + (3*f*x)/2] + 1062*Sin[2*e + (5
*f*x)/2] + 630*Sin[3*e + (5*f*x)/2] - 108*Sin[3*e + (7*f*x)/2] - 315*Sin[4*
e + (7*f*x)/2] + 47*Sin[4*e + (9*f*x)/2]))/(80640*c^5*f)
```

Maple [A] time = 0.102, size = 52, normalized size = 0.4

$$\frac{a^2}{4fc^5}\left(\frac{1}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}-\frac{2}{7}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}+\frac{1}{9}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x)
```

```
[Out] 1/4/f*a^2/c^5*(1/5/tan(1/2*f*x+1/2*e)^5-2/7/tan(1/2*f*x+1/2*e)^7+1/9/tan(1/
2*f*x+1/2*e)^9)
```

Maxima [B] time = 1.05711, size = 363, normalized size = 3.

$$-\frac{\frac{a^2\left(\frac{180 \sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378 \sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420 \sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{315 \sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5 \sin(fx+e)^9}+\frac{10a^2\left(\frac{18 \sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42 \sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{63 \sin(fx+e)^8}{(\cos(fx+e)+1)^8}-7\right)(\cos(fx\right.}{c^5 \sin(fx+e)^9}}{5040 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] -1/5040*(a^2*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^
9) + 10*a^2*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x +
e) + 1)^9/(c^5*sin(f*x + e)^9) + 7*a^2*(18*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 - 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5)*(cos(f*x + e) + 1)^9/(c^5
*sin(f*x + e)^9))/f
```

Fricas [A] time = 0.45639, size = 342, normalized size = 2.83

$$\frac{47 a^2 \cos(fx+e)^5+127 a^2 \cos(fx+e)^4+101 a^2 \cos(fx+e)^3+11 a^2 \cos(fx+e)^2-8 a^2 \cos(fx+e)+2 a^2}{315\left(c^5 f \cos(fx+e)^4-4 c^5 f \cos(fx+e)^3+6 c^5 f \cos(fx+e)^2-4 c^5 f \cos(fx+e)+c^5 f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/315*(47*a^2*cos(f*x + e)^5 + 127*a^2*cos(f*x + e)^4 + 101*a^2*cos(f*x + e
)^3 + 11*a^2*cos(f*x + e)^2 - 8*a^2*cos(f*x + e) + 2*a^2)/((c^5*f*cos(f*x +
 e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x +
 e) + c^5*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2\left(\int \frac{\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int \frac{2\sec^2\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+f\right.}\right.}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(
e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(2*sec
(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10
*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(
e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 +
 5*sec(e + f*x) - 1), x))/c**5
```

Giac [A] time = 1.26366, size = 81, normalized size = 0.67

$$\frac{63\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-90\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35\,a^2}{1260\,c^5 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/1260*(63*a^2*tan(1/2*f*x + 1/2*e)^4 - 90*a^2*tan(1/2*f*x + 1/2*e)^2 + 35*
a^2)/(c^5*f*tan(1/2*f*x + 1/2*e)^9)
```

3.20 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx$

Optimal. Leaf size=163

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{1155f\left(c^2-c^2\sec(e+fx)\right)^3}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{231c^2f(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{33cf(c-c\sec(e+fx))^5}-\frac{\tan(e+fx)}{11f(c-}$$

```
[Out] -((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(11*f*(c - c*Sec[e + f*x])^6) - ((a
+ a*Sec[e + f*x])^2*Tan[e + f*x])/(33*c*f*(c - c*Sec[e + f*x])^5) - (2*(a +
 a*Sec[e + f*x])^2*Tan[e + f*x])/(231*c^2*f*(c - c*Sec[e + f*x])^4) - (2*(a
 + a*Sec[e + f*x])^2*Tan[e + f*x])/(1155*f*(c^2 - c^2*Sec[e + f*x])^3)
```

Rubi [A] time = 0.314677, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3951, 3950}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{1155f\left(c^2-c^2\sec(e+fx)\right)^3}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{231c^2f(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{33cf(c-c\sec(e+fx))^5}-\frac{\tan(e+fx)}{11f(c-}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^6,x]
```

```
[Out] -((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(11*f*(c - c*Sec[e + f*x])^6) - ((a
+ a*Sec[e + f*x])^2*Tan[e + f*x])/(33*c*f*(c - c*Sec[e + f*x])^5) - (2*(a +
 a*Sec[e + f*x])^2*Tan[e + f*x])/(231*c^2*f*(c - c*Sec[e + f*x])^4) - (2*(a
 + a*Sec[e + f*x])^2*Tan[e + f*x])/(1155*f*(c^2 - c^2*Sec[e + f*x])^3)
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} + \frac{3\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}dx}{11c}$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{(a+a\sec(e+fx))^2\tan(e+fx)}{33cf(c-c\sec(e+fx))^5} + \frac{2\int \mathrm{s}\ldots}{\ldots}$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{(a+a\sec(e+fx))^2\tan(e+fx)}{33cf(c-c\sec(e+fx))^5} - \frac{2(a+\ldots}{2\ldots}$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{(a+a\sec(e+fx))^2\tan(e+fx)}{33cf(c-c\sec(e+fx))^5} - \frac{2(a+\ldots}{2\ldots}$$

Mathematica [A] time = 0.703718, size = 167, normalized size = 1.02

$$-\frac{a^2\csc\left(\frac{e}{2}\right)\left(37422\sin\left(e+\frac{fx}{2}\right)-27060\sin\left(e+\frac{3fx}{2}\right)-23100\sin\left(2e+\frac{3fx}{2}\right)+11220\sin\left(2e+\frac{5fx}{2}\right)+13860\sin\left(3e\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^6,x]
```

```
[Out] -(a^2*Csc[e/2]*Csc[(e + f*x)/2]^11*(32802*Sin[(f*x)/2] + 37422*Sin[e + (f*x
)/2] - 27060*Sin[e + (3*f*x)/2] - 23100*Sin[2*e + (3*f*x)/2] + 11220*Sin[2*
e + (5*f*x)/2] + 13860*Sin[3*e + (5*f*x)/2] - 4895*Sin[3*e + (7*f*x)/2] - 3
465*Sin[4*e + (7*f*x)/2] + 517*Sin[4*e + (9*f*x)/2] + 1155*Sin[5*e + (9*f*x
)/2] - 152*Sin[5*e + (11*f*x)/2]))/(1182720*c^6*f)
```

Maple [A] time = 0.115, size = 65, normalized size = 0.4

$$\frac{a^2}{8fc^6}\left(-\frac{1}{11}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-11}+\frac{1}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}-\frac{3}{7}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}+\frac{1}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x)
```

```
[Out] 1/8/f*a^2/c^6*(-1/11/tan(1/2*f*x+1/2*e)^11+1/5/tan(1/2*f*x+1/2*e)^5-3/7/tan
(1/2*f*x+1/2*e)^7+1/3/tan(1/2*f*x+1/2*e)^9)
```

Maxima [B] time = 1.05086, size = 525, normalized size = 3.22

$$\frac{a^2\left(\frac{385\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{990\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{1386\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{1155\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{3465\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-315\right)(\cos(fx+e)+1)^{11}}{c^6\sin(fx+e)^{11}}+\frac{6a^2\left(\frac{385\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{330\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{462\ldots}{(\cos\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/110880*(a^2*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 990*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 1386*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1155*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^10/(cos(f*x + e) + 1)^
10 - 315)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11) + 6*a^2*(385*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)^8/(cos(f*x + e) +
1)^8 - 1155*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 105)*(cos(f*x + e) + 1)
^11/(c^6*sin(f*x + e)^11) + 5*a^2*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
- 990*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1386*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 693*sin(f*x + e)^10/
(cos(f*x + e) + 1)^10 - 63)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11))/f
```

Fricas [A] time = 0.462013, size = 414, normalized size = 2.54

$$\frac{152\,a^2\cos\left(fx+e\right)^6+395\,a^2\cos\left(fx+e\right)^5+289\,a^2\cos\left(fx+e\right)^4+15\,a^2\cos\left(fx+e\right)^3-19\,a^2\cos\left(fx+e\right)^2+10\,a^2\cos}{1155\left(c^6f\cos\left(fx+e\right)^5-5\,c^6f\cos\left(fx+e\right)^4+10\,c^6f\cos\left(fx+e\right)^3-10\,c^6f\cos\left(fx+e\right)^2+5\,c^6f\cos\left(fx+e\right)-c^6f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/1155*(152*a^2*cos(f*x + e)^6 + 395*a^2*cos(f*x + e)^5 + 289*a^2*cos(f*x +
 e)^4 + 15*a^2*cos(f*x + e)^3 - 19*a^2*cos(f*x + e)^2 + 10*a^2*cos(f*x + e)
 - 2*a^2)/((c^6*f*cos(f*x + e)^5 - 5*c^6*f*cos(f*x + e)^4 + 10*c^6*f*cos(f*
x + e)^3 - 10*c^6*f*cos(f*x + e)^2 + 5*c^6*f*cos(f*x + e) - c^6*f)*sin(f*x
+ e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\left(\int\frac{\sec\left(e+fx\right)}{\sec^6\left(e+fx\right)-6\sec^5\left(e+fx\right)+15\sec^4\left(e+fx\right)-20\sec^3\left(e+fx\right)+15\sec^2\left(e+fx\right)-6\sec\left(e+fx\right)+1}\,dx+\int\frac{2\sec^2\left(e\right.}{\sec^6\left(e+fx\right)-6\sec^5\left(e+fx\right)+15\sec^4\left(e+fx\right)-20}\right.}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**6,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*sec(e
 + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x) + 1),
 x) + Integral(2*sec(e + f*x)**2/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*
sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x)
+ 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 +
15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*
x) + 1), x))/c**6
```

Giac [A] time = 1.32409, size = 104, normalized size = 0.64

$$\frac{231\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-495\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+385\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-105\,a^2}{9240\,c^6f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] 1/9240*(231*a^2*tan(1/2*f*x + 1/2*e)^6 - 495*a^2*tan(1/2*f*x + 1/2*e)^4 + 3
85*a^2*tan(1/2*f*x + 1/2*e)^2 - 105*a^2)/(c^6*f*tan(1/2*f*x + 1/2*e)^11)
```

3.21 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx$

Optimal. Leaf size=227

$$\frac{a^3c^6\tan^9(e+fx)}{9f}+\frac{4a^3c^6\tan^7(e+fx)}{7f}+\frac{55a^3c^6\tanh^{-1}(\sin(e+fx))}{128f}-\frac{3a^3c^6\tan^5(e+fx)\sec^3(e+fx)}{8f}+\frac{5a^3c^6\tan^3(e$$

```
[Out] (55*a^3*c^6*ArcTanh[Sin[e + f*x]])/(128*f) - (25*a^3*c^6*Sec[e + f*x]*Tan[e
 + f*x])/(128*f) - (15*a^3*c^6*Sec[e + f*x]^3*Tan[e + f*x])/(64*f) + (5*a^3
*c^6*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) + (5*a^3*c^6*Sec[e + f*x]^3*Tan[e
+ f*x]^3)/(16*f) - (a^3*c^6*Sec[e + f*x]*Tan[e + f*x]^5)/(6*f) - (3*a^3*c^6
*Sec[e + f*x]^3*Tan[e + f*x]^5)/(8*f) + (4*a^3*c^6*Tan[e + f*x]^7)/(7*f) +
(a^3*c^6*Tan[e + f*x]^9)/(9*f)
```

Rubi [A] time = 0.335235, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3958, 2611, 3770, 2607, 30, 3768, 14}

$$\frac{a^3c^6\tan^9(e+fx)}{9f}+\frac{4a^3c^6\tan^7(e+fx)}{7f}+\frac{55a^3c^6\tanh^{-1}(\sin(e+fx))}{128f}-\frac{3a^3c^6\tan^5(e+fx)\sec^3(e+fx)}{8f}+\frac{5a^3c^6\tan^3(e$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6,x]
```



```
[Out] (55*a^3*c^6*ArcTanh[Sin[e + f*x]])/(128*f) - (25*a^3*c^6*Sec[e + f*x]*Tan[e
 + f*x])/(128*f) - (15*a^3*c^6*Sec[e + f*x]^3*Tan[e + f*x])/(64*f) + (5*a^3
*c^6*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) + (5*a^3*c^6*Sec[e + f*x]^3*Tan[e
+ f*x]^3)/(16*f) - (a^3*c^6*Sec[e + f*x]*Tan[e + f*x]^5)/(6*f) - (3*a^3*c^6
*Sec[e + f*x]^3*Tan[e + f*x]^5)/(8*f) + (4*a^3*c^6*Tan[e + f*x]^7)/(7*f) +
(a^3*c^6*Tan[e + f*x]^9)/(9*f)
```


Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx = -\left(\left(a^3c^3\right)\int \left(c^3\sec(e+fx)\tan^6(e+fx)-3c^3\sec^2(e+fx\right.\right.$$

$$= -\left(\left(a^3c^6\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\right)+\left(a^3c^6\right)\int \sec^4($$

$$= -\frac{a^3c^6\sec(e+fx)\tan^5(e+fx)}{6f}-\frac{3a^3c^6\sec^3(e+fx)\tan^5(}{8f}$$

$$= \frac{5a^3c^6\sec(e+fx)\tan^3(e+fx)}{24f}+\frac{5a^3c^6\sec^3(e+fx)\tan^3(}{16f}$$

$$= -\frac{5a^3c^6\sec(e+fx)\tan(e+fx)}{16f}-\frac{15a^3c^6\sec^3(e+fx)\tan(}{64f}$$

$$= \frac{5a^3c^6\tanh^{-1}(\sin(e+fx))}{16f}-\frac{25a^3c^6\sec(e+fx)\tan(e+f}{128f}$$

$$= \frac{55a^3c^6\tanh^{-1}(\sin(e+fx))}{128f}-\frac{25a^3c^6\sec(e+fx)\tan(e+}{128f}$$

Mathematica [A] time = 3.39876, size = 122, normalized size = 0.54

$$\frac{a^3c^6\left(443520\tanh^{-1}(\sin(e+fx))-(-88704\sin(e+fx)+88074\sin(2(e+fx))+37632\sin(3(e+fx))-2142\sin(4(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6,x]
```

```
[Out] (a^3*c^6*(443520*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^9*(-88704*Sin[e + f*x
] + 88074*Sin[2*(e + f*x)] + 37632*Sin[3*(e + f*x)] - 2142*Sin[4*(e + f*x)]
 + 2304*Sin[5*(e + f*x)] + 39858*Sin[6*(e + f*x)] - 7488*Sin[7*(e + f*x)] +
 4599*Sin[8*(e + f*x)] + 1856*Sin[9*(e + f*x)])))/(1032192*f)
```

Maple [A] time = 0.143, size = 242, normalized size = 1.1

$$-\frac{29\,a^3c^6\tan\left(fx+e\right)}{63\,f}+\frac{a^3c^6\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^8}{9\,f}+\frac{8\,a^3c^6\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^6}{63\,f}-\frac{22\,a^3c^6\tan\left(fx+e\right)\left(\cdots\right.}{21\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x)
```

```
[Out] -29/63/f*a^3*c^6*tan(f*x+e)+1/9/f*a^3*c^6*tan(f*x+e)*sec(f*x+e)^8+8/63/f*a^
3*c^6*tan(f*x+e)*sec(f*x+e)^6-22/21/f*a^3*c^6*tan(f*x+e)*sec(f*x+e)^4+80/63
/f*a^3*c^6*tan(f*x+e)*sec(f*x+e)^2+55/128/f*a^3*c^6*ln(sec(f*x+e)+tan(f*x+e
))+43/48/f*a^3*c^6*tan(f*x+e)*sec(f*x+e)^5-73/192*a^3*c^6*sec(f*x+e)^3*tan(
f*x+e)/f-73/128*a^3*c^6*sec(f*x+e)*tan(f*x+e)/f-3/8/f*a^3*c^6*tan(f*x+e)*se
c(f*x+e)^7
```

Maxima [B] time = 1.00825, size = 598, normalized size = 2.63

$$\frac{256\left(35\,\tan\left(fx+e\right)^9+180\,\tan\left(fx+e\right)^7+378\,\tan\left(fx+e\right)^5+420\,\tan\left(fx+e\right)^3+315\,\tan\left(fx+e\right)\right)a^3c^6-32256\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/80640*(256*(35*tan(f*x + e)^9 + 180*tan(f*x + e)^7 + 378*tan(f*x + e)^5 +
 420*tan(f*x + e)^3 + 315*tan(f*x + e))*a^3*c^6 - 32256*(3*tan(f*x + e)^5 +
 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^6 + 215040*(tan(f*x + e)^3 + 3*
tan(f*x + e))*a^3*c^6 + 315*a^3*c^6*(2*(105*sin(f*x + e)^7 - 385*sin(f*x +
e)^5 + 511*sin(f*x + e)^3 - 279*sin(f*x + e))/(sin(f*x + e)^8 - 4*sin(f*x +
 e)^6 + 6*sin(f*x + e)^4 - 4*sin(f*x + e)^2 + 1) - 105*log(sin(f*x + e) + 1
) + 105*log(sin(f*x + e) - 1)) - 6720*a^3*c^6*(2*(15*sin(f*x + e)^5 - 40*si
n(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(
f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x + e) - 1)) + 30
240*a^3*c^6*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(
f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 8064
0*a^3*c^6*log(sec(f*x + e) + tan(f*x + e)) - 241920*a^3*c^6*tan(f*x + e))/f
```

Fricas [A] time = 0.532053, size = 544, normalized size = 2.4

$$\frac{3465\,a^3c^6\cos\left(fx+e\right)^9\log\left(\sin\left(fx+e\right)+1\right)-3465\,a^3c^6\cos\left(fx+e\right)^9\log\left(-\sin\left(fx+e\right)+1\right)-2\left(3712\,a^3c^6\cos\left(fx+\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/16128*(3465*a^3*c^6*cos(f*x + e)^9*log(sin(f*x + e) + 1) - 3465*a^3*c^6*c
os(f*x + e)^9*log(-sin(f*x + e) + 1) - 2*(3712*a^3*c^6*cos(f*x + e)^8 + 459
9*a^3*c^6*cos(f*x + e)^7 - 10240*a^3*c^6*cos(f*x + e)^6 + 3066*a^3*c^6*cos(
f*x + e)^5 + 8448*a^3*c^6*cos(f*x + e)^4 - 7224*a^3*c^6*cos(f*x + e)^3 - 10
24*a^3*c^6*cos(f*x + e)^2 + 3024*a^3*c^6*cos(f*x + e) - 896*a^3*c^6)*sin(f*
x + e))/(f*cos(f*x + e)^9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3c^6\left(\int \sec\left(e+fx\right)dx + \int -3\sec^2\left(e+fx\right)dx + \int 8\sec^4\left(e+fx\right)dx + \int -6\sec^5\left(e+fx\right)dx + \int -6\sec^6\left(e+fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**6,x)
```

```
[Out] a**3*c**6*(Integral(sec(e + f*x), x) + Integral(-3*sec(e + f*x)**2, x) + In
tegral(8*sec(e + f*x)**4, x) + Integral(-6*sec(e + f*x)**5, x) + Integral(-
6*sec(e + f*x)**6, x) + Integral(8*sec(e + f*x)**7, x) + Integral(-3*sec(e
+ f*x)**9, x) + Integral(sec(e + f*x)**10, x))
```

Giac [A] time = 1.39888, size = 333, normalized size = 1.47

$$3465\,a^3c^6\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right) - 3465\,a^3c^6\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right) - \frac{2\left(3465\,a^3c^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{17} - 30030\,a^3c^6\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] 1/8064*(3465*a^3*c^6*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3465*a^3*c^6*log(
abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(3465*a^3*c^6*tan(1/2*f*x + 1/2*e)^17 -
30030*a^3*c^6*tan(1/2*f*x + 1/2*e)^15 + 115038*a^3*c^6*tan(1/2*f*x + 1/2*e)
^13 + 334602*a^3*c^6*tan(1/2*f*x + 1/2*e)^11 - 360448*a^3*c^6*tan(1/2*f*x +
 1/2*e)^9 + 255222*a^3*c^6*tan(1/2*f*x + 1/2*e)^7 - 115038*a^3*c^6*tan(1/2*
f*x + 1/2*e)^5 + 30030*a^3*c^6*tan(1/2*f*x + 1/2*e)^3 - 3465*a^3*c^6*tan(1/
2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^9)/f
```

3.22 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$

Optimal. Leaf size=206

$$\frac{2a^3c^5\tan^7(e+fx)}{7f}+\frac{45a^3c^5\tanh^{-1}(\sin(e+fx))}{128f}-\frac{a^3c^5\tan^5(e+fx)\sec^3(e+fx)}{8f}+\frac{5a^3c^5\tan^3(e+fx)\sec^3(e+fx)}{48f}-$$

```
[Out] (45*a^3*c^5*ArcTanh[Sin[e + f*x]])/(128*f) - (35*a^3*c^5*Sec[e + f*x]*Tan[e
 + f*x])/(128*f) - (5*a^3*c^5*Sec[e + f*x]^3*Tan[e + f*x])/(64*f) + (5*a^3*
c^5*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) + (5*a^3*c^5*Sec[e + f*x]^3*Tan[e +
 f*x]^3)/(48*f) - (a^3*c^5*Sec[e + f*x]*Tan[e + f*x]^5)/(6*f) - (a^3*c^5*Se
c[e + f*x]^3*Tan[e + f*x]^5)/(8*f) + (2*a^3*c^5*Tan[e + f*x]^7)/(7*f)
```

Rubi [A] time = 0.301823, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3958, 2611, 3770, 2607, 30, 3768}

$$\frac{2a^3c^5\tan^7(e+fx)}{7f}+\frac{45a^3c^5\tanh^{-1}(\sin(e+fx))}{128f}-\frac{a^3c^5\tan^5(e+fx)\sec^3(e+fx)}{8f}+\frac{5a^3c^5\tan^3(e+fx)\sec^3(e+fx)}{48f}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```



```
[Out] (45*a^3*c^5*ArcTanh[Sin[e + f*x]])/(128*f) - (35*a^3*c^5*Sec[e + f*x]*Tan[e
 + f*x])/(128*f) - (5*a^3*c^5*Sec[e + f*x]^3*Tan[e + f*x])/(64*f) + (5*a^3*
c^5*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) + (5*a^3*c^5*Sec[e + f*x]^3*Tan[e +
 f*x]^3)/(48*f) - (a^3*c^5*Sec[e + f*x]*Tan[e + f*x]^5)/(6*f) - (a^3*c^5*Se
c[e + f*x]^3*Tan[e + f*x]^5)/(8*f) + (2*a^3*c^5*Tan[e + f*x]^7)/(7*f)
```


Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx &= -\left(\left(a^3c^3\right)\int \left(c^2\sec(e+fx)\tan^6(e+fx)-2c^2\sec^2(e+fx\right.\right.\\ &= -\left(\left(a^3c^5\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\right)-\left(a^3c^5\right)\int \sec^3(\\ &= -\frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{6f}-\frac{a^3c^5\sec^3(e+fx)\tan^5(e}{8f}\\ &= \frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{24f}+\frac{5a^3c^5\sec^3(e+fx)\tan^3(}{48f}\\ &= -\frac{5a^3c^5\sec(e+fx)\tan(e+fx)}{16f}-\frac{5a^3c^5\sec^3(e+fx)\tan(e}{64f}\\ &= \frac{5a^3c^5\tanh^{-1}(\sin(e+fx))}{16f}-\frac{35a^3c^5\sec(e+fx)\tan(e+f}{128f}\\ &= \frac{45a^3c^5\tanh^{-1}(\sin(e+fx))}{128f}-\frac{35a^3c^5\sec(e+fx)\tan(e+}{128f}\end{aligned}$$

Mathematica [A] time = 2.3447, size = 111, normalized size = 0.54

$$-\frac{a^3c^5\left((5705\sin(e+fx)-1792\sin(2(e+fx))+21\sin(3(e+fx))+1792\sin(4(e+fx))+2065\sin(5(e+fx))-768\right.}{57344f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -(a^3*c^5*(-20160*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]^8*(5705*Sin[e + f*x]
 - 1792*Sin[2*(e + f*x)] + 21*Sin[3*(e + f*x)] + 1792*Sin[4*(e + f*x)] + 20
65*Sin[5*(e + f*x)] - 768*Sin[6*(e + f*x)] + 581*Sin[7*(e + f*x)] + 128*Sin
[8*(e + f*x)])))/(57344*f)
```

Maple [A] time = 0.035, size = 217, normalized size = 1.1

$$-\frac{2a^3c^5\tan\left(fx+e\right)}{7f}-\frac{6a^3c^5\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^4}{7f}+\frac{6a^3c^5\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^2}{7f}-\frac{83a^3c^5\sec\left(fx+e\right.}{128f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x)
```

```
[Out] -2/7/f*a^3*c^5*tan(f*x+e)-6/7/f*a^3*c^5*tan(f*x+e)*sec(f*x+e)^4+6/7/f*a^3*c
^5*tan(f*x+e)*sec(f*x+e)^2-83/128*a^3*c^5*sec(f*x+e)*tan(f*x+e)/f+45/128/f*
a^3*c^5*ln(sec(f*x+e)+tan(f*x+e))+3/16/f*a^3*c^5*tan(f*x+e)*sec(f*x+e)^5+15
/64*a^3*c^5*sec(f*x+e)^3*tan(f*x+e)/f+2/7/f*a^3*c^5*tan(f*x+e)*sec(f*x+e)^6
-1/8/f*a^3*c^5*tan(f*x+e)*sec(f*x+e)^7
```

Maxima [B] time = 1.01107, size = 551, normalized size = 2.67

$$\frac{1536\left(5\tan\left(fx+e\right)^{7}+21\tan\left(fx+e\right)^{5}+35\tan\left(fx+e\right)^{3}+35\tan\left(fx+e\right)\right)a^{3}c^{5}-10752\left(3\tan\left(fx+e\right)^{5}+10\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] 1/26880*(1536*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 3
5*tan(f*x + e))*a^3*c^5 - 10752*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*
tan(f*x + e))*a^3*c^5 + 53760*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^5 + 3
5*a^3*c^5*(2*(105*sin(f*x + e)^7 - 385*sin(f*x + e)^5 + 511*sin(f*x + e)^3
- 279*sin(f*x + e))/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x + e)^4 -
 4*sin(f*x + e)^2 + 1) - 105*log(sin(f*x + e) + 1) + 105*log(sin(f*x + e) -
 1)) - 560*a^3*c^5*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x +
 e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(si
n(f*x + e) + 1) + 15*log(sin(f*x + e) - 1)) + 13440*a^3*c^5*(2*sin(f*x + e)
/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 26
880*a^3*c^5*log(sec(f*x + e) + tan(f*x + e)) - 53760*a^3*c^5*tan(f*x + e))/
f
```

Fricas [A] time = 0.522448, size = 489, normalized size = 2.37

$$\frac{315\,a^{3}c^{5}\cos\left(fx+e\right)^{8}\log\left(\sin\left(fx+e\right)+1\right)-315\,a^{3}c^{5}\cos\left(fx+e\right)^{8}\log\left(-\sin\left(fx+e\right)+1\right)-2\left(256\,a^{3}c^{5}\cos\left(fx+e\right)^{7}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/1792*(315*a^3*c^5*cos(f*x + e)^8*log(sin(f*x + e) + 1) - 315*a^3*c^5*cos(
f*x + e)^8*log(-sin(f*x + e) + 1) - 2*(256*a^3*c^5*cos(f*x + e)^7 + 581*a^3
*c^5*cos(f*x + e)^6 - 768*a^3*c^5*cos(f*x + e)^5 - 210*a^3*c^5*cos(f*x + e)
^4 + 768*a^3*c^5*cos(f*x + e)^3 - 168*a^3*c^5*cos(f*x + e)^2 - 256*a^3*c^5*
cos(f*x + e) + 112*a^3*c^5)*sin(f*x + e))/(f*cos(f*x + e)^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^{3}c^{5}\left(\int -\sec\left(e+fx\right)dx+\int 2\sec^{2}\left(e+fx\right)dx+\int 2\sec^{3}\left(e+fx\right)dx+\int -6\sec^{4}\left(e+fx\right)dx+\int 6\sec^{6}\left(e+fx\right)d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**3*c**5*(Integral(-sec(e + f*x), x) + Integral(2*sec(e + f*x)**2, x) + I
ntegral(2*sec(e + f*x)**3, x) + Integral(-6*sec(e + f*x)**4, x) + Integral(
6*sec(e + f*x)**6, x) + Integral(-2*sec(e + f*x)**7, x) + Integral(-2*sec(e
 + f*x)**8, x) + Integral(sec(e + f*x)**9, x))
```

Giac [A] time = 1.42727, size = 306, normalized size = 1.49

$$\frac{315\, a^3 c^5 \log\left(\left|\tan\left(\frac{1}{2}\, fx + \frac{1}{2}\, e\right) + 1\right|\right) - 315\, a^3 c^5 \log\left(\left|\tan\left(\frac{1}{2}\, fx + \frac{1}{2}\, e\right) - 1\right|\right) - \frac{2\left(315\, a^3 c^5 \tan\left(\frac{1}{2}\, fx + \frac{1}{2}\, e\right)^{15} - 2415\, a^3 c^5 \tan\left(\frac{1}{2}\, fx + \frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/896*(315*a^3*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 315*a^3*c^5*log(abs
(tan(1/2*f*x + 1/2*e) - 1)) - 2*(315*a^3*c^5*tan(1/2*f*x + 1/2*e)^15 - 2415
*a^3*c^5*tan(1/2*f*x + 1/2*e)^13 + 8043*a^3*c^5*tan(1/2*f*x + 1/2*e)^11 + 1
7609*a^3*c^5*tan(1/2*f*x + 1/2*e)^9 - 15159*a^3*c^5*tan(1/2*f*x + 1/2*e)^7
+ 8043*a^3*c^5*tan(1/2*f*x + 1/2*e)^5 - 2415*a^3*c^5*tan(1/2*f*x + 1/2*e)^3
 + 315*a^3*c^5*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^8)/f
```

3.23 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$

Optimal. Leaf size=121

$$\frac{a^3c^4\tan^7(e+fx)}{7f}+\frac{5a^3c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^3c^4\tan^5(e+fx)\sec(e+fx)}{6f}+\frac{5a^3c^4\tan^3(e+fx)\sec(e+fx)}{24f}-\frac{5a^3c}{}$$

```
[Out] (5*a^3*c^4*ArcTanh[Sin[e + f*x]])/(16*f) - (5*a^3*c^4*Sec[e + f*x]*Tan[e +
f*x])/(16*f) + (5*a^3*c^4*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) - (a^3*c^4*Se
c[e + f*x]*Tan[e + f*x]^5)/(6*f) + (a^3*c^4*Tan[e + f*x]^7)/(7*f)
```

Rubi [A] time = 0.178182, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3958, 2611, 3770, 2607, 30}

$$\frac{a^3c^4\tan^7(e+fx)}{7f}+\frac{5a^3c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^3c^4\tan^5(e+fx)\sec(e+fx)}{6f}+\frac{5a^3c^4\tan^3(e+fx)\sec(e+fx)}{24f}-\frac{5a^3c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (5*a^3*c^4*ArcTanh[Sin[e + f*x]])/(16*f) - (5*a^3*c^4*Sec[e + f*x]*Tan[e +
f*x])/(16*f) + (5*a^3*c^4*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) - (a^3*c^4*Se
c[e + f*x]*Tan[e + f*x]^5)/(6*f) + (a^3*c^4*Tan[e + f*x]^7)/(7*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = -\left(\left(a^3c^3\right)\int \left(c\sec(e+fx)\tan^6(e+fx)-c\sec^2(e+fx)\,\mathrm{ta}\right.\right.$$
$$= -\left(\left(a^3c^4\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\right)+\left(a^3c^4\right)\int \sec^2($$
$$= -\frac{a^3c^4\sec(e+fx)\tan^5(e+fx)}{6f}+\frac{1}{6}\left(5a^3c^4\right)\int \sec(e+fx)$$
$$= \frac{5a^3c^4\sec(e+fx)\tan^3(e+fx)}{24f}-\frac{a^3c^4\sec(e+fx)\tan^5(e+}{6f}$$
$$= -\frac{5a^3c^4\sec(e+fx)\tan(e+fx)}{16f}+\frac{5a^3c^4\sec(e+fx)\tan^3(e}{24f}$$
$$= \frac{5a^3c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{5a^3c^4\sec(e+fx)\tan(e+fx}{16f}$$

Mathematica [A] time = 1.71463, size = 102, normalized size = 0.84

$$\frac{a^3c^4\left(3360\tanh^{-1}(\sin(e+fx))-(-840\sin(e+fx)+595\sin(2(e+fx))+504\sin(3(e+fx))+196\sin(4(e+fx))-\right.}{10752f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*c^4*(3360*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^7*(-840*Sin[e + f*x] +
595*Sin[2*(e + f*x)] + 504*Sin[3*(e + f*x)] + 196*Sin[4*(e + f*x)] - 168*Si
n[5*(e + f*x)] + 231*Sin[6*(e + f*x)] + 24*Sin[7*(e + f*x)])))/(10752*f)
```

Maple [A] time = 0.029, size = 192, normalized size = 1.6

$$\frac{13\,a^3c^4\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^3}{24\,f}-\frac{11\,a^3c^4\sec\left(fx+e\right)\tan\left(fx+e\right)}{16\,f}+\frac{5\,a^3c^4\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{16\,f}-\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x)
```

```
[Out] 13/24/f*a^3*c^4*tan(f*x+e)*sec(f*x+e)^3-11/16*a^3*c^4*sec(f*x+e)*tan(f*x+e)
/f+5/16/f*a^3*c^4*ln(sec(f*x+e)+tan(f*x+e))-1/7/f*a^3*c^4*tan(f*x+e)+3/7/f*
a^3*c^4*tan(f*x+e)*sec(f*x+e)^2-3/7/f*a^3*c^4*tan(f*x+e)*sec(f*x+e)^4-1/6/f
*a^3*c^4*tan(f*x+e)*sec(f*x+e)^5+1/7/f*a^3*c^4*tan(f*x+e)*sec(f*x+e)^6
```

Maxima [B] time = 1.02888, size = 497, normalized size = 4.11

$$96\left(5\tan\left(fx+e\right)^7+21\tan\left(fx+e\right)^5+35\tan\left(fx+e\right)^3+35\tan\left(fx+e\right)\right)a^3c^4-672\left(3\tan\left(fx+e\right)^5+10\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/3360*(96*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 35*t
an(f*x + e))*a^3*c^4 - 672*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f
*x + e))*a^3*c^4 + 3360*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^4 + 35*a^3*
c^4*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x +
 e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1)
 + 15*log(sin(f*x + e) - 1)) - 630*a^3*c^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x
 + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) +
3*log(sin(f*x + e) - 1)) + 2520*a^3*c^4*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1
) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 3360*a^3*c^4*log(sec(f
*x + e) + tan(f*x + e)) - 3360*a^3*c^4*tan(f*x + e))/f
```

Fricas [A] time = 0.511063, size = 444, normalized size = 3.67

$$\frac{105\,a^3c^4\cos\left(fx+e\right)^7\log\left(\sin\left(fx+e\right)+1\right)-105\,a^3c^4\cos\left(fx+e\right)^7\log\left(-\sin\left(fx+e\right)+1\right)-2\left(48\,a^3c^4\cos\left(fx+e\right)^6+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/672*(105*a^3*c^4*cos(f*x + e)^7*log(sin(f*x + e) + 1) - 105*a^3*c^4*cos(f
*x + e)^7*log(-sin(f*x + e) + 1) - 2*(48*a^3*c^4*cos(f*x + e)^6 + 231*a^3*c
^4*cos(f*x + e)^5 - 144*a^3*c^4*cos(f*x + e)^4 - 182*a^3*c^4*cos(f*x + e)^3
 + 144*a^3*c^4*cos(f*x + e)^2 + 56*a^3*c^4*cos(f*x + e) - 48*a^3*c^4)*sin(f
*x + e))/(f*cos(f*x + e)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3c^4\left(\int\sec\left(e+fx\right)dx+\int-\sec^2\left(e+fx\right)dx+\int-3\sec^3\left(e+fx\right)dx+\int3\sec^4\left(e+fx\right)dx+\int3\sec^5\left(e+fx\right)dx+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**3*c**4*(Integral(sec(e + f*x), x) + Integral(-sec(e + f*x)**2, x) + Inte
gral(-3*sec(e + f*x)**3, x) + Integral(3*sec(e + f*x)**4, x) + Integral(3*s
ec(e + f*x)**5, x) + Integral(-3*sec(e + f*x)**6, x) + Integral(-sec(e + f*
x)**7, x) + Integral(sec(e + f*x)**8, x))
```

Giac [A] time = 1.37582, size = 279, normalized size = 2.31

$$\frac{105\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-105\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(105\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}-700\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}\cdots\right.}{\cdots}}{336\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/336*(105*a^3*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 105*a^3*c^4*log(abs
(tan(1/2*f*x + 1/2*e) - 1)) - 2*(105*a^3*c^4*tan(1/2*f*x + 1/2*e)^13 - 700*
a^3*c^4*tan(1/2*f*x + 1/2*e)^11 + 1981*a^3*c^4*tan(1/2*f*x + 1/2*e)^9 + 307
2*a^3*c^4*tan(1/2*f*x + 1/2*e)^7 - 1981*a^3*c^4*tan(1/2*f*x + 1/2*e)^5 + 70
0*a^3*c^4*tan(1/2*f*x + 1/2*e)^3 - 105*a^3*c^4*tan(1/2*f*x + 1/2*e))/(tan(1
/2*f*x + 1/2*e)^2 - 1)^7)/f
```

3.24 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=100

$$\frac{5a^3c^3\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^3c^3\tan^5(e+fx)\sec(e+fx)}{6f}+\frac{5a^3c^3\tan^3(e+fx)\sec(e+fx)}{24f}-\frac{5a^3c^3\tan(e+fx)\sec(e+fx)}{16f}$$

```
[Out] (5*a^3*c^3*ArcTanh[Sin[e + f*x]])/(16*f) - (5*a^3*c^3*Sec[e + f*x]*Tan[e +
f*x])/(16*f) + (5*a^3*c^3*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) - (a^3*c^3*Se
c[e + f*x]*Tan[e + f*x]^5)/(6*f)
```

Rubi [A] time = 0.131113, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3958, 2611, 3770}

$$\frac{5a^3c^3\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^3c^3\tan^5(e+fx)\sec(e+fx)}{6f}+\frac{5a^3c^3\tan^3(e+fx)\sec(e+fx)}{24f}-\frac{5a^3c^3\tan(e+fx)\sec(e+fx)}{16f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (5*a^3*c^3*ArcTanh[Sin[e + f*x]])/(16*f) - (5*a^3*c^3*Sec[e + f*x]*Tan[e +
f*x])/(16*f) + (5*a^3*c^3*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) - (a^3*c^3*Se
c[e + f*x]*Tan[e + f*x]^5)/(6*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = -\left(\left(a^3c^3\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\right)$$
$$= -\frac{a^3c^3\sec(e+fx)\tan^5(e+fx)}{6f} + \frac{1}{6}\left(5a^3c^3\right)\int \sec(e+fx)$$
$$= \frac{5a^3c^3\sec(e+fx)\tan^3(e+fx)}{24f} - \frac{a^3c^3\sec(e+fx)\tan^5(e+}{6f}$$
$$= -\frac{5a^3c^3\sec(e+fx)\tan(e+fx)}{16f} + \frac{5a^3c^3\sec(e+fx)\tan^3(}{24f}$$
$$= \frac{5a^3c^3\tanh^{-1}(\sin(e+fx))}{16f} - \frac{5a^3c^3\sec(e+fx)\tan(e+fx}{16f}$$

Mathematica [A] time = 0.248216, size = 60, normalized size = 0.6

$$-\frac{a^3c^3\left((28\cos(2(e+fx))+33\cos(4(e+fx))+59)\tan(e+fx)\sec^5(e+fx)-120\tanh^{-1}(\sin(e+fx))\right)}{384f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -(a^3*c^3*(-120*ArcTanh[Sin[e + f*x]] + (59 + 28*Cos[2*(e + f*x)] + 33*Cos[
4*(e + f*x)])*Sec[e + f*x]^5*Tan[e + f*x]))/(384*f)
```

Maple [A] time = 0.024, size = 100, normalized size = 1.

$$-\frac{11\,a^3c^3\sec\left(fx+e\right)\tan\left(fx+e\right)}{16\,f} + \frac{5\,a^3c^3\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{16\,f} + \frac{13\,a^3c^3\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^3}{24\,f} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x)
```

```
[Out] -11/16*a^3*c^3*sec(f*x+e)*tan(f*x+e)/f+5/16/f*a^3*c^3*ln(sec(f*x+e)+tan(f*x
+e))+13/24/f*a^3*c^3*tan(f*x+e)*sec(f*x+e)^3-1/6/f*a^3*c^3*tan(f*x+e)*sec(f
*x+e)^5
```

Maxima [B] time = 0.984666, size = 329, normalized size = 3.29

$$a^3c^3\left(\frac{2\left(15\sin\left(fx+e\right)^5-40\sin\left(fx+e\right)^3+33\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^6-3\sin\left(fx+e\right)^4+3\sin\left(fx+e\right)^2-1} - 15\log\left(\sin\left(fx+e\right)+1\right) + 15\log\left(\sin\left(fx+e\right)-1\right)\right) - 18\,a^3c^3\left(\frac{2\left(3\sin(\right.}{\sin(fx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/96*(a^3*c^3*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/
(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x
```

```
 + e) + 1) + 15*log(sin(f*x + e) - 1)) - 18*a^3*c^3*(2*(3*sin(f*x + e)^3 -
5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e
) + 1) + 3*log(sin(f*x + e) - 1)) + 72*a^3*c^3*(2*sin(f*x + e)/(sin(f*x + e
)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 96*a^3*c^3*log(
sec(f*x + e) + tan(f*x + e)))/f
```

Fricas [A] time = 0.495505, size = 285, normalized size = 2.85

$$\frac{15\,a^3c^3\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-15\,a^3c^3\cos\left(fx+e\right)^6\log\left(-\sin\left(fx+e\right)+1\right)-2\left(33\,a^3c^3\cos\left(fx+e\right)^4-2\right.}{96\,f\cos\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/96*(15*a^3*c^3*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 15*a^3*c^3*cos(f*x
+ e)^6*log(-sin(f*x + e) + 1) - 2*(33*a^3*c^3*cos(f*x + e)^4 - 26*a^3*c^3*c
os(f*x + e)^2 + 8*a^3*c^3)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^3c^3\left(\int -\sec\left(e+fx\right)dx+\int 3\sec^3\left(e+fx\right)dx+\int -3\sec^5\left(e+fx\right)dx+\int \sec^7\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**3*c**3*(Integral(-sec(e + f*x), x) + Integral(3*sec(e + f*x)**3, x) + I
ntegral(-3*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**7, x))
```

Giac [A] time = 1.36111, size = 147, normalized size = 1.47

$$\frac{15\,a^3c^3\log\left(\sin\left(fx+e\right)+1\right)-15\,a^3c^3\log\left(-\sin\left(fx+e\right)+1\right)+\frac{2\left(33\,a^3c^3\sin\left(fx+e\right)^5-40\,a^3c^3\sin\left(fx+e\right)^3+15\,a^3c^3\sin\left(fx+e\right)\right)}{\left(\sin\left(fx+e\right)^2-1\right)^3}}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/96*(15*a^3*c^3*log(sin(f*x + e) + 1) - 15*a^3*c^3*log(-sin(f*x + e) + 1)
+ 2*(33*a^3*c^3*sin(f*x + e)^5 - 40*a^3*c^3*sin(f*x + e)^3 + 15*a^3*c^3*sin
(f*x + e))/(sin(f*x + e)^2 - 1)^3)/f
```

3.25 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=94

$$\frac{a^3c^2\tan^5(e+fx)}{5f}+\frac{3a^3c^2\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^3c^2\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^3c^2\tan(e+fx)\sec(e+fx)}{8f}$$

```
[Out] (3*a^3*c^2*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^3*c^2*Sec[e + f*x]*Tan[e + f
*x])/(8*f) + (a^3*c^2*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f) + (a^3*c^2*Tan[e +
 f*x]^5)/(5*f)
```

Rubi [A] time = 0.147801, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3958, 2611, 3770, 2607, 30}

$$\frac{a^3c^2\tan^5(e+fx)}{5f}+\frac{3a^3c^2\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^3c^2\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^3c^2\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (3*a^3*c^2*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^3*c^2*Sec[e + f*x]*Tan[e + f
*x])/(8*f) + (a^3*c^2*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f) + (a^3*c^2*Tan[e +
 f*x]^5)/(5*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = \left(a^2c^2\right)\int \left(a\sec(e+fx)\tan^4(e+fx)+a\sec^2(e+fx)\tan^4(e\right.$$
$$= \left(a^3c^2\right)\int \sec(e+fx)\tan^4(e+fx)\,dx + \left(a^3c^2\right)\int \sec^2(e+fx)$$
$$= \frac{a^3c^2\sec(e+fx)\tan^3(e+fx)}{4f} - \frac{1}{4}\left(3a^3c^2\right)\int \sec(e+fx)\tan^2$$
$$= -\frac{3a^3c^2\sec(e+fx)\tan(e+fx)}{8f} + \frac{a^3c^2\sec(e+fx)\tan^3(e+f}{4f}$$
$$= \frac{3a^3c^2\tanh^{-1}(\sin(e+fx))}{8f} - \frac{3a^3c^2\sec(e+fx)\tan(e+fx)}{8f} +$$

Mathematica [A] time = 0.848097, size = 81, normalized size = 0.86

$$\frac{a^3c^2\left(120\tanh^{-1}(\sin(e+fx))+(40\sin(e+fx)-10\sin(2(e+fx))-20\sin(3(e+fx))-25\sin(4(e+fx))+4\sin(5(e+\right.}{320f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*c^2*(120*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]^5*(40*Sin[e + f*x] - 10*
Sin[2*(e + f*x)] - 20*Sin[3*(e + f*x)] - 25*Sin[4*(e + f*x)] + 4*Sin[5*(e +
 f*x)])))/(320*f)
```

Maple [A] time = 0.027, size = 142, normalized size = 1.5

$$-\frac{5a^3c^2\sec(fx+e)\tan(fx+e)}{8f} + \frac{3a^3c^2\ln(\sec(fx+e)+\tan(fx+e))}{8f} + \frac{a^3c^2\tan(fx+e)}{5f} - \frac{2a^3c^2\tan(fx+e)(\mathrm{se}}{5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x)
```

```
[Out] -5/8*a^3*c^2*sec(f*x+e)*tan(f*x+e)/f+3/8/f*a^3*c^2*ln(sec(f*x+e)+tan(f*x+e)
)+1/5/f*a^3*c^2*tan(f*x+e)-2/5/f*a^3*c^2*tan(f*x+e)*sec(f*x+e)^2+1/4/f*a^3*
c^2*tan(f*x+e)*sec(f*x+e)^3+1/5/f*a^3*c^2*tan(f*x+e)*sec(f*x+e)^4
```

Maxima [B] time = 0.987646, size = 306, normalized size = 3.26

$$16\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)a^3c^2-160\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3c^2-15a^3c^2\left(\frac{2(3}{\sin}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^2
- 160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^2 - 15*a^3*c^2*(2*(3*sin(f*x
+ e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(si
n(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 120*a^3*c^2*(2*sin(f*x + e)/(s
in(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a
^3*c^2*log(sec(f*x + e) + tan(f*x + e)) + 240*a^3*c^2*tan(f*x + e))/f
```

Fricas [A] time = 0.493915, size = 356, normalized size = 3.79

$$\frac{15\,a^3c^2\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\,a^3c^2\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)+2\left(8\,a^3c^2\cos\left(fx+e\right)^4-\right.}{80\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/80*(15*a^3*c^2*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 15*a^3*c^2*cos(f*x
+ e)^5*log(-sin(f*x + e) + 1) + 2*(8*a^3*c^2*cos(f*x + e)^4 - 25*a^3*c^2*co
s(f*x + e)^3 - 16*a^3*c^2*cos(f*x + e)^2 + 10*a^3*c^2*cos(f*x + e) + 8*a^3*
c^2)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3c^2\left(\int\sec\left(e+fx\right)dx+\int\sec^2\left(e+fx\right)dx+\int-2\sec^3\left(e+fx\right)dx+\int-2\sec^4\left(e+fx\right)dx+\int\sec^5\left(e+fx\right)dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**3*c**2*(Integral(sec(e + f*x), x) + Integral(sec(e + f*x)**2, x) + Integ
ral(-2*sec(e + f*x)**3, x) + Integral(-2*sec(e + f*x)**4, x) + Integral(sec
(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Giac [A] time = 1.35731, size = 225, normalized size = 2.39

$$\frac{15\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(15\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9-70\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+12\right.}{\left(t\right.}}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/40*(15*a^3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^3*c^2*log(abs(ta
n(1/2*f*x + 1/2*e) - 1)) - 2*(15*a^3*c^2*tan(1/2*f*x + 1/2*e)^9 - 70*a^3*c^
2*tan(1/2*f*x + 1/2*e)^7 + 128*a^3*c^2*tan(1/2*f*x + 1/2*e)^5 + 70*a^3*c^2*
tan(1/2*f*x + 1/2*e)^3 - 15*a^3*c^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/
2*e)^2 - 1)^5)/f
```

3.26 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=86

$$-\frac{2a^3c\tan^3(e+fx)}{3f}+\frac{5a^3c\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^3c\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{3a^3c\tan(e+fx)\sec(e+fx)}{8f}$$

```
[Out] (5*a^3*c*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^3*c*Sec[e + f*x]*Tan[e + f*x])
/(8*f) - (a^3*c*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) - (2*a^3*c*Tan[e + f*x]^
3)/(3*f)
```

Rubi [A] time = 0.15574, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3958, 2611, 3770, 2607, 30, 3768}

$$-\frac{2a^3c\tan^3(e+fx)}{3f}+\frac{5a^3c\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^3c\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{3a^3c\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (5*a^3*c*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^3*c*Sec[e + f*x]*Tan[e + f*x])
/(8*f) - (a^3*c*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) - (2*a^3*c*Tan[e + f*x]^
3)/(3*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = -\left((ac)\int \left(a^2\sec(e+fx)\tan^2(e+fx)+2a^2\sec^2(e+fx)\tan^2\right.\right.$$
$$= -\left(\left(a^3c\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)-\left(a^3c\right)\int \sec^3(e+fx$$
$$= -\frac{a^3c\sec(e+fx)\tan(e+fx)}{2f}-\frac{a^3c\sec^3(e+fx)\tan(e+fx)}{4f}+$$
$$= \frac{a^3c\tanh^{-1}(\sin(e+fx))}{2f}-\frac{3a^3c\sec(e+fx)\tan(e+fx)}{8f}-\frac{a^3c}{}$$
$$= \frac{5a^3c\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3a^3c\sec(e+fx)\tan(e+fx)}{8f}-\frac{a^3}{}$$

Mathematica [A] time = 0.616023, size = 70, normalized size = 0.81

$$\frac{a^3c\left(60\tanh^{-1}(\sin(e+fx))-(33\sin(e+fx)+16\sin(2(e+fx))+9\sin(3(e+fx))-8\sin(4(e+fx)))\sec^4(e+fx)\right)}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*c*(60*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^4*(33*Sin[e + f*x] + 16*Sin
[2*(e + f*x)] + 9*Sin[3*(e + f*x)] - 8*Sin[4*(e + f*x)])))/(96*f)
```

Maple [A] time = 0.024, size = 107, normalized size = 1.2

$$\frac{2a^3c\tan\left(fx+e\right)}{3f}+\frac{5a^3c\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{8f}-\frac{2a^3c\tan\left(fx+e\right)\left(\sec\left(fx+e\right)\right)^2}{3f}-\frac{a^3c\left(\sec\left(fx+e\right)\right)^3\tan}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x)
```

```
[Out] 2/3/f*a^3*c*tan(f*x+e)+5/8/f*a^3*c*ln(sec(f*x+e)+tan(f*x+e))-2/3/f*a^3*c*ta
n(f*x+e)*sec(f*x+e)^2-1/4*a^3*c*sec(f*x+e)^3*tan(f*x+e)/f-3/8*a^3*c*sec(f*x
+e)*tan(f*x+e)/f
```

Maxima [A] time = 1.01203, size = 180, normalized size = 2.09

$$-\frac{32\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)a^{3}c-3\,a^{3}c\left(\frac{2\left(3\,\sin\left(fx+e\right)^{3}-5\,\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^{4}-2\,\sin\left(fx+e\right)^{2}+1}-3\,\log\left(\sin\left(fx+e\right)+1\right)+3\,\log\left(\sin\left(fx\right.\right.\right.}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/48*(32*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c - 3*a^3*c*(2*(3*sin(f*x +
 e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin
(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 48*a^3*c*log(sec(f*x + e) + tan
(f*x + e)) - 96*a^3*c*tan(f*x + e))/f
```

Fricas [A] time = 0.487776, size = 302, normalized size = 3.51

$$\frac{15\,a^{3}c\cos\left(fx+e\right)^{4}\log\left(\sin\left(fx+e\right)+1\right)-15\,a^{3}c\cos\left(fx+e\right)^{4}\log\left(-\sin\left(fx+e\right)+1\right)+2\left(16\,a^{3}c\cos\left(fx+e\right)^{3}-9\right.}{48\,f\cos\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/48*(15*a^3*c*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 15*a^3*c*cos(f*x + e)
^4*log(-sin(f*x + e) + 1) + 2*(16*a^3*c*cos(f*x + e)^3 - 9*a^3*c*cos(f*x +
e)^2 - 16*a^3*c*cos(f*x + e) - 6*a^3*c)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^{3}c\left(\int -\sec\left(e+fx\right)dx+\int -2\sec^{2}\left(e+fx\right)dx+\int 2\sec^{4}\left(e+fx\right)dx+\int \sec^{5}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**3*c*(Integral(-sec(e + f*x), x) + Integral(-2*sec(e + f*x)**2, x) + Int
egral(2*sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))
```

Giac [A] time = 1.31749, size = 182, normalized size = 2.12

$$\frac{15\,a^{3}c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,a^{3}c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(15\,a^{3}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}-55\,a^{3}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+73\,a^{3}c\,\mathrm{t}\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right.}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/24*(15*a^3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^3*c*log(abs(tan(1/
2*f*x + 1/2*e) - 1)) - 2*(15*a^3*c*tan(1/2*f*x + 1/2*e)^7 - 55*a^3*c*tan(1/
2*f*x + 1/2*e)^5 + 73*a^3*c*tan(1/2*f*x + 1/2*e)^3 + 15*a^3*c*tan(1/2*f*x +
 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

3.27 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)} dx$

Optimal. Leaf size=100

$$-\frac{10a^3\tan(e+fx)}{cf}-\frac{15a^3\tanh^{-1}(\sin(e+fx))}{2cf}-\frac{5a^3\tan(e+fx)\sec(e+fx)}{2cf}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))}$$

```
[Out] (-15*a^3*ArcTanh[Sin[e + f*x]])/(2*c*f) - (10*a^3*Tan[e + f*x])/(c*f) - (5*
a^3*Sec[e + f*x]*Tan[e + f*x])/(2*c*f) - (2*a*(a + a*Sec[e + f*x])^2*Tan[e
+ f*x])/(f*(c - c*Sec[e + f*x]))
```

Rubi [A] time = 0.128658, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3957, 3788, 3767, 8, 4046, 3770}

$$-\frac{10a^3\tan(e+fx)}{cf}-\frac{15a^3\tanh^{-1}(\sin(e+fx))}{2cf}-\frac{5a^3\tan(e+fx)\sec(e+fx)}{2cf}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x]),x]
```


```
[Out] (-15*a^3*ArcTanh[Sin[e + f*x]])/(2*c*f) - (10*a^3*Tan[e + f*x])/(c*f) - (5*
a^3*Sec[e + f*x]*Tan[e + f*x])/(2*c*f) - (2*a*(a + a*Sec[e + f*x])^2*Tan[e
+ f*x])/(f*(c - c*Sec[e + f*x]))
```


Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{(5a)\int \sec(e+fx)(a+a\sec(e+fx))^2\,dx}{c}$$
$$= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{(5a)\int \sec(e+fx)\left(a^2+a^2\sec^2(e+fx)\right)}{c}$$
$$= -\frac{5a^3\sec(e+fx)\tan(e+fx)}{2cf} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{\left(15a^3\right)\int s}{}$$
$$= -\frac{15a^3\tanh^{-1}(\sin(e+fx))}{2cf} - \frac{10a^3\tan(e+fx)}{cf} - \frac{5a^3\sec(e+fx)\tan(e+fx)}{2cf} -$$

Mathematica [B] time = 3.04039, size = 287, normalized size = 2.87

$$\frac{a^3\cos^2(e+fx)\tan\left(\frac{1}{2}(e+fx)\right)\sec^4\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^3\left(32\csc\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sec\left(\frac{1}{2}(e+fx)\right)+\tan\left(\frac{1}{2}(e+fx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*Cos[e + f*x]^2*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x])^3*Tan[(e + f*x)/2
]*(32*Csc[e/2]*Sec[(e + f*x)/2]*Sin[(f*x)/2] + (-30*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]] + 30*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^(-2) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^(
-2) + (16*Sin[f*x])/((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))*Tan[(e
+ f*x)/2]))/(16*f*(c - c*Sec[e + f*x]))
```

Maple [A] time = 0.076, size = 166, normalized size = 1.7

$$\frac{a^3}{2fc}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-2}+\frac{7a^3}{2fc}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1}-\frac{15a^3}{2fc}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-\frac{a^3}{2fc}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{-2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x)
```

```
[Out] 1/2/f*a^3/c/(tan(1/2*f*x+1/2*e)+1)^2+7/2/f*a^3/c/(tan(1/2*f*x+1/2*e)+1)-15/
2/f*a^3/c*ln(tan(1/2*f*x+1/2*e)+1)-1/2/f*a^3/c/(tan(1/2*f*x+1/2*e)-1)^2+7/2
```

```
/f*a^3/c/(tan(1/2*f*x+1/2*e)-1)+15/2/f*a^3/c*ln(tan(1/2*f*x+1/2*e)-1)+8/f*a
^3/c/tan(1/2*f*x+1/2*e)
```

Maxima [B] time = 1.01378, size = 522, normalized size = 5.22

$$-\frac{a^3\left(\frac{2\left(\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}\right)+6a^3\left(\frac{\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\log\left(\right.\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(a^3*(2*(5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 1)/(c*sin(f*x + e)/(cos(f*x + e) + 1) - 2*c*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*log(si
n(f*x + e)/(cos(f*x + e) + 1) + 1)/c - 3*log(sin(f*x + e)/(cos(f*x + e) + 1
) - 1)/c) + 6*a^3*((3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c*sin(f*x +
 e)/(cos(f*x + e) + 1) - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + log(sin(f
*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1
)/c) + 6*a^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)
/(cos(f*x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - 2*a^3*(
cos(f*x + e) + 1)/(c*sin(f*x + e)))/f
```

Fricas [A] time = 0.484043, size = 320, normalized size = 3.2

$$-\frac{15\,a^3\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-15\,a^3\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-48}{4\,cf\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/4*(15*a^3*cos(f*x + e)^2*log(sin(f*x + e) + 1)*sin(f*x + e) - 15*a^3*cos
(f*x + e)^2*log(-sin(f*x + e) + 1)*sin(f*x + e) - 48*a^3*cos(f*x + e)^3 - 3
4*a^3*cos(f*x + e)^2 + 16*a^3*cos(f*x + e) + 2*a^3)/(c*f*cos(f*x + e)^2*sin
(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3\left(\int\frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{3\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{3\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{\sec^4\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

```
[Out] -a**3*(Integral(sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(3*sec(e + f*
x)**2/(sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**3/(sec(e + f*x) - 1
), x) + Integral(sec(e + f*x)**4/(sec(e + f*x) - 1), x))/c
```

Giac [A] time = 1.27316, size = 167, normalized size = 1.67

$$-\frac{\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c}-\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c}-\frac{16\,a^3}{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}-\frac{2\left(7\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-9\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2c}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/2*(15*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c - 15*a^3*log(abs(tan(1/2*
f*x + 1/2*e) - 1))/c - 16*a^3/(c*tan(1/2*f*x + 1/2*e)) - 2*(7*a^3*tan(1/2*f
*x + 1/2*e)^3 - 9*a^3*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2
*c))/f
```

3.28 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=119

$$\frac{5a^3\tan(e+fx)}{c^2f}+\frac{5a^3\tanh^{-1}(\sin(e+fx))}{c^2f}+\frac{10\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3f\left(c^2-c^2\sec(e+fx)\right)}-\frac{2a\tan(e+fx)(a\sec(e+fx)+}{3f(c-c\sec(e+fx))^2}$$

```
[Out] (5*a^3*ArcTanh[Sin[e + f*x]])/(c^2*f) + (5*a^3*Tan[e + f*x])/(c^2*f) - (2*a
*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(c - c*Sec[e + f*x])^2) + (10*(a
^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*(c^2 - c^2*Sec[e + f*x]))
```

Rubi [A] time = 0.183051, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3957, 3787, 3770, 3767, 8}

$$\frac{5a^3\tan(e+fx)}{c^2f}+\frac{5a^3\tanh^{-1}(\sin(e+fx))}{c^2f}+\frac{10\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3f\left(c^2-c^2\sec(e+fx)\right)}-\frac{2a\tan(e+fx)(a\sec(e+fx)+}{3f(c-c\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (5*a^3*ArcTanh[Sin[e + f*x]])/(c^2*f) + (5*a^3*Tan[e + f*x])/(c^2*f) - (2*a
*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(c - c*Sec[e + f*x])^2) + (10*(a
^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*(c^2 - c^2*Sec[e + f*x]))
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} - \frac{(5a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx}{3c}$$

$$= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{10\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{3f\left(c^2-c^2\sec(e+fx)\right)} + \frac{(5}{}$$

$$= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{10\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{3f\left(c^2-c^2\sec(e+fx)\right)} + \frac{(5}{}$$

$$= \frac{5a^3\tanh^{-1}(\sin(e+fx))}{c^2f} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{10\left(a^3+a^3\sec\right.}{3f\left(c^2-\right.}$$

$$= \frac{5a^3\tanh^{-1}(\sin(e+fx))}{c^2f} + \frac{5a^3\tan(e+fx)}{c^2f} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx}{3f(c-c\sec(e+fx))^2}$$

Mathematica [B] time = 3.24675, size = 402, normalized size = 3.38

$$\frac{a^3(\cos(e+fx)+1)^3\tan\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)\left(-48\sin(e)\csc^3\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sin^7\left(\frac{1}{2}(e+fx)\right)\csc^4(e+fx)+\frac{1}{16}\csc\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*(1 + Cos[e + f*x])^3*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*(((-74 + 42*C
os[e] - 76*Cos[f*x] + 120*Cos[e + f*x] - 46*Cos[2*(e + f*x)] - 76*Cos[2*e +
 f*x] + 23*Cos[e + 2*f*x] + 23*Cos[3*e + 2*f*x])*Csc[e/2]^3*Sec[(e + f*x)/2
]^5*Sin[(f*x)/2])/16 - 48*Csc[e/2]^3*Csc[e + f*x]^4*Sin[e]*Sin[(f*x)/2]*Sin
[(e + f*x)/2]^7 + Cos[e]*Cos[e + f*x]*Csc[e/2]^2*Sec[(e + f*x)/2]^4*(4*Cot[
e/2] + 15*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]])*Sin[(e + f*x)/2]^2)*Tan[(e + f*x)/2] - 4*(-1 + 5*Cos[e
 + f*x])*Cot[e/2]^2*Csc[e/2]*Sec[(e + f*x)/2]^3*Sin[(f*x)/2]*Tan[(e + f*x)/
2]^2))/(6*c^2*f*(-1 + Cos[e + f*x])^2*(-1 + Cot[e/2])*(1 + Cot[e/2])*(-1 +
Tan[(e + f*x)/2])*(1 + Tan[(e + f*x)/2]))
```

Maple [A] time = 0.078, size = 140, normalized size = 1.2

$$-\frac{a^3}{fc^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1} + 5\,\frac{a^3\ln\left(\tan\left(1/2\,fx+e/2\right)+1\right)}{fc^2} - \frac{a^3}{fc^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{-1} - 5\,\frac{a^3\ln\left(\tan\left(1/2\,fx+e/2\right)\right.}{fc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x)
```

```
[Out] -1/f*a^3/c^2/(tan(1/2*f*x+1/2*e)+1)+5/f*a^3/c^2*ln(tan(1/2*f*x+1/2*e)+1)-1/
f*a^3/c^2/(tan(1/2*f*x+1/2*e)-1)-5/f*a^3/c^2*ln(tan(1/2*f*x+1/2*e)-1)-4/3/f
*a^3/c^2/tan(1/2*f*x+1/2*e)^3-8/f*a^3/c^2/tan(1/2*f*x+1/2*e)
```

Maxima [B] time = 1.03476, size = 471, normalized size = 3.96

$$-\frac{a^3\left(\frac{\frac{14\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{27\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+1}{\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{c^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2}\right)-3a^3\left(\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2}\cdots\right.}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(a^3*((14*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 27*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + 1)/(c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - c^2*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1
)/c^2 + 12*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^2) - 3*a^3*(6*log(sin
(f*x + e)/(cos(f*x + e) + 1) + 1)/c^2 - 6*log(sin(f*x + e)/(cos(f*x + e) +
1) - 1)/c^2 - (9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1
)^3/(c^2*sin(f*x + e)^3)) + 3*a^3*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3) - a^3*(3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3))/f
```

Fricas [A] time = 0.48671, size = 410, normalized size = 3.45

$$-\frac{46\,a^3\cos\left(fx+e\right)^3-22\,a^3\cos\left(fx+e\right)^2-62\,a^3\cos\left(fx+e\right)+6\,a^3-15\left(a^3\cos\left(fx+e\right)^2-a^3\cos\left(fx+e\right)\right)\log\left(\sin\cdots\right.}{6\left(c^2f\cos\left(fx+e\right)^2-c^2f\cos\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(46*a^3*cos(f*x + e)^3 - 22*a^3*cos(f*x + e)^2 - 62*a^3*cos(f*x + e) +
 6*a^3 - 15*(a^3*cos(f*x + e)^2 - a^3*cos(f*x + e))*log(sin(f*x + e) + 1)*s
in(f*x + e) + 15*(a^3*cos(f*x + e)^2 - a^3*cos(f*x + e))*log(-sin(f*x + e)
+ 1)*sin(f*x + e))/((c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e))*sin(f*x + e
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3\left(\int\frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int\frac{3\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int\frac{3\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int\frac{\sec^4\left(e+fx\right)}{\sec^2\left(e+fx\right)-2\sec\left(e+fx\right)+1\cdots}\right.}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + In
tegral(3*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
```

```
ral(3*sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integral
(sec(e + f*x)**4/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Giac [A] time = 1.32433, size = 165, normalized size = 1.39

$$\frac{\frac{15\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c^{2}}-\frac{15\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c^{2}}-\frac{6\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)c^{2}}-\frac{4\left(6\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a^{3}\right)}{c^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(15*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^2 - 15*a^3*log(abs(tan(1/2
*f*x + 1/2*e) - 1))/c^2 - 6*a^3*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)
^2 - 1)*c^2) - 4*(6*a^3*tan(1/2*f*x + 1/2*e)^2 + a^3)/(c^2*tan(1/2*f*x + 1/
2*e)^3))/f
```

3.29 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=132

$$-\frac{a^3\tanh^{-1}(\sin(e+fx))}{c^3 f}-\frac{2a^3\tan(e+fx)}{f\left(c^3-c^3\sec(e+fx)\right)}+\frac{2\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3cf(c-c\sec(e+fx))^2}-\frac{2a\tan(e+fx)(a\sec(e+f}{5f(c-c\sec(e+fx)}$$

```
[Out] -((a^3*ArcTanh[Sin[e + f*x]])/(c^3*f)) - (2*a*(a + a*Sec[e + f*x])^2*Tan[e
+ f*x])/(5*f*(c - c*Sec[e + f*x])^3) + (2*(a^3 + a^3*Sec[e + f*x])*Tan[e +
f*x])/(3*c*f*(c - c*Sec[e + f*x])^2) - (2*a^3*Tan[e + f*x])/(f*(c^3 - c^3*S
ec[e + f*x]))
```

Rubi [A] time = 0.21442, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3957, 3770}

$$-\frac{a^3\tanh^{-1}(\sin(e+fx))}{c^3 f}-\frac{2a^3\tan(e+fx)}{f\left(c^3-c^3\sec(e+fx)\right)}+\frac{2\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3cf(c-c\sec(e+fx))^2}-\frac{2a\tan(e+fx)(a\sec(e+f}{5f(c-c\sec(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -((a^3*ArcTanh[Sin[e + f*x]])/(c^3*f)) - (2*a*(a + a*Sec[e + f*x])^2*Tan[e
+ f*x])/(5*f*(c - c*Sec[e + f*x])^3) + (2*(a^3 + a^3*Sec[e + f*x])*Tan[e +
f*x])/(3*c*f*(c - c*Sec[e + f*x])^2) - (2*a^3*Tan[e + f*x])/(f*(c^3 - c^3*S
ec[e + f*x]))
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} - \frac{a\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}dx}{c}$$

$$= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} + \frac{2\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{3cf(c-c\sec(e+fx))^2} + \frac{a^2}{\ }$$

$$= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} + \frac{2\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{3cf(c-c\sec(e+fx))^2} - \frac{\ }{f(}$$

$$= -\frac{a^3\tanh^{-1}(\sin(e+fx))}{c^3f} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} + \frac{2\left(a^3+a^3\sec(\right.}{3cf(c-c}$$

Mathematica [A] time = 0.108118, size = 139, normalized size = 1.05

$$-\frac{a^3\left(-\frac{26\cot\left(\frac{1}{2}(e+fx)\right)}{15f} - \frac{2\cot\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)}{5f} + \frac{2\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{15f} - \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\right.}{f}\right.}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -((a^3*((-26*Cot[(e + f*x)/2])/(15*f) + (2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2
]^2)/(15*f) - (2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4)/(5*f) - Log[Cos[(e +
f*x)/2] - Sin[(e + f*x)/2]]/f + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]/f)
)/c^3)
```

Maple [A] time = 0.09, size = 113, normalized size = 0.9

$$-\frac{a^3}{fc^3}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right) + \frac{a^3}{fc^3}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right) + \frac{2a^3}{5fc^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5} + \frac{2a^3}{3fc^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3} + 2\frac{\ }{fc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)
```

```
[Out] -1/f*a^3/c^3*ln(tan(1/2*f*x+1/2*e)+1)+1/f*a^3/c^3*ln(tan(1/2*f*x+1/2*e)-1)+
2/5/f*a^3/c^3/tan(1/2*f*x+1/2*e)^5+2/3/f*a^3/c^3/tan(1/2*f*x+1/2*e)^3+2/f*a
^3/c^3/tan(1/2*f*x+1/2*e)
```

Maxima [B] time = 1.02943, size = 417, normalized size = 3.16

$$-\frac{a^3\left(\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^3} - \frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^3} - \frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}\right) - \frac{3a^3\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+3\right.}{c^3\sin(fx+e)^5}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(a^3*(60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^3 - 60*log(sin(f*
x + e)/(cos(f*x + e) + 1) - 1)/c^3 - (20*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1)^5/(c^3*
sin(f*x + e)^5)) - 3*a^3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^
5) + a^3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) + 9*a^3*(5*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x +
e)^5))/f
```

Fricas [A] time = 0.484952, size = 446, normalized size = 3.38

$$\frac{52\,a^3\cos\left(fx+e\right)^3-44\,a^3\cos\left(fx+e\right)^2-4\,a^3\cos\left(fx+e\right)+92\,a^3-15\left(a^3\cos\left(fx+e\right)^2-2\,a^3\cos\left(fx+e\right)+a^3\right)\mathrm{lo}}{30\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(52*a^3*cos(f*x + e)^3 - 44*a^3*cos(f*x + e)^2 - 4*a^3*cos(f*x + e) +
92*a^3 - 15*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*log(sin(f*x + e
) + 1)*sin(f*x + e) + 15*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*lo
g(-sin(f*x + e) + 1)*sin(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x
 + e) + c^3*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3\left(\int\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int\frac{3\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int\frac{3\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)+3\sec\left(e\right.}\right.}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) - 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e +
f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**3/(sec(e + f*x
)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)*
*4/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Giac [A] time = 1.28838, size = 144, normalized size = 1.09

$$-\frac{\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c^3}-\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c^3}-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+5\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,a^3\right)}{c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/15*(15*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^3 - 15*a^3*log(abs(tan(1
/2*f*x + 1/2*e) - 1))/c^3 - 2*(15*a^3*tan(1/2*f*x + 1/2*e)^4 + 5*a^3*tan(1/
2*f*x + 1/2*e)^2 + 3*a^3)/(c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

3.30 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=38

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{7f(c-c\sec(e+fx))^4}$$

```
[Out] -((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*(c - c*Sec[e + f*x])^4)
```

Rubi [A] time = 0.0724353, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{7f(c-c\sec(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*(c - c*Sec[e + f*x])^4)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{7f(c-c\sec(e+fx))^4}$$

Mathematica [A] time = 0.151558, size = 25, normalized size = 0.66

$$-\frac{a^3\cot^7\left(\frac{1}{2}(e+fx)\right)}{7c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -(a^3*Cot[(e + f*x)/2]^7)/(7*c^4*f)
```

Maple [A] time = 0.095, size = 23, normalized size = 0.6

$$-\frac{a^3}{7fc^4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x)
```

```
[Out] -1/7/f*a^3/c^4/tan(1/2*f*x+1/2*e)^7
```

Maxima [B] time = 1.058, size = 481, normalized size = 12.66

$$-\frac{\frac{a^3\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{35\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+5\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}-\frac{a^3\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}-\frac{a^3\left(21\ \cdots\right.}{(\mathrm{c}\cdots}}{280\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] -1/280*(a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5)*(cos(f*x +
e) + 1)^7/(c^4*sin(f*x + e)^7) - a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 - 15)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) - a^3*(21*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x
+ e)^7) + a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15)*(cos(f*
x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f
```

Fricas [B] time = 0.449165, size = 261, normalized size = 6.87

$$\frac{a^3\cos\left(fx+e\right)^4+4\,a^3\cos\left(fx+e\right)^3+6\,a^3\cos\left(fx+e\right)^2+4\,a^3\cos\left(fx+e\right)+a^3}{7\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f\cos\left(fx+e\right)-c^4f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/7*(a^3*cos(f*x + e)^4 + 4*a^3*cos(f*x + e)^3 + 6*a^3*cos(f*x + e)^2 + 4*a
^3*cos(f*x + e) + a^3)/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*
c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3\left(\int\frac{\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int\frac{3\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int\frac{\cdots}{\sec^4\left(e+fx\right)\cdots}\right.}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e
+ f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f
*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) +
Integral(3*sec(e + f*x)**3/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e +
 f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x)
**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

Giac [A] time = 1.34222, size = 31, normalized size = 0.82

$$-\frac{a^3}{7\,c^4 f \tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] -1/7*a^3/(c^4*f*tan(1/2*f*x + 1/2*e)^7)
```

3.31 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=80

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{63cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{9f(c-c\sec(e+fx))^5}$$

```
[Out] -((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(9*f*(c - c*Sec[e + f*x])^5) - ((a +
 a*Sec[e + f*x])^3*Tan[e + f*x])/(63*c*f*(c - c*Sec[e + f*x])^4)
```

Rubi [A] time = 0.152401, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3951, 3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{63cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{9f(c-c\sec(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(9*f*(c - c*Sec[e + f*x])^5) - ((a +
 a*Sec[e + f*x])^3*Tan[e + f*x])/(63*c*f*(c - c*Sec[e + f*x])^4)
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx &= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{9f(c-c\sec(e+fx))^5}+\frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx}{9c}\\ &= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{9f(c-c\sec(e+fx))^5}-\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{63cf(c-c\sec(e+fx))^4}\end{aligned}$$

Mathematica [A] time = 0.385381, size = 141, normalized size = 1.76

$$-\frac{a^3\csc\left(\frac{e}{2}\right)\left(315\sin\left(e+\frac{fx}{2}\right)-189\sin\left(e+\frac{3fx}{2}\right)-483\sin\left(2e+\frac{3fx}{2}\right)+225\sin\left(2e+\frac{5fx}{2}\right)+63\sin\left(3e+\frac{5fx}{2}\right)-9\sin\left(3\right.\right.}{16128c^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -(a^3*Csc[e/2]*Csc[(e + f*x)/2]^9*(693*Sin[(f*x)/2] + 315*Sin[e + (f*x)/2]
- 189*Sin[e + (3*f*x)/2] - 483*Sin[2*e + (3*f*x)/2] + 225*Sin[2*e + (5*f*x)
/2] + 63*Sin[3*e + (5*f*x)/2] - 9*Sin[3*e + (7*f*x)/2] - 63*Sin[4*e + (7*f*
x)/2] + 8*Sin[4*e + (9*f*x)/2]))/(16128*c^5*f)
```

Maple [A] time = 0.113, size = 39, normalized size = 0.5

$$\frac{a^3}{2fc^5}\left(-\frac{1}{7}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}+\frac{1}{9}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x)
```

```
[Out] 1/2/f*a^3/c^5*(-1/7/tan(1/2*f*x+1/2*e)^7+1/9/tan(1/2*f*x+1/2*e)^9)
```

Maxima [B] time = 1.06921, size = 482, normalized size = 6.02

$$-\frac{\frac{a^3\left(\frac{180\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{315\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}+\frac{15a^3\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{63\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-7\right)(\cos \cdots}{c^5\sin(fx+e)^9}}{5040 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] -1/5040*(a^3*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^
9) + 15*a^3*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x +
e) + 1)^9/(c^5*sin(f*x + e)^9) - 5*a^3*(18*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 63*sin(f*x + e)^8/(cos(f*x +
 e) + 1)^8 + 7)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) + 21*a^3*(18*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 -
5)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9))/f
```

Fricas [A] time = 0.456998, size = 333, normalized size = 4.16

$$\frac{8a^3\cos(fx+e)^5+31a^3\cos(fx+e)^4+44a^3\cos(fx+e)^3+26a^3\cos(fx+e)^2+4a^3\cos(fx+e)-a^3}{63\left(c^5f\cos(fx+e)^4-4c^5f\cos(fx+e)^3+6c^5f\cos(fx+e)^2-4c^5f\cos(fx+e)+c^5f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/63*(8*a^3*cos(f*x + e)^5 + 31*a^3*cos(f*x + e)^4 + 44*a^3*cos(f*x + e)^3
+ 26*a^3*cos(f*x + e)^2 + 4*a^3*cos(f*x + e) - a^3)/((c^5*f*cos(f*x + e)^4
- 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) +
c^5*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3\left(\int \frac{\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(
e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(3*sec
(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10
*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**3/(se
c(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2
 + 5*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x)**5 - 5*
sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x)
- 1), x))/c**5
```

Giac [A] time = 1.25216, size = 58, normalized size = 0.72

$$-\frac{9\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-7\,a^3}{126\,c^5 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] -1/126*(9*a^3*tan(1/2*f*x + 1/2*e)^2 - 7*a^3)/(c^5*f*tan(1/2*f*x + 1/2*e)^9
)
```

3.32 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx$

Optimal. Leaf size=121

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{693c^2f(c-c\sec(e+fx))^4}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{99cf(c-c\sec(e+fx))^5}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}$$

```
[Out] -((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(11*f*(c - c*Sec[e + f*x])^6) - (2*(
a + a*Sec[e + f*x])^3*Tan[e + f*x])/(99*c*f*(c - c*Sec[e + f*x])^5) - (2*(a
 + a*Sec[e + f*x])^3*Tan[e + f*x])/(693*c^2*f*(c - c*Sec[e + f*x])^4)
```

Rubi [A] time = 0.230325, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3951, 3950}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{693c^2f(c-c\sec(e+fx))^4}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{99cf(c-c\sec(e+fx))^5}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^6,x]
```

```
[Out] -((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(11*f*(c - c*Sec[e + f*x])^6) - (2*(
a + a*Sec[e + f*x])^3*Tan[e + f*x])/(99*c*f*(c - c*Sec[e + f*x])^5) - (2*(a
 + a*Sec[e + f*x])^3*Tan[e + f*x])/(693*c^2*f*(c - c*Sec[e + f*x])^4)
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx=-\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{11f(c-c\sec(e+fx))^6}+\frac{2\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx}{11c}$$

$$=-\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{11f(c-c\sec(e+fx))^6}-\frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{99cf(c-c\sec(e+fx))^5}+\frac{2\int$$

$$=-\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{11f(c-c\sec(e+fx))^6}-\frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{99cf(c-c\sec(e+fx))^5}-\frac{2(a$$

Mathematica [A] time = 0.677084, size = 167, normalized size = 1.38

$$-\frac{a^3 \csc\left(\frac{e}{2}\right)\left(21252\sin\left(e+\frac{fx}{2}\right)-15444\sin\left(e+\frac{3fx}{2}\right)-10626\sin\left(2e+\frac{3fx}{2}\right)+4950\sin\left(2e+\frac{5fx}{2}\right)+8085\sin\left(3e+\frac{5fx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^6,x]
```

```
[Out] -(a^3*Csc[e/2]*Csc[(e + f*x)/2]^11*(15246*Sin[(f*x)/2] + 21252*Sin[e + (f*x
)/2] - 15444*Sin[e + (3*f*x)/2] - 10626*Sin[2*e + (3*f*x)/2] + 4950*Sin[2*e
 + (5*f*x)/2] + 8085*Sin[3*e + (5*f*x)/2] - 2959*Sin[3*e + (7*f*x)/2] - 138
6*Sin[4*e + (7*f*x)/2] + 176*Sin[4*e + (9*f*x)/2] + 693*Sin[5*e + (9*f*x)/2
] - 79*Sin[5*e + (11*f*x)/2]))/(709632*c^6*f)
```

Maple [A] time = 0.125, size = 52, normalized size = 0.4

$$\frac{a^3}{4fc^6}\left(-\frac{1}{11}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-11}-\frac{1}{7}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}+\frac{2}{9}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x)
```

```
[Out] 1/4/f*a^3/c^6*(-1/11/tan(1/2*f*x+1/2*e)^11-1/7/tan(1/2*f*x+1/2*e)^7+2/9/tan
(1/2*f*x+1/2*e)^9)
```

Maxima [B] time = 1.09299, size = 699, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/110880*(3*a^3*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 990*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - 1386*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1155*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^10/(cos(f*x + e) + 1
)^10 - 315)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11) + 9*a^3*(385*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4
62*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)^8/(cos(f*x + e)
+ 1)^8 - 1155*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 105)*(cos(f*x + e) +
1)^11/(c^6*sin(f*x + e)^11) + 5*a^3*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - 990*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1386*sin(f*x + e)^6/(cos(f*x
+ e) + 1)^6 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 693*sin(f*x + e)^1
0/(cos(f*x + e) + 1)^10 - 63)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11) -
 a^3*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 990*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 1386*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 315)
*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11))/f
```

Fricas [A] time = 0.466873, size = 413, normalized size = 3.41

$$\frac{79\,a^3\cos\left(fx+e\right)^6+298\,a^3\cos\left(fx+e\right)^5+404\,a^3\cos\left(fx+e\right)^4+216\,a^3\cos\left(fx+e\right)^3+19\,a^3\cos\left(fx+e\right)^2-10\,a^3 \cdots}{693\left(c^6f\cos\left(fx+e\right)^5-5\,c^6f\cos\left(fx+e\right)^4+10\,c^6f\cos\left(fx+e\right)^3-10\,c^6f\cos\left(fx+e\right)^2+5\,c^6f\cos\left(fx+e\right)-c^6 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/693*(79*a^3*cos(f*x + e)^6 + 298*a^3*cos(f*x + e)^5 + 404*a^3*cos(f*x + e
)^4 + 216*a^3*cos(f*x + e)^3 + 19*a^3*cos(f*x + e)^2 - 10*a^3*cos(f*x + e)
+ 2*a^3)/((c^6*f*cos(f*x + e)^5 - 5*c^6*f*cos(f*x + e)^4 + 10*c^6*f*cos(f*x
 + e)^3 - 10*c^6*f*cos(f*x + e)^2 + 5*c^6*f*cos(f*x + e) - c^6*f)*sin(f*x +
 e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29698, size = 81, normalized size = 0.67

$$-\frac{99\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-154\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+63\,a^3}{2772\,c^6f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] -1/2772*(99*a^3*tan(1/2*f*x + 1/2*e)^4 - 154*a^3*tan(1/2*f*x + 1/2*e)^2 + 6
3*a^3)/(c^6*f*tan(1/2*f*x + 1/2*e)^11)
```

3.33 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx$

Optimal. Leaf size=162

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{3003c^3f(c-c\sec(e+fx))^4}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{429c^2f(c-c\sec(e+fx))^5}-\frac{3\tan(e+fx)(a\sec(e+fx)+a)^3}{143cf(c-c\sec(e+fx))^6}-\frac{\tan(e+f}{13f(c}$$

```
[Out] -((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(13*f*(c - c*Sec[e + f*x])^7) - (3*(
a + a*Sec[e + f*x])^3*Tan[e + f*x])/(143*c*f*(c - c*Sec[e + f*x])^6) - (2*(
a + a*Sec[e + f*x])^3*Tan[e + f*x])/(429*c^2*f*(c - c*Sec[e + f*x])^5) - (2
*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(3003*c^3*f*(c - c*Sec[e + f*x])^4)
```

Rubi [A] time = 0.316715, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3951, 3950}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{3003c^3f(c-c\sec(e+fx))^4}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{429c^2f(c-c\sec(e+fx))^5}-\frac{3\tan(e+fx)(a\sec(e+fx)+a)^3}{143cf(c-c\sec(e+fx))^6}-\frac{\tan(e+f}{13f(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^7,x]
```

```
[Out] -((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(13*f*(c - c*Sec[e + f*x])^7) - (3*(
a + a*Sec[e + f*x])^3*Tan[e + f*x])/(143*c*f*(c - c*Sec[e + f*x])^6) - (2*(
a + a*Sec[e + f*x])^3*Tan[e + f*x])/(429*c^2*f*(c - c*Sec[e + f*x])^5) - (2
*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(3003*c^3*f*(c - c*Sec[e + f*x])^4)
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7} + \frac{3\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx}{13c}$$

$$= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7} - \frac{3(a+a\sec(e+fx))^3\tan(e+fx)}{143cf(c-c\sec(e+fx))^6} + \frac{6\int}{}$$

$$= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7} - \frac{3(a+a\sec(e+fx))^3\tan(e+fx)}{143cf(c-c\sec(e+fx))^6} - \frac{2(a}{}$$

$$= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7} - \frac{3(a+a\sec(e+fx))^3\tan(e+fx)}{143cf(c-c\sec(e+fx))^6} - \frac{2(a}{}$$

Mathematica [A] time = 0.575218, size = 193, normalized size = 1.19

$$-\frac{a^3\csc\left(\frac{e}{2}\right)\left(246246\sin\left(e+\frac{fx}{2}\right)-182754\sin\left(e+\frac{3fx}{2}\right)-216216\sin\left(2e+\frac{3fx}{2}\right)+122551\sin\left(2e+\frac{5fx}{2}\right)+99099\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^7,x]
```

```
[Out] -(a^3*Csc[e/2]*Csc[(e + f*x)/2]^13*(285714*Sin[(f*x)/2] + 246246*Sin[e + (f
*x)/2] - 182754*Sin[e + (3*f*x)/2] - 216216*Sin[2*e + (3*f*x)/2] + 122551*S
in[2*e + (5*f*x)/2] + 99099*Sin[3*e + (5*f*x)/2] - 37609*Sin[3*e + (7*f*x)/
2] - 51051*Sin[4*e + (7*f*x)/2] + 15171*Sin[4*e + (9*f*x)/2] + 9009*Sin[5*e
 + (9*f*x)/2] - 1027*Sin[5*e + (11*f*x)/2] - 3003*Sin[6*e + (11*f*x)/2] + 3
10*Sin[6*e + (13*f*x)/2]))/(12300288*c^7*f)
```

Maple [A] time = 0.135, size = 65, normalized size = 0.4

$$\frac{a^3}{8fc^7}\left(\frac{1}{13}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-13}-\frac{3}{11}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-11}-\frac{1}{7}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}+\frac{1}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x)
```

```
[Out] 1/8/f*a^3/c^7*(1/13/tan(1/2*f*x+1/2*e)^13-3/11/tan(1/2*f*x+1/2*e)^11-1/7/ta
n(1/2*f*x+1/2*e)^7+1/3/tan(1/2*f*x+1/2*e)^9)
```

Maxima [B] time = 1.1346, size = 698, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x, algorithm="ma
xima")
```

```
[Out] -1/960960*(a^3*(8190*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5005*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 - 25740*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 9009
```

```
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 30030*sin(f*x + e)^10/(cos(f*x + e)
+ 1)^10 - 45045*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 3465)*(cos(f*x + e)
 + 1)^13/(c^7*sin(f*x + e)^13) + 5*a^3*(1638*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 5005*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 8580*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 - 9009*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 6006*sin(f*x
+ e)^10/(cos(f*x + e) + 1)^10 - 3003*sin(f*x + e)^12/(cos(f*x + e) + 1)^12
- 231)*(cos(f*x + e) + 1)^13/(c^7*sin(f*x + e)^13) + 35*a^3*(468*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 715*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1287*
sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 1716*sin(f*x + e)^10/(cos(f*x + e) +
1)^10 + 1287*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 99)*(cos(f*x + e) + 1)
^13/(c^7*sin(f*x + e)^13) + 77*a^3*(65*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 117*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)^12/(cos(f*x +
e) + 1)^12 - 15)*(cos(f*x + e) + 1)^13/(c^7*sin(f*x + e)^13))/f
```

Fricas [A] time = 0.474635, size = 486, normalized size = 3.

$$\frac{310\,a^3\cos\left(fx+e\right)^7+1143\,a^3\cos\left(fx+e\right)^6+1492\,a^3\cos\left(fx+e\right)^5+736\,a^3\cos\left(fx+e\right)^4+34\,a^3\cos\left(fx+e\right)^3-29\,a^3}{3003\left(c^7f\cos\left(fx+e\right)^6-6\,c^7f\cos\left(fx+e\right)^5+15\,c^7f\cos\left(fx+e\right)^4-20\,c^7f\cos\left(fx+e\right)^3+15\,c^7f\cos\left(fx+e\right)^2-6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x, algorithm="fr
icas")
```

```
[Out] 1/3003*(310*a^3*cos(f*x + e)^7 + 1143*a^3*cos(f*x + e)^6 + 1492*a^3*cos(f*x
 + e)^5 + 736*a^3*cos(f*x + e)^4 + 34*a^3*cos(f*x + e)^3 - 29*a^3*cos(f*x +
 e)^2 + 12*a^3*cos(f*x + e) - 2*a^3)/((c^7*f*cos(f*x + e)^6 - 6*c^7*f*cos(f
*x + e)^5 + 15*c^7*f*cos(f*x + e)^4 - 20*c^7*f*cos(f*x + e)^3 + 15*c^7*f*co
s(f*x + e)^2 - 6*c^7*f*cos(f*x + e) + c^7*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35317, size = 104, normalized size = 0.64

$$-\frac{429\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-1001\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+819\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-231\,a^3}{24024\,c^7f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x, algorithm="gi
ac")
```

```
[Out] -1/24024*(429*a^3*tan(1/2*f*x + 1/2*e)^6 - 1001*a^3*tan(1/2*f*x + 1/2*e)^4
+ 819*a^3*tan(1/2*f*x + 1/2*e)^2 - 231*a^3)/(c^7*f*tan(1/2*f*x + 1/2*e)^13)
```

3.34 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=121

$$\frac{7c^4\tan^3(e+fx)}{3af}+\frac{28c^4\tan(e+fx)}{af}-\frac{35c^4\tanh^{-1}(\sin(e+fx))}{2af}-\frac{21c^4\tan(e+fx)\sec(e+fx)}{2af}+\frac{2c\tan(e+fx)(c-c\ldots}{f(a\sec(e+f\ldots}$$

```
[Out] (-35*c^4*ArcTanh[Sin[e + f*x]])/(2*a*f) + (28*c^4*Tan[e + f*x])/(a*f) - (21
*c^4*Sec[e + f*x]*Tan[e + f*x])/(2*a*f) + (2*c*(c - c*Sec[e + f*x])^3*Tan[e
 + f*x])/(f*(a + a*Sec[e + f*x])) + (7*c^4*Tan[e + f*x]^3)/(3*a*f)
```

Rubi [A] time = 0.155514, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3957, 3791, 3770, 3767, 8, 3768}

$$\frac{7c^4\tan^3(e+fx)}{3af}+\frac{28c^4\tan(e+fx)}{af}-\frac{35c^4\tanh^{-1}(\sin(e+fx))}{2af}-\frac{21c^4\tan(e+fx)\sec(e+fx)}{2af}+\frac{2c\tan(e+fx)(c-c\ldots}{f(a\sec(e+f\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```


```
[Out] (-35*c^4*ArcTanh[Sin[e + f*x]])/(2*a*f) + (28*c^4*Tan[e + f*x])/(a*f) - (21
*c^4*Sec[e + f*x]*Tan[e + f*x])/(2*a*f) + (2*c*(c - c*Sec[e + f*x])^3*Tan[e
 + f*x])/(f*(a + a*Sec[e + f*x])) + (7*c^4*Tan[e + f*x]^3)/(3*a*f)
```


Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(7c)\int \sec(e+fx)(c-c\sec(e+fx))^3\,dx}{a}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(7c)\int \left(c^3\sec(e+fx)-3c^3\sec^2(e+fx)\right.}{a}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{\left(7c^4\right)\int \sec(e+fx)\,dx}{a} + \frac{\left(7c^4\right)\int \sec^4(e}{a}$$

$$= -\frac{7c^4\tanh^{-1}(\sin(e+fx))}{af} - \frac{21c^4\sec(e+fx)\tan(e+fx)}{2af} + \frac{2c(c-c\sec(e+}{f(a+a\sec}$$

$$= -\frac{35c^4\tanh^{-1}(\sin(e+fx))}{2af} + \frac{28c^4\tan(e+fx)}{af} - \frac{21c^4\sec(e+fx)\tan(e+fx}{2af}$$

Mathematica [B] time = 6.4092, size = 1036, normalized size = 8.56

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (35*Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*Log[Cos[e/2 +
(f*x)/2] - Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^4)/(16*f*(a + a*Sec[e +
 f*x])) - (35*Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*Log[
Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^4)/(16*f*(a +
 a*Sec[e + f*x])) + (2*Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]*Csc[e/2 + (f*x)/2]
^7*Sec[e/2]*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(f*(a + a*Sec[e + f*x])) +
 (Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f
*x])^4*Sin[(f*x)/2])/(48*f*(a + a*Sec[e + f*x])*(Cos[e/2] - Sin[e/2])*(Cos[
e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^3*Cot[e/2 + (f*x)/2
]^2*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^4*(-7*Cos[e/2] + 8*Sin[e/2]))
/(48*f*(a + a*Sec[e + f*x])*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin
[e/2 + (f*x)/2])^2) + (35*Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*
x)/2]^6*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(24*f*(a + a*Sec[e + f*x])*(Co
s[e/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])) + (Cos[e + f*
x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^4*Sin[(
f*x)/2])/(48*f*(a + a*Sec[e + f*x])*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/
2] + Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2
+ (f*x)/2]^6*(c - c*Sec[e + f*x])^4*(7*Cos[e/2] + 8*Sin[e/2]))/(48*f*(a + a
*Sec[e + f*x])*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/
2])^2) + (35*Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*(c -
c*Sec[e + f*x])^4*Sin[(f*x)/2])/(24*f*(a + a*Sec[e + f*x])*(Cos[e/2] + Sin[
e/2])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]))
```

Maple [A] time = 0.086, size = 212, normalized size = 1.8

$$16\,\frac{c^4\tan\left(1/2\,fx+e/2\right)}{fa}-\frac{c^4}{3\,fa}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-3}+3\,\frac{c^4}{fa\left(\tan\left(1/2\,fx+e/2\right)+1\right)^2}-\frac{29\,c^4}{2\,fa}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x)
```

```
[Out] 16/f*c^4/a*tan(1/2*f*x+1/2*e)-1/3/f*c^4/a/(tan(1/2*f*x+1/2*e)+1)^3+3/f*c^4/
a/(tan(1/2*f*x+1/2*e)+1)^2-29/2/f*c^4/a/(tan(1/2*f*x+1/2*e)+1)-35/2/f*c^4/a
*ln(tan(1/2*f*x+1/2*e)+1)-1/3/f*c^4/a/(tan(1/2*f*x+1/2*e)-1)^3-3/f*c^4/a/(t
an(1/2*f*x+1/2*e)-1)^2-29/2/f*c^4/a/(tan(1/2*f*x+1/2*e)-1)+35/2/f*c^4/a*ln(
tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.05054, size = 798, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/6*(c^4*(2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 16*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a - 3*a*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - a*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6) - 9*log(sin(f*x + e)/(cos(f*x + e) + 1) +
1)/a + 9*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 6*sin(f*x + e)/(a*(co
s(f*x + e) + 1))) + 12*c^4*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e)
+ 1) + 1)/a + 3*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x + e)
/(a*(cos(f*x + e) + 1))) - 36*c^4*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)
/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(c
os(f*x + e) + 1))) - 24*c^4*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - l
og(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) +
 1))) + 6*c^4*sin(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 0.492197, size = 386, normalized size = 3.19

$$-\frac{105\left(c^4\cos\left(fx+e\right)^4+c^4\cos\left(fx+e\right)^3\right)\log\left(\sin\left(fx+e\right)+1\right)-105\left(c^4\cos\left(fx+e\right)^4+c^4\cos\left(fx+e\right)^3\right)\log\left(-\sin\left(f\ldots\right.\right.}{12\left(af\cos\left(fx+e\right)^4+af\cos\left(\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/12*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*log(sin(f*x + e) + 1)
- 105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) - 2*
```

```
(166*c^4*cos(f*x + e)^3 + 55*c^4*cos(f*x + e)^2 - 13*c^4*cos(f*x + e) + 2*c
^4)*sin(f*x + e))/(a*f*cos(f*x + e)^4 + a*f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^4\left(\int \frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx + \int -\frac{4\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx + \int \frac{6\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx + \int -\frac{4\sec^4\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx + \int \frac{\sec^5\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**4/(a+a*sec(f*x+e)),x)
```

```
[Out] c**4*(Integral(sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(-4*sec(e + f*
x)**2/(sec(e + f*x) + 1), x) + Integral(6*sec(e + f*x)**3/(sec(e + f*x) + 1
), x) + Integral(-4*sec(e + f*x)**4/(sec(e + f*x) + 1), x) + Integral(sec(e
 + f*x)**5/(sec(e + f*x) + 1), x))/a
```

Giac [A] time = 1.32156, size = 188, normalized size = 1.55

$$-\frac{\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a} - \frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a} - \frac{96\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a} + \frac{2\left(87\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-136\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+57\,c^4\,\mathrm{ta}\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3 a}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/6*(105*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 105*c^4*log(abs(tan(1/
2*f*x + 1/2*e) - 1))/a - 96*c^4*tan(1/2*f*x + 1/2*e)/a + 2*(87*c^4*tan(1/2*
f*x + 1/2*e)^5 - 136*c^4*tan(1/2*f*x + 1/2*e)^3 + 57*c^4*tan(1/2*f*x + 1/2*
e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a))/f
```

3.35 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=100

$$\frac{10c^3\tan(e+fx)}{af}-\frac{15c^3\tanh^{-1}(\sin(e+fx))}{2af}-\frac{5c^3\tan(e+fx)\sec(e+fx)}{2af}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}$$

```
[Out] (-15*c^3*ArcTanh[Sin[e + f*x]])/(2*a*f) + (10*c^3*Tan[e + f*x])/(a*f) - (5*
c^3*Sec[e + f*x]*Tan[e + f*x])/(2*a*f) + (2*c*(c - c*Sec[e + f*x])^2*Tan[e
+ f*x])/(f*(a + a*Sec[e + f*x]))
```

Rubi [A] time = 0.130097, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3957, 3788, 3767, 8, 4046, 3770}

$$\frac{10c^3\tan(e+fx)}{af}-\frac{15c^3\tanh^{-1}(\sin(e+fx))}{2af}-\frac{5c^3\tan(e+fx)\sec(e+fx)}{2af}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-15*c^3*ArcTanh[Sin[e + f*x]])/(2*a*f) + (10*c^3*Tan[e + f*x])/(a*f) - (5*
c^3*Sec[e + f*x]*Tan[e + f*x])/(2*a*f) + (2*c*(c - c*Sec[e + f*x])^2*Tan[e
+ f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(5c)\int \sec(e+fx)(c-c\sec(e+fx))^2\,dx}{a}$$

$$= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(5c)\int \sec(e+fx)\left(c^2+c^2\sec^2(e+fx)\right)}{a}$$

$$= -\frac{5c^3\sec(e+fx)\tan(e+fx)}{2af} + \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{\left(15c^3\right)\int}{}$$

$$= -\frac{15c^3\tanh^{-1}(\sin(e+fx))}{2af} + \frac{10c^3\tan(e+fx)}{af} - \frac{5c^3\sec(e+fx)\tan(e+fx)}{2af}$$

Mathematica [B] time = 3.17075, size = 287, normalized size = 2.87

$$\frac{\cos^2(e+fx)\cot\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)(c-c\sec(e+fx))^3\left(\cot\left(\frac{1}{2}(e+fx)\right)\left(-\frac{}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}\right.\right.}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]^2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4*(c - c*Sec[e + f*x])^3*
(-32*Csc[(e + f*x)/2]*Sec[e/2]*Sin[(f*x)/2] + Cot[(e + f*x)/2]*(-30*Log[Cos
[(e + f*x)/2] - Sin[(e + f*x)/2]] + 30*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)
/2]] + (Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(-2) - (Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^(-2) - (16*Sin[f*x])/((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[
e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])))))/(16*a*f*(1 + Sec[e + f*x]))
```

Maple [A] time = 0.078, size = 164, normalized size = 1.6

$$8\,\frac{c^3\tan\left(1/2\,fx+e/2\right)}{fa} + \frac{c^3}{2\,fa}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-2} - \frac{9\,c^3}{2\,fa}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1} - \frac{15\,c^3}{2\,fa}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x)
```

```
[Out] 8/f*c^3/a*tan(1/2*f*x+1/2*e)+1/2/f*c^3/a/(tan(1/2*f*x+1/2*e)+1)^2-9/2/f*c^3
/a/(tan(1/2*f*x+1/2*e)+1)-15/2/f*c^3/a*ln(tan(1/2*f*x+1/2*e)+1)-1/2/f*c^3/a
/(tan(1/2*f*x+1/2*e)-1)^2-9/2/f*c^3/a/(tan(1/2*f*x+1/2*e)-1)+15/2/f*c^3/a*l
```

```
n(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.01553, size = 521, normalized size = 5.21

$$c^3\left(\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a-\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}+\frac{2\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-6c^3\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(f}{\cos(fx}\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(c^3*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a + 3*lo
g(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x + e)/(a*(cos(f*x + e)
+ 1))) - 6*c^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x +
e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1)))
- 6*c^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + 2*c^3*sin(f*
x + e)/(a*(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 0.482304, size = 344, normalized size = 3.44

$$-\frac{15\left(c^3\cos\left(fx+e\right)^3+c^3\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(c^3\cos\left(fx+e\right)^3+c^3\cos\left(fx+e\right)^2\right)\log\left(-\sin\left(fx+\right.\right.}{4\left(af\cos\left(fx+e\right)^3+af\cos\left(fx+e\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/4*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*log(sin(f*x + e) + 1) -
15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) - 2*(24
*c^3*cos(f*x + e)^2 + 7*c^3*cos(f*x + e) - c^3)*sin(f*x + e))/(a*f*cos(f*x
+ e)^3 + a*f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3\left(\int -\frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int -\frac{3\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^4\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e)),x)
```

```
[Out] -c**3*(Integral(-sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(3*sec(e + f
*x)**2/(sec(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**3/(sec(e + f*x) +
 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x) + 1), x))/a
```

Giac [A] time = 1.39285, size = 165, normalized size = 1.65

$$-\frac{\frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}-\frac{16\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a}+\frac{2\left(9\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-7\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/2*(15*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 15*c^3*log(abs(tan(1/2*
f*x + 1/2*e) - 1))/a - 16*c^3*tan(1/2*f*x + 1/2*e)/a + 2*(9*c^3*tan(1/2*f*x
 + 1/2*e)^3 - 7*c^3*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a
))/f
```

3.36 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)} dx$

Optimal. Leaf size=74

$$\frac{3c^2\tan(e+fx)}{af}-\frac{3c^2\tanh^{-1}(\sin(e+fx))}{af}+\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}$$

```
[Out] (-3*c^2*ArcTanh[Sin[e + f*x]])/(a*f) + (3*c^2*Tan[e + f*x])/(a*f) + (2*(c^2
 - c^2*Sec[e + f*x])*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rubi [A] time = 0.102935, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3957, 3787, 3770, 3767, 8}

$$\frac{3c^2\tan(e+fx)}{af}-\frac{3c^2\tanh^{-1}(\sin(e+fx))}{af}+\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```


```
[Out] (-3*c^2*ArcTanh[Sin[e + f*x]])/(a*f) + (3*c^2*Tan[e + f*x])/(a*f) + (2*(c^2
 - c^2*Sec[e + f*x])*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```


Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx &= \frac{2\left(c^2-c^2\sec(e+fx)\right)\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{(3c)\int \sec(e+fx)(c-c\sec(e+fx))\,dx}{a}\\ &= \frac{2\left(c^2-c^2\sec(e+fx)\right)\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{\left(3c^2\right)\int \sec(e+fx)\,dx}{a}+\frac{\left(3c^2\right)\int \sec^2(e}{a}\\ &= -\frac{3c^2\tanh^{-1}(\sin(e+fx))}{af}+\frac{2\left(c^2-c^2\sec(e+fx)\right)\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{\left(3c^2\right)\operatorname{Subs}}{}\\ &= -\frac{3c^2\tanh^{-1}(\sin(e+fx))}{af}+\frac{3c^2\tan(e+fx)}{af}+\frac{2\left(c^2-c^2\sec(e+fx)\right)\tan(e+}{f(a+a\sec(e+fx))}\end{aligned}$$

Mathematica [B] time = 1.57997, size = 220, normalized size = 2.97

$$\frac{2c^2\sin\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(4\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\csc\left(\frac{1}{2}(e+fx)\right)+\cot\left(\frac{1}{2}(e+fx)\right)\left(\frac{}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)(\sin}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*c^2*Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]*(4*Csc[(e + f*x)/2]*S
ec[e/2]*Sin[(f*x)/2] + Cot[(e + f*x)/2]*(3*Log[Cos[(e + f*x)/2] - Sin[(e +
f*x)/2]] - 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + Sin[f*x]/((Cos[e/2]
 - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])))))/(a*f*(1 + Sec[e + f*x]))
```

Maple [A] time = 0.059, size = 116, normalized size = 1.6

$$4\,\frac{c^2\tan\left(1/2\,fx+e/2\right)}{fa}-\frac{c^2}{fa}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1}-3\,\frac{c^2\ln\left(\tan\left(1/2\,fx+e/2\right)+1\right)}{fa}-\frac{c^2}{fa}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{-1}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x)
```

```
[Out] 4/f*c^2/a*tan(1/2*f*x+1/2*e)-1/f*c^2/a/(tan(1/2*f*x+1/2*e)+1)-3/f*c^2/a*ln(
tan(1/2*f*x+1/2*e)+1)-1/f*c^2/a/(tan(1/2*f*x+1/2*e)-1)+3/f*c^2/a*ln(tan(1/2
*f*x+1/2*e)-1)
```

Maxima [B] time = 0.961418, size = 302, normalized size = 4.08

$$-\frac{c^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+2c^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin}{\cos}\right.}{}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -(c^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f
*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + 2*c^2*(
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e)
 + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - c^2*sin(f*x + e)/(a*(
cos(f*x + e) + 1)))/f
```

Fricas [A] time = 0.478713, size = 301, normalized size = 4.07

$$-\frac{3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\log\left(-\sin\left(fx+e\right)\cdots\right.}{2\left(af\cos\left(fx+e\right)^2+af\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/2*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*log(sin(f*x + e) + 1) - 3*(
c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*log(-sin(f*x + e) + 1) - 2*(5*c^2*co
s(f*x + e) + c^2)*sin(f*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2\left(\int\frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int-\frac{2\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\frac{\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e)),x)
```

```
[Out] c**2*(Integral(sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(-2*sec(e + f*
x)**2/(sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) + 1),
 x))/a
```

Giac [A] time = 1.32756, size = 138, normalized size = 1.86

$$-\frac{\frac{3c^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a}-\frac{3c^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a}-\frac{4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a}+\frac{2c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -(3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 3*c^2*log(abs(tan(1/2*f*x +
1/2*e) - 1))/a - 4*c^2*tan(1/2*f*x + 1/2*e)/a + 2*c^2*tan(1/2*f*x + 1/2*e)/
((tan(1/2*f*x + 1/2*e)^2 - 1)*a))/f
```

3.37 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=41

$$\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{c\tanh^{-1}(\sin(e+fx))}{af}$$

```
[Out] -((c*ArcTanh[Sin[e + f*x]])/(a*f)) + (2*c*Tan[e + f*x])/(f*(a + a*Sec[e + f
*x]))
```

Rubi [A] time = 0.0533329, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3957, 3770}

$$\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{c\tanh^{-1}(\sin(e+fx))}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*ArcTanh[Sin[e + f*x]])/(a*f)) + (2*c*Tan[e + f*x])/(f*(a + a*Sec[e + f
*x]))
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx &= \frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{c\int \sec(e+fx)\,dx}{a}\\ &= -\frac{c\tanh^{-1}(\sin(e+fx))}{af}+\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}\end{aligned}$$

Mathematica [A] time = 0.0686182, size = 77, normalized size = 1.88

$$-\frac{c\left(-\frac{2\tan\left(\frac{1}{2}(e+fx)\right)}{f}-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*(-(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f) + Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]/f - (2*Tan[(e + f*x)/2])/f))/a)
```

Maple [A] time = 0.055, size = 61, normalized size = 1.5

$$2\,\frac{c\tan\left(1/2\,fx+e/2\right)}{fa}+\frac{c}{fa}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\frac{c}{fa}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] 2/f*c/a*tan(1/2*f*x+1/2*e)+1/f*c/a*ln(tan(1/2*f*x+1/2*e)-1)-1/f*c/a*ln(tan(
1/2*f*x+1/2*e)+1)
```

Maxima [B] time = 0.954331, size = 136, normalized size = 3.32

$$-\frac{c\left(\frac{\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{a}-\frac{\sin\left(fx+e\right)}{a\left(\cos\left(fx+e\right)+1\right)}\right)-\frac{c\sin\left(fx+e\right)}{a\left(\cos\left(fx+e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] -(c*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - c*sin(f*x + e)/(
a*(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 0.469585, size = 190, normalized size = 4.63

$$-\frac{\left(c\cos\left(fx+e\right)+c\right)\log\left(\sin\left(fx+e\right)+1\right)-\left(c\cos\left(fx+e\right)+c\right)\log\left(-\sin\left(fx+e\right)+1\right)-4\,c\sin\left(fx+e\right)}{2\left(af\cos\left(fx+e\right)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] -1/2*((c*cos(f*x + e) + c)*log(sin(f*x + e) + 1) - (c*cos(f*x + e) + c)*log
(-sin(f*x + e) + 1) - 4*c*sin(f*x + e))/(a*f*cos(f*x + e) + a*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c\left(\int -\frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**
2/(sec(e + f*x) + 1), x))/a
```

Giac [A] time = 1.30494, size = 82, normalized size = 2.

$$-\frac{\frac{c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a}-\frac{c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a}-\frac{2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] -(c*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - c*log(abs(tan(1/2*f*x + 1/2*e) -
 1))/a - 2*c*tan(1/2*f*x + 1/2*e)/a)/f
```

3.38 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=16

$$\frac{\csc(e+fx)}{acf}$$

```
[Out] Csc[e + f*x]/(a*c*f)
```

Rubi [A] time = 0.0890242, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3958, 2606, 8}

$$\frac{\csc(e+fx)}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])),x]
```

```
[Out] Csc[e + f*x]/(a*c*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx &= -\frac{\int \cot(e+fx)\csc(e+fx)\,dx}{ac}\\ &= \frac{\text{Subst}(\int 1\,dx, x, \csc(e+fx))}{acf}\\ &= \frac{\csc(e+fx)}{acf}\end{aligned}$$

Mathematica [A] time = 0.0284897, size = 16, normalized size = 1.

$$\frac{\csc(e+fx)}{acf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])),x]
```

```
[Out] Csc[e + f*x]/(a*c*f)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{\sec(fx+e)}{(a+a\sec(fx+e))(c-c\sec(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

Maxima [A] time = 0.961591, size = 24, normalized size = 1.5

$$\frac{1}{acf\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] 1/(a*c*f*sin(f*x + e))
```

Fricas [A] time = 0.433743, size = 32, normalized size = 2.

$$\frac{1}{acf\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/(a*c*f*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sec(e+fx)}{\sec^2(e+fx)-1}\,dx}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**2 - 1), x)/(a*c)
```

Giac [A] time = 1.23839, size = 26, normalized size = 1.62

$$\frac{1}{acf\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/(a*c*f*sin(f*x + e))
```

3.39 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=59

$$-\frac{\cot^3(e+fx)}{3ac^2f}-\frac{\csc^3(e+fx)}{3ac^2f}+\frac{\csc(e+fx)}{ac^2f}$$

```
[Out] -Cot[e + f*x]^3/(3*a*c^2*f) + Csc[e + f*x]/(a*c^2*f) - Csc[e + f*x]^3/(3*a*
c^2*f)
```

Rubi [A] time = 0.136684, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3958, 2606, 2607, 30}

$$-\frac{\cot^3(e+fx)}{3ac^2f}-\frac{\csc^3(e+fx)}{3ac^2f}+\frac{\csc(e+fx)}{ac^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] -Cot[e + f*x]^3/(3*a*c^2*f) + Csc[e + f*x]/(a*c^2*f) - Csc[e + f*x]^3/(3*a*
c^2*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx = \frac{\int \left(a\cot^3(e+fx)\csc(e+fx)+a\cot^2(e+fx)\csc^2(e+fx)\right)dx}{a^2c^2}$$
$$= \frac{\int \cot^3(e+fx)\csc(e+fx)\,dx}{ac^2} + \frac{\int \cot^2(e+fx)\csc^2(e+fx)\,dx}{ac^2}$$
$$= \frac{\text{Subst}\left(\int x^2\,dx, x, -\cot(e+fx)\right)}{ac^2f} - \frac{\text{Subst}\left(\int \left(-1+x^2\right)dx, x, \csc(e+\right.}{ac^2f}$$
$$= -\frac{\cot^3(e+fx)}{3ac^2f} + \frac{\csc(e+fx)}{ac^2f} - \frac{\csc^3(e+fx)}{3ac^2f}$$

Mathematica [A] time = 0.458726, size = 81, normalized size = 1.37

$$\frac{\csc(e)(-10\sin(e+fx)+5\sin(2(e+fx))-6\sin(2e+fx)+2\sin(e+2fx)+6\sin(e)+2\sin(fx))\csc^2\left(\frac{1}{2}(e+fx)\right)\text{cs}}{24ac^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (Csc[e]*Csc[(e + f*x)/2]^2*Csc[e + f*x]*(6*Sin[e] + 2*Sin[f*x] - 10*Sin[e +
 f*x] + 5*Sin[2*(e + f*x)] - 6*Sin[2*e + f*x] + 2*Sin[e + 2*f*x]))/(24*a*c^
2*f)
```

Maple [A] time = 0.049, size = 48, normalized size = 0.8

$$\frac{1}{4fc^2a}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}+2\left(\tan\left(1/2\,fx+e/2\right)\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x)
```

```
[Out] 1/4/f/c^2/a*(tan(1/2*f*x+1/2*e)-1/3/tan(1/2*f*x+1/2*e)^3+2/tan(1/2*f*x+1/2*
e))
```

Maxima [A] time = 0.991779, size = 104, normalized size = 1.76

$$\frac{\frac{\left(\frac{6\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{ac^2\sin(fx+e)^3}+\frac{3\sin(fx+e)}{ac^2(\cos(fx+e)+1)}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/12*((6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a*c
^2*sin(f*x + e)^3) + 3*sin(f*x + e)/(a*c^2*(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 0.435448, size = 123, normalized size = 2.08

$$\frac{\cos\left(fx+e\right)^{2}+2\,\cos\left(fx+e\right)-2}{3\,\left(ac^{2}f\cos\left(fx+e\right)-ac^{2}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*(cos(f*x + e)^2 + 2*cos(f*x + e) - 2)/((a*c^2*f*cos(f*x + e) - a*c^2*f)
*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec\left(e+fx\right)}{\sec^{3}\left(e+fx\right)-\sec^{2}\left(e+fx\right)-\sec\left(e+fx\right)+1}\,dx}{ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**3 - sec(e + f*x)**2 - sec(e + f*x) + 1
), x)/(a*c**2)
```

Giac [A] time = 1.24125, size = 80, normalized size = 1.36

$$\frac{\frac{3\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{ac^{2}}+\frac{6\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1}{ac^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] 1/12*(3*tan(1/2*f*x + 1/2*e)/(a*c^2) + (6*tan(1/2*f*x + 1/2*e)^2 - 1)/(a*c^
2*tan(1/2*f*x + 1/2*e)^3))/f
```

3.40 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=78

$$\frac{2\cot^5(e+fx)}{5ac^3f}+\frac{2\csc^5(e+fx)}{5ac^3f}-\frac{\csc^3(e+fx)}{ac^3f}+\frac{\csc(e+fx)}{ac^3f}$$

```
[Out] (2*Cot[e + f*x]^5)/(5*a*c^3*f) + Csc[e + f*x]/(a*c^3*f) - Csc[e + f*x]^3/(a
*c^3*f) + (2*Csc[e + f*x]^5)/(5*a*c^3*f)
```

Rubi [A] time = 0.179323, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3958, 2606, 194, 2607, 30, 14}

$$\frac{2\cot^5(e+fx)}{5ac^3f}+\frac{2\csc^5(e+fx)}{5ac^3f}-\frac{\csc^3(e+fx)}{ac^3f}+\frac{\csc(e+fx)}{ac^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] (2*Cot[e + f*x]^5)/(5*a*c^3*f) + Csc[e + f*x]/(a*c^3*f) - Csc[e + f*x]^3/(a
*c^3*f) + (2*Csc[e + f*x]^5)/(5*a*c^3*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = -\frac{\int \left(a^2\cot^5(e+fx)\csc(e+fx)+2a^2\cot^4(e+fx)\csc^2(e+fx)+a^2\cot^3\right.}{a^3c^3}$$
$$= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{ac^3} - \frac{\int \cot^3(e+fx)\csc^3(e+fx)\,dx}{ac^3} - \frac{2\int \mathrm{co}}{}$$
$$= \frac{\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)dx, x, \csc(e+fx)\right)}{ac^3f} + \frac{\mathrm{Subst}\left(\int \left(-1+x^2\right)^2 dx, x, \mathrm{c}\right.}{ac^3f}$$
$$= \frac{2\cot^5(e+fx)}{5ac^3f} + \frac{\mathrm{Subst}\left(\int \left(1-2x^2+x^4\right)dx, x, \csc(e+fx)\right)}{ac^3f} + \frac{\mathrm{Subst}\left(\int\right.}{}$$
$$= \frac{2\cot^5(e+fx)}{5ac^3f} + \frac{\csc(e+fx)}{ac^3f} - \frac{\csc^3(e+fx)}{ac^3f} + \frac{2\csc^5(e+fx)}{5ac^3f}$$

Mathematica [A] time = 0.866247, size = 107, normalized size = 1.37

$$-\frac{\csc(e)(65\sin(e+fx)-52\sin(2(e+fx))+13\sin(3(e+fx))+40\sin(2e+fx)-12\sin(e+2fx)-20\sin(3e+2fx)+8}{320ac^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] -(Csc[e]*Csc[(e + f*x)/2]^4*Csc[e + f*x]*(-40*Sin[e] + 65*Sin[e + f*x] - 52
*Sin[2*(e + f*x)] + 13*Sin[3*(e + f*x)] + 40*Sin[2*e + f*x] - 12*Sin[e + 2*
f*x] - 20*Sin[3*e + 2*f*x] + 8*Sin[2*e + 3*f*x]))/(320*a*c^3*f)
```

Maple [A] time = 0.055, size = 61, normalized size = 0.8

$$\frac{1}{8fac^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}+3\left(\tan\left(1/2\,fx+e/2\right)\right)^{-1}+\frac{1}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x)
```

```
[Out] 1/8/f/a/c^3*(tan(1/2*f*x+1/2*e)-1/tan(1/2*f*x+1/2*e)^3+3/tan(1/2*f*x+1/2*e)
+1/5/tan(1/2*f*x+1/2*e)^5)
```

Maxima [A] time = 0.983902, size = 131, normalized size = 1.68

$$-\frac{\frac{\left(\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)(\cos(fx+e)+1)^5}{ac^3\sin(fx+e)^5}-\frac{5\sin(fx+e)}{ac^3(\cos(fx+e)+1)}}{40f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] -1/40*((5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(a*c^3*sin(f*x + e)^5) - 5*sin(f*x +
e)/(a*c^3*(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 0.445246, size = 185, normalized size = 2.37

$$\frac{2\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}-4\cos\left(fx+e\right)+2}{5\left(ac^{3}f\cos\left(fx+e\right)^{2}-2\,ac^{3}f\cos\left(fx+e\right)+ac^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/5*(2*cos(f*x + e)^3 + cos(f*x + e)^2 - 4*cos(f*x + e) + 2)/((a*c^3*f*cos(
f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{\sec\left(e+fx\right)}{\sec^{4}\left(e+fx\right)-2\sec^{3}\left(e+fx\right)+2\sec\left(e+fx\right)-1}\,dx}{ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*sec(e + f*x
) - 1), x)/(a*c**3)
```

Giac [A] time = 1.3243, size = 99, normalized size = 1.27

$$\frac{\frac{5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac^{3}}+\frac{15\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1}{ac^{3}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}}}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/40*(5*tan(1/2*f*x + 1/2*e)/(a*c^3) + (15*tan(1/2*f*x + 1/2*e)^4 - 5*tan(1
/2*f*x + 1/2*e)^2 + 1)/(a*c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

3.41 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=120

$$-\frac{4\cot^7(e+fx)}{7ac^4f}-\frac{\cot^5(e+fx)}{5ac^4f}-\frac{4\csc^7(e+fx)}{7ac^4f}+\frac{9\csc^5(e+fx)}{5ac^4f}-\frac{2\csc^3(e+fx)}{ac^4f}+\frac{\csc(e+fx)}{ac^4f}$$

```
[Out] -Cot[e + f*x]^5/(5*a*c^4*f) - (4*Cot[e + f*x]^7)/(7*a*c^4*f) + Csc[e + f*x]
/(a*c^4*f) - (2*Csc[e + f*x]^3)/(a*c^4*f) + (9*Csc[e + f*x]^5)/(5*a*c^4*f)
- (4*Csc[e + f*x]^7)/(7*a*c^4*f)
```

Rubi [A] time = 0.229039, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3958, 2606, 194, 2607, 30, 270, 14}

$$-\frac{4\cot^7(e+fx)}{7ac^4f}-\frac{\cot^5(e+fx)}{5ac^4f}-\frac{4\csc^7(e+fx)}{7ac^4f}+\frac{9\csc^5(e+fx)}{5ac^4f}-\frac{2\csc^3(e+fx)}{ac^4f}+\frac{\csc(e+fx)}{ac^4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] -Cot[e + f*x]^5/(5*a*c^4*f) - (4*Cot[e + f*x]^7)/(7*a*c^4*f) + Csc[e + f*x]
/(a*c^4*f) - (2*Csc[e + f*x]^3)/(a*c^4*f) + (9*Csc[e + f*x]^5)/(5*a*c^4*f)
- (4*Csc[e + f*x]^7)/(7*a*c^4*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx = \frac{\int \left(a^3\cot^7(e+fx)\csc(e+fx)+3a^3\cot^6(e+fx)\csc^2(e+fx)+3a^3 \text{c}\right.}{a^4c^4}$$

$$= \frac{\int \cot^7(e+fx)\csc(e+fx)\,dx}{ac^4}+\frac{\int \cot^4(e+fx)\csc^4(e+fx)\,dx}{ac^4}+\frac{3\int}{}$$

$$= -\frac{\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx,x,\csc(e+fx)\right)}{ac^4f}+\frac{\text{Subst}\left(\int x^4\left(1+x^2\right)\,dx,\right.}{ac^4f}$$

$$= -\frac{3\cot^7(e+fx)}{7ac^4f}-\frac{\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx,x,\csc(e+fx)\right)}{ac^4f}$$

$$= -\frac{\cot^5(e+fx)}{5ac^4f}-\frac{4\cot^7(e+fx)}{7ac^4f}+\frac{\csc(e+fx)}{ac^4f}-\frac{2\csc^3(e+fx)}{ac^4f}+\frac{9\,\text{cs}}{}$$

Mathematica [A] time = 0.877835, size = 145, normalized size = 1.21

$$\csc(e)(-1946\sin(e+fx)+1946\sin(2(e+fx))-834\sin(3(e+fx))+139\sin(4(e+fx))-1400\sin(2e+fx)+616\,\text{s}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (Csc[e]*Csc[(e + f*x)/2]^6*Csc[e + f*x]*(840*Sin[e] - 56*Sin[f*x] - 1946*Si
n[e + f*x] + 1946*Sin[2*(e + f*x)] - 834*Sin[3*(e + f*x)] + 139*Sin[4*(e +
f*x)] - 1400*Sin[2*e + f*x] + 616*Sin[e + 2*f*x] + 840*Sin[3*e + 2*f*x] - 3
44*Sin[2*e + 3*f*x] - 280*Sin[4*e + 3*f*x] + 104*Sin[3*e + 4*f*x]))/(17920*
a*c^4*f)
```

Maple [A] time = 0.059, size = 74, normalized size = 0.6

$$\frac{1}{16fac^4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2\left(\tan\left(1/2\,fx+e/2\right)\right)^{-3}+4\left(\tan\left(1/2\,fx+e/2\right)\right)^{-1}+\frac{4}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}-\frac{1}{7}\left(\tan\left(\frac{fx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x)
```

```
[Out] 1/16/f/a/c^4*(tan(1/2*f*x+1/2*e)-2/tan(1/2*f*x+1/2*e)^3+4/tan(1/2*f*x+1/2*e
)+4/5/tan(1/2*f*x+1/2*e)^5-1/7/tan(1/2*f*x+1/2*e)^7)
```

Maxima [A] time = 1.01181, size = 158, normalized size = 1.32

$$\frac{\frac{\left(\frac{28\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{70\,\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{140\,\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-5\right)\left(\cos\left(fx+e\right)+1\right)^{7}}{ac^{4}\sin\left(fx+e\right)^{7}}+\frac{35\,\sin\left(fx+e\right)}{ac^{4}\left(\cos\left(fx+e\right)+1\right)}}{560\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/560*((28*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 70*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + 140*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) +
 1)^7/(a*c^4*sin(f*x + e)^7) + 35*sin(f*x + e)/(a*c^4*(cos(f*x + e) + 1)))/
f
```

Fricas [A] time = 0.451931, size = 255, normalized size = 2.12

$$\frac{13\,\cos\left(fx+e\right)^{4}-4\,\cos\left(fx+e\right)^{3}-20\,\cos\left(fx+e\right)^{2}+24\,\cos\left(fx+e\right)-8}{35\left(ac^{4}f\cos\left(fx+e\right)^{3}-3\,ac^{4}f\cos\left(fx+e\right)^{2}+3\,ac^{4}f\cos\left(fx+e\right)-ac^{4}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/35*(13*cos(f*x + e)^4 - 4*cos(f*x + e)^3 - 20*cos(f*x + e)^2 + 24*cos(f*x
 + e) - 8)/((a*c^4*f*cos(f*x + e)^3 - 3*a*c^4*f*cos(f*x + e)^2 + 3*a*c^4*f*
cos(f*x + e) - a*c^4*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec\left(e+fx\right)}{\sec^{5}\left(e+fx\right)-3\sec^{4}\left(e+fx\right)+2\sec^{3}\left(e+fx\right)+2\sec^{2}\left(e+fx\right)-3\sec\left(e+fx\right)+1}\,dx}{ac^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**5 - 3*sec(e + f*x)**4 + 2*sec(e + f*x)
**3 + 2*sec(e + f*x)**2 - 3*sec(e + f*x) + 1), x)/(a*c**4)
```

Giac [A] time = 1.21872, size = 117, normalized size = 0.98

$$\frac{\frac{35\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac^4}+\frac{140\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-70\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+28\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-5}{ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7}}{560f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] 1/560*(35*tan(1/2*f*x + 1/2*e)/(a*c^4) + (140*tan(1/2*f*x + 1/2*e)^6 - 70*t
an(1/2*f*x + 1/2*e)^4 + 28*tan(1/2*f*x + 1/2*e)^2 - 5)/(a*c^4*tan(1/2*f*x +
 1/2*e)^7))/f
```

3.42 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=164

$$-\frac{7c^5\tan^3(e+fx)}{a^2f}-\frac{84c^5\tan(e+fx)}{a^2f}+\frac{105c^5\tanh^{-1}(\sin(e+fx))}{2a^2f}+\frac{63c^5\tan(e+fx)\sec(e+fx)}{2a^2f}-\frac{6c^2\tan(e+fx)(c}{f\left(a^2\sec(e\right.}$$

```
[Out] (105*c^5*ArcTanh[Sin[e + f*x]])/(2*a^2*f) - (84*c^5*Tan[e + f*x])/(a^2*f) +
 (63*c^5*Sec[e + f*x]*Tan[e + f*x])/(2*a^2*f) - (6*c^2*(c - c*Sec[e + f*x])
^3*Tan[e + f*x])/(f*(a^2 + a^2*Sec[e + f*x])) + (2*c*(c - c*Sec[e + f*x])^4
*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (7*c^5*Tan[e + f*x]^3)/(a^2*f
)
```

Rubi [A] time = 0.246943, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3957, 3791, 3770, 3767, 8, 3768}

$$-\frac{7c^5\tan^3(e+fx)}{a^2f}-\frac{84c^5\tan(e+fx)}{a^2f}+\frac{105c^5\tanh^{-1}(\sin(e+fx))}{2a^2f}+\frac{63c^5\tan(e+fx)\sec(e+fx)}{2a^2f}-\frac{6c^2\tan(e+fx)(c}{f\left(a^2\sec(e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```


```
[Out] (105*c^5*ArcTanh[Sin[e + f*x]])/(2*a^2*f) - (84*c^5*Tan[e + f*x])/(a^2*f) +
 (63*c^5*Sec[e + f*x]*Tan[e + f*x])/(2*a^2*f) - (6*c^2*(c - c*Sec[e + f*x])
^3*Tan[e + f*x])/(f*(a^2 + a^2*Sec[e + f*x])) + (2*c*(c - c*Sec[e + f*x])^4
*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (7*c^5*Tan[e + f*x]^3)/(a^2*f
)
```


Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(3c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx}{a}$$

$$= -\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(}{}$$

$$= -\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(}{}$$

$$= -\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(}{}$$

$$= \frac{21c^5\tanh^{-1}(\sin(e+fx))}{a^2f} + \frac{63c^5\sec(e+fx)\tan(e+fx)}{2a^2f} - \frac{6c^2(c-c\sec(e+}{f\left(a^2+a^2\mathrm{s}\right.}$$

$$= \frac{105c^5\tanh^{-1}(\sin(e+fx))}{2a^2f} - \frac{84c^5\tan(e+fx)}{a^2f} + \frac{63c^5\sec(e+fx)\tan(e+fx}{2a^2f}$$

Mathematica [B] time = 1.27436, size = 380, normalized size = 2.32

$$\frac{\cot\left(\frac{1}{2}(e+fx)\right)\csc^6\left(\frac{1}{2}(e+fx)\right)(c-c\sec(e+fx))^5\left(\sec\left(\frac{e}{2}\right)\sec(e)\left(-2901\sin\left(e-\frac{fx}{2}\right)+1197\sin\left(e+\frac{fx}{2}\right)-3027\sin\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^6*(c - c*Sec[e + f*x])^5*(20160*Cos[e +
f*x]^3*Cot[(e + f*x)/2]^3*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[C
os[(e + f*x)/2] + Sin[(e + f*x)/2]]) + Csc[(e + f*x)/2]^3*Sec[e/2]*Sec[e]*(
-1323*Sin[(f*x)/2] + 3247*Sin[(3*f*x)/2] - 2901*Sin[e - (f*x)/2] + 1197*Sin
[e + (f*x)/2] - 3027*Sin[2*e + (f*x)/2] - 273*Sin[e + (3*f*x)/2] + 1827*Sin
[2*e + (3*f*x)/2] - 1693*Sin[3*e + (3*f*x)/2] + 1995*Sin[e + (5*f*x)/2] - 1
17*Sin[2*e + (5*f*x)/2] + 1143*Sin[3*e + (5*f*x)/2] - 969*Sin[4*e + (5*f*x)
/2] + 1173*Sin[2*e + (7*f*x)/2] + 117*Sin[3*e + (7*f*x)/2] + 747*Sin[4*e +
(7*f*x)/2] - 309*Sin[5*e + (7*f*x)/2] + 494*Sin[3*e + (9*f*x)/2] + 142*Sin[
4*e + (9*f*x)/2] + 352*Sin[5*e + (9*f*x)/2])))/(3072*a^2*f*(1 + Sec[e + f*x
])^2)
```

Maple [A] time = 0.102, size = 234, normalized size = 1.4

$$-\frac{16c^5}{3fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-64\frac{c^5\tan\left(1/2\,fx+e/2\right)}{fa^2}+\frac{c^5}{3fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-3}-4\frac{c^5}{fa^2\left(\tan\left(1/2\,fx+e/2\right)+1\right)^2}+\frac{5}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -16/3/f*c^5/a^2*tan(1/2*f*x+1/2*e)^3-64/f*c^5/a^2*tan(1/2*f*x+1/2*e)+1/3/f*
c^5/a^2/(tan(1/2*f*x+1/2*e)+1)^3-4/f*c^5/a^2/(tan(1/2*f*x+1/2*e)+1)^2+55/2/
f*c^5/a^2/(tan(1/2*f*x+1/2*e)+1)+105/2/f*c^5/a^2*ln(tan(1/2*f*x+1/2*e)+1)+1
/3/f*c^5/a^2/(tan(1/2*f*x+1/2*e)-1)^3+4/f*c^5/a^2/(tan(1/2*f*x+1/2*e)-1)^2+
55/2/f*c^5/a^2/(tan(1/2*f*x+1/2*e)-1)-105/2/f*c^5/a^2*ln(tan(1/2*f*x+1/2*e)
-1)
```

Maxima [B] time = 1.04855, size = 1033, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(c^5*(4*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^2 - 3*a^2*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (27*sin(f*x + e)/(cos(f*x + e)
+ 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 30*log(sin(f*x + e)/(cos(
f*x + e) + 1) + 1)/a^2 + 30*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) +
 5*c^5*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) + 1)
+ 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 10*c^5*((15*s
in(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 -
 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 10*c^5*((9*sin(f*x + e)/(cos(f*x + e
) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos
(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) +
 5*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
1)^3)/a^2 - c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/a^2)/f
```

Fricas [A] time = 0.499922, size = 521, normalized size = 3.18

$$\frac{315\left(c^5\cos\left(fx+e\right)^5+2c^5\cos\left(fx+e\right)^4+c^5\cos\left(fx+e\right)^3\right)\log\left(\sin\left(fx+e\right)+1\right)-315\left(c^5\cos\left(fx+e\right)^5+2c^5\cos\left(fx\right.\right.}{12\left(a^2f\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(315*(c^5*cos(f*x + e)^5 + 2*c^5*cos(f*x + e)^4 + c^5*cos(f*x + e)^3)*
log(sin(f*x + e) + 1) - 315*(c^5*cos(f*x + e)^5 + 2*c^5*cos(f*x + e)^4 + c^
5*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) - 2*(494*c^5*cos(f*x + e)^4 + 679*
c^5*cos(f*x + e)^3 + 102*c^5*cos(f*x + e)^2 - 17*c^5*cos(f*x + e) + 2*c^5)*
sin(f*x + e))/(a^2*f*cos(f*x + e)^5 + 2*a^2*f*cos(f*x + e)^4 + a^2*f*cos(f*
x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^5\left(\int -\frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{5\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int -\frac{10\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{10\sec^4(e+fx)}{\sec^2(e+fx)+2\sec(e\cdots}\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c**5*(Integral(-sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(5*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(-10*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(10*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integr
al(-5*sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral
(sec(e + f*x)**6/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A] time = 1.36791, size = 221, normalized size = 1.35

$$\frac{\frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2} - \frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2} + \frac{2\left(165\,c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-280\,c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+123\,c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3a^2} - \frac{32\left(a^4\cdots\right.}{\cdots}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(315*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 315*c^5*log(abs(tan(1
/2*f*x + 1/2*e) - 1))/a^2 + 2*(165*c^5*tan(1/2*f*x + 1/2*e)^5 - 280*c^5*tan
(1/2*f*x + 1/2*e)^3 + 123*c^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^
2 - 1)^3*a^2) - 32*(a^4*c^5*tan(1/2*f*x + 1/2*e)^3 + 12*a^4*c^5*tan(1/2*f*x
 + 1/2*e))/a^6)/f
```

3.43 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2} dx$

Optimal. Leaf size=150

$$-\frac{70c^4\tan(e+fx)}{3a^2f}+\frac{35c^4\tanh^{-1}(\sin(e+fx))}{2a^2f}+\frac{35c^4\tan(e+fx)\sec(e+fx)}{6a^2f}-\frac{14\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)^2}{3f\left(a^2\sec(e+fx)+a^2\right)}+$$

```
[Out] (35*c^4*ArcTanh[Sin[e + f*x]])/(2*a^2*f) - (70*c^4*Tan[e + f*x])/(3*a^2*f)
+ (35*c^4*Sec[e + f*x]*Tan[e + f*x])/(6*a^2*f) + (2*c*(c - c*Sec[e + f*x])^
3*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (14*(c^2 - c^2*Sec[e + f*x])
^2*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rubi [A] time = 0.215091, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3957, 3788, 3767, 8, 4046, 3770}

$$-\frac{70c^4\tan(e+fx)}{3a^2f}+\frac{35c^4\tanh^{-1}(\sin(e+fx))}{2a^2f}+\frac{35c^4\tan(e+fx)\sec(e+fx)}{6a^2f}-\frac{14\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)^2}{3f\left(a^2\sec(e+fx)+a^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```


```
[Out] (35*c^4*ArcTanh[Sin[e + f*x]])/(2*a^2*f) - (70*c^4*Tan[e + f*x])/(3*a^2*f)
+ (35*c^4*Sec[e + f*x]*Tan[e + f*x])/(6*a^2*f) + (2*c*(c - c*Sec[e + f*x])^
3*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (14*(c^2 - c^2*Sec[e + f*x])
^2*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```


Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(7c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx}{3a}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \cdots$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \cdots$$

$$= \frac{35c^4\sec(e+fx)\tan(e+fx)}{6a^2f} + \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{14\left(c^2-\cdots\right.}{3f\cdots}$$

$$= \frac{35c^4\tanh^{-1}(\sin(e+fx))}{2a^2f} - \frac{70c^4\tan(e+fx)}{3a^2f} + \frac{35c^4\sec(e+fx)\tan(e+fx)}{6a^2f} \cdots$$

Mathematica [B] time = 1.94691, size = 349, normalized size = 2.33

$$\frac{c^4\sin^3\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left(-32\tan\left(\frac{e}{2}\right)\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)-32\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\mathrm{cs}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^4*Cos[(e + f*x)/2]*Sec[e + f*x]^2*Sin[(e + f*x)/2]^3*(-256*Cot[(e + f*x)
/2]^2*Csc[(e + f*x)/2]*Sec[e/2]*Sin[(f*x)/2] - 32*Csc[(e + f*x)/2]^3*Sec[e/
2]*Sin[(f*x)/2] + 3*Cot[(e + f*x)/2]^3*(-70*Log[Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2]] + 70*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^(-2) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^(-2) - (2
4*Sin[f*x])/((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))) - 32*Cot[(e + f
*x)/2]*Csc[(e + f*x)/2]^2*Tan[e/2]))/(3*a^2*f*(1 + Sec[e + f*x])^2)
```

Maple [A] time = 0.092, size = 186, normalized size = 1.2

$$-\frac{8c^4}{3fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3 - 24\frac{c^4\tan\left(1/2\,fx+e/2\right)}{fa^2} - \frac{c^4}{2fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-2} + \frac{13c^4}{2fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -8/3/f*c^4/a^2*tan(1/2*f*x+1/2*e)^3-24/f*c^4/a^2*tan(1/2*f*x+1/2*e)-1/2/f*c
^4/a^2/(tan(1/2*f*x+1/2*e)+1)^2+13/2/f*c^4/a^2/(tan(1/2*f*x+1/2*e)+1)+35/2/
f*c^4/a^2*ln(tan(1/2*f*x+1/2*e)+1)+1/2/f*c^4/a^2/(tan(1/2*f*x+1/2*e)-1)^2+1
3/2/f*c^4/a^2/(tan(1/2*f*x+1/2*e)-1)-35/2/f*c^4/a^2*ln(tan(1/2*f*x+1/2*e)-1
)
```

Maxima [B] time = 1.02256, size = 717, normalized size = 4.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(c^4*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(
f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) +
1) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 4*c^4*((15
*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2
 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/(c
os(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 6*c^4*((9*sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(co
s(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2)
+ 4*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 - c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)/a^2)/f
```

Fricas [A] time = 0.494155, size = 487, normalized size = 3.25

$$\frac{105\left(c^4\cos\left(fx+e\right)^4+2\,c^4\cos\left(fx+e\right)^3+c^4\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-105\left(c^4\cos\left(fx+e\right)^4+2\,c^4\cos\left(fx\right.\right.}{12\left(a^2f\cos\left(fx+e\right)^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(105*(c^4*cos(f*x + e)^4 + 2*c^4*cos(f*x + e)^3 + c^4*cos(f*x + e)^2)*
log(sin(f*x + e) + 1) - 105*(c^4*cos(f*x + e)^4 + 2*c^4*cos(f*x + e)^3 + c^
4*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) - 2*(164*c^4*cos(f*x + e)^3 + 229*
c^4*cos(f*x + e)^2 + 30*c^4*cos(f*x + e) - 3*c^4)*sin(f*x + e))/(a^2*f*cos(
f*x + e)^4 + 2*a^2*f*cos(f*x + e)^3 + a^2*f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^4\left(\int \frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int -\frac{4\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{6\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int -\frac{4\sec^4(e+fx)}{\sec^2(e+fx)+2\sec(e+f}\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)
```

```
[Out] c**4*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(-4*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(6*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integra
l(-4*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(
sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A] time = 1.27273, size = 198, normalized size = 1.32

$$\frac{\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2} - \frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2} + \frac{6\left(13\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-11\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2a^2} - \frac{16\left(a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+9\,a^4c^4\right.}{a^6}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(105*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 105*c^4*log(abs(tan(1
/2*f*x + 1/2*e) - 1))/a^2 + 6*(13*c^4*tan(1/2*f*x + 1/2*e)^3 - 11*c^4*tan(1
/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a^2) - 16*(a^4*c^4*tan(1/2
*f*x + 1/2*e)^3 + 9*a^4*c^4*tan(1/2*f*x + 1/2*e))/a^6)/f
```

3.44 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=119

$$-\frac{5c^3\tan(e+fx)}{a^2f}+\frac{5c^3\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{10\tan(e+fx)\left(c^3-c^3\sec(e+fx)\right)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}$$

```
[Out] (5*c^3*ArcTanh[Sin[e + f*x]])/(a^2*f) - (5*c^3*Tan[e + f*x])/(a^2*f) + (2*c
*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (10*(c
^3 - c^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rubi [A] time = 0.185887, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3957, 3787, 3770, 3767, 8}

$$-\frac{5c^3\tan(e+fx)}{a^2f}+\frac{5c^3\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{10\tan(e+fx)\left(c^3-c^3\sec(e+fx)\right)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (5*c^3*ArcTanh[Sin[e + f*x]])/(a^2*f) - (5*c^3*Tan[e + f*x])/(a^2*f) + (2*c
*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (10*(c
^3 - c^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(5c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx}{3a}$$

$$= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{10\left(c^3-c^3\sec(e+fx)\right)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(5$$

$$= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{10\left(c^3-c^3\sec(e+fx)\right)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(5$$

$$= \frac{5c^3\tanh^{-1}(\sin(e+fx))}{a^2f} + \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{10\left(c^3-c^3\text{se}\right.}{3f\left(a^2\right.}$$

$$= \frac{5c^3\tanh^{-1}(\sin(e+fx))}{a^2f} - \frac{5c^3\tan(e+fx)}{a^2f} + \frac{2c(c-c\sec(e+fx))^2\tan(e+f}{3f(a+a\sec(e+fx))^2}$$

Mathematica [B] time = 4.25574, size = 485, normalized size = 4.08

$$c^3(\cos(e+fx)-1)^3\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)\left(-\frac{1}{16}\sec^3\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)(-80\cos(e+fx)-40\cos(2(e+fx))+66\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^3*(-1 + Cos[e + f*x])^3*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2*(-15*Cos[e]*
Cot[(e + f*x)/2]^5*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]])*Sec[e/2]^2 + 2*Cot[(e + f*x)/2]*Csc[(e + f*x)
/2]^2*Sec[e/2]^3*(-Sin[e/2] + Sin[(3*e)/2]) + 20*Cot[(e + f*x)/2]^2*Csc[(e
+ f*x)/2]*Sec[e/2]*Sin[(f*x)/2] - 26*Cot[(e + f*x)/2]^4*Csc[(e + f*x)/2]*Se
c[e/2]*Sin[(f*x)/2] - ((-40 + 40*Cos[e] + 78*Cos[f*x] - 80*Cos[e + f*x] - 4
0*Cos[2*(e + f*x)] + 66*Cos[2*e + f*x] + 23*Cos[e + 2*f*x] + 17*Cos[3*e + 2
*f*x])*Csc[(e + f*x)/2]^5*Sec[e/2]^3*Sin[(f*x)/2])/16 - Cot[(e + f*x)/2]^3*
(15*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[
(e + f*x)/2]]) - 4*Csc[(e + f*x)/2]^2*Tan[e/2])*(-1 + Tan[e/2]^2)))/(6*a^2*
f*(1 + Cos[e + f*x])^2*(-1 + Cot[(e + f*x)/2])*(1 + Cot[(e + f*x)/2])*(-1 +
 Tan[e/2])*(1 + Tan[e/2]))
```

Maple [A] time = 0.074, size = 136, normalized size = 1.1

$$-\frac{4c^3}{3fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3 - 8\frac{c^3\tan\left(1/2\,fx+e/2\right)}{fa^2} + \frac{c^3}{fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1} + 5\frac{c^3\ln\left(\tan\left(1/2\,fx+e/2\right)+1\right)}{fa^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -4/3/f*c^3/a^2*tan(1/2*f*x+1/2*e)^3-8/f*c^3/a^2*tan(1/2*f*x+1/2*e)+1/f*c^3/
a^2/(tan(1/2*f*x+1/2*e)+1)+5/f*c^3/a^2*ln(tan(1/2*f*x+1/2*e)+1)+1/f*c^3/a^2
```

```
/(tan(1/2*f*x+1/2*e)-1)-5/f*c^3/a^2*ln(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.01091, size = 460, normalized size = 3.87

$$-\frac{c^3\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}+\frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)+3c^3\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\cdots}{a^2}\right.}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(c^3*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(s
in(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 3*c^3*((9*sin(f*x +
e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(si
n(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) +
 1) - 1)/a^2) + 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(
cos(f*x + e) + 1)^3)/a^2 - c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [A] time = 0.484874, size = 440, normalized size = 3.7

$$\frac{15\left(c^3\cos(fx+e)^3+2c^3\cos(fx+e)^2+c^3\cos(fx+e)\right)\log(\sin(fx+e)+1)-15\left(c^3\cos(fx+e)^3+2c^3\cos(fx+e\cdots\right.}{6\left(a^2f\cos(fx+e)^3+2a^2f\cos(fx\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(15*(c^3*cos(f*x + e)^3 + 2*c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*log(
sin(f*x + e) + 1) - 15*(c^3*cos(f*x + e)^3 + 2*c^3*cos(f*x + e)^2 + c^3*cos
(f*x + e))*log(-sin(f*x + e) + 1) - 2*(23*c^3*cos(f*x + e)^2 + 34*c^3*cos(f
*x + e) + 3*c^3)*sin(f*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)
^2 + a^2*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3\left(\int -\frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\frac{3\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int -\frac{3\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\frac{\sec^4(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)\cdots}\right.}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c**3*(Integral(-sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(3*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(-3*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integ
ral(sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A] time = 1.3827, size = 171, normalized size = 1.44

$$\frac{\frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}-\frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}+\frac{6\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^2}-\frac{4\left(a^4c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+6\,a^4c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(15*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 15*c^3*log(abs(tan(1/2
*f*x + 1/2*e) - 1))/a^2 + 6*c^3*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)
^2 - 1)*a^2) - 4*(a^4*c^3*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c^3*tan(1/2*f*x +
1/2*e))/a^6)/f
```

3.45 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=88

$$\frac{c^2\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{2c^2\tan(e+fx)}{f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f(a\sec(e+fx)+a)^2}$$

```
[Out] (c^2*ArcTanh[Sin[e + f*x]])/(a^2*f) - (2*c^2*Tan[e + f*x])/(f*(a^2 + a^2*Se
c[e + f*x])) + (2*(c^2 - c^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a + a*Sec[e
+ f*x])^2)
```

Rubi [A] time = 0.131277, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3957, 3770}

$$\frac{c^2\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{2c^2\tan(e+fx)}{f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^2*ArcTanh[Sin[e + f*x]])/(a^2*f) - (2*c^2*Tan[e + f*x])/(f*(a^2 + a^2*Se
c[e + f*x])) + (2*(c^2 - c^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a + a*Sec[e
+ f*x])^2)
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx &= \frac{2\left(c^2-c^2\sec(e+fx)\right)\tan(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{c\int\frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx}{a}\\ &= -\frac{2c^2\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)}+\frac{2\left(c^2-c^2\sec(e+fx)\right)\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{c^2\int\sec(e+fx}{a^2}\\ &= \frac{c^2\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{2c^2\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)}+\frac{2\left(c^2-c^2\sec(e+fx)\right)\tan(e}{3f(a+a\sec(e+fx))^2}\end{aligned}$$

Mathematica [A] time = 0.0938139, size = 109, normalized size = 1.24

$$\frac{c^2\left(-\frac{4\tan\left(\frac{1}{2}(e+fx)\right)}{3f}-\frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)}{3f}-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{f}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^2*(-(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f) + Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]/f - (4*Tan[(e + f*x)/2])/(3*f) - (2*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2])/(3*f)))/a^2
```

Maple [A] time = 0.07, size = 89, normalized size = 1.

$$-\frac{2c^2}{3fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-2\frac{c^2\tan\left(1/2\,fx+e/2\right)}{fa^2}-\frac{c^2}{fa^2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\frac{c^2}{fa^2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -2/3/f*c^2/a^2*tan(1/2*f*x+1/2*e)^3-2/f*c^2/a^2*tan(1/2*f*x+1/2*e)-1/f*c^2/
a^2*ln(tan(1/2*f*x+1/2*e)-1)+1/f*c^2/a^2*ln(tan(1/2*f*x+1/2*e)+1)
```

Maxima [B] time = 0.99547, size = 265, normalized size = 3.01

$$-\frac{c^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+\frac{2c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}-\frac{c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(\ldots}{(\cos(f\ldots}\right.}{a^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 2*c^2*(3*sin(f*x + e)/(cos(f*x + e)
 + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - c^2*(3*sin(f*x + e)/(cos
(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [A] time = 0.477391, size = 340, normalized size = 3.86

$$\frac{3\left(c^2\cos\left(fx+e\right)^2+2c^2\cos\left(fx+e\right)+c^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(c^2\cos\left(fx+e\right)^2+2c^2\cos\left(fx+e\right)+c^2\right)\log\left(\ldots\right.}{6\left(a^2f\cos\left(fx+e\right)^2+2a^2f\cos\left(fx+e\right)+a^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*log(sin(f*x + e) + 1
) - 3*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*log(-sin(f*x + e) + 1
) - 8*(c^2*cos(f*x + e) + 2*c^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^
2*f*cos(f*x + e) + a^2*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2\left(\int \frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int -\frac{2\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)
```

```
[Out] c**2*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(-2*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A] time = 1.313, size = 126, normalized size = 1.43

$$\frac{\frac{3c^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2}-\frac{3c^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2}-\frac{2\left(a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+3a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^6}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 3*c^2*log(abs(tan(1/2*f
*x + 1/2*e) - 1))/a^2 - 2*(a^4*c^2*tan(1/2*f*x + 1/2*e)^3 + 3*a^4*c^2*tan(1
/2*f*x + 1/2*e))/a^6)/f
```

3.46 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=36

$$\frac{\tan(e+fx)(c-c\sec(e+fx))}{3f(a\sec(e+fx)+a)^2}$$

```
[Out] ((c - c*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rubi [A] time = 0.047863, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3950}

$$\frac{\tan(e+fx)(c-c\sec(e+fx))}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] ((c - c*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=\frac{(c-c\sec(e+fx))\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

Mathematica [A] time = 0.0882635, size = 23, normalized size = 0.64

$$-\frac{c\tan^3\left(\frac{1}{2}(e+fx)\right)}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -(c*Tan[(e + f*x)/2]^3)/(3*a^2*f)
```

Maple [A] time = 0.066, size = 21, normalized size = 0.6

$$-\frac{c}{3fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -1/3/f*c/a^2*tan(1/2*f*x+1/2*e)^3
```

Maxima [B] time = 0.978455, size = 127, normalized size = 3.53

$$-\frac{\frac{c\left(\frac{3\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)}{a^{2}}-\frac{c\left(\frac{3\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)}{a^{2}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/6*(c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 - c*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/a^2)/f
```

Fricas [A] time = 0.436839, size = 126, normalized size = 3.5

$$\frac{\left(c\cos\left(fx+e\right)-c\right)\sin\left(fx+e\right)}{3\left(a^{2}f\cos\left(fx+e\right)^{2}+2\,a^{2}f\cos\left(fx+e\right)+a^{2}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*(c*cos(f*x + e) - c)*sin(f*x + e)/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f
*x + e) + a^2*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c\left(\int -\frac{\sec\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^{2}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c*(Integral(-sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

<u>Giac</u> [A] time = 1.35652, size = 28, normalized size = 0.78

$$-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}{3a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -1/3*c*tan(1/2*f*x + 1/2*e)^3/(a^2*f)
```

3.47 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=59

$$\frac{\cot^3(e+fx)}{3a^2cf}-\frac{\csc^3(e+fx)}{3a^2cf}+\frac{\csc(e+fx)}{a^2cf}$$

```
[Out] Cot[e + f*x]^3/(3*a^2*c*f) + Csc[e + f*x]/(a^2*c*f) - Csc[e + f*x]^3/(3*a^2
*c*f)
```

Rubi [A] time = 0.136156, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3958, 2606, 2607, 30}

$$\frac{\cot^3(e+fx)}{3a^2cf}-\frac{\csc^3(e+fx)}{3a^2cf}+\frac{\csc(e+fx)}{a^2cf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

```
[Out] Cot[e + f*x]^3/(3*a^2*c*f) + Csc[e + f*x]/(a^2*c*f) - Csc[e + f*x]^3/(3*a^2
*c*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{\int \left(c\cot^3(e+fx)\csc(e+fx)-c\cot^2(e+fx)\csc^2(e+fx)\right)dx}{a^2c^2}$$
$$= \frac{\int \cot^3(e+fx)\csc(e+fx)\,dx}{a^2c} - \frac{\int \cot^2(e+fx)\csc^2(e+fx)\,dx}{a^2c}$$
$$= -\frac{\text{Subst}\left(\int x^2\,dx, x, -\cot(e+fx)\right)}{a^2cf} - \frac{\text{Subst}\left(\int \left(-1+x^2\right)dx, x, \csc(e\cdots\right)}{a^2cf}$$
$$= \frac{\cot^3(e+fx)}{3a^2cf} + \frac{\csc(e+fx)}{a^2cf} - \frac{\csc^3(e+fx)}{3a^2cf}$$

Mathematica [A] time = 0.570834, size = 83, normalized size = 1.41

$$-\frac{\csc(e)\sin^2\left(\frac{1}{2}(e+fx)\right)(10\sin(e+fx)+5\sin(2(e+fx))-6\sin(2e+fx)-2\sin(e+2fx)-6\sin(e)+2\sin(fx))\,\text{cs}\cdots}{6a^2cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

```
[Out] -(Csc[e]*Csc[e + f*x]^3*Sin[(e + f*x)/2]^2*(-6*Sin[e] + 2*Sin[f*x] + 10*Sin
[e + f*x] + 5*Sin[2*(e + f*x)] - 6*Sin[2*e + f*x] - 2*Sin[e + 2*f*x]))/(6*a
^2*c*f)
```

Maple [A] time = 0.048, size = 48, normalized size = 0.8

$$\frac{1}{4\,fa^2c}\left(-\frac{1}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+2\,\tan\left(1/2\,fx+e/2\right)+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x)
```

```
[Out] 1/4/f/a^2/c*(-1/3*tan(1/2*f*x+1/2*e)^3+2*tan(1/2*f*x+1/2*e)+1/tan(1/2*f*x+1
/2*e))
```

Maxima [A] time = 0.988689, size = 103, normalized size = 1.75

$$\frac{\frac{\frac{6\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2c}+\frac{3(\cos(fx+e)+1)}{a^2c\sin(fx+e)}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*((6*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1
)^3)/(a^2*c) + 3*(cos(f*x + e) + 1)/(a^2*c*sin(f*x + e)))/f
```

Fricas [A] time = 0.438719, size = 124, normalized size = 2.1

$$-\frac{\cos\left(fx+e\right)^2-2\,\cos\left(fx+e\right)-2}{3\left(a^2cf\cos\left(fx+e\right)+a^2cf\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/3*(cos(f*x + e)^2 - 2*cos(f*x + e) - 2)/((a^2*c*f*cos(f*x + e) + a^2*c*f
)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+\sec^2\left(e+fx\right)-\sec\left(e+fx\right)-1}\,dx}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**3 + sec(e + f*x)**2 - sec(e + f*x) -
1), x)/(a**2*c)
```

Giac [A] time = 1.22861, size = 97, normalized size = 1.64

$$\frac{\frac{3}{a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}-\frac{a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-6\,a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6c^3}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/12*(3/(a^2*c*tan(1/2*f*x + 1/2*e)) - (a^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 6*
a^4*c^2*tan(1/2*f*x + 1/2*e))/(a^6*c^3))/f
```

3.48 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=38

$$\frac{\csc(e+fx)}{a^2c^2f}-\frac{\csc^3(e+fx)}{3a^2c^2f}$$

```
[Out] Csc[e + f*x]/(a^2*c^2*f) - Csc[e + f*x]^3/(3*a^2*c^2*f)
```

Rubi [A] time = 0.0981878, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3958, 2606}

$$\frac{\csc(e+fx)}{a^2c^2f}-\frac{\csc^3(e+fx)}{3a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] Csc[e + f*x]/(a^2*c^2*f) - Csc[e + f*x]^3/(3*a^2*c^2*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx &= \frac{\int \cot^3(e+fx)\csc(e+fx)\,dx}{a^2c^2}\\ &= -\frac{\text{Subst}\left(\int \left(-1+x^2\right)dx, x, \csc(e+fx)\right)}{a^2c^2f}\\ &= \frac{\csc(e+fx)}{a^2c^2f}-\frac{\csc^3(e+fx)}{3a^2c^2f}\end{aligned}$$

Mathematica [A] time = 0.0495572, size = 33, normalized size = 0.87

$$\frac{\frac{\csc(e+fx)}{f}-\frac{\csc^3(e+fx)}{3f}}{a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (Csc[e + f*x]/f - Csc[e + f*x]^3/(3*f))/(a^2*c^2)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{\sec(fx+e)}{(a+a\sec(fx+e))^2\,(c-c\sec(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)
```

```
[Out] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x)
```

Maxima [A] time = 0.960094, size = 42, normalized size = 1.11

$$\frac{3\,\sin(fx+e)^2-1}{3\,a^2c^2f\,\sin(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/3*(3*sin(f*x + e)^2 - 1)/(a^2*c^2*f*sin(f*x + e)^3)
```

Fricas [A] time = 0.438563, size = 111, normalized size = 2.92

$$\frac{3\,\cos(fx+e)^2-2}{3\left(a^2c^2f\cos(fx+e)^2-a^2c^2f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/3*(3*cos(f*x + e)^2 - 2)/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x
+ e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(e+fx)}{\sec^4(e+fx)-2\sec^2(e+fx)+1}\,dx}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**4 - 2*sec(e + f*x)**2 + 1), x)/(a**2*c
**2)
```

Giac [A] time = 1.23126, size = 45, normalized size = 1.18

$$\frac{3 \sin\left(fx+e\right)^2 - 1}{3 a^2 c^2 f \sin\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*sin(f*x + e)^2 - 1)/(a^2*c^2*f*sin(f*x + e)^3)
```

3.49 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=80

$$\frac{\cot^5(e+fx)}{5a^2c^3f}+\frac{\csc^5(e+fx)}{5a^2c^3f}-\frac{2\csc^3(e+fx)}{3a^2c^3f}+\frac{\csc(e+fx)}{a^2c^3f}$$

```
[Out] Cot[e + f*x]^5/(5*a^2*c^3*f) + Csc[e + f*x]/(a^2*c^3*f) - (2*Csc[e + f*x]^3
)/(3*a^2*c^3*f) + Csc[e + f*x]^5/(5*a^2*c^3*f)
```

Rubi [A] time = 0.146201, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3958, 2606, 194, 2607, 30}

$$\frac{\cot^5(e+fx)}{5a^2c^3f}+\frac{\csc^5(e+fx)}{5a^2c^3f}-\frac{2\csc^3(e+fx)}{3a^2c^3f}+\frac{\csc(e+fx)}{a^2c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] Cot[e + f*x]^5/(5*a^2*c^3*f) + Csc[e + f*x]/(a^2*c^3*f) - (2*Csc[e + f*x]^3
)/(3*a^2*c^3*f) + Csc[e + f*x]^5/(5*a^2*c^3*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = -\frac{\int \left(a\cot^5(e+fx)\csc(e+fx)+a\cot^4(e+fx)\csc^2(e+fx)\right)dx}{a^3c^3}$$
$$= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^2c^3} - \frac{\int \cot^4(e+fx)\csc^2(e+fx)\,dx}{a^2c^3}$$
$$= -\frac{\text{Subst}\left(\int x^4\,dx, x, -\cot(e+fx)\right)}{a^2c^3f} + \frac{\text{Subst}\left(\int \left(-1+x^2\right)^2 dx, x, \csc(\right.}{a^2c^3f}$$
$$= \frac{\cot^5(e+fx)}{5a^2c^3f} + \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right) dx, x, \csc(e+fx)\right)}{a^2c^3f}$$
$$= \frac{\cot^5(e+fx)}{5a^2c^3f} + \frac{\csc(e+fx)}{a^2c^3f} - \frac{2\csc^3(e+fx)}{3a^2c^3f} + \frac{\csc^5(e+fx)}{5a^2c^3f}$$

Mathematica [A] time = 0.983502, size = 147, normalized size = 1.84

$$-\frac{\csc(e)(534\sin(e+fx)-178\sin(2(e+fx))-178\sin(3(e+fx))+89\sin(4(e+fx))+40\sin(2e+fx)-168\sin(e+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] -(Csc[e]*Csc[(e + f*x)/2]^2*Csc[e + f*x]^3*(-200*Sin[e] + 104*Sin[f*x] + 53
4*Sin[e + f*x] - 178*Sin[2*(e + f*x)] - 178*Sin[3*(e + f*x)] + 89*Sin[4*(e
+ f*x)] + 40*Sin[2*e + f*x] - 168*Sin[e + 2*f*x] + 120*Sin[3*e + 2*f*x] + 7
2*Sin[2*e + 3*f*x] - 120*Sin[4*e + 3*f*x] + 24*Sin[3*e + 4*f*x]))/(1920*a^2
*c^3*f)
```

Maple [A] time = 0.055, size = 76, normalized size = 1.

$$\frac{1}{16fa^2c^3}\left(-\frac{1}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+4\tan\left(1/2\,fx+e/2\right)-\frac{4}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}+6\left(\tan\left(1/2\,fx+e/2\right)\right)^{-1}+\frac{1}{5}\left(\tan\left(\frac{f}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x)
```

```
[Out] 1/16/f/a^2/c^3*(-1/3*tan(1/2*f*x+1/2*e)^3+4*tan(1/2*f*x+1/2*e)-4/3/tan(1/2*
f*x+1/2*e)^3+6/tan(1/2*f*x+1/2*e)+1/5/tan(1/2*f*x+1/2*e)^5)
```

Maxima [A] time = 0.983762, size = 163, normalized size = 2.04

$$\frac{\frac{5\left(\frac{12\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^3}-\frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{90\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{a^2c^3\sin(fx+e)^5}}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/240*(5*(12*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/(a^2*c^3) - (20*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 90*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(a^2*c^3*sin(f*x + e)^
5))/f
```

Fricas [A] time = 0.452245, size = 259, normalized size = 3.24

$$\frac{3\cos\left(fx+e\right)^{4}+12\cos\left(fx+e\right)^{3}-12\cos\left(fx+e\right)^{2}-8\cos\left(fx+e\right)+8}{15\left(a^{2}c^{3}f\cos\left(fx+e\right)^{3}-a^{2}c^{3}f\cos\left(fx+e\right)^{2}-a^{2}c^{3}f\cos\left(fx+e\right)+a^{2}c^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(3*cos(f*x + e)^4 + 12*cos(f*x + e)^3 - 12*cos(f*x + e)^2 - 8*cos(f*x
+ e) + 8)/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f
*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sec\left(e+fx\right)}{\sec^{5}\left(e+fx\right)-\sec^{4}\left(e+fx\right)-2\sec^{3}\left(e+fx\right)+2\sec^{2}\left(e+fx\right)+\sec\left(e+fx\right)-1}\,dx}{a^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**5 - sec(e + f*x)**4 - 2*sec(e + f*x)*
*3 + 2*sec(e + f*x)**2 + sec(e + f*x) - 1), x)/(a**2*c**3)
```

Giac [A] time = 1.28363, size = 136, normalized size = 1.7

$$\frac{\frac{90\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-20\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+3}{a^{2}c^{3}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}}-\frac{5\left(a^{4}c^{6}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-12a^{4}c^{6}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{6}c^{9}}}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/240*((90*tan(1/2*f*x + 1/2*e)^4 - 20*tan(1/2*f*x + 1/2*e)^2 + 3)/(a^2*c^3
*tan(1/2*f*x + 1/2*e)^5) - 5*(a^4*c^6*tan(1/2*f*x + 1/2*e)^3 - 12*a^4*c^6*t
an(1/2*f*x + 1/2*e))/(a^6*c^9))/f
```

3.50 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=98

$$-\frac{2\cot^7(e+fx)}{7a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}+\frac{\csc^5(e+fx)}{a^2c^4f}-\frac{4\csc^3(e+fx)}{3a^2c^4f}+\frac{\csc(e+fx)}{a^2c^4f}$$

```
[Out] (-2*Cot[e + f*x]^7)/(7*a^2*c^4*f) + Csc[e + f*x]/(a^2*c^4*f) - (4*Csc[e + f
*x]^3)/(3*a^2*c^4*f) + Csc[e + f*x]^5/(a^2*c^4*f) - (2*Csc[e + f*x]^7)/(7*a
^2*c^4*f)
```

Rubi [A] time = 0.188573, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3958, 2606, 194, 2607, 30, 270}

$$-\frac{2\cot^7(e+fx)}{7a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}+\frac{\csc^5(e+fx)}{a^2c^4f}-\frac{4\csc^3(e+fx)}{3a^2c^4f}+\frac{\csc(e+fx)}{a^2c^4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (-2*Cot[e + f*x]^7)/(7*a^2*c^4*f) + Csc[e + f*x]/(a^2*c^4*f) - (4*Csc[e + f
*x]^3)/(3*a^2*c^4*f) + Csc[e + f*x]^5/(a^2*c^4*f) - (2*Csc[e + f*x]^7)/(7*a
^2*c^4*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{\int \left(a^2\cot^7(e+fx)\csc(e+fx)+2a^2\cot^6(e+fx)\csc^2(e+fx)+a^2\cot^5\right.}{a^4c^4}$$

$$= \frac{\int \cot^7(e+fx)\csc(e+fx)\,dx}{a^2c^4} + \frac{\int \cot^5(e+fx)\csc^3(e+fx)\,dx}{a^2c^4} + \frac{2\int \mathrm{c}}{}$$

$$= -\frac{\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)^2 dx, x, \csc(e+fx)\right)}{a^2c^4f} - \frac{\mathrm{Subst}\left(\int \left(-1+x^2\right)^3 dx,\right.}{a^2c^4f}$$

$$= -\frac{2\cot^7(e+fx)}{7a^2c^4f} - \frac{\mathrm{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right) dx, x, \csc(e+fx)\right)}{a^2c^4f} -$$

$$= -\frac{2\cot^7(e+fx)}{7a^2c^4f} + \frac{\csc(e+fx)}{a^2c^4f} - \frac{4\csc^3(e+fx)}{3a^2c^4f} + \frac{\csc^5(e+fx)}{a^2c^4f} - \frac{2\csc^7}{7a}$$

Mathematica [A] time = 0.916271, size = 179, normalized size = 1.83

$$\frac{\csc(e)(-182\sin(e+fx)+104\sin(2(e+fx))+39\sin(3(e+fx))-52\sin(4(e+fx))+13\sin(5(e+fx))-56\sin(2e+f}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (Csc[e]*Csc[(e + f*x)/2]^4*Csc[e + f*x]^3*(42*Sin[e] - 28*Sin[f*x] - 182*Si
n[e + f*x] + 104*Sin[2*(e + f*x)] + 39*Sin[3*(e + f*x)] - 52*Sin[4*(e + f*x
)] + 13*Sin[5*(e + f*x)] - 56*Sin[2*e + f*x] + 76*Sin[e + 2*f*x] - 28*Sin[3
*e + 2*f*x] - 24*Sin[2*e + 3*f*x] + 42*Sin[4*e + 3*f*x] - 3*Sin[3*e + 4*f*x
] - 21*Sin[5*e + 4*f*x] + 6*Sin[4*e + 5*f*x]))/(1344*a^2*c^4*f)
```

Maple [A] time = 0.058, size = 87, normalized size = 0.9

$$\frac{1}{32fa^2c^4}\left(-\frac{1}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+5\tan\left(1/2fx+e/2\right)-\frac{10}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}+10\left(\tan\left(1/2fx+e/2\right)\right)^{-1}+\left(\tan\left(\frac{fx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x)
```

```
[Out] 1/32/f/a^2/c^4*(-1/3*tan(1/2*f*x+1/2*e)^3+5*tan(1/2*f*x+1/2*e)-10/3/tan(1/2
*f*x+1/2*e)^3+10/tan(1/2*f*x+1/2*e)+1/tan(1/2*f*x+1/2*e)^5-1/7/tan(1/2*f*x+
1/2*e)^7)
```

Maxima [A] time = 1.01934, size = 189, normalized size = 1.93

$$\frac{\frac{7\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^4}+\frac{\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{70\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{210\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-3\right)(\cos(fx+e)+1)^7}{a^2c^4\sin(fx+e)^7}}{672f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/672*(7*(15*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/(a^2*c^4) + (21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 70*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 210*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(
cos(f*x + e) + 1)^7/(a^2*c^4*sin(f*x + e)^7))/f
```

Fricas [A] time = 0.457526, size = 290, normalized size = 2.96

$$\frac{6\cos(fx+e)^5+9\cos(fx+e)^4-24\cos(fx+e)^3+4\cos(fx+e)^2+16\cos(fx+e)-8}{21\left(a^2c^4f\cos(fx+e)^4-2a^2c^4f\cos(fx+e)^3+2a^2c^4f\cos(fx+e)-a^2c^4f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/21*(6*cos(f*x + e)^5 + 9*cos(f*x + e)^4 - 24*cos(f*x + e)^3 + 4*cos(f*x +
 e)^2 + 16*cos(f*x + e) - 8)/((a^2*c^4*f*cos(f*x + e)^4 - 2*a^2*c^4*f*cos(f
*x + e)^3 + 2*a^2*c^4*f*cos(f*x + e) - a^2*c^4*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sec(e+fx)}{\sec^6(e+fx)-2\sec^5(e+fx)-\sec^4(e+fx)+4\sec^3(e+fx)-\sec^2(e+fx)-2\sec(e+fx)+1}\,dx}{a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**6 - 2*sec(e + f*x)**5 - sec(e + f*x)**
4 + 4*sec(e + f*x)**3 - sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x)/(a**2*c**
4)
```

Giac [A] time = 1.23031, size = 155, normalized size = 1.58

$$\frac{\frac{210\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-70\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+21\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3}{a^2c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7}-\frac{7\left(a^4c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-15a^4c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^6c^{12}}}{672f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/672*((210*tan(1/2*f*x + 1/2*e)^6 - 70*tan(1/2*f*x + 1/2*e)^4 + 21*tan(1/2
*f*x + 1/2*e)^2 - 3)/(a^2*c^4*tan(1/2*f*x + 1/2*e)^7) - 7*(a^4*c^8*tan(1/2*
f*x + 1/2*e)^3 - 15*a^4*c^8*tan(1/2*f*x + 1/2*e))/(a^6*c^12))/f
```

3.51 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=141

$$\frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}-\frac{13\csc^7(e+fx)}{7a^2c^5f}+\frac{3\csc^5(e+fx)}{a^2c^5f}-\frac{7\csc^3(e+fx)}{3a^2c^5f}+\frac{\csc(e+fx)}{a^2c^5f}$$

```
[Out] Cot[e + f*x]^7/(7*a^2*c^5*f) + (4*Cot[e + f*x]^9)/(9*a^2*c^5*f) + Csc[e + f
*x]/(a^2*c^5*f) - (7*Csc[e + f*x]^3)/(3*a^2*c^5*f) + (3*Csc[e + f*x]^5)/(a^
2*c^5*f) - (13*Csc[e + f*x]^7)/(7*a^2*c^5*f) + (4*Csc[e + f*x]^9)/(9*a^2*c^
5*f)
```

Rubi [A] time = 0.245591, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3958, 2606, 194, 2607, 30, 270, 14}

$$\frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}-\frac{13\csc^7(e+fx)}{7a^2c^5f}+\frac{3\csc^5(e+fx)}{a^2c^5f}-\frac{7\csc^3(e+fx)}{3a^2c^5f}+\frac{\csc(e+fx)}{a^2c^5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```


```
[Out] Cot[e + f*x]^7/(7*a^2*c^5*f) + (4*Cot[e + f*x]^9)/(9*a^2*c^5*f) + Csc[e + f
*x]/(a^2*c^5*f) - (7*Csc[e + f*x]^3)/(3*a^2*c^5*f) + (3*Csc[e + f*x]^5)/(a^
2*c^5*f) - (13*Csc[e + f*x]^7)/(7*a^2*c^5*f) + (4*Csc[e + f*x]^9)/(9*a^2*c^
5*f)
```


Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx &= -\frac{\int \left(a^3\cot^9(e+fx)\csc(e+fx)+3a^3\cot^8(e+fx)\csc^2(e+fx)+3a^3\,\text{co}\right.}{a^5c^5}\\ &= -\frac{\int \cot^9(e+fx)\csc(e+fx)\,dx}{a^2c^5}-\frac{\int \cot^6(e+fx)\csc^4(e+fx)\,dx}{a^2c^5}-\frac{3\int \text{c}}{}\\ &= \frac{\text{Subst}\left(\int \left(-1+x^2\right)^4\,dx,x,\csc(e+fx)\right)}{a^2c^5f}-\frac{\text{Subst}\left(\int x^6\left(1+x^2\right)\,dx,x,-\right.}{a^2c^5f}\\ &= \frac{\cot^9(e+fx)}{3a^2c^5f}+\frac{\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)\,dx,x,\csc(e+fx)\right)}{a^2c^5f}\\ &= \frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{\csc(e+fx)}{a^2c^5f}-\frac{7\csc^3(e+fx)}{3a^2c^5f}+\frac{3\csc^5(}{a^2}\end{aligned}$$

Mathematica [A] time = 1.2793, size = 211, normalized size = 1.5

$$-\frac{\csc(e)(36252\sin(e+fx)-27189\sin(2(e+fx))-2014\sin(3(e+fx))+12084\sin(4(e+fx))-6042\sin(5(e+fx))+1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] -(Csc[e]*Csc[(e + f*x)/2]^6*Csc[e + f*x]^3*(-9408*Sin[e] + 9792*Sin[f*x] +
36252*Sin[e + f*x] - 27189*Sin[2*(e + f*x)] - 2014*Sin[3*(e + f*x)] + 12084
*Sin[4*(e + f*x)] - 6042*Sin[5*(e + f*x)] + 1007*Sin[6*(e + f*x)] + 12096*S
in[2*e + f*x] - 14400*Sin[e + 2*f*x] - 2016*Sin[3*e + 2*f*x] + 7520*Sin[2*e
 + 3*f*x] - 8736*Sin[4*e + 3*f*x] + 1248*Sin[3*e + 4*f*x] + 6048*Sin[5*e +
4*f*x] - 1632*Sin[4*e + 5*f*x] - 2016*Sin[6*e + 5*f*x] + 608*Sin[5*e + 6*f*
x]))/(516096*a^2*c^5*f)
```

Maple [A] time = 0.066, size = 102, normalized size = 0.7

$$\frac{1}{64\,fa^2c^5}\left(-\frac{1}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+6\,\tan\left(1/2\,fx+e/2\right)-\frac{20}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}+15\,\left(\tan\left(1/2\,fx+e/2\right)\right)^{-1}+3\,\left(\tan\left(1/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x)
```

```
[Out] 1/64/f/a^2/c^5*(-1/3*tan(1/2*f*x+1/2*e)^3+6*tan(1/2*f*x+1/2*e)-20/3/tan(1/2
*f*x+1/2*e)^3+15/tan(1/2*f*x+1/2*e)+3/tan(1/2*f*x+1/2*e)^5-6/7/tan(1/2*f*x+
1/2*e)^7+1/9/tan(1/2*f*x+1/2*e)^9)
```

Maxima [A] time = 1.0005, size = 217, normalized size = 1.54

$$\frac{\frac{21\left(\frac{18\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^5}-\frac{\left(\frac{54\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{189\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{945\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-7\right)(\cos(fx+e)+1)^9}{a^2c^5\sin(fx+e)^9}}{4032f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] 1/4032*(21*(18*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/(a^2*c^5) - (54*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 189*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 9
45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(a^2*c^5*s
in(f*x + e)^9))/f
```

Fricas [A] time = 0.468487, size = 400, normalized size = 2.84

$$\frac{19\cos\left(fx+e\right)^6+6\cos\left(fx+e\right)^5-66\cos\left(fx+e\right)^4+56\cos\left(fx+e\right)^3+24\cos\left(fx+e\right)^2-48\cos\left(fx\right.}{63\left(a^2c^5f\cos\left(fx+e\right)^5-3a^2c^5f\cos\left(fx+e\right)^4+2a^2c^5f\cos\left(fx+e\right)^3+2a^2c^5f\cos\left(fx+e\right)^2-3a^2c^5f\cos\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/63*(19*cos(f*x + e)^6 + 6*cos(f*x + e)^5 - 66*cos(f*x + e)^4 + 56*cos(f*x
 + e)^3 + 24*cos(f*x + e)^2 - 48*cos(f*x + e) + 16)/((a^2*c^5*f*cos(f*x + e
)^5 - 3*a^2*c^5*f*cos(f*x + e)^4 + 2*a^2*c^5*f*cos(f*x + e)^3 + 2*a^2*c^5*f
*cos(f*x + e)^2 - 3*a^2*c^5*f*cos(f*x + e) + a^2*c^5*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{\sec(e+fx)}{\sec^7(e+fx)-3\sec^6(e+fx)+\sec^5(e+fx)+5\sec^4(e+fx)-5\sec^3(e+fx)-\sec^2(e+fx)+3\sec(e+fx)-1}\,dx}{a^2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**7 - 3*sec(e + f*x)**6 + sec(e + f*x)*
*5 + 5*sec(e + f*x)**4 - 5*sec(e + f*x)**3 - sec(e + f*x)**2 + 3*sec(e + f*
```

```
x) - 1), x)/(a**2*c**5)
```

Giac [A] time = 1.3399, size = 174, normalized size = 1.23

$$\frac{\frac{945 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{8}-420 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{6}+189 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{4}-54 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{2}+7}{a^{2}c^{5} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{9}}-\frac{21 \left(a^{4}c^{10} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{3}-18 a^{4}c^{10} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)\right)}{a^{6}c^{15}}}{4032 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/4032*((945*tan(1/2*f*x + 1/2*e)^8 - 420*tan(1/2*f*x + 1/2*e)^6 + 189*tan(
1/2*f*x + 1/2*e)^4 - 54*tan(1/2*f*x + 1/2*e)^2 + 7)/(a^2*c^5*tan(1/2*f*x +
1/2*e)^9) - 21*(a^4*c^10*tan(1/2*f*x + 1/2*e)^3 - 18*a^4*c^10*tan(1/2*f*x +
 1/2*e))/(a^6*c^15))/f
```

3.52 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=215

$$\frac{77c^6\tan^3(e+fx)}{5a^3f}+\frac{924c^6\tan(e+fx)}{5a^3f}-\frac{231c^6\tanh^{-1}(\sin(e+fx))}{2a^3f}-\frac{693c^6\tan(e+fx)\sec(e+fx)}{10a^3f}+\frac{66\tan(e+f}{5f\left(a^3\right.}$$

```
[Out] (-231*c^6*ArcTanh[Sin[e + f*x]])/(2*a^3*f) + (924*c^6*Tan[e + f*x])/(5*a^3*
f) - (693*c^6*Sec[e + f*x]*Tan[e + f*x])/(10*a^3*f) - (22*c^2*(c - c*Sec[e
+ f*x])^4*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + (2*c*(c - c*Sec[e
 + f*x])^5*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (66*(c^2 - c^2*Sec[
e + f*x])^3*Tan[e + f*x])/(5*f*(a^3 + a^3*Sec[e + f*x])) + (77*c^6*Tan[e +
f*x]^3)/(5*a^3*f)
```

Rubi [A] time = 0.336145, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3957, 3791, 3770, 3767, 8, 3768}

$$\frac{77c^6\tan^3(e+fx)}{5a^3f}+\frac{924c^6\tan(e+fx)}{5a^3f}-\frac{231c^6\tanh^{-1}(\sin(e+fx))}{2a^3f}-\frac{693c^6\tan(e+fx)\sec(e+fx)}{10a^3f}+\frac{66\tan(e+f}{5f\left(a^3\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```


```
[Out] (-231*c^6*ArcTanh[Sin[e + f*x]])/(2*a^3*f) + (924*c^6*Tan[e + f*x])/(5*a^3*
f) - (693*c^6*Sec[e + f*x]*Tan[e + f*x])/(10*a^3*f) - (22*c^2*(c - c*Sec[e
+ f*x])^4*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + (2*c*(c - c*Sec[e
 + f*x])^5*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (66*(c^2 - c^2*Sec[
e + f*x])^3*Tan[e + f*x])/(5*f*(a^3 + a^3*Sec[e + f*x])) + (77*c^6*Tan[e +
f*x]^3)/(5*a^3*f)
```


Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(11c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx}{5a}$$

$$= -\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(33}{}$$

$$= -\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{66}{}$$

$$= -\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{66}{}$$

$$= -\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{66}{}$$

$$= -\frac{231c^6\tanh^{-1}(\sin(e+fx))}{5a^3f} - \frac{693c^6\sec(e+fx)\tan(e+fx)}{10a^3f} - \frac{22c^2(c-c\sec(e}{15af(a+a}$$

$$= -\frac{231c^6\tanh^{-1}(\sin(e+fx))}{2a^3f} + \frac{924c^6\tan(e+fx)}{5a^3f} - \frac{693c^6\sec(e+fx)\tan(e+f}{10a^3f}$$

Mathematica [A] time = 2.19962, size = 406, normalized size = 1.89

$$c^6\cos\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\left(\sec\left(\frac{e}{2}\right)\sec(e)\left(-130340\sin\left(e-\frac{fx}{2}\right)+75600\sin\left(e+\frac{fx}{2}\right)-120176\sin\left(2e+\frac{fx}{2}\right)-3423\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^6*Cos[(e + f*x)/2]*Sec[e + f*x]^3*(887040*Cos[(e + f*x)/2]^5*(Log[Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])
+ Sec[e/2]*Sec[e]*Sec[e + f*x]^3*(-65436*Sin[(f*x)/2] + 127498*Sin[(3*f*x)/
2] - 130340*Sin[e - (f*x)/2] + 75600*Sin[e + (f*x)/2] - 120176*Sin[2*e + (f
*x)/2] - 34230*Sin[e + (3*f*x)/2] + 82278*Sin[2*e + (3*f*x)/2] - 79450*Sin[
3*e + (3*f*x)/2] + 91670*Sin[e + (5*f*x)/2] - 14730*Sin[2*e + (5*f*x)/2] +
```

```
61920*Sin[3*e + (5*f*x)/2] - 44480*Sin[4*e + (5*f*x)/2] + 53593*Sin[2*e + (
7*f*x)/2] - 1735*Sin[3*e + (7*f*x)/2] + 38123*Sin[4*e + (7*f*x)/2] - 17205*
Sin[5*e + (7*f*x)/2] + 23735*Sin[3*e + (9*f*x)/2] + 2455*Sin[4*e + (9*f*x)/
2] + 17785*Sin[5*e + (9*f*x)/2] - 3495*Sin[6*e + (9*f*x)/2] + 5446*Sin[4*e
+ (11*f*x)/2] + 1190*Sin[5*e + (11*f*x)/2] + 4256*Sin[6*e + (11*f*x)/2])))/
(960*a^3*f*(1 + Sec[e + f*x])^3)
```

Maple [A] time = 0.123, size = 256, normalized size = 1.2

$$\frac{16\,c^6}{5\,fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5+\frac{64\,c^6}{3\,fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+160\,\frac{c^6\tan\left(1/2\,fx+e/2\right)}{fa^3}-\frac{c^6}{3\,fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-3}+5\,\frac{}{fa^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 16/5/f*c^6/a^3*tan(1/2*f*x+1/2*e)^5+64/3/f*c^6/a^3*tan(1/2*f*x+1/2*e)^3+160
/f*c^6/a^3*tan(1/2*f*x+1/2*e)-1/3/f*c^6/a^3/(tan(1/2*f*x+1/2*e)+1)^3+5/f*c^
6/a^3/(tan(1/2*f*x+1/2*e)+1)^2-89/2/f*c^6/a^3/(tan(1/2*f*x+1/2*e)+1)-231/2/
f*c^6/a^3*ln(tan(1/2*f*x+1/2*e)+1)-1/3/f*c^6/a^3/(tan(1/2*f*x+1/2*e)-1)^3-5
/f*c^6/a^3/(tan(1/2*f*x+1/2*e)-1)^2-89/2/f*c^6/a^3/(tan(1/2*f*x+1/2*e)-1)+2
31/2/f*c^6/a^3*ln(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.1138, size = 1262, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^6*(20*(33*sin(f*x + e)/(cos(f*x + e) + 1) - 76*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 51*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3 - 3*a^3*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (735*sin(f*x + e)/(cos(f*x +
e) + 1) + 50*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5)/a^3 - 690*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 690
*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 6*c^6*(60*(5*sin(f*x + e)/
(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^3 - 2*a^3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 390*log(sin(f*x + e)/
(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/
a^3) + 45*c^6*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 6
0*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f
*x + e) + 1) - 1)/a^3) + 20*c^6*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x +
e)/(cos(f*x + e) + 1) - 1)/a^3) + 15*c^6*(15*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5)/a^3 + c^6*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/
```

```
(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 18*c^6*
(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a
^3)/f
```

Fricas [A] time = 0.516365, size = 663, normalized size = 3.08

$$-\frac{3465\left(c^6\cos\left(fx+e\right)^6+3\,c^6\cos\left(fx+e\right)^5+3\,c^6\cos\left(fx+e\right)^4+c^6\cos\left(fx+e\right)^3\right)\log\left(\sin\left(fx+e\right)+1\right)-3465\left(c^6\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/60*(3465*(c^6*cos(f*x + e)^6 + 3*c^6*cos(f*x + e)^5 + 3*c^6*cos(f*x + e)
^4 + c^6*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 3465*(c^6*cos(f*x + e)^6 +
 3*c^6*cos(f*x + e)^5 + 3*c^6*cos(f*x + e)^4 + c^6*cos(f*x + e)^3)*log(-sin
(f*x + e) + 1) - 2*(5446*c^6*cos(f*x + e)^5 + 12843*c^6*cos(f*x + e)^4 + 87
11*c^6*cos(f*x + e)^3 + 815*c^6*cos(f*x + e)^2 - 105*c^6*cos(f*x + e) + 10*
c^6)*sin(f*x + e))/(a^3*f*cos(f*x + e)^6 + 3*a^3*f*cos(f*x + e)^5 + 3*a^3*f
*cos(f*x + e)^4 + a^3*f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**6/(a+a*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33142, size = 250, normalized size = 1.16

$$-\frac{\frac{3465\,c^6\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{3465\,c^6\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}+\frac{10\left(267\,c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-472\,c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+213\,c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3a^3}-\frac{32\left(3\right.}{}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/30*(3465*c^6*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 3465*c^6*log(abs(t
an(1/2*f*x + 1/2*e) - 1))/a^3 + 10*(267*c^6*tan(1/2*f*x + 1/2*e)^5 - 472*c^
6*tan(1/2*f*x + 1/2*e)^3 + 213*c^6*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/
2*e)^2 - 1)^3*a^3) - 32*(3*a^12*c^6*tan(1/2*f*x + 1/2*e)^5 + 20*a^12*c^6*ta
n(1/2*f*x + 1/2*e)^3 + 150*a^12*c^6*tan(1/2*f*x + 1/2*e))/a^15)/f
```

3.53 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=193

$$\frac{42c^5\tan(e+fx)}{a^3f}-\frac{63c^5\tanh^{-1}(\sin(e+fx))}{2a^3f}-\frac{21c^5\tan(e+fx)\sec(e+fx)}{2a^3f}+\frac{42c\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)^2}{5f\left(a^3\sec(e+fx)+a^3\right)}$$

```
[Out] (-63*c^5*ArcTanh[Sin[e + f*x]])/(2*a^3*f) + (42*c^5*Tan[e + f*x])/(a^3*f) -
 (21*c^5*Sec[e + f*x]*Tan[e + f*x])/(2*a^3*f) - (6*c^2*(c - c*Sec[e + f*x])
^3*Tan[e + f*x])/(5*a*f*(a + a*Sec[e + f*x])^2) + (2*c*(c - c*Sec[e + f*x])
^4*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (42*c*(c^2 - c^2*Sec[e + f*
x])^2*Tan[e + f*x])/(5*f*(a^3 + a^3*Sec[e + f*x]))
```

Rubi [A] time = 0.309145, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3957, 3788, 3767, 8, 4046, 3770}

$$\frac{42c^5\tan(e+fx)}{a^3f}-\frac{63c^5\tanh^{-1}(\sin(e+fx))}{2a^3f}-\frac{21c^5\tan(e+fx)\sec(e+fx)}{2a^3f}+\frac{42c\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)^2}{5f\left(a^3\sec(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```


```
[Out] (-63*c^5*ArcTanh[Sin[e + f*x]])/(2*a^3*f) + (42*c^5*Tan[e + f*x])/(a^3*f) -
 (21*c^5*Sec[e + f*x]*Tan[e + f*x])/(2*a^3*f) - (6*c^2*(c - c*Sec[e + f*x])
^3*Tan[e + f*x])/(5*a*f*(a + a*Sec[e + f*x])^2) + (2*c*(c - c*Sec[e + f*x])
^4*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (42*c*(c^2 - c^2*Sec[e + f*
x])^2*Tan[e + f*x])/(5*f*(a^3 + a^3*Sec[e + f*x]))
```


Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(9c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx}{5a}$$

$$= -\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{5af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(21c}{}$$

$$= \frac{42c^3(c-c\sec(e+fx))^2\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)} - \frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{5af(a+a\sec(e+fx))^2} + \frac{2c(}{}$$

$$= \frac{42c^3(c-c\sec(e+fx))^2\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)} - \frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{5af(a+a\sec(e+fx))^2} + \frac{2c(}{}$$

$$= -\frac{21c^5\sec(e+fx)\tan(e+fx)}{2a^3f} + \frac{42c^3(c-c\sec(e+fx))^2\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)} - \frac{6c^2(c-}{5a}$$

$$= -\frac{63c^5\tanh^{-1}(\sin(e+fx))}{2a^3f} + \frac{42c^5\tan(e+fx)}{a^3f} - \frac{21c^5\sec(e+fx)\tan(e+fx)}{2a^3f} +$$

Mathematica [A] time = 1.37743, size = 380, normalized size = 1.97

$$\frac{\cot\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)(c-c\sec(e+fx))^5\left(\sec\left(\frac{e}{2}\right)\sec(e)\left(7351\sin\left(e-\frac{fx}{2}\right)-5271\sin\left(e+\frac{fx}{2}\right)+5545\sin\left(2e\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4*(c - c*Sec[e + f*x])^5*(-40320*Cos[e +
 f*x]^2*Cot[(e + f*x)/2]^5*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + Csc[(e + f*x)/2]^5*Sec[e/2]*Sec[e]*
(3465*Sin[(f*x)/2] - 6115*Sin[(3*f*x)/2] + 7351*Sin[e - (f*x)/2] - 5271*Sin
[e + (f*x)/2] + 5545*Sin[2*e + (f*x)/2] + 2205*Sin[e + (3*f*x)/2] - 4515*Si
n[2*e + (3*f*x)/2] + 3805*Sin[3*e + (3*f*x)/2] - 4407*Sin[e + (5*f*x)/2] +
585*Sin[2*e + (5*f*x)/2] - 3447*Sin[3*e + (5*f*x)/2] + 1545*Sin[4*e + (5*f*
x)/2] - 2155*Sin[2*e + (7*f*x)/2] - 75*Sin[3*e + (7*f*x)/2] - 1755*Sin[4*e
+ (7*f*x)/2] + 325*Sin[5*e + (7*f*x)/2] - 496*Sin[3*e + (9*f*x)/2] - 80*Sin
[4*e + (9*f*x)/2] - 416*Sin[5*e + (9*f*x)/2])))/(5120*a^3*f*(1 + Sec[e + f*
x])^3)
```

Maple [A] time = 0.103, size = 208, normalized size = 1.1

$$\frac{8c^5}{5fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5+8\frac{c^5\left(\tan\left(1/2fx+e/2\right)\right)^3}{fa^3}+48\frac{c^5\tan\left(1/2fx+e/2\right)}{fa^3}+\frac{c^5}{2fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-2}-\frac{17c^5}{2fa^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 8/5/f*c^5/a^3*tan(1/2*f*x+1/2*e)^5+8/f*c^5/a^3*tan(1/2*f*x+1/2*e)^3+48/f*c^
5/a^3*tan(1/2*f*x+1/2*e)+1/2/f*c^5/a^3/(tan(1/2*f*x+1/2*e)+1)^2-17/2/f*c^5/
a^3/(tan(1/2*f*x+1/2*e)+1)-63/2/f*c^5/a^3*ln(tan(1/2*f*x+1/2*e)+1)-1/2/f*c^
5/a^3/(tan(1/2*f*x+1/2*e)-1)^2-17/2/f*c^5/a^3/(tan(1/2*f*x+1/2*e)-1)+63/2/f
*c^5/a^3*ln(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.06249, size = 918, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^5*(60*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/(a^3 - 2*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 40
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5)/a^3 - 390*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*x
 + e)/(cos(f*x + e) + 1) - 1)/a^3) + 15*c^5*(40*sin(f*x + e)/((a^3 - a^3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(
cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5
/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^
3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 10*c^5*((105*sin(f*x
 + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 10*c^5*(15*
sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^5*(15*sin(f*x + e)/(cos(f*x
+ e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5)/a^3 - 15*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A] time = 0.503679, size = 620, normalized size = 3.21

$$-\frac{315\left(c^5\cos\left(fx+e\right)^5+3c^5\cos\left(fx+e\right)^4+3c^5\cos\left(fx+e\right)^3+c^5\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-315\left(c^5\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/20*(315*(c^5*cos(f*x + e)^5 + 3*c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^
3 + c^5*cos(f*x + e)^2)*log(sin(f*x + e) + 1) - 315*(c^5*cos(f*x + e)^5 + 3
*c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + c^5*cos(f*x + e)^2)*log(-sin(f
*x + e) + 1) - 2*(496*c^5*cos(f*x + e)^4 + 1163*c^5*cos(f*x + e)^3 + 801*c^
5*cos(f*x + e)^2 + 65*c^5*cos(f*x + e) - 5*c^5)*sin(f*x + e))/(a^3*f*cos(f*
x + e)^5 + 3*a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + a^3*f*cos(f*x
+ e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^5\left(\int -\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{5\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int -\frac{10\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c**5*(Integral(-sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x) + Integral(5*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
 f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-10*sec(e + f*x)**3/(sec(e +
f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(10*sec(e +
 f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + I
ntegral(-5*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
 f*x) + 1), x) + Integral(sec(e + f*x)**6/(sec(e + f*x)**3 + 3*sec(e + f*x)
**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A] time = 1.35371, size = 225, normalized size = 1.17

$$-\frac{\frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}+\frac{10\left(17\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2a^3}-\frac{16\left(a^{12}c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+5\,a^{12}c^{\cdots}\right.}{\cdots}}{10\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/10*(315*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 315*c^5*log(abs(tan
(1/2*f*x + 1/2*e) - 1))/a^3 + 10*(17*c^5*tan(1/2*f*x + 1/2*e)^3 - 15*c^5*ta
n(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a^3) - 16*(a^12*c^5*tan
(1/2*f*x + 1/2*e)^5 + 5*a^12*c^5*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c^5*tan(1
/2*f*x + 1/2*e))/a^15)/f
```

3.54 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=164

$$\frac{7c^4\tan(e+fx)}{a^3f}-\frac{7c^4\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{14\tan(e+fx)\left(c^4-c^4\sec(e+fx)\right)}{3f\left(a^3\sec(e+fx)+a^3\right)}-\frac{14\tan(e+fx)\left(c^2-c^2\sec(e+f\right.}{15af(a\sec(e+fx)+a)^2}$$

```
[Out] (-7*c^4*ArcTanh[Sin[e + f*x]])/(a^3*f) + (7*c^4*Tan[e + f*x])/(a^3*f) + (2*
c*(c - c*Sec[e + f*x])^3*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) - (14*(
c^2 - c^2*Sec[e + f*x])^2*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + (
14*(c^4 - c^4*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a^3 + a^3*Sec[e + f*x]))
```

Rubi [A] time = 0.275221, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3957, 3787, 3770, 3767, 8}

$$\frac{7c^4\tan(e+fx)}{a^3f}-\frac{7c^4\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{14\tan(e+fx)\left(c^4-c^4\sec(e+fx)\right)}{3f\left(a^3\sec(e+fx)+a^3\right)}-\frac{14\tan(e+fx)\left(c^2-c^2\sec(e+f\right.}{15af(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-7*c^4*ArcTanh[Sin[e + f*x]])/(a^3*f) + (7*c^4*Tan[e + f*x])/(a^3*f) + (2*
c*(c - c*Sec[e + f*x])^3*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) - (14*(
c^2 - c^2*Sec[e + f*x])^2*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + (
14*(c^4 - c^4*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a^3 + a^3*Sec[e + f*x]))
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(7c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx}{5a}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(7c^2}{}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{14\left(\right.}{}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{14\left(\right.}{}$$

$$= -\frac{7c^4\tanh^{-1}(\sin(e+fx))}{a^3f} + \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14\left(c^2-c^2\sec(\right.}{15af(a+}$$

$$= -\frac{7c^4\tanh^{-1}(\sin(e+fx))}{a^3f} + \frac{7c^4\tan(e+fx)}{a^3f} + \frac{2c(c-c\sec(e+fx))^3\tan(e+fx}{5f(a+a\sec(e+fx))^3}$$

Mathematica [B] time = 6.31824, size = 826, normalized size = 5.04

$$\frac{2\cos(e+fx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)\sec\left(\frac{e}{2}\right)(c-c\sec(e+fx))^4\sin\left(\frac{fx}{2}\right)\csc^7\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f(\sec(e+fx)a+a)^3} + \frac{2\cos(e+fx)\cot^2\left(\frac{e}{2}+\frac{fx}{2}\right)(c-c\sec(e+}{5f(\sec(e+fx)a-}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (7*Cos[e + f*x]*Cot[e/2 + (f*x)/2]^6*Csc[e/2 + (f*x)/2]^2*Log[Cos[e/2 + (f*
x)/2] - Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^4)/(2*f*(a + a*Sec[e + f*x
])^3) - (7*Cos[e + f*x]*Cot[e/2 + (f*x)/2]^6*Csc[e/2 + (f*x)/2]^2*Log[Cos[e
/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^4)/(2*f*(a + a*Sec
[e + f*x])^3) + (76*Cos[e + f*x]*Cot[e/2 + (f*x)/2]^5*Csc[e/2 + (f*x)/2]^3*
Sec[e/2]*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(15*f*(a + a*Sec[e + f*x])^3)
 + (8*Cos[e + f*x]*Cot[e/2 + (f*x)/2]^3*Csc[e/2 + (f*x)/2]^5*Sec[e/2]*(c -
c*Sec[e + f*x])^4*Sin[(f*x)/2])/(15*f*(a + a*Sec[e + f*x])^3) + (2*Cos[e +
f*x]*Cot[e/2 + (f*x)/2]*Csc[e/2 + (f*x)/2]^7*Sec[e/2]*(c - c*Sec[e + f*x])^
4*Sin[(f*x)/2])/(5*f*(a + a*Sec[e + f*x])^3) + (Cos[e + f*x]*Cot[e/2 + (f*x
)/2]^6*Csc[e/2 + (f*x)/2]^2*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(2*f*(a +
a*Sec[e + f*x])^3*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*
x)/2])) + (Cos[e + f*x]*Cot[e/2 + (f*x)/2]^6*Csc[e/2 + (f*x)/2]^2*(c - c*Se
c[e + f*x])^4*Sin[(f*x)/2])/(2*f*(a + a*Sec[e + f*x])^3*(Cos[e/2] + Sin[e/2
])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])) + (8*Cos[e + f*x]*Cot[e/2 + (
f*x)/2]^4*Csc[e/2 + (f*x)/2]^4*(c - c*Sec[e + f*x])^4*Tan[e/2])/(15*f*(a +
a*Sec[e + f*x])^3) + (2*Cos[e + f*x]*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2
]^6*(c - c*Sec[e + f*x])^4*Tan[e/2])/(5*f*(a + a*Sec[e + f*x])^3)
```

Maple [A] time = 0.085, size = 160, normalized size = 1.

$$\frac{4c^4}{5fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5+\frac{8c^4}{3fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+12\frac{c^4\tan\left(1/2fx+e/2\right)}{fa^3}-\frac{c^4}{fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1}-7\frac{c^4\ln\left(t\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 4/5/f*c^4/a^3*tan(1/2*f*x+1/2*e)^5+8/3/f*c^4/a^3*tan(1/2*f*x+1/2*e)^3+12/f*
c^4/a^3*tan(1/2*f*x+1/2*e)-1/f*c^4/a^3/(tan(1/2*f*x+1/2*e)+1)-7/f*c^4/a^3*l
n(tan(1/2*f*x+1/2*e)+1)-1/f*c^4/a^3/(tan(1/2*f*x+1/2*e)-1)+7/f*c^4/a^3*ln(t
an(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.0405, size = 635, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*c^4*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a^3) + 4*c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a
^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)/a^3) + 6*c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) +
10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5)/a^3 + c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 12*c^4*(5*s
in(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/
f
```

Fricas [A] time = 0.489546, size = 579, normalized size = 3.53

$$-\frac{105\left(c^4\cos\left(fx+e\right)^4+3c^4\cos\left(fx+e\right)^3+3c^4\cos\left(fx+e\right)^2+c^4\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-105\left(c^4\cos\right.}{30\left(a^3f\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(105*(c^4*cos(f*x + e)^4 + 3*c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^
2 + c^4*cos(f*x + e))*log(sin(f*x + e) + 1) - 105*(c^4*cos(f*x + e)^4 + 3*c
^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e))*log(-sin(f*x +
 e) + 1) - 2*(167*c^4*cos(f*x + e)^3 + 381*c^4*cos(f*x + e)^2 + 277*c^4*cos
(f*x + e) + 15*c^4)*sin(f*x + e))/(a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x +
```

```
e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^4\left(\int\frac{\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int-\frac{4\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int\frac{6\sec^3(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**4*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(-4*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(6*sec(e + f*x)**3/(sec(e + f*x
)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-4*sec(e + f*
x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Inte
gral(sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x)
+ 1), x))/a**3
```

Giac [A] time = 1.37334, size = 200, normalized size = 1.22

$$-\frac{\frac{105c^4\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^3}-\frac{105c^4\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^3}+\frac{30c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a^3}-\frac{4\left(3a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+10a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+45a^{1}\right.}{a^{15}}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/15*(105*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 105*c^4*log(abs(tan
(1/2*f*x + 1/2*e) - 1))/a^3 + 30*c^4*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1
/2*e)^2 - 1)*a^3) - 4*(3*a^12*c^4*tan(1/2*f*x + 1/2*e)^5 + 10*a^12*c^4*tan(
1/2*f*x + 1/2*e)^3 + 45*a^12*c^4*tan(1/2*f*x + 1/2*e))/a^15)/f
```

3.55 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=131

$$-\frac{c^3\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{2c^3\tan(e+fx)}{f\left(a^3\sec(e+fx)+a^3\right)}-\frac{2\tan(e+fx)\left(c^3-c^3\sec(e+fx)\right)}{3af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c-c\sec(e}{5f(a\sec(e+fx)+a}$$

```
[Out] -((c^3*ArcTanh[Sin[e + f*x]])/(a^3*f)) + (2*c^3*Tan[e + f*x])/(f*(a^3 + a^3
*Sec[e + f*x])) + (2*c*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec
[e + f*x])^3) - (2*(c^3 - c^3*Sec[e + f*x])*Tan[e + f*x])/(3*a*f*(a + a*Sec
[e + f*x])^2)
```

Rubi [A] time = 0.214818, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3957, 3770}

$$-\frac{c^3\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{2c^3\tan(e+fx)}{f\left(a^3\sec(e+fx)+a^3\right)}-\frac{2\tan(e+fx)\left(c^3-c^3\sec(e+fx)\right)}{3af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c-c\sec(e}{5f(a\sec(e+fx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -((c^3*ArcTanh[Sin[e + f*x]])/(a^3*f)) + (2*c^3*Tan[e + f*x])/(f*(a^3 + a^3
*Sec[e + f*x])) + (2*c*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec
[e + f*x])^3) - (2*(c^3 - c^3*Sec[e + f*x])*Tan[e + f*x])/(3*a*f*(a + a*Sec
[e + f*x])^2)
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}dx}{a}$$

$$= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{2\left(c^3-c^3\sec(e+fx)\right)\tan(e+fx)}{3af(a+a\sec(e+fx))^2} + \frac{c^2\int \frac{\text{se}}{}}{}$$

$$= \frac{2c^3\tan(e+fx)}{f\left(a^3+a^3\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{2\left(c^3-c^3\sec(e+\right.}{3af(a+a\,\text{se}}$$

$$= -\frac{c^3\tanh^{-1}(\sin(e+fx))}{a^3f} + \frac{2c^3\tan(e+fx)}{f\left(a^3+a^3\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^2\tan(}{5f(a+a\sec(e+fx))}$$

Mathematica [A] time = 0.119014, size = 139, normalized size = 1.06

$$-\frac{c^3\left(-\frac{26\tan\left(\frac{1}{2}(e+fx)\right)}{15f} - \frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sec^4\left(\frac{1}{2}(e+fx)\right)}{5f} + \frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)}{15f} - \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\right.}{f}\right.}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -((c^3*(-(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f) + Log[Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]]/f - (26*Tan[(e + f*x)/2])/(15*f) + (2*Sec[(e + f*x)/2
]^2*Tan[(e + f*x)/2])/(15*f) - (2*Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2])/(5*f
)))/a^3)
```

Maple [A] time = 0.084, size = 111, normalized size = 0.9

$$\frac{2c^3}{5fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5 + \frac{2c^3}{3fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3 + 2\frac{c^3\tan\left(1/2\,fx+e/2\right)}{fa^3} + \frac{c^3}{fa^3}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right) - \frac{c^3}{fa^3}\ln\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 2/5/f*c^3/a^3*tan(1/2*f*x+1/2*e)^5+2/3/f*c^3/a^3*tan(1/2*f*x+1/2*e)^3+2/f*c
^3/a^3*tan(1/2*f*x+1/2*e)+1/f*c^3/a^3*ln(tan(1/2*f*x+1/2*e)-1)-1/f*c^3/a^3*
ln(tan(1/2*f*x+1/2*e)+1)
```

Maxima [B] time = 1.02398, size = 410, normalized size = 3.13

$$\frac{c^3\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3} - \frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3} + \frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right) + \frac{3c^3\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)}\right.}{a^3}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x
 + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1)
- 1)/a^3) + 3*c^3*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^3*(15*
sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 9*c^3*(5*sin(f*x + e)/(cos(f*x
 + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A] time = 0.485504, size = 479, normalized size = 3.66

$$-\frac{15\left(c^3\cos\left(fx+e\right)^3+3\,c^3\cos\left(fx+e\right)^2+3\,c^3\cos\left(fx+e\right)+c^3\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(c^3\cos\left(fx+e\right)^3+3\,c^3\right.}{30\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x + e) +
 c^3)*log(sin(f*x + e) + 1) - 15*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2
 + 3*c^3*cos(f*x + e) + c^3)*log(-sin(f*x + e) + 1) - 4*(13*c^3*cos(f*x + e
)^2 + 24*c^3*cos(f*x + e) + 23*c^3)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3
*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c^3\left(\int -\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int -\frac{3\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\,\mathrm{s}\right.}\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c**3*(Integral(-sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
 f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**3/(sec(e + f
*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x
)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A] time = 1.26328, size = 154, normalized size = 1.18

$$-\frac{\frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}-\frac{2\left(3\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+5\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/15*(15*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 15*c^3*log(abs(tan(1
/2*f*x + 1/2*e) - 1))/a^3 - 2*(3*a^12*c^3*tan(1/2*f*x + 1/2*e)^5 + 5*a^12*c
^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^3*tan(1/2*f*x + 1/2*e))/a^15)/f
```

3.56 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=38

$$\frac{\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}$$

```
[Out] ((c - c*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3)
```

Rubi [A] time = 0.0737364, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3950}

$$\frac{\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]

[Out] ((c - c*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3)
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

Mathematica [A] time = 0.113791, size = 25, normalized size = 0.66

$$\frac{c^2\tan^5\left(\frac{1}{2}(e+fx)\right)}{5a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]

[Out] (c^2*Tan[(e + f*x)/2]^5)/(5*a^3*f)
```

Maple [A] time = 0.082, size = 23, normalized size = 0.6

$$\frac{c^2}{5fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 1/5/f*c^2/a^3*tan(1/2*f*x+1/2*e)^5
```

Maxima [B] time = 1.00711, size = 250, normalized size = 6.58

$$\frac{\frac{c^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{c^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{6c^2\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^2*(15*sin(f*x
+ e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 6*c^2*(5*sin(f*x + e)/(cos(f*x + e) +
1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [B] time = 0.441037, size = 196, normalized size = 5.16

$$\frac{\left(c^2\cos\left(fx+e\right)^2-2\,c^2\cos\left(fx+e\right)+c^2\right)\sin\left(fx+e\right)}{5\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/5*(c^2*cos(f*x + e)^2 - 2*c^2*cos(f*x + e) + c^2)*sin(f*x + e)/(a^3*f*cos
(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2\left(\int\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int-\frac{2\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\frac{\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+f\right.}\right.}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**2*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(-2*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)*
```

```
*3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A] time = 1.35964, size = 31, normalized size = 0.82

$$\frac{c^2 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^5}{5 a^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/5*c^2*tan(1/2*f*x + 1/2*e)^5/(a^3*f)
```

3.57 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=76

$$\frac{\tan(e+fx)(c-c\sec(e+fx))}{15af(a\sec(e+fx)+a)^2}+\frac{\tan(e+fx)(c-c\sec(e+fx))}{5f(a\sec(e+fx)+a)^3}$$

```
[Out] ((c - c*Sec[e + f*x])*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((c - c*
Sec[e + f*x])*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2)
```

Rubi [A] time = 0.0987711, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3951, 3950}

$$\frac{\tan(e+fx)(c-c\sec(e+fx))}{15af(a\sec(e+fx)+a)^2}+\frac{\tan(e+fx)(c-c\sec(e+fx))}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((c - c*Sec[e + f*x])*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((c - c*
Sec[e + f*x])*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2)
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx &= \frac{(c-c\sec(e+fx))\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx}{5a}\\ &= \frac{(c-c\sec(e+fx))\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{(c-c\sec(e+fx))\tan(e+fx)}{15af(a+a\sec(e+fx))^2}\end{aligned}$$

Mathematica [A] time = 0.328775, size = 87, normalized size = 1.14

$$\frac{c\sec\left(\frac{e}{2}\right)\left(-15\sin\left(e+\frac{fx}{2}\right)+5\sin\left(e+\frac{3fx}{2}\right)-15\sin\left(2e+\frac{3fx}{2}\right)+4\sin\left(2e+\frac{5fx}{2}\right)+25\sin\left(\frac{fx}{2}\right)\right)\sec^5\left(\frac{1}{2}(e+fx)\right)}{240a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c*Sec[e/2]*Sec[(e + f*x)/2]^5*(25*Sin[(f*x)/2] - 15*Sin[e + (f*x)/2] + 5*S
in[e + (3*f*x)/2] - 15*Sin[2*e + (3*f*x)/2] + 4*Sin[2*e + (5*f*x)/2]))/(240
*a^3*f)
```

Maple [A] time = 0.069, size = 37, normalized size = 0.5

$$\frac{c}{2fa^3}\left(\frac{1}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5-\frac{1}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 1/2/f*c/a^3*(1/5*tan(1/2*f*x+1/2*e)^5-1/3*tan(1/2*f*x+1/2*e)^3)
```

Maxima [A] time = 1.0071, size = 155, normalized size = 2.04

$$\frac{\frac{c\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{3c\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/60*(c*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 3*c*(5*sin(f*x + e
)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A] time = 0.438331, size = 192, normalized size = 2.53

$$\frac{\left(4c\cos\left(fx+e\right)^2-3c\cos\left(fx+e\right)-c\right)\sin\left(fx+e\right)}{15\left(a^3f\cos\left(fx+e\right)^3+3a^3f\cos\left(fx+e\right)^2+3a^3f\cos\left(fx+e\right)+a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*(4*c*cos(f*x + e)^2 - 3*c*cos(f*x + e) - c)*sin(f*x + e)/(a^3*f*cos(f*
x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c\left(\int -\frac{\sec \left(e+fx\right)}{\sec^3 \left(e+fx\right)+3\sec^2 \left(e+fx\right)+3\sec \left(e+fx\right)+1}\,dx+\int \frac{\sec^2 \left(e+fx\right)}{\sec^3 \left(e+fx\right)+3\sec^2 \left(e+fx\right)+3\sec \left(e+fx\right)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c*(Integral(-sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
 f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)
**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A] time = 1.27811, size = 53, normalized size = 0.7

$$\frac{3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-5\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}{30\,a^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/30*(3*c*tan(1/2*f*x + 1/2*e)^5 - 5*c*tan(1/2*f*x + 1/2*e)^3)/(a^3*f)
```

3.58 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=78

$$-\frac{2\cot^5(e+fx)}{5a^3cf}+\frac{2\csc^5(e+fx)}{5a^3cf}-\frac{\csc^3(e+fx)}{a^3cf}+\frac{\csc(e+fx)}{a^3cf}$$

```
[Out] (-2*Cot[e + f*x]^5)/(5*a^3*c*f) + Csc[e + f*x]/(a^3*c*f) - Csc[e + f*x]^3/(
a^3*c*f) + (2*Csc[e + f*x]^5)/(5*a^3*c*f)
```

Rubi [A] time = 0.180993, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3958, 2606, 194, 2607, 30, 14}

$$-\frac{2\cot^5(e+fx)}{5a^3cf}+\frac{2\csc^5(e+fx)}{5a^3cf}-\frac{\csc^3(e+fx)}{a^3cf}+\frac{\csc(e+fx)}{a^3cf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

```
[Out] (-2*Cot[e + f*x]^5)/(5*a^3*c*f) + Csc[e + f*x]/(a^3*c*f) - Csc[e + f*x]^3/(
a^3*c*f) + (2*Csc[e + f*x]^5)/(5*a^3*c*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{\int \left(c^2\cot^5(e+fx)\csc(e+fx)-2c^2\cot^4(e+fx)\csc^2(e+fx)+c^2\cot^3(\right.}{a^3c^3}$$

$$= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^3c} - \frac{\int \cot^3(e+fx)\csc^3(e+fx)\,dx}{a^3c} + \frac{2\int \mathrm{c}}{}$$

$$= \frac{\operatorname{Subst}\left(\int x^2\left(-1+x^2\right)dx, x, \csc(e+fx)\right)}{a^3cf} + \frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)^2 dx, x, \mathrm{c}\right.}{a^3cf}$$

$$= -\frac{2\cot^5(e+fx)}{5a^3cf} + \frac{\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)dx, x, \csc(e+fx)\right)}{a^3cf} + \frac{\operatorname{Subst}\left(\right.}{}$$

$$= -\frac{2\cot^5(e+fx)}{5a^3cf} + \frac{\csc(e+fx)}{a^3cf} - \frac{\csc^3(e+fx)}{a^3cf} + \frac{2\csc^5(e+fx)}{5a^3cf}$$

Mathematica [A] time = 0.773441, size = 109, normalized size = 1.4

$$-\frac{\csc(e)\sin^4\left(\frac{1}{2}(e+fx)\right)(65\sin(e+fx)+52\sin(2(e+fx))+13\sin(3(e+fx))-40\sin(2e+fx)-12\sin(e+2fx)-20}{20a^3cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

```
[Out] -(Csc[e]*Csc[e + f*x]^5*Sin[(e + f*x)/2]^4*(-40*Sin[e] + 65*Sin[e + f*x] +
52*Sin[2*(e + f*x)] + 13*Sin[3*(e + f*x)] - 40*Sin[2*e + f*x] - 12*Sin[e +
2*f*x] - 20*Sin[3*e + 2*f*x] - 8*Sin[2*e + 3*f*x]))/(20*a^3*c*f)
```

Maple [A] time = 0.053, size = 61, normalized size = 0.8

$$\frac{1}{8fa^3c}\left(\frac{1}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5-\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+3\tan\left(1/2\,fx+e/2\right)+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x)
```

```
[Out] 1/8/f/a^3/c*(1/5*tan(1/2*f*x+1/2*e)^5-tan(1/2*f*x+1/2*e)^3+3*tan(1/2*f*x+1/
2*e)+1/tan(1/2*f*x+1/2*e))
```

Maxima [A] time = 0.971455, size = 128, normalized size = 1.64

$$\frac{\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3c}+\frac{5\left(\cos(fx+e)+1\right)}{a^3c\sin(fx+e)}}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/40*((15*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c) + 5*(cos(f*x + e) + 1
)/(a^3*c*sin(f*x + e)))/f
```

Fricas [A] time = 0.443794, size = 186, normalized size = 2.38

$$-\frac{2\cos\left(fx+e\right)^{3}-\cos\left(fx+e\right)^{2}-4\cos\left(fx+e\right)-2}{5\left(a^{3}cf\cos\left(fx+e\right)^{2}+2\,a^{3}cf\cos\left(fx+e\right)+a^{3}cf\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/5*(2*cos(f*x + e)^3 - cos(f*x + e)^2 - 4*cos(f*x + e) - 2)/((a^3*c*f*cos
(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{\sec\left(e+fx\right)}{\sec^{4}\left(e+fx\right)+2\sec^{3}\left(e+fx\right)-2\sec\left(e+fx\right)-1}\,dx}{a^{3}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**4 + 2*sec(e + f*x)**3 - 2*sec(e + f*x
) - 1), x)/(a**3*c)
```

Giac [A] time = 1.28059, size = 123, normalized size = 1.58

$$\frac{\frac{5}{a^{3}c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\frac{a^{12}c^{4}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-5\,a^{12}c^{4}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+15\,a^{12}c^{4}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{15}c^{5}}}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/40*(5/(a^3*c*tan(1/2*f*x + 1/2*e)) + (a^12*c^4*tan(1/2*f*x + 1/2*e)^5 - 5
*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^4*tan(1/2*f*x + 1/2*e))/(a^15*
c^5))/f
```

3.59 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=80

$$-\frac{\cot^5(e+fx)}{5a^3c^2f}+\frac{\csc^5(e+fx)}{5a^3c^2f}-\frac{2\csc^3(e+fx)}{3a^3c^2f}+\frac{\csc(e+fx)}{a^3c^2f}$$

```
[Out] -Cot[e + f*x]^5/(5*a^3*c^2*f) + Csc[e + f*x]/(a^3*c^2*f) - (2*Csc[e + f*x]^
3)/(3*a^3*c^2*f) + Csc[e + f*x]^5/(5*a^3*c^2*f)
```

Rubi [A] time = 0.141616, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3958, 2606, 194, 2607, 30}

$$-\frac{\cot^5(e+fx)}{5a^3c^2f}+\frac{\csc^5(e+fx)}{5a^3c^2f}-\frac{2\csc^3(e+fx)}{3a^3c^2f}+\frac{\csc(e+fx)}{a^3c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] -Cot[e + f*x]^5/(5*a^3*c^2*f) + Csc[e + f*x]/(a^3*c^2*f) - (2*Csc[e + f*x]^
3)/(3*a^3*c^2*f) + Csc[e + f*x]^5/(5*a^3*c^2*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx &= -\frac{\int \left(c\cot^5(e+fx)\csc(e+fx)-c\cot^4(e+fx)\csc^2(e+fx)\right)dx}{a^3c^3}\\ &= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^3c^2}+\frac{\int \cot^4(e+fx)\csc^2(e+fx)\,dx}{a^3c^2}\\ &= \frac{\text{Subst}\left(\int x^4\,dx,x,-\cot(e+fx)\right)}{a^3c^2f}+\frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\csc(e\right.}{a^3c^2f}\\ &= -\frac{\cot^5(e+fx)}{5a^3c^2f}+\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx,x,\csc(e+fx)\right)}{a^3c^2f}\\ &= -\frac{\cot^5(e+fx)}{5a^3c^2f}+\frac{\csc(e+fx)}{a^3c^2f}-\frac{2\csc^3(e+fx)}{3a^3c^2f}+\frac{\csc^5(e+fx)}{5a^3c^2f}\end{aligned}$$

Mathematica [A] time = 0.767076, size = 147, normalized size = 1.84

$$\csc(e)\sin^2\left(\frac{1}{2}(e+fx)\right)(-534\sin(e+fx)-178\sin(2(e+fx))+178\sin(3(e+fx))+89\sin(4(e+fx))+40\sin(2e+$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (Csc[e]*Csc[e + f*x]^5*Sin[(e + f*x)/2]^2*(200*Sin[e] + 104*Sin[f*x] - 534*
Sin[e + f*x] - 178*Sin[2*(e + f*x)] + 178*Sin[3*(e + f*x)] + 89*Sin[4*(e +
f*x)] + 40*Sin[2*e + f*x] + 168*Sin[e + 2*f*x] - 120*Sin[3*e + 2*f*x] + 72*
Sin[2*e + 3*f*x] - 120*Sin[4*e + 3*f*x] - 24*Sin[3*e + 4*f*x]))/(480*a^3*c^
2*f)
```

Maple [A] time = 0.055, size = 76, normalized size = 1.

$$\frac{1}{16fa^3c^2}\left(\frac{1}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5-\frac{4}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+6\tan\left(1/2\,fx+e/2\right)-\frac{1}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}+4\left(\tan\left(1/2\,fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x)
```

```
[Out] 1/16/f/a^3/c^2*(1/5*tan(1/2*f*x+1/2*e)^5-4/3*tan(1/2*f*x+1/2*e)^3+6*tan(1/2
*f*x+1/2*e)-1/3/tan(1/2*f*x+1/2*e)^3+4/tan(1/2*f*x+1/2*e))
```

Maxima [A] time = 1.01112, size = 162, normalized size = 2.02

$$\frac{\frac{\frac{90\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3c^2}+\frac{5\left(\frac{12\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{a^3c^2\sin(fx+e)^3}}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/240*((90*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^2) + 5*(12*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a^3*c^2*sin(f*x + e
)^3))/f
```

Fricas [A] time = 0.447618, size = 261, normalized size = 3.26

$$-\frac{3\cos\left(fx+e\right)^{4}-12\cos\left(fx+e\right)^{3}-12\cos\left(fx+e\right)^{2}+8\cos\left(fx+e\right)+8}{15\left(a^{3}c^{2}f\cos\left(fx+e\right)^{3}+a^{3}c^{2}f\cos\left(fx+e\right)^{2}-a^{3}c^{2}f\cos\left(fx+e\right)-a^{3}c^{2}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] -1/15*(3*cos(f*x + e)^4 - 12*cos(f*x + e)^3 - 12*cos(f*x + e)^2 + 8*cos(f*x
 + e) + 8)/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*
f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec\left(e+fx\right)}{\sec^{5}\left(e+fx\right)+\sec^{4}\left(e+fx\right)-2\sec^{3}\left(e+fx\right)-2\sec^{2}\left(e+fx\right)+\sec\left(e+fx\right)+1}\,dx}{a^{3}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**5 + sec(e + f*x)**4 - 2*sec(e + f*x)**
3 - 2*sec(e + f*x)**2 + sec(e + f*x) + 1), x)/(a**3*c**2)
```

Giac [A] time = 1.28554, size = 146, normalized size = 1.82

$$\frac{\frac{5\left(12\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)}{a^{3}c^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}}+\frac{3a^{12}c^{8}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-20a^{12}c^{8}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+90a^{12}c^{8}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{15}c^{10}}}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/240*(5*(12*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^3*c^2*tan(1/2*f*x + 1/2*e)^3) +
 (3*a^12*c^8*tan(1/2*f*x + 1/2*e)^5 - 20*a^12*c^8*tan(1/2*f*x + 1/2*e)^3 +
90*a^12*c^8*tan(1/2*f*x + 1/2*e))/(a^15*c^10))/f
```

3.60 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=59

$$\frac{\csc^5(e+fx)}{5a^3c^3f}-\frac{2\csc^3(e+fx)}{3a^3c^3f}+\frac{\csc(e+fx)}{a^3c^3f}$$

```
[Out] Csc[e + f*x]/(a^3*c^3*f) - (2*Csc[e + f*x]^3)/(3*a^3*c^3*f) + Csc[e + f*x]^
5/(5*a^3*c^3*f)
```

Rubi [A] time = 0.108673, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3958, 2606, 194}

$$\frac{\csc^5(e+fx)}{5a^3c^3f}-\frac{2\csc^3(e+fx)}{3a^3c^3f}+\frac{\csc(e+fx)}{a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] Csc[e + f*x]/(a^3*c^3*f) - (2*Csc[e + f*x]^3)/(3*a^3*c^3*f) + Csc[e + f*x]^
5/(5*a^3*c^3*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^3c^3}$$
$$= \frac{\text{Subst}\left(\int \left(-1+x^2\right)^2 dx, x, \csc(e+fx)\right)}{a^3c^3f}$$
$$= \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right) dx, x, \csc(e+fx)\right)}{a^3c^3f}$$
$$= \frac{\csc(e+fx)}{a^3c^3f} - \frac{2\csc^3(e+fx)}{3a^3c^3f} + \frac{\csc^5(e+fx)}{5a^3c^3f}$$

Mathematica [A] time = 0.0681945, size = 50, normalized size = 0.85

$$-\frac{-\frac{\csc^5(e+fx)}{5f} + \frac{2\csc^3(e+fx)}{3f} - \frac{\csc(e+fx)}{f}}{a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] -((-(Csc[e + f*x]/f) + (2*Csc[e + f*x]^3)/(3*f) - Csc[e + f*x]^5/(5*f))/(a^
3*c^3))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{\sec(fx+e)}{\left(a+a\sec(fx+e)\right)^3\left(c-c\sec(fx+e)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)
```

```
[Out] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x)
```

Maxima [A] time = 0.960382, size = 55, normalized size = 0.93

$$\frac{15\sin\left(fx+e\right)^4 - 10\sin\left(fx+e\right)^2 + 3}{15\,a^3c^3f\sin\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/15*(15*sin(f*x + e)^4 - 10*sin(f*x + e)^2 + 3)/(a^3*c^3*f*sin(f*x + e)^5)
```

Fricas [A] time = 0.455858, size = 180, normalized size = 3.05

$$\frac{15 \cos\left(fx+e\right)^{4} - 20 \cos\left(fx+e\right)^{2} + 8}{15 \left(a^{3} c^{3} f \cos\left(fx+e\right)^{4} - 2 a^{3} c^{3} f \cos\left(fx+e\right)^{2} + a^{3} c^{3} f\right) \sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(15*cos(f*x + e)^4 - 20*cos(f*x + e)^2 + 8)/((a^3*c^3*f*cos(f*x + e)^4
 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sec\left(e+fx\right)}{\sec^{6}\left(e+fx\right)-3\sec^{4}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)-1}\, dx}{a^{3} c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**6 - 3*sec(e + f*x)**4 + 3*sec(e + f*x
)**2 - 1), x)/(a**3*c**3)
```

Giac [A] time = 1.2356, size = 59, normalized size = 1.

$$\frac{15 \sin\left(fx+e\right)^{4} - 10 \sin\left(fx+e\right)^{2} + 3}{15 a^{3} c^{3} f \sin\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/15*(15*sin(f*x + e)^4 - 10*sin(f*x + e)^2 + 3)/(a^3*c^3*f*sin(f*x + e)^5)
```

3.61 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=99

$$-\frac{\cot^7(e+fx)}{7a^3c^4f}-\frac{\csc^7(e+fx)}{7a^3c^4f}+\frac{3\csc^5(e+fx)}{5a^3c^4f}-\frac{\csc^3(e+fx)}{a^3c^4f}+\frac{\csc(e+fx)}{a^3c^4f}$$

```
[Out] -Cot[e + f*x]^7/(7*a^3*c^4*f) + Csc[e + f*x]/(a^3*c^4*f) - Csc[e + f*x]^3/(
a^3*c^4*f) + (3*Csc[e + f*x]^5)/(5*a^3*c^4*f) - Csc[e + f*x]^7/(7*a^3*c^4*f
)
```

Rubi [A] time = 0.148994, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3958, 2606, 194, 2607, 30}

$$-\frac{\cot^7(e+fx)}{7a^3c^4f}-\frac{\csc^7(e+fx)}{7a^3c^4f}+\frac{3\csc^5(e+fx)}{5a^3c^4f}-\frac{\csc^3(e+fx)}{a^3c^4f}+\frac{\csc(e+fx)}{a^3c^4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] -Cot[e + f*x]^7/(7*a^3*c^4*f) + Csc[e + f*x]/(a^3*c^4*f) - Csc[e + f*x]^3/(
a^3*c^4*f) + (3*Csc[e + f*x]^5)/(5*a^3*c^4*f) - Csc[e + f*x]^7/(7*a^3*c^4*f
)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\int \left(a\cot^7(e+fx)\csc(e+fx)+a\cot^6(e+fx)\csc^2(e+fx)\right)dx}{a^4c^4}$$
$$= \frac{\int \cot^7(e+fx)\csc(e+fx)\,dx}{a^3c^4} + \frac{\int \cot^6(e+fx)\csc^2(e+fx)\,dx}{a^3c^4}$$
$$= \frac{\text{Subst}\left(\int x^6\,dx, x, -\cot(e+fx)\right)}{a^3c^4f} - \frac{\text{Subst}\left(\int \left(-1+x^2\right)^3 dx, x, \csc(e\right.}{a^3c^4f}$$
$$= -\frac{\cot^7(e+fx)}{7a^3c^4f} - \frac{\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)dx, x, \csc(e+fx)\right)}{a^3c^4f}$$
$$= -\frac{\cot^7(e+fx)}{7a^3c^4f} + \frac{\csc(e+fx)}{a^3c^4f} - \frac{\csc^3(e+fx)}{a^3c^4f} + \frac{3\csc^5(e+fx)}{5a^3c^4f} - \frac{\csc^7}{7}$$

Mathematica [B] time = 1.25931, size = 211, normalized size = 2.13

$$\frac{\csc(e)(-7620\sin(e+fx)+1905\sin(2(e+fx))+3810\sin(3(e+fx))-1524\sin(4(e+fx))-762\sin(5(e+fx))+38}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (Csc[e]*Csc[(e + f*x)/2]^2*Csc[e + f*x]^5*(2912*Sin[e] + 416*Sin[f*x] - 762
0*Sin[e + f*x] + 1905*Sin[2*(e + f*x)] + 3810*Sin[3*(e + f*x)] - 1524*Sin[4
*(e + f*x)] - 762*Sin[5*(e + f*x)] + 381*Sin[6*(e + f*x)] - 2016*Sin[2*e +
f*x] + 2080*Sin[e + 2*f*x] - 1680*Sin[3*e + 2*f*x] + 240*Sin[2*e + 3*f*x] +
 560*Sin[4*e + 3*f*x] - 880*Sin[3*e + 4*f*x] + 560*Sin[5*e + 4*f*x] + 400*S
in[4*e + 5*f*x] - 560*Sin[6*e + 5*f*x] + 80*Sin[5*e + 6*f*x]))/(35840*a^3*c
^4*f)
```

Maple [A] time = 0.063, size = 102, normalized size = 1.

$$\frac{1}{64\,fa^3c^4}\left(\frac{1}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5 - 2\left(\tan\left(1/2\,fx+e/2\right)\right)^3 + 15\tan\left(1/2\,fx+e/2\right) - 5\left(\tan\left(1/2\,fx+e/2\right)\right)^{-3} + 20\left(\tan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x)
```

```
[Out] 1/64/f/a^3/c^4*(1/5*tan(1/2*f*x+1/2*e)^5-2*tan(1/2*f*x+1/2*e)^3+15*tan(1/2*
f*x+1/2*e)-5/tan(1/2*f*x+1/2*e)^3+20/tan(1/2*f*x+1/2*e)+6/5/tan(1/2*f*x+1/2
*e)^5-1/7/tan(1/2*f*x+1/2*e)^7)
```

Maxima [A] time = 0.989402, size = 215, normalized size = 2.17

$$\frac{\frac{7\left(\frac{75\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{10\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)}{a^3c^4}+\frac{\left(\frac{42\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{175\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{700\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}-5\right)\left(\cos\left(fx+e\right)+1\right)^7}{a^3c^4\sin\left(fx+e\right)^7}}{2240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/2240*(7*(75*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^4) + (42*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 700
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(a^3*c^4*sin
(f*x + e)^7))/f
```

Fricas [A] time = 0.478234, size = 394, normalized size = 3.98

$$\frac{5\cos\left(fx+e\right)^6+30\cos\left(fx+e\right)^5-30\cos\left(fx+e\right)^4-40\cos\left(fx+e\right)^3+40\cos\left(fx+e\right)^2+16\cos\left(fx+e\right)}{35\left(a^3c^4f\cos\left(fx+e\right)^5-a^3c^4f\cos\left(fx+e\right)^4-2\,a^3c^4f\cos\left(fx+e\right)^3+2\,a^3c^4f\cos\left(fx+e\right)^2+a^3c^4f\cos\left(fx+e\right)-a^3c^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/35*(5*cos(f*x + e)^6 + 30*cos(f*x + e)^5 - 30*cos(f*x + e)^4 - 40*cos(f*x
 + e)^3 + 40*cos(f*x + e)^2 + 16*cos(f*x + e) - 16)/((a^3*c^4*f*cos(f*x + e
)^5 - a^3*c^4*f*cos(f*x + e)^4 - 2*a^3*c^4*f*cos(f*x + e)^3 + 2*a^3*c^4*f*c
os(f*x + e)^2 + a^3*c^4*f*cos(f*x + e) - a^3*c^4*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec\left(e+fx\right)}{\sec^7\left(e+fx\right)-\sec^6\left(e+fx\right)-3\sec^5\left(e+fx\right)+3\sec^4\left(e+fx\right)+3\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)-\sec\left(e+fx\right)+1}\,dx}{a^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**7 - sec(e + f*x)**6 - 3*sec(e + f*x)**
5 + 3*sec(e + f*x)**4 + 3*sec(e + f*x)**3 - 3*sec(e + f*x)**2 - sec(e + f*x
) + 1), x)/(a**3*c**4)
```

Giac [A] time = 1.36433, size = 182, normalized size = 1.84

$$\frac{\frac{700\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-175\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+42\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-5}{a^3c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7}+\frac{7\left(a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-10\,a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+75\,a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{15}c^{20}}}{2240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/2240*((700*tan(1/2*f*x + 1/2*e)^6 - 175*tan(1/2*f*x + 1/2*e)^4 + 42*tan(1
/2*f*x + 1/2*e)^2 - 5)/(a^3*c^4*tan(1/2*f*x + 1/2*e)^7) + 7*(a^12*c^16*tan(
1/2*f*x + 1/2*e)^5 - 10*a^12*c^16*tan(1/2*f*x + 1/2*e)^3 + 75*a^12*c^16*tan
(1/2*f*x + 1/2*e))/(a^15*c^20))/f
```

3.62 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=120

$$\frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}-\frac{\csc^7(e+fx)}{a^3c^5f}+\frac{9\csc^5(e+fx)}{5a^3c^5f}-\frac{5\csc^3(e+fx)}{3a^3c^5f}+\frac{\csc(e+fx)}{a^3c^5f}$$

```
[Out] (2*Cot[e + f*x]^9)/(9*a^3*c^5*f) + Csc[e + f*x]/(a^3*c^5*f) - (5*Csc[e + f*
x]^3)/(3*a^3*c^5*f) + (9*Csc[e + f*x]^5)/(5*a^3*c^5*f) - Csc[e + f*x]^7/(a^
3*c^5*f) + (2*Csc[e + f*x]^9)/(9*a^3*c^5*f)
```

Rubi [A] time = 0.204163, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3958, 2606, 194, 2607, 30, 270}

$$\frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}-\frac{\csc^7(e+fx)}{a^3c^5f}+\frac{9\csc^5(e+fx)}{5a^3c^5f}-\frac{5\csc^3(e+fx)}{3a^3c^5f}+\frac{\csc(e+fx)}{a^3c^5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] (2*Cot[e + f*x]^9)/(9*a^3*c^5*f) + Csc[e + f*x]/(a^3*c^5*f) - (5*Csc[e + f*
x]^3)/(3*a^3*c^5*f) + (9*Csc[e + f*x]^5)/(5*a^3*c^5*f) - Csc[e + f*x]^7/(a^
3*c^5*f) + (2*Csc[e + f*x]^9)/(9*a^3*c^5*f)
```

Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = -\frac{\int \left(a^2\cot^9(e+fx)\csc(e+fx)+2a^2\cot^8(e+fx)\csc^2(e+fx)+a^2\right.}{a^5c^5}$$

$$= -\frac{\int \cot^9(e+fx)\csc(e+fx)\,dx}{a^3c^5} - \frac{\int \cot^7(e+fx)\csc^3(e+fx)\,dx}{a^3c^5} - 2$$

$$= \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx, x, \csc(e+fx)\right)}{a^3c^5f} + \frac{\text{Subst}\left(\int \left(-1+x^2\right)^4\,d\right.}{a^3c^5f}$$

$$= \frac{2\cot^9(e+fx)}{9a^3c^5f} + \frac{\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)dx, x, \csc(e+\right.}{a^3c^5f}$$

$$= \frac{2\cot^9(e+fx)}{9a^3c^5f} + \frac{\csc(e+fx)}{a^3c^5f} - \frac{5\csc^3(e+fx)}{3a^3c^5f} + \frac{9\csc^5(e+fx)}{5a^3c^5f} - \text{cs}$$

Mathematica [B] time = 1.56247, size = 257, normalized size = 2.14

$$-\frac{\csc(e)(76455\sin(e+fx)-33980\sin(2(e+fx))-32281\sin(3(e+fx))+27184\sin(4(e+fx))+1699\sin(5(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] -(Csc[e]*Sec[e + f*x]^7*(-33024*Sin[e] + 6144*Sin[f*x] + 76455*Sin[e + f*x]
 - 33980*Sin[2*(e + f*x)] - 32281*Sin[3*(e + f*x)] + 27184*Sin[4*(e + f*x)]
 + 1699*Sin[5*(e + f*x)] - 6796*Sin[6*(e + f*x)] + 1699*Sin[7*(e + f*x)] +
22656*Sin[2*e + f*x] - 17216*Sin[e + 2*f*x] + 4416*Sin[3*e + 2*f*x] + 3200*
Sin[2*e + 3*f*x] - 15360*Sin[4*e + 3*f*x] + 12160*Sin[3*e + 4*f*x] - 1920*S
in[5*e + 4*f*x] - 5120*Sin[4*e + 5*f*x] + 5760*Sin[6*e + 5*f*x] + 320*Sin[5
*e + 6*f*x] - 2880*Sin[7*e + 6*f*x] + 640*Sin[6*e + 7*f*x])*Tan[e + f*x])/(
184320*a^3*c^5*f*(-1 + Sec[e + f*x])^5*(1 + Sec[e + f*x])^3)
```

Maple [A] time = 0.065, size = 115, normalized size = 1.

$$\frac{1}{128\,fa^3c^5}\left(\frac{1}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5-\frac{7}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+21\,\tan\left(1/2\,fx+e/2\right)-\frac{35}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}+35\,\left(\tan\left(1/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x)
```

```
[Out] 1/128/f/a^3/c^5*(1/5*tan(1/2*f*x+1/2*e)^5-7/3*tan(1/2*f*x+1/2*e)^3+21*tan(1
/2*f*x+1/2*e)-35/3/tan(1/2*f*x+1/2*e)^3+35/tan(1/2*f*x+1/2*e)+21/5/tan(1/2*
```

```
f*x+1/2*e)^5-1/tan(1/2*f*x+1/2*e)^7+1/9/tan(1/2*f*x+1/2*e)^9)
```

Maxima [A] time = 1.03045, size = 244, normalized size = 2.03

$$\frac{\frac{3\left(\frac{315\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{35\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)}{a^{3}c^{5}}-\frac{\left(\frac{45\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{189\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{525\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\frac{1575\sin\left(fx+e\right)^{8}}{\left(\cos\left(fx+e\right)+1\right)^{8}}-5\right)\left(\cos\left(fx+e\right)+1\right)^{9}}{a^{3}c^{5}\sin\left(fx+e\right)^{9}}}{5760\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] 1/5760*(3*(315*sin(f*x + e)/(cos(f*x + e) + 1) - 35*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^5) - (45*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 189*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
525*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1575*sin(f*x + e)^8/(cos(f*x + e)
 + 1)^8 - 5)*(cos(f*x + e) + 1)^9/(a^3*c^5*sin(f*x + e)^9))/f
```

Fricas [A] time = 0.476116, size = 462, normalized size = 3.85

$$\frac{10\cos\left(fx+e\right)^{7}+25\cos\left(fx+e\right)^{6}-60\cos\left(fx+e\right)^{5}-10\cos\left(fx+e\right)^{4}+80\cos\left(fx+e\right)^{3}-24\cos\left(fx+\right.}{45\left(a^{3}c^{5}f\cos\left(fx+e\right)^{6}-2\,a^{3}c^{5}f\cos\left(fx+e\right)^{5}-a^{3}c^{5}f\cos\left(fx+e\right)^{4}+4\,a^{3}c^{5}f\cos\left(fx+e\right)^{3}-a^{3}c^{5}f\cos\left(fx+e\right)^{2}-2\,a^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/45*(10*cos(f*x + e)^7 + 25*cos(f*x + e)^6 - 60*cos(f*x + e)^5 - 10*cos(f*
x + e)^4 + 80*cos(f*x + e)^3 - 24*cos(f*x + e)^2 - 32*cos(f*x + e) + 16)/((
a^3*c^5*f*cos(f*x + e)^6 - 2*a^3*c^5*f*cos(f*x + e)^5 - a^3*c^5*f*cos(f*x +
 e)^4 + 4*a^3*c^5*f*cos(f*x + e)^3 - a^3*c^5*f*cos(f*x + e)^2 - 2*a^3*c^5*f
*cos(f*x + e) + a^3*c^5*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40693, size = 203, normalized size = 1.69

$$\frac{\frac{1575 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^8-525 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^6+189 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^4-45 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^2+5}{a^3c^5 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^9}+\frac{3\left(3 a^{12}c^{20} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^5-35 a^{12}c^{20} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^3+315 \ldots}{a^{15}c^{25}}}{5760 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/5760*((1575*tan(1/2*f*x + 1/2*e)^8 - 525*tan(1/2*f*x + 1/2*e)^6 + 189*tan
(1/2*f*x + 1/2*e)^4 - 45*tan(1/2*f*x + 1/2*e)^2 + 5)/(a^3*c^5*tan(1/2*f*x +
 1/2*e)^9) + 3*(3*a^12*c^20*tan(1/2*f*x + 1/2*e)^5 - 35*a^12*c^20*tan(1/2*f
*x + 1/2*e)^3 + 315*a^12*c^20*tan(1/2*f*x + 1/2*e))/(a^15*c^25))/f
```

3.63 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$

Optimal. Leaf size=162

$$-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}-\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}+\frac{17\csc^9(e+fx)}{9a^3c^6f}-\frac{4\csc^7(e+fx)}{a^3c^6f}+\frac{22\csc^5(e+fx)}{5a^3c^6f}-\frac{8\csc^3(e+fx)}{3a^3c^6f}$$

```
[Out] -Cot[e + f*x]^9/(9*a^3*c^6*f) - (4*Cot[e + f*x]^11)/(11*a^3*c^6*f) + Csc[e
+ f*x]/(a^3*c^6*f) - (8*Csc[e + f*x]^3)/(3*a^3*c^6*f) + (22*Csc[e + f*x]^5)
/(5*a^3*c^6*f) - (4*Csc[e + f*x]^7)/(a^3*c^6*f) + (17*Csc[e + f*x]^9)/(9*a^
3*c^6*f) - (4*Csc[e + f*x]^11)/(11*a^3*c^6*f)
```

Rubi [A] time = 0.259866, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3958, 2606, 194, 2607, 30, 270, 14}

$$-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}-\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}+\frac{17\csc^9(e+fx)}{9a^3c^6f}-\frac{4\csc^7(e+fx)}{a^3c^6f}+\frac{22\csc^5(e+fx)}{5a^3c^6f}-\frac{8\csc^3(e+fx)}{3a^3c^6f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```


```
[Out] -Cot[e + f*x]^9/(9*a^3*c^6*f) - (4*Cot[e + f*x]^11)/(11*a^3*c^6*f) + Csc[e
+ f*x]/(a^3*c^6*f) - (8*Csc[e + f*x]^3)/(3*a^3*c^6*f) + (22*Csc[e + f*x]^5)
/(5*a^3*c^6*f) - (4*Csc[e + f*x]^7)/(a^3*c^6*f) + (17*Csc[e + f*x]^9)/(9*a^
3*c^6*f) - (4*Csc[e + f*x]^11)/(11*a^3*c^6*f)
```


Rule 3958

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{\int \left(a^3\cot^{11}(e+fx)\csc(e+fx)+3a^3\cot^{10}(e+fx)\csc^2(e+fx)+3a\right.}{a^6c^6}$$

$$= \frac{\int \cot^{11}(e+fx)\csc(e+fx)\,dx}{a^3c^6}+\frac{\int \cot^8(e+fx)\csc^4(e+fx)\,dx}{a^3c^6}+\frac{3}{}$$

$$= -\frac{\text{Subst}\left(\int \left(-1+x^2\right)^5 dx, x, \csc(e+fx)\right)}{a^3c^6f}+\frac{\text{Subst}\left(\int x^8\left(1+x^2\right) dx,\right.}{a^3c^6f}$$

$$= -\frac{3\cot^{11}(e+fx)}{11a^3c^6f}-\frac{\text{Subst}\left(\int \left(-1+5x^2-10x^4+10x^6-5x^8+x^{10}\right) d\right.}{a^3c^6f}$$

$$= -\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{\csc(e+fx)}{a^3c^6f}-\frac{8\csc^3(e+fx)}{3a^3c^6f}+\frac{2}{}$$

Mathematica [A] time = 2.15696, size = 289, normalized size = 1.78

$$\frac{\csc(e)(-3440690\sin(e+fx)+2064414\sin(2(e+fx))+1063486\sin(3(e+fx))-1563950\sin(4(e+fx))+312790\,s}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

```
[Out] (Csc[e]*Sec[e + f*x]^8*(1119360*Sin[e] - 260480*Sin[f*x] - 3440690*Sin[e +
f*x] + 2064414*Sin[2*(e + f*x)] + 1063486*Sin[3*(e + f*x)] - 1563950*Sin[4*
(e + f*x)] + 312790*Sin[5*(e + f*x)] + 312790*Sin[6*(e + f*x)] - 187674*Sin
[7*(e + f*x)] + 31279*Sin[8*(e + f*x)] - 1499520*Sin[2*e + f*x] + 1051776*S
in[e + 2*f*x] + 4224*Sin[3*e + 2*f*x] - 85376*Sin[2*e + 3*f*x] + 629376*Sin
[4*e + 3*f*x] - 483200*Sin[3*e + 4*f*x] - 316800*Sin[5*e + 4*f*x] + 392320*
Sin[4*e + 5*f*x] - 232320*Sin[6*e + 5*f*x] - 30080*Sin[5*e + 6*f*x] + 19008
0*Sin[7*e + 6*f*x] - 32640*Sin[6*e + 7*f*x] - 63360*Sin[8*e + 7*f*x] + 1600
0*Sin[7*e + 8*f*x])*Tan[e + f*x])/(8110080*a^3*c^6*f*(-1 + Sec[e + f*x])^6*
(1 + Sec[e + f*x])^3)
```

Maple [A] time = 0.068, size = 128, normalized size = 0.8

$$\frac{1}{256\,fa^3c^6}\left(\frac{1}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5-\frac{8}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+28\,\tan\left(1/2\,fx+e/2\right)-\frac{70}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-3}-\frac{1}{11}\left(\tan\left(\frac{fx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x)
```

```
[Out] 1/256/f/a^3/c^6*(1/5*tan(1/2*f*x+1/2*e)^5-8/3*tan(1/2*f*x+1/2*e)^3+28*tan(1
/2*f*x+1/2*e)-70/3/tan(1/2*f*x+1/2*e)^3-1/11/tan(1/2*f*x+1/2*e)^11+56/tan(1
/2*f*x+1/2*e)+56/5/tan(1/2*f*x+1/2*e)^5-4/tan(1/2*f*x+1/2*e)^7+8/9/tan(1/2*
f*x+1/2*e)^9)
```

Maxima [A] time = 1.01738, size = 270, normalized size = 1.67

$$\frac{\frac{33\left(\frac{420\sin(fx+e)}{\cos(fx+e)+1}-\frac{40\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^6}+\frac{\left(\frac{440\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1980\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{5544\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{11550\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{27720\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-45\right)(\cos(fx+e)+1}{a^3c^6\sin(fx+e)^{11}}}{126720f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/126720*(33*(420*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^6) + (440*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1980*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + 5544*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 11550*sin(f*x + e)^8/(cos(f
*x + e) + 1)^8 + 27720*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 45)*(cos(f*x
 + e) + 1)^11/(a^3*c^6*sin(f*x + e)^11))/f
```

Fricas [A] time = 0.491307, size = 540, normalized size = 3.33

$$\frac{125\cos(fx+e)^8+120\cos(fx+e)^7-680\cos(fx+e)^6+400\cos(fx+e)^5+720\cos(fx+e)^4-832\cos}{495\left(a^3c^6f\cos(fx+e)^7-3a^3c^6f\cos(fx+e)^6+a^3c^6f\cos(fx+e)^5+5a^3c^6f\cos(fx+e)^4-5a^3c^6f\cos(fx+e)^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/495*(125*cos(f*x + e)^8 + 120*cos(f*x + e)^7 - 680*cos(f*x + e)^6 + 400*c
os(f*x + e)^5 + 720*cos(f*x + e)^4 - 832*cos(f*x + e)^3 - 64*cos(f*x + e)^2
 + 384*cos(f*x + e) - 128)/((a^3*c^6*f*cos(f*x + e)^7 - 3*a^3*c^6*f*cos(f*x
 + e)^6 + a^3*c^6*f*cos(f*x + e)^5 + 5*a^3*c^6*f*cos(f*x + e)^4 - 5*a^3*c^6
*f*cos(f*x + e)^3 - a^3*c^6*f*cos(f*x + e)^2 + 3*a^3*c^6*f*cos(f*x + e) - a
^3*c^6*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26152, size = 221, normalized size = 1.36

$$\frac{\frac{27720 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{10}-11550 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{8}+5544 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{6}-1980 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{4}+440 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{2}-45}{a^{3}c^{6} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{11}}+\frac{33\left(3 a^{12}c^{24} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{5}\right.}{}}{126720 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] 1/126720*((27720*tan(1/2*f*x + 1/2*e)^10 - 11550*tan(1/2*f*x + 1/2*e)^8 + 5
544*tan(1/2*f*x + 1/2*e)^6 - 1980*tan(1/2*f*x + 1/2*e)^4 + 440*tan(1/2*f*x
+ 1/2*e)^2 - 45)/(a^3*c^6*tan(1/2*f*x + 1/2*e)^11) + 33*(3*a^12*c^24*tan(1/
2*f*x + 1/2*e)^5 - 40*a^12*c^24*tan(1/2*f*x + 1/2*e)^3 + 420*a^12*c^24*tan(
1/2*f*x + 1/2*e))/(a^15*c^30))/f
```

3.64 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=163

$$-\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)}{315f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{105f}-\frac{8c^2\tan(e+fx)(a\sec(}{}$$

```
[Out] (-256*c^4*(a + a*Sec[e + f*x])*Tan[e + f*x])/(315*f*Sqrt[c - c*Sec[e + f*x]
]) - (64*c^3*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(1
05*f) - (8*c^2*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x]
)/(21*f) - (2*c*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x
])/(9*f)
```

Rubi [A] time = 0.277657, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3955, 3953}

$$-\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)}{315f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{105f}-\frac{8c^2\tan(e+fx)(a\sec(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(7/2),x]
```


```
[Out] (-256*c^4*(a + a*Sec[e + f*x])*Tan[e + f*x])/(315*f*Sqrt[c - c*Sec[e + f*x]
]) - (64*c^3*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(1
05*f) - (8*c^2*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x]
)/(21*f) - (2*c*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x
])/(9*f)
```


Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx = -\frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{9f} +$$
$$= -\frac{8c^2(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{21f} -$$
$$= -\frac{64c^3(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{105f} -$$
$$= -\frac{256c^4(a+a\sec(e+fx))\tan(e+fx)}{315f\sqrt{c-c\sec(e+fx)}} - \frac{64c^3(a+a\sec(e$$

Mathematica [A] time = 0.890051, size = 86, normalized size = 0.53

$$\frac{ac^3\cos^2\left(\frac{1}{2}(e+fx)\right)(1617\cos(e+fx)-642\cos(2(e+fx))+319\cos(3(e+fx))-782)\cot\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)}{315f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a*c^3*Cos[(e + f*x)/2]^2*(-782 + 1617*Cos[e + f*x] - 642*Cos[2*(e + f*x)]
+ 319*Cos[3*(e + f*x)])*Cot[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[c - c*Sec[e +
f*x]])/(315*f)
```

Maple [A] time = 0.186, size = 83, normalized size = 0.5

$$\frac{2a(\sin(fx+e))^3\left(319(\cos(fx+e))^3-321(\cos(fx+e))^2+165\cos(fx+e)-35\right)}{315f(-1+\cos(fx+e))^5\cos(fx+e)}\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] 2/315*a/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*sin(f*x+e)^3*(319*cos(f*x+e)
^3-321*cos(f*x+e)^2+165*cos(f*x+e)-35)/(-1+cos(f*x+e))^5/cos(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.48252, size = 298, normalized size = 1.83

$$\frac{2\left(319\,ac^3\cos\left(fx+e\right)^5+317\,ac^3\cos\left(fx+e\right)^4-158\,ac^3\cos\left(fx+e\right)^3-26\,ac^3\cos\left(fx+e\right)^2+95\,ac^3\cos\left(fx+e\right)-35\right.}{315\,f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x, algorithm="
fricas")
```

```
[Out] 2/315*(319*a*c^3*cos(f*x + e)^5 + 317*a*c^3*cos(f*x + e)^4 - 158*a*c^3*cos(
f*x + e)^3 - 26*a*c^3*cos(f*x + e)^2 + 95*a*c^3*cos(f*x + e) - 35*a*c^3)*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.53527, size = 150, normalized size = 0.92

$$\frac{32\,\sqrt{2}\left(105\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^3c^3+189\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^4+135\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^5+35\,c^6\right)ac^2}{315\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{9}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x, algorithm="
giac")
```

```
[Out] 32/315*sqrt(2)*(105*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^3 + 189*(c*tan(1/2*f
*x + 1/2*e)^2 - c)^2*c^4 + 135*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^5 + 35*c^6)
*a*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(9/2)*f)
```

3.65 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=122

$$-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)}{105f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{35f}-\frac{2c\tan(e+fx)(a\sec$$

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])*Tan[e + f*x])/(105*f*Sqrt[c - c*Sec[e + f*x]]
) - (16*c^2*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(35
*f) - (2*c*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(7
*f)
```

Rubi [A] time = 0.200355, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3955, 3953}

$$-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)}{105f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{35f}-\frac{2c\tan(e+fx)(a\sec$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])*Tan[e + f*x])/(105*f*Sqrt[c - c*Sec[e + f*x]]
) - (16*c^2*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(35
*f) - (2*c*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(7
*f)
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx=-\frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{7f}+$$

$$=-\frac{16c^2(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{35f}-$$

$$=-\frac{64c^3(a+a\sec(e+fx))\tan(e+fx)}{105f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2(a+a\sec(e+$$

Mathematica [A] time = 0.479334, size = 76, normalized size = 0.62

$$\frac{2ac^2\cos^2\left(\frac{1}{2}(e+fx)\right)(-108\cos(e+fx)+71\cos(2(e+fx))+101)\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx)}}{105f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*a*c^2*Cos[(e + f*x)/2]^2*(101 - 108*Cos[e + f*x] + 71*Cos[2*(e + f*x)])*
Cot[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[c - c*Sec[e + f*x]])/(105*f)
```

Maple [A] time = 0.18, size = 73, normalized size = 0.6

$$\frac{2\,a\left(\sin\left(fx+e\right)\right)^{3}\left(71\,\left(\cos\left(fx+e\right)\right)^{2}-54\,\cos\left(fx+e\right)+15\right)}{105\,f\left(-1+\cos\left(fx+e\right)\right)^{4}\cos\left(fx+e\right)}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 2/105*a/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*sin(f*x+e)^3*(71*cos(f*x+e)^
2-54*cos(f*x+e)+15)/(-1+cos(f*x+e))^4/cos(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.469479, size = 259, normalized size = 2.12

$$\frac{2\left(71\,ac^{2}\cos\left(fx+e\right)^{4}+88\,ac^{2}\cos\left(fx+e\right)^{3}-22\,ac^{2}\cos\left(fx+e\right)^{2}-24\,ac^{2}\cos\left(fx+e\right)+15\,ac^{2}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{105\,f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x, algorithm="
fricas")
```

```
[Out] 2/105*(71*a*c^2*cos(f*x + e)^4 + 88*a*c^2*cos(f*x + e)^3 - 22*a*c^2*cos(f*x
 + e)^2 - 24*a*c^2*cos(f*x + e) + 15*a*c^2)*sqrt((c*cos(f*x + e) - c)/cos(f
```

```
*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.83178, size = 113, normalized size = 0.93

$$\frac{16\sqrt{2}\left(35\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{2}c^{3}+42\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)c^{4}+15c^{5}\right)ac}{105\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{7}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x, algorithm="
giac")
```

```
[Out] 16/105*sqrt(2)*(35*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^3 + 42*(c*tan(1/2*f*x
 + 1/2*e)^2 - c)*c^4 + 15*c^5)*a*c/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(7/2)*f)
```

3.66 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=81

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)}{15f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{5f}$$

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])*Tan[e + f*x])/(15*f*Sqrt[c - c*Sec[e + f*x]])
- (2*c*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(5*f)
```

Rubi [A] time = 0.127802, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3955, 3953}

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)}{15f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])*Tan[e + f*x])/(15*f*Sqrt[c - c*Sec[e + f*x]])
- (2*c*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(5*f)
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx=-\frac{2c(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f}+\frac{1}{5}(4c$$

$$=-\frac{8c^2(a+a\sec(e+fx))\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}}-\frac{2c(a+a\sec(e+fx))\sqrt{}}{}$$

Mathematica [A] time = 0.272285, size = 64, normalized size = 0.79

$$\frac{4ac\cos^2\left(\frac{1}{2}(e+fx)\right)(7\cos(e+fx)-3)\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx)}}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*a*c*Cos[(e + f*x)/2]^2*(-3 + 7*Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e + f*
x]^2*Sqrt[c - c*Sec[e + f*x]])/(15*f)
```

Maple [A] time = 0.171, size = 63, normalized size = 0.8

$$\frac{2 a \left(7 \cos \left(f x+e\right)-3\right) \left(\sin \left(f x+e\right)\right)^{3}}{15 f \left(-1+\cos \left(f x+e\right)\right)^{3} \cos \left(f x+e\right)} \left(\frac{c \left(-1+\cos \left(f x+e\right)\right)}{\cos \left(f x+e\right)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/15*a/f*(7*cos(f*x+e)-3)*sin(f*x+e)^3*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)
/(-1+cos(f*x+e))^3/cos(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.466482, size = 205, normalized size = 2.53

$$\frac{2 \left(7 a c \cos \left(f x+e\right)^{3}+11 a c \cos \left(f x+e\right)^{2}+a c \cos \left(f x+e\right)-3 a c\right) \sqrt{\frac{c \cos\left(f x+e\right)-c}{\cos\left(f x+e\right)}}}{15 f \cos \left(f x+e\right)^{2} \sin \left(f x+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x, algorithm="
fricas")
```

```
[Out] 2/15*(7*a*c*cos(f*x + e)^3 + 11*a*c*cos(f*x + e)^2 + a*c*cos(f*x + e) - 3*a
*c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^2*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.49286, size = 82, normalized size = 1.01

$$\frac{8\sqrt{2}\left(5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^4+3c^5\right)a}{15\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x, algorithm="
giac")
```

```
[Out] 8/15*sqrt(2)*(5*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^4 + 3*c^5)*a/((c*tan(1/2*f
*x + 1/2*e)^2 - c)^(5/2)*c*f)
```

3.67 $\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=39

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (-2*c*(a + a*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.0591537, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3953}

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)}{3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*c*(a + a*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.175435, size = 51, normalized size = 1.31

$$\frac{4a\cos^2\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (4*a*Cos[(e + f*x)/2]^2*Cot[(e + f*x)/2]*Sec[e + f*x]*Sqrt[c - c*Sec[e + f*
x]])/(3*f)
```

Maple [A] time = 0.221, size = 53, normalized size = 1.4

$$\frac{2\,a\left(\sin\left(fx+e\right)\right)^{3}}{3\,f\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^{2}}\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/3*a/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^3/cos(f*x+e)/(-1+co
s(f*x+e))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.454288, size = 158, normalized size = 4.05

$$\frac{2\left(a\cos\left(fx+e\right)^{2}+2\,a\cos\left(fx+e\right)+a\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\,f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] 2/3*(a*cos(f*x + e)^2 + 2*a*cos(f*x + e) + a)*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)dx+\int\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(sqrt(-c*s
ec(e + f*x) + c)*sec(e + f*x)**2, x))
```

Giac [A] time = 1.36078, size = 43, normalized size = 1.1

$$\frac{4\sqrt{2}ac^{2}}{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] 4/3*sqrt(2)*a*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*f)
```

3.68 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=77

$$\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{2\sqrt{2}a\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f}$$

```
[Out] (-2*Sqrt[2]*a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x
]])])/(Sqrt[c]*f) + (2*a*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.106849, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3956, 3795, 203}

$$\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{2\sqrt{2}a\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[2]*a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x
]])])/(Sqrt[c]*f) + (2*a*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3956

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*d*Cot[e +
 f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]),
 x] + Dist[(2*c*(2*n - 1))/(2*n - 1), Int[(Csc[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx$$
$$= \frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{2c+x^2}\,dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{f}$$
$$= -\frac{2\sqrt{2}a\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f} + \frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.616451, size = 132, normalized size = 1.71

$$-\frac{i\sqrt{2}a\left(-1+e^{i(e+fx)}\right)\left(\sqrt{2}\left(1+e^{i(e+fx)}\right)-2\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)\right)}{f\left(1+e^{2i(e+fx)}\right)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] ((-I)*Sqrt[2]*a*(-1 + E^(I*(e + f*x)))*(Sqrt[2]*(1 + E^(I*(e + f*x))) - 2*S
qrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1
+ E^((2*I)*(e + f*x))])]))/((1 + E^((2*I)*(e + f*x)))*f*Sqrt[c - c*Sec[e +
f*x]])
```

Maple [A] time = 0.239, size = 85, normalized size = 1.1

$$-2\,\frac{a\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}\left(\arctan\left(\frac{1}{\sqrt{-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}}\right)\sqrt{-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}-1\right)\frac{1}{\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -2*a/f*(arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))*(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)-1)*sin(f*x+e)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/cos(f*x
+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)\sec\left(fx+e\right)}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

Fricas [A] time = 0.58336, size = 686, normalized size = 8.91

$$\left[\frac{\sqrt{2}ac\sqrt{-\frac{1}{c}}\log\left(-\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{c}}-\left(3\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-2\left(a\cos\left(fx+e\right)+a\right)\sqrt{\frac{c}{}}}{cf\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] [(sqrt(2)*a*c*sqrt(-1/c)*log(-(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) - (3*cos(f*x + e) + 1)*sin
(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(a*cos(f*x +
 e) + a)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*sin(f*x + e)), 2*(sq
rt(2)*a*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(
f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - (a*cos(f*x + e) + a)*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{\sec\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sec(e + f*x)/sqrt(-c*sec(e + f*x) + c), x) + Integral(sec(e + f
*x)**2/sqrt(-c*sec(e + f*x) + c), x))
```

Giac [C] time = 1.82451, size = 228, normalized size = 2.96

$$-\frac{2\left(ac\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}}{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)-\frac{\left(-i\sqrt{2}a\sqrt{-c}\arctan(\cdots\right.}{}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] -2*(a*c*(sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/(c^(3/2
)*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e))) + sqrt(2)/(sqr
t(c*tan(1/2*f*x + 1/2*e)^2 - c)*c*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1
/2*f*x + 1/2*e)))) - (-I*sqrt(2)*a*sqrt(-c)*arctan(-I) + sqrt(2)*a*sqrt(-c)
)*sgn(tan(1/2*f*x + 1/2*e))/c)/f
```

3.69 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{a\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}c^{3/2}f}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

```
[Out] (a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(Sqrt
[2]*c^(3/2)*f) - (a*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.114423, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3957, 3795, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}c^{3/2}f}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(Sqrt
[2]*c^(3/2)*f) - (a*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{a\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{2c}$$
$$= -\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{2c+x^2}\,dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{cf}$$
$$= \frac{a\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}c^{3/2}f} - \frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

Mathematica [C] time = 1.10001, size = 246, normalized size = 3.24

$$\frac{a\left(\frac{i\sqrt{2}\left(-1+e^{i(e+fx)}\right)^3\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)}{\left(1+e^{2i(e+fx)}\right)^{3/2}} + 8\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sin^3\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx) - 8\cos\left(\frac{e}{2}\right)\cos\left(\frac{fx}{2}\right)\sin^3\left(\frac{1}{2}(e+\right.\right.}{2cf(\sec(e+fx)-1)\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*((I*Sqrt[2]*(-1 + E^(I*(e + f*x)))^3*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt
[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/(1 + E^((2*I)*(e + f*x)))^(3/2) - 4*Cs
c[e/2]*Sec[e + f*x]^2*Sin[(f*x)/2]*Sin[(e + f*x)/2] + 4*Cot[e/2]*Sec[e + f*
x]^2*Sin[(e + f*x)/2]^2 - 8*Cos[e/2]*Cos[(f*x)/2]*Sec[e + f*x]^2*Sin[(e + f
*x)/2]^3 + 8*Sec[e + f*x]^2*Sin[e/2]*Sin[(f*x)/2]*Sin[(e + f*x)/2]^3))/(2*c
*f*(-1 + Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] time = 0.199, size = 164, normalized size = 2.2

$$2\,\frac{a\left(-1+\cos\left(fx+e\right)\right)^2}{f\left(\sin\left(fx+e\right)\right)^3}\left(\sqrt{-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\cos\left(fx+e\right)+\cos\left(fx+e\right)\arctan\left(\frac{1}{\sqrt{-2\,\frac{\cos(fx+e)}{1+\cos(fx+e)}}}\right)+\sqrt{-2\,\frac{\cos}{1+\cos}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 2*a/f*(-1+cos(f*x+e))^2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)+co
s(f*x+e)*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))+(-2*cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)-arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)))/(c*(-1+co
s(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)^3/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2), x)
```

Fricas [B] time = 0.606074, size = 864, normalized size = 11.37

$$\left[\frac{\sqrt{2}\left(ac\cos\left(fx+e\right)-ac\right)\sqrt{-\frac{1}{c}}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{c}}+\left(3\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+4\left(a\cos\cdots\right.}{4\left(c^{2}f\cos\left(fx+e\right)-c^{2}f\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/4*(sqrt(2)*(a*c*cos(f*x + e) - a*c)*sqrt(-1/c)*log((2*sqrt(2)*(cos(f*x +
 e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) +
(3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f
*x + e) + 4*(a*cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), -1/2*(sqrt(2)*(a
*c*cos(f*x + e) - a*c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e
))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e)/sqrt(c) - 2*(a*cos(f*x
 + e)^2 + a*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*f*
cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{\sec\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int\frac{\sec^{2}\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sqr
t(-c*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**2/(-c*sqrt(-c*sec(e +
f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x))
```

Giac [A] time = 1.59768, size = 142, normalized size = 1.87

$$\frac{\sqrt{2}a\left(\frac{\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}}{c^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}\right)}{2f\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
giac")
```

```
[Out] 1/2*sqrt(2)*a*(arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(3/2) +
 sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/(c^2*tan(1/2*f*x + 1/2*e)^2))/(f*sgn(ta
n(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)))
```

3.70 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{a\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f}+\frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}$$

```
[Out] (a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(8*Sq
rt[2]*c^(5/2)*f) - (a*Tan[e + f*x])/(2*f*(c - c*Sec[e + f*x])^(5/2)) + (a*T
an[e + f*x])/(8*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.154883, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3957, 3796, 3795, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f}+\frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(8*Sq
rt[2]*c^(5/2)*f) - (a*Tan[e + f*x])/(2*f*(c - c*Sec[e + f*x])^(5/2)) + (a*T
an[e + f*x])/(8*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} - \frac{a\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}\,dx}{4c}$$

$$= -\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}} - \frac{a\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{16c^2}$$

$$= -\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{2c+x^2}\,dx, x$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f} - \frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/}}$$

Mathematica [C] time = 1.20918, size = 309, normalized size = 2.73

$$a\left(-\frac{i\sqrt{2}\left(-1+e^{i(e+fx)}\right)^5\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)}{\left(1+e^{2i(e+fx)}\right)^{5/2}} + 48\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sin^5\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx) - 48\cos\left(\frac{e}{2}\right)\cos\left(\frac{fx}{2}\right)\sin^5\left(\frac{1}{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*(((-I)*Sqrt[2]*(-1 + E^(I*(e + f*x)))^5*ArcTanh[(1 + E^(I*(e + f*x)))/(S
qrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/(1 + E^((2*I)*(e + f*x)))^(5/2) + 1
6*Csc[e/2]*Sec[e + f*x]^3*Sin[(f*x)/2]*Sin[(e + f*x)/2] - 16*Cot[e/2]*Sec[e
 + f*x]^3*Sin[(e + f*x)/2]^2 - 56*Csc[e/2]*Sec[e + f*x]^3*Sin[(f*x)/2]*Sin[
(e + f*x)/2]^3 + 56*Cot[e/2]*Sec[e + f*x]^3*Sin[(e + f*x)/2]^4 - 48*Cos[e/2
]*Cos[(f*x)/2]*Sec[e + f*x]^3*Sin[(e + f*x)/2]^5 + 48*Sec[e + f*x]^3*Sin[e/
2]*Sin[(f*x)/2]*Sin[(e + f*x)/2]^5))/(16*c^2*f*(-1 + Sec[e + f*x])^2*Sqrt[c
 - c*Sec[e + f*x]])
```

Maple [B] time = 0.214, size = 308, normalized size = 2.7

$$-\frac{a\left(-1+\cos\left(fx+e\right)\right)^3}{2f\left(\sin\left(fx+e\right)\right)^5}\left(\left(\cos\left(fx+e\right)\right)^2\left(-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{3}{2}} + 4\cos\left(fx+e\right)\left(-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{3/2} - \left(\cos\left(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/2*a/f*(-1+cos(f*x+e))^3*(cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/
2)+4*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-cos(f*x+e)^2*(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)-cos(f*x+e)^2*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x
```

```
+e)))^(1/2))+3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+2*(-2*cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*cos(f*x+e)+2*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2))-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-arctan(1/(-2*cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)^5/(-2
*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.619489, size = 1042, normalized size = 9.22

$$\left[-\frac{\sqrt{2}\left(a\cos\left(fx+e\right)^{2}-2\,a\cos\left(fx+e\right)+a\right)\sqrt{-c}\log\left(-\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}-\left(3\,c\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\mathrm{si}}{32\left(c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)+c^{3}f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
fricas")
```

```
[Out] [-1/32*(sqrt(2)*(a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)*sqrt(-c)*log(-(2*
sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/
cos(f*x + e)) - (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*si
n(f*x + e)))*sin(f*x + e) - 4*(3*a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 + a*
cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^3*f*cos(f*x + e)
^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), -1/16*(sqrt(2)*(a*cos(f*x
 + e)^2 - 2*a*cos(f*x + e) + a)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(
3*a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3
*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(5/2),x)
```

[Out] Timed out

Giac [A] time = 1.88553, size = 178, normalized size = 1.58

$$\frac{\sqrt{2}a\left(\frac{\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}c}{c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}\right)}{16\,cf\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
giac")
```

```
[Out] 1/16*sqrt(2)*a*(arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(3/2)
+ ((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c
)*c)/(c^3*tan(1/2*f*x + 1/2*e)^4))/(c*f*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn
(tan(1/2*f*x + 1/2*e)))
```

3.71 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=171

$$-\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)^2}{1155f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{231f}-\frac{8c^2\tan(e+fx)(a\sec$$

```
[Out] (-256*c^4*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(1155*f*Sqrt[c - c*Sec[e + f
*x]]) - (64*c^3*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x
])/(231*f) - (8*c^2*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e
 + f*x])/(33*f) - (2*c*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)*Ta
n[e + f*x])/(11*f)
```

Rubi [A] time = 0.447941, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)^2}{1155f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{231f}-\frac{8c^2\tan(e+fx)(a\sec$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(7/2),x]
```


```
[Out] (-256*c^4*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(1155*f*Sqrt[c - c*Sec[e + f
*x]]) - (64*c^3*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x
])/(231*f) - (8*c^2*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e
 + f*x])/(33*f) - (2*c*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)*Ta
n[e + f*x])/(11*f)
```


Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx = -\frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{11f}$$
$$= -\frac{8c^2(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\tan(e+fx}{33f}$$
$$= -\frac{64c^3(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{231f}$$
$$= -\frac{256c^4(a+a\sec(e+fx))^2\tan(e+fx)}{1155f\sqrt{c-c\sec(e+fx)}} - \frac{64c^3(a+a\sec}{}$$

Mathematica [A] time = 1.59485, size = 88, normalized size = 0.51

$$\frac{2a^2c^3\cos^4\left(\frac{1}{2}(e+fx)\right)(3419\cos(e+fx)-1510\cos(2(e+fx))+533\cos(3(e+fx))-1930)\cot\left(\frac{1}{2}(e+fx)\right)\sec^5(e+}{1155f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*a^2*c^3*Cos[(e + f*x)/2]^4*(-1930 + 3419*Cos[e + f*x] - 1510*Cos[2*(e +
f*x)] + 533*Cos[3*(e + f*x)])*Cot[(e + f*x)/2]*Sec[e + f*x]^5*Sqrt[c - c*Se
c[e + f*x]])/(1155*f)
```

Maple [A] time = 0.2, size = 85, normalized size = 0.5

$$-\frac{2\,a^2\left(\sin\left(fx+e\right)\right)^5\left(533\,\left(\cos\left(fx+e\right)\right)^3-755\,\left(\cos\left(fx+e\right)\right)^2+455\,\cos\left(fx+e\right)-105\right)}{1155\,f\left(-1+\cos\left(fx+e\right)\right)^6\left(\cos\left(fx+e\right)\right)^2}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] -2/1155*a^2/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*sin(f*x+e)^5*(533*cos(f*
x+e)^3-755*cos(f*x+e)^2+455*cos(f*x+e)-105)/(-1+cos(f*x+e))^6/cos(f*x+e)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.489223, size = 359, normalized size = 2.1

$$\frac{2\left(533\,a^2c^3\cos\left(fx+e\right)^6+844\,a^2c^3\cos\left(fx+e\right)^5-211\,a^2c^3\cos\left(fx+e\right)^4-472\,a^2c^3\cos\left(fx+e\right)^3+295\,a^2c^3\cos\left(fx+\right.\right.}{1155\,f\cos\left(fx+e\right)^5\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x, algorithm
="fricas")
```

```
[Out] 2/1155*(533*a^2*c^3*cos(f*x + e)^6 + 844*a^2*c^3*cos(f*x + e)^5 - 211*a^2*c
^3*cos(f*x + e)^4 - 472*a^2*c^3*cos(f*x + e)^3 + 295*a^2*c^3*cos(f*x + e)^2
 + 140*a^2*c^3*cos(f*x + e) - 105*a^2*c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*
x + e))/(f*cos(f*x + e)^5*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.10614, size = 153, normalized size = 0.89

$$-\frac{64\,\sqrt{2}\left(231\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^3c^4+495\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^5+385\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^6+105\,c^7\right)a^2}{1155\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{11}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x, algorithm
="giac")
```

```
[Out] -64/1155*sqrt(2)*(231*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^4 + 495*(c*tan(1/2
*f*x + 1/2*e)^2 - c)^2*c^5 + 385*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^6 + 105*c
^7)*a^2*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(11/2)*f)
```

3.72 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=128

$$-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^2}{315f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{63f}-2c\tan(e+fx)(a\sec\ldots$$

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(315*f*Sqrt[c - c*Sec[e + f*x
]]) - (16*c^2*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])
/(63*f) - (2*c*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*
x])/(9*f)
```

Rubi [A] time = 0.327305, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^2}{315f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{63f}-2c\tan(e+fx)(a\sec\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(315*f*Sqrt[c - c*Sec[e + f*x
]]) - (16*c^2*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])
/(63*f) - (2*c*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*
x])/(9*f)
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{9f} + \cdots$$
$$= -\frac{16c^2(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{63f} - \cdots$$
$$= -\frac{64c^3(a+a\sec(e+fx))^2\tan(e+fx)}{315f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a+a\sec(e+\cdots}{\cdots}$$

Mathematica [A] time = 1.19244, size = 78, normalized size = 0.61

$$\frac{4a^2c^2\cos^4\left(\frac{1}{2}(e+fx)\right)(-220\cos(e+fx)+107\cos(2(e+fx))+177)\cot\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{c-c\sec(e+fx)}}{315f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (4*a^2*c^2*Cos[(e + f*x)/2]^4*(177 - 220*Cos[e + f*x] + 107*Cos[2*(e + f*x)
])*Cot[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[c - c*Sec[e + f*x]])/(315*f)
```

Maple [A] time = 0.19, size = 75, normalized size = 0.6

$$-\frac{2a^2\left(107\left(\cos\left(fx+e\right)\right)^2-110\cos\left(fx+e\right)+35\right)\left(\sin\left(fx+e\right)\right)^5}{315f\left(-1+\cos\left(fx+e\right)\right)^5\left(\cos\left(fx+e\right)\right)^2}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -2/315*a^2/f*(107*cos(f*x+e)^2-110*cos(f*x+e)+35)*sin(f*x+e)^5*(c*(-1+cos(f
*x+e))/cos(f*x+e))^(5/2)/(-1+cos(f*x+e))^5/cos(f*x+e)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.482101, size = 313, normalized size = 2.45

$$\frac{2\left(107a^2c^2\cos\left(fx+e\right)^5+211a^2c^2\cos\left(fx+e\right)^4+26a^2c^2\cos\left(fx+e\right)^3-118a^2c^2\cos\left(fx+e\right)^2-5a^2c^2\cos\left(fx+e\right)+\cdots\right.}{315f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] 2/315*(107*a^2*c^2*cos(f*x + e)^5 + 211*a^2*c^2*cos(f*x + e)^4 + 26*a^2*c^2
*cos(f*x + e)^3 - 118*a^2*c^2*cos(f*x + e)^2 - 5*a^2*c^2*cos(f*x + e) + 35*
a^2*c^2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x
+ e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.65187, size = 116, normalized size = 0.91

$$-\frac{32\sqrt{2}\left(63\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^4+90\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^5+35c^6\right)a^2c}{315\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{9}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] -32/315*sqrt(2)*(63*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^4 + 90*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*c^5 + 35*c^6)*a^2*c/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(9/2)
*f)
```

3.73 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=85

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^2}{35f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{7f}$$

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(35*f*Sqrt[c - c*Sec[e + f*x]]
) - (2*c*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(7*f
)
```

Rubi [A] time = 0.207793, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^2}{35f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(35*f*Sqrt[c - c*Sec[e + f*x]]
) - (2*c*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(7*f
)
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx = -\frac{2c(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{7f}+\frac{1}{7}($$

$$= -\frac{8c^2(a+a\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{c-c\sec(e+fx)}}-\frac{2c(a+a\sec(e+fx)}{}$$

Mathematica [A] time = 0.73391, size = 66, normalized size = 0.78

$$\frac{8a^2c\cos^4\left(\frac{1}{2}(e+fx)\right)(9\cos(e+fx)-5)\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx)}}{35f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (8*a^2*c*Cos[(e + f*x)/2]^4*(-5 + 9*Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e +
f*x]^3*Sqrt[c - c*Sec[e + f*x]])/(35*f)
```

Maple [A] time = 0.187, size = 65, normalized size = 0.8

$$-\frac{2a^2(9\cos(fx+e)-5)(\sin(fx+e))^5}{35f(-1+\cos(fx+e))^4(\cos(fx+e))^2}\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -2/35*a^2/f*(9*cos(f*x+e)-5)*sin(f*x+e)^5*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3
/2)/(-1+cos(f*x+e))^4/cos(f*x+e)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.471815, size = 254, normalized size = 2.99

$$\frac{2\left(9a^2c\cos(fx+e)^4+22a^2c\cos(fx+e)^3+12a^2c\cos(fx+e)^2-6a^2c\cos(fx+e)-5a^2c\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{35f\cos(fx+e)^3\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2/35*(9*a^2*c*cos(f*x + e)^4 + 22*a^2*c*cos(f*x + e)^3 + 12*a^2*c*cos(f*x +
 e)^2 - 6*a^2*c*cos(f*x + e) - 5*a^2*c)*sqrt((c*cos(f*x + e) - c)/cos(f*x +
```

```
e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.94869, size = 85, normalized size = 1.

$$-\frac{16\sqrt{2}\left(7\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^5+5c^6\right)a^2}{35\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{7}{2}}cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] -16/35*sqrt(2)*(7*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^5 + 5*c^6)*a^2/((c*tan(1
/2*f*x + 1/2*e)^2 - c)^(7/2)*c*f)
```

3.74 $\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=41

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (-2*c*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.0967487, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3953}

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*c*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.42574, size = 55, normalized size = 1.34

$$\frac{8a^2\cos^4\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (8*a^2*Cos[(e + f*x)/2]^4*Cot[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[c - c*Sec[e
+ f*x]])/(5*f)
```

Maple [A] time = 0.253, size = 55, normalized size = 1.3

$$-\frac{2a^2\left(\sin\left(fx+e\right)\right)^5}{5f\left(\cos\left(fx+e\right)\right)^2\left(-1+\cos\left(fx+e\right)\right)^3}\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/5*a^2/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^5/cos(f*x+e)^2/(
-1+cos(f*x+e))^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.465069, size = 200, normalized size = 4.88

$$\frac{2\left(a^{2}\cos\left(fx+e\right)^{3}+3\,a^{2}\cos\left(fx+e\right)^{2}+3\,a^{2}\cos\left(fx+e\right)+a^{2}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{5\,f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] 2/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) + a^2)*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^2*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)dx+\int2\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)dx+\int\sqrt{-c\sec\left(e+fx\right)+c}\sec^{3}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(2*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(sqrt(-c*sec(e + f*x) +
 c)*sec(e + f*x)**3, x))
```

Giac [A] time = 1.42619, size = 46, normalized size = 1.12

$$-\frac{8\sqrt{2}a^{2}c^{3}}{5\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{5}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] -8/5*sqrt(2)*a^2*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*f)
```

3.75 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=117

$$-\frac{4\sqrt{2}a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f}-\frac{2a^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3cf}+\frac{16a^2\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (-4*Sqrt[2]*a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f
*x]])])/(Sqrt[c]*f) + (16*a^2*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
- (2*a^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(3*c*f)
```

Rubi [A] time = 0.213263, antiderivative size = 123, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3956, 3795, 203}

$$-\frac{4\sqrt{2}a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f}+\frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-4*Sqrt[2]*a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f
*x]])])/(Sqrt[c]*f) + (4*a^2*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]]) + (
2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3956

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*d*Cot[e +
 f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]),
 x] + Dist[(2*c*(2*n - 1))/(2*n - 1), Int[(Csc[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}dx = \frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} + \left(4a^2\right)\int \frac{\sec}{\sqrt{c-c}}$$

$$= \frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} - \frac{\left(8a^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{4\sqrt{2}a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f} + \frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2\left(a^2+a^2\sec(e+\right.}{3f\sqrt{c-c\sec}}$$

Mathematica [C] time = 0.889227, size = 173, normalized size = 1.48

$$\frac{4a^2e^{-\frac{1}{2}i(e+fx)}\sin\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+7)-3\sqrt{\right.}{3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (4*a^2*Sec[e + f*x]*((-3*Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 +
 E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/E^((I/2)*(e + f
*x)) + Cos[(e + f*x)/2]*(7 + Sec[e + f*x]))*(Cos[(e + f*x)/2] + I*Sin[(e +
f*x)/2])*Sin[(e + f*x)/2])/(3*E^((I/2)*(e + f*x))*f*Sqrt[c - c*Sec[e + f*x]
])
```

Maple [A] time = 0.25, size = 145, normalized size = 1.2

$$\frac{2a^2\sin\left(fx+e\right)}{3f\left(\cos\left(fx+e\right)\right)^2}\left(3\cos\left(fx+e\right)\arctan\left(\frac{1}{\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}}\right)\left(-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{3/2}+3\arctan\left(\frac{1}{\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/3*a^2/f*(3*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))*(-2*
cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+3*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+7*cos(f*x+e)+1)*sin(f*x+e)/cos(
f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^2\sec\left(fx+e\right)}{\sqrt{-c\sec\left(fx+e\right)+c}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

Fricas [A] time = 0.596823, size = 851, normalized size = 7.27

$$\left[\frac{2\left(3\sqrt{2}a^2c\sqrt{-\frac{1}{c}}\cos\left(fx+e\right)\log\left(-\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{c}}-\left(3\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-\left(7a^2c\right.\right.}{3cf\cos\left(fx+e\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*a^2*c*sqrt(-1/c)*cos(f*x + e)*log(-(2*sqrt(2)*(cos(f*x + e)
^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) - (3*
cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x
+ e) - (7*a^2*cos(f*x + e)^2 + 8*a^2*cos(f*x + e) + a^2)*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e)))/(c*f*cos(f*x + e)*sin(f*x + e)), 2/3*(6*sqrt(2)*a^2*
sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)
/(sqrt(c)*sin(f*x + e)))*cos(f*x + e)*sin(f*x + e) - (7*a^2*cos(f*x + e)^2
+ 8*a^2*cos(f*x + e) + a^2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*c
os(f*x + e)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{\sec\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int\frac{2\sec^2\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int\frac{\sec^3\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/sqrt(-c*sec(e + f*x) + c), x) + Integral(2*sec(
e + f*x)**2/sqrt(-c*sec(e + f*x) + c), x) + Integral(sec(e + f*x)**3/sqrt(-
c*sec(e + f*x) + c), x))
```

Giac [C] time = 1.95031, size = 267, normalized size = 2.28

$$-\frac{2\left(2\,a^2c^2\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-4c\right)}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)+\frac{\left(6i\sqrt{2}a^2\right.}{}\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] -2/3*(2*a^2*c^2*(3*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c
))/(c^(5/2)*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e))) + sq
rt(2)*(3*c*tan(1/2*f*x + 1/2*e)^2 - 4*c)/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3
/2)*c^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)))) + (6*I*
sqrt(2)*a^2*sqrt(-c)*arctan(-I) - 8*sqrt(2)*a^2*sqrt(-c))*sgn(tan(1/2*f*x +
 1/2*e))/c)/f
```

3.76 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{3\sqrt{2}a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{2a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}-\frac{2a^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

```
[Out] (3*Sqrt[2]*a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*
x]])])/(c^(3/2)*f) - (2*a^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2)) -
(2*a^2*Tan[e + f*x])/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.22811, antiderivative size = 124, normalized size of antiderivative = 1.1, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3957, 3956, 3795, 203}

$$\frac{3\sqrt{2}a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{3a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (3*Sqrt[2]*a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*
x]])])/(c^(3/2)*f) - ((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[
e + f*x])^(3/2)) - (3*a^2*Tan[e + f*x])/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3956

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*d*Cot[e +
 f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]),
 x] + Dist[(2*c*(2*n - 1))/(2*n - 1), Int[(Csc[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{(3a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx}{2c}$$

$$= -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{3a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{\left(3a^2\right)\int \frac{\sec(e\cdots}{\sqrt{c-c\sec\cdots}}}{c}$$

$$= -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{3a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} + \frac{\left(6a^2\right)\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{3\sqrt{2}a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{3a^2\,\cdots}{cf\sqrt{c-\cdots}}$$

Mathematica [C] time = 1.43093, size = 184, normalized size = 1.63

$$\frac{a^2e^{-2i(e+fx)}\sin\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left(4\left(1+e^{3i(e+fx)}\right)-3\sqrt{2}\left(-1+e^{i(e+fx)}\right)^2\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)\right)\left(\cdots\right.}{2cf(\sec(e+fx)-1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(3/2),
x]
```

```
[Out] (a^2*(4*(1 + E^((3*I)*(e + f*x))) - 3*Sqrt[2]*(-1 + E^(I*(e + f*x)))^2*Sqrt
[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E
^((2*I)*(e + f*x))])])*Sec[e + f*x]^2*(Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2
])*Sin[(e + f*x)/2])/(2*c*E^((2*I)*(e + f*x))*f*(-1 + Sec[e + f*x])*Sqrt[c
- c*Sec[e + f*x]])
```

Maple [A] time = 0.218, size = 144, normalized size = 1.3

$$\frac{a^2\sin(fx+e)}{f(\cos(fx+e))^2}\left(3\arctan\left(\frac{1}{\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}}\right)\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}\cos(fx+e)-3\arctan\left(\frac{1}{\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}}\right)\sqrt{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] a^2/f*(3*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))*(-2*cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*cos(f*x+e)-3*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-4*cos(f*x+e)+2)*sin(f*x+e)/cos(f*x+
e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.611561, size = 907, normalized size = 8.03

$$\left[\frac{3\sqrt{2}\left(a^2c\cos\left(fx+e\right)-a^2c\right)\sqrt{-\frac{1}{c}}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{c}}+\left(3\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+4\left(\right.}{2\left(c^2f\cos\left(fx+e\right)-c^2f\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/2*(3*sqrt(2)*(a^2*c*cos(f*x + e) - a^2*c)*sqrt(-1/c)*log((2*sqrt(2)*(cos
(f*x + e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1
/c) + (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))
*sin(f*x + e) + 4*(2*a^2*cos(f*x + e)^2 + a^2*cos(f*x + e) - a^2)*sqrt((c*c
os(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
, -(3*sqrt(2)*(a^2*c*cos(f*x + e) - a^2*c)*arctan(sqrt(2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e)/sq
rt(c) - 2*(2*a^2*cos(f*x + e)^2 + a^2*cos(f*x + e) - a^2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{\sec\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int\frac{2\sec^2\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*
sqrt(-c*sec(e + f*x) + c)), x) + Integral(2*sec(e + f*x)**2/(-c*sqrt(-c*sec
(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x) + Integral(s
ec(e + f*x)**3/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e
```

```
+ f*x) + c)), x))
```

Giac [A] time = 1.70779, size = 234, normalized size = 2.07

$$\frac{a^2c\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)}{\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}+\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c\right)c^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.\right.}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] a^2*c*(3*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/(c^(5/2
)*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e))) + sqrt(2)*(3*c
*tan(1/2*f*x + 1/2*e)^2 - c)/(((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2) + sqrt(
c*tan(1/2*f*x + 1/2*e)^2 - c)*c)*c^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(ta
n(1/2*f*x + 1/2*e))))/f
```

3.77 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{3a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f}+\frac{5a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}}-\frac{a^2\tan(e+fx)}{f(c-c\sec(e+fx))^{5/2}}$$

```
[Out] (-3*a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/
(4*Sqrt[2]*c^(5/2)*f) - (a^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(5/2)) +
 (5*a^2*Tan[e + f*x])/(4*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.237584, antiderivative size = 130, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3957, 3795, 203}

$$-\frac{3a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f}+\frac{3a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{2f(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-3*a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/
(4*Sqrt[2]*c^(5/2)*f) - ((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(2*f*(c - c
*Sec[e + f*x])^(5/2)) + (3*a^2*Tan[e + f*x])/(4*c*f*(c - c*Sec[e + f*x])^(3
/2))
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} - \frac{(3a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx}{4c}$$

$$= -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{3a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} + \frac{\left(3a^2\right)\int \frac{s}{\sqrt{c-}}}{8}$$

$$= -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{3a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} - \frac{\left(3a^2\right)\text{Subs}}{}$$

$$= -\frac{3a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f} - \frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{3a^2}{4cf(c-}$$

Mathematica [C] time = 2.41095, size = 359, normalized size = 3.07

$$-\frac{a^2\csc\left(\frac{e}{2}\right)e^{-\frac{1}{2}i(e+fx)}\tan\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}(\sec(e+fx)+1)^2\left(3\sin\left(\frac{e}{2}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e}}\right.}{4c^2f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(5/2),
x]
```

```
[Out] -(a^2*Csc[e/2]*Sec[(e + f*x)/2]^3*Sqrt[Sec[e + f*x]]*(1 + Sec[e + f*x])^2*(
((-1 + E^(I*e))*(Cos[(f*x)/2] + I*Sin[(f*x)/2])*((-9*I)*E^(I*e)*(1 + E^(I*e
))*Cos[(f*x)/2] + I*(1 + E^((3*I)*e))*Cos[(3*f*x)/2] - 9*E^(I*e)*Sin[(f*x)/
2] + 9*E^((2*I)*e)*Sin[(f*x)/2] + Sin[(3*f*x)/2] - E^((3*I)*e)*Sin[(3*f*x)/
2]))/(16*E^(((3*I)/2)*e)*Sqrt[Sec[e + f*x]]) + 3*Sqrt[E^(I*(e + f*x))/(1 +
E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e +
f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])]*Sin[e/2]*Sin[(e + f*x)/2]^4
)*Tan[(e + f*x)/2])/(4*c^2*E^((I/2)*(e + f*x))*f*(-1 + Sec[e + f*x])^2*Sqrt
[c - c*Sec[e + f*x]])
```

Maple [B] time = 0.227, size = 230, normalized size = 2.

$$-\frac{a^2\left(-1+\cos\left(fx+e\right)\right)^3}{f\left(\sin\left(fx+e\right)\right)^5}\left(\left(\cos\left(fx+e\right)\right)^2\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}+3\left(\cos\left(fx+e\right)\right)^2\arctan\left(\frac{1}{\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}}\right)-4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -a^2/f*(-1+cos(f*x+e))^3*(cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
+3*cos(f*x+e)^2*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))-4*(-2*cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)-6*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/
(1+cos(f*x+e)))^(1/2))-5*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+3*arctan(1/(-
2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/s
in(f*x+e)^5/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)
```

Maima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.625488, size = 1076, normalized size = 9.2

$$\left[-\frac{3\sqrt{2}\left(a^2\cos\left(fx+e\right)^2-2a^2\cos\left(fx+e\right)+a^2\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right.}{16\left(c^3f\cos\left(fx+e\right)^2-2c^3f\cos\left(fx+e\right)+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/16*(3*sqrt(2)*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(-c)*
log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e)
- 1)*sin(f*x + e)))*sin(f*x + e) + 4*(a^2*cos(f*x + e)^3 - 4*a^2*cos(f*x +
e)^2 - 5*a^2*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^3*f
*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/8*(3*sqrt(
2)*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(c)*arctan(sqrt(2)*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))
*sin(f*x + e) - 2*(a^2*cos(f*x + e)^3 - 4*a^2*cos(f*x + e)^2 - 5*a^2*cos(f*
x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2
*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{\sec\left(e+fx\right)}{c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)-2c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c^2\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int\frac{}{c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2
 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x
) + c)), x) + Integral(2*sec(e + f*x)**2/(c**2*sqrt(-c*sec(e + f*x) + c)*se
```

```
c(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-
c*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**3/(c**2*sqrt(-c*sec(e + f
*x) + c)*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) +
c**2*sqrt(-c*sec(e + f*x) + c)), x))
```

Giac [A] time = 2.19636, size = 181, normalized size = 1.55

$$-\frac{\sqrt{2}a^2\left(\frac{3\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}+5\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}c}{c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}\right)}{8f\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/8*sqrt(2)*a^2*(3*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(5
/2) + (3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2) + 5*sqrt(c*tan(1/2*f*x + 1/2*
e)^2 - c)*c)/(c^4*tan(1/2*f*x + 1/2*e)^4))/(f*sgn(tan(1/2*f*x + 1/2*e)^2 -
1)*sgn(tan(1/2*f*x + 1/2*e)))
```

3.78 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=164

$$-\frac{a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f}-\frac{a^2\tan(e+fx)}{16c^2f(c-c\sec(e+fx))^{3/2}}+\frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^{7/2}}$$

```
[Out] -(a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(1
6*Sqrt[2]*c^(7/2)*f) - ((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*(c - c*
Sec[e + f*x])^(7/2)) + (a^2*Tan[e + f*x])/(4*c*f*(c - c*Sec[e + f*x])^(5/2)
) - (a^2*Tan[e + f*x])/(16*c^2*f*(c - c*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.281871, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3957, 3796, 3795, 203}

$$-\frac{a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f}-\frac{a^2\tan(e+fx)}{16c^2f(c-c\sec(e+fx))^{3/2}}+\frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(7/2),x]
```


```
[Out] -(a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(1
6*Sqrt[2]*c^(7/2)*f) - ((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*(c - c*
Sec[e + f*x])^(7/2)) + (a^2*Tan[e + f*x])/(4*c*f*(c - c*Sec[e + f*x])^(5/2)
) - (a^2*Tan[e + f*x])/(16*c^2*f*(c - c*Sec[e + f*x])^(3/2))
```


Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} - \frac{a\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx}{2c}$$

$$= -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} + \frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}} + \frac{a^2\int \frac{\sec(}{(c-c\sec}}{8c}$$

$$= -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} + \frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}} - \frac{a^2\tan}{16c^2f(c-c}$$

$$= -\frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} + \frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}} - \frac{a^2\tan}{16c^2f(c-c}$$

$$= -\frac{a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f} - \frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} + \frac{a^2\tan}{4cf(c-c}$$

Mathematica [C] time = 5.69167, size = 398, normalized size = 2.43

$$\frac{a^2\csc\left(\frac{e}{2}\right)e^{-\frac{1}{2}i(e+fx)}\tan\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{3}{2}}(e+fx)(\sec(e+fx)+1)^2\left(e^{\frac{ie}{2}}\sqrt{\sec(e+fx)}\left(e^{\frac{ifx}{2}}\sin\left(\frac{fx}{2}\right)\right(1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(7/2),
x]
```

```
[Out] (a^2*Csc[e/2]*Sec[(e + f*x)/2]^3*Sec[e + f*x]^(3/2)*(1 + Sec[e + f*x])^2*(-
3*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*
x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])]
*Sin[e/2]*Sin[(e + f*x)/2]^6 + E^((I/2)*e)*Sqrt[Sec[e + f*x]]*(-4*I + (4*I)
*E^(I*f*x) - (E^((I/2)*f*x)*Cos[e/2]*(-57 + 36*Cos[e + f*x] - 43*Cos[2*(e +
 f*x)])*Sin[(e + f*x)/2])/8 - (7*E^((I/2)*f*x)*Csc[(f*x)/2]*Sin[e]*Sin[f*x]
*Sin[(e + f*x)/2]^6)/2 + E^((I/2)*f*x)*Sin[(f*x)/2]*Sin[(e + f*x)/2]^2*(34
- 43*Sin[(e + f*x)/2]^2 + 14*Sin[e/2]^2*Sin[(e + f*x)/2]^4)))*Tan[(e + f*x)
/2])/(24*c^3*E^((I/2)*(e + f*x))*f*(-1 + Sec[e + f*x])^3*Sqrt[c - c*Sec[e +
 f*x]])
```

Maple [B] time = 0.236, size = 402, normalized size = 2.5

$$-\frac{a^2\left(-1+\cos\left(fx+e\right)\right)^4}{6f\left(\sin\left(fx+e\right)\right)^7}\left(5\left(-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{3/2}\left(\cos\left(fx+e\right)\right)^3+15\left(\cos\left(fx+e\right)\right)^2\left(-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{3/2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] -1/6*a^2/f*(-1+cos(f*x+e))^4*(5*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)*cos(f*
x+e)^3+15*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+3*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^3+3*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e
)))^(1/2))*cos(f*x+e)^3+27*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-
9*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-9*cos(f*x+e)^2*arctan(1
/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))+17*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3
/2)+9*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)+9*cos(f*x+e)*arctan(1
/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))-3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)-3*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos
(f*x+e))^(7/2)/sin(f*x+e)^7/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.664644, size = 1281, normalized size = 7.81

$$\left[-\frac{3\sqrt{2}\left(a^2\cos\left(fx+e\right)^3-3a^2\cos\left(fx+e\right)^2+3a^2\cos\left(fx+e\right)-a^2\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+(3}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right.}{192\left(c^4f\cos\left(fx+e\right)^3-3c^4f\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorithm
="fricas")
```

```
[Out] [-1/192*(3*sqrt(2)*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f
*x + e) - a^2)*sqrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt
(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f
*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 4*(7*a^2*cos(f*x
 + e)^4 + 29*a^2*cos(f*x + e)^3 + 25*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*
f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/96*(3*sqr
t(2)*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) - a^2)
*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e
)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) + 2*(7*a^2*cos(f*x + e)^4 + 29*a^2*c
os(f*x + e)^3 + 25*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e))*sqrt((c*cos(f*x
 + e) - c)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 +
 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.7277, size = 219, normalized size = 1.34

$$-\frac{\sqrt{2}a^2\left(\frac{3\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}+8\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c-3\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}c^2}{c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6}\right)}{96cf\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorithm
="giac")
```

```
[Out] -1/96*sqrt(2)*a^2*(3*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(
5/2) + (3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2) + 8*(c*tan(1/2*f*x + 1/2*e)^
2 - c)^(3/2)*c - 3*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2)/(c^5*tan(1/2*f*x
 + 1/2*e)^6))/(c*f*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)
))
```

3.79 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=171

$$-\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)^3}{3003f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{429f}-\frac{24c^2\tan(e+fx)(a\,\mathrm{s}}{}$$

```
[Out] (-256*c^4*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(3003*f*Sqrt[c - c*Sec[e + f
*x]]) - (64*c^3*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x
])/(429*f) - (24*c^2*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)*Tan[
e + f*x])/(143*f) - (2*c*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)*
Tan[e + f*x])/(13*f)
```

Rubi [A] time = 0.443859, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)^3}{3003f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{429f}-\frac{24c^2\tan(e+fx)(a\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-256*c^4*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(3003*f*Sqrt[c - c*Sec[e + f
*x]]) - (64*c^3*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x
])/(429*f) - (24*c^2*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)*Tan[
e + f*x])/(143*f) - (2*c*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)*
Tan[e + f*x])/(13*f)
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx = -\frac{2c(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{13f}$$

$$= -\frac{24c^2(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\tan(e+f}{143f}$$

$$= -\frac{64c^3(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{429f}$$

$$= -\frac{256c^4(a+a\sec(e+fx))^3\tan(e+fx)}{3003f\sqrt{c-c\sec(e+fx)}} - \frac{64c^3(a+a\sec}{}$$

Mathematica [A] time = 2.59016, size = 88, normalized size = 0.51

$$\frac{4a^3c^3\cos^6\left(\frac{1}{2}(e+fx)\right)(6285\cos(e+fx)-2842\cos(2(e+fx))+835\cos(3(e+fx))-3766)\cot\left(\frac{1}{2}(e+fx)\right)\sec^6(e+}{3003f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (4*a^3*c^3*Cos[(e + f*x)/2]^6*(-3766 + 6285*Cos[e + f*x] - 2842*Cos[2*(e +
f*x)] + 835*Cos[3*(e + f*x)])*Cot[(e + f*x)/2]*Sec[e + f*x]^6*Sqrt[c - c*Se
c[e + f*x]])/(3003*f)
```

Maple [A] time = 0.218, size = 85, normalized size = 0.5

$$\frac{2a^3\left(835\left(\cos\left(fx+e\right)\right)^3-1421\left(\cos\left(fx+e\right)\right)^2+945\cos\left(fx+e\right)-231\right)\left(\sin\left(fx+e\right)\right)^7}{3003f\left(-1+\cos\left(fx+e\right)\right)^7\left(\cos\left(fx+e\right)\right)^3}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] 2/3003*a^3/f*(835*cos(f*x+e)^3-1421*cos(f*x+e)^2+945*cos(f*x+e)-231)*sin(f*
x+e)^7*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)/(-1+cos(f*x+e))^7/cos(f*x+e)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.504792, size = 400, normalized size = 2.34

$$\frac{2\left(835\,a^3c^3\cos\left(fx+e\right)^7+1919\,a^3c^3\cos\left(fx+e\right)^6+271\,a^3c^3\cos\left(fx+e\right)^5-1637\,a^3c^3\cos\left(fx+e\right)^4-103\,a^3c^3\cos\left(fx\right.\right.}{3003\,f\cos\left(fx+e\right)^6\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x, algorithm
="fricas")
```

```
[Out] 2/3003*(835*a^3*c^3*cos(f*x + e)^7 + 1919*a^3*c^3*cos(f*x + e)^6 + 271*a^3*
c^3*cos(f*x + e)^5 - 1637*a^3*c^3*cos(f*x + e)^4 - 103*a^3*c^3*cos(f*x + e)
^3 + 973*a^3*c^3*cos(f*x + e)^2 + 21*a^3*c^3*cos(f*x + e) - 231*a^3*c^3)*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^6*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.74682, size = 153, normalized size = 0.89

$$\frac{128\,\sqrt{2}\left(429\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^3c^5+1001\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^6+819\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^7+231\,c^8\right)a}{3003\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{13}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x, algorithm
="giac")
```

```
[Out] 128/3003*sqrt(2)*(429*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^5 + 1001*(c*tan(1/
2*f*x + 1/2*e)^2 - c)^2*c^6 + 819*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^7 + 231*
c^8)*a^3*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(13/2)*f)
```

3.80 $\int \sec(e + fx)(a + a\sec(e + fx))^3(c - c\sec(e + fx))^{5/2}\,dx$

Optimal. Leaf size=128

$$-\frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{99f}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^3}{693f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3 \ldots}{}$$

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(693*f*Sqrt[c - c*Sec[e + f*x
]]) - (16*c^2*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])
/(99*f) - (2*c*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*
x])/(11*f)
```

Rubi [A] time = 0.323442, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{99f}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^3}{693f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3 \ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(693*f*Sqrt[c - c*Sec[e + f*x
]]) - (16*c^2*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])
/(99*f) - (2*c*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*
x])/(11*f)
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2c(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{11f} +$$
$$= -\frac{16c^2(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{99f} -$$
$$= -\frac{64c^3(a+a\sec(e+fx))^3\tan(e+fx)}{693f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a+a\sec(e+$$

Mathematica [A] time = 1.54198, size = 78, normalized size = 0.61

$$\frac{8a^3c^2\cos^6\left(\frac{1}{2}(e+fx)\right)(-364\cos(e+fx)+151\cos(2(e+fx))+277)\cot\left(\frac{1}{2}(e+fx)\right)\sec^5(e+fx)\sqrt{c-c\sec(e+fx)}}{693f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (8*a^3*c^2*Cos[(e + f*x)/2]^6*(277 - 364*Cos[e + f*x] + 151*Cos[2*(e + f*x)
])*Cot[(e + f*x)/2]*Sec[e + f*x]^5*Sqrt[c - c*Sec[e + f*x]])/(693*f)
```

Maple [A] time = 0.208, size = 75, normalized size = 0.6

$$\frac{2a^3\left(151\left(\cos(fx+e)\right)^2-182\cos(fx+e)+63\right)\left(\sin(fx+e)\right)^7}{693f\left(-1+\cos(fx+e)\right)^6\left(\cos(fx+e)\right)^3}\left(\frac{c\left(-1+\cos(fx+e)\right)}{\cos(fx+e)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 2/693*a^3/f*(151*cos(f*x+e)^2-182*cos(f*x+e)+63)*sin(f*x+e)^7*(c*(-1+cos(f*
x+e))/cos(f*x+e))^(5/2)/(-1+cos(f*x+e))^6/cos(f*x+e)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.488722, size = 355, normalized size = 2.77

$$\frac{2\left(151a^3c^2\cos(fx+e)^6+422a^3c^2\cos(fx+e)^5+241a^3c^2\cos(fx+e)^4-236a^3c^2\cos(fx+e)^3-199a^3c^2\cos(fx+\right.}{693f\cos(fx+e)^5\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] 2/693*(151*a^3*c^2*cos(f*x + e)^6 + 422*a^3*c^2*cos(f*x + e)^5 + 241*a^3*c^
2*cos(f*x + e)^4 - 236*a^3*c^2*cos(f*x + e)^3 - 199*a^3*c^2*cos(f*x + e)^2
+ 70*a^3*c^2*cos(f*x + e) + 63*a^3*c^2)*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))/(f*cos(f*x + e)^5*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 3.54377, size = 116, normalized size = 0.91

$$\frac{64\sqrt{2}\left(99\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^5+154\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^6+63c^7\right)a^3c}{693\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{11}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] 64/693*sqrt(2)*(99*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^5 + 154*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*c^6 + 63*c^7)*a^3*c/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(11/2
)*f)
```

3.81 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=85

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^3}{63f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{9f}$$

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(63*f*Sqrt[c - c*Sec[e + f*x]]
) - (2*c*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(9*f
)
```

Rubi [A] time = 0.208853, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^3}{63f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(63*f*Sqrt[c - c*Sec[e + f*x]]
) - (2*c*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(9*f
)
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx &= -\frac{2c(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{9f}+\frac{1}{9}(\\ &= -\frac{8c^2(a+a\sec(e+fx))^3\tan(e+fx)}{63f\sqrt{c-c\sec(e+fx)}}-\frac{2c(a+a\sec(e+fx)}{}\end{aligned}$$

Mathematica [A] time = 1.02977, size = 66, normalized size = 0.78

$$\frac{16a^3c\cos^6\left(\frac{1}{2}(e+fx)\right)(11\cos(e+fx)-7)\cot\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{c-c\sec(e+fx)}}{63f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (16*a^3*c*Cos[(e + f*x)/2]^6*(-7 + 11*Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e
+ f*x]^4*Sqrt[c - c*Sec[e + f*x]])/(63*f)
```

Maple [A] time = 0.199, size = 65, normalized size = 0.8

$$\frac{2a^3(11\cos(fx+e)-7)(\sin(fx+e))^7}{63f(-1+\cos(fx+e))^5(\cos(fx+e))^3}\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/63*a^3/f*(11*cos(f*x+e)-7)*sin(f*x+e)^7*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3
/2)/(-1+cos(f*x+e))^5/cos(f*x+e)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.479186, size = 290, normalized size = 3.41

$$\frac{2\left(11a^3c\cos(fx+e)^5+37a^3c\cos(fx+e)^4+38a^3c\cos(fx+e)^3+2a^3c\cos(fx+e)^2-17a^3c\cos(fx+e)-7a^3c\right.}{63f\cos(fx+e)^4\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2/63*(11*a^3*c*cos(f*x + e)^5 + 37*a^3*c*cos(f*x + e)^4 + 38*a^3*c*cos(f*x
+ e)^3 + 2*a^3*c*cos(f*x + e)^2 - 17*a^3*c*cos(f*x + e) - 7*a^3*c)*sqrt((c*
```

```
cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.44048, size = 85, normalized size = 1.

$$\frac{32\sqrt{2}\left(9\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^6+7c^7\right)a^3}{63\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{9}{2}}cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] 32/63*sqrt(2)*(9*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^6 + 7*c^7)*a^3/((c*tan(1/
2*f*x + 1/2*e)^2 - c)^(9/2)*c*f)
```

3.82 $\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=41

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3}{7f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (-2*c*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.09561, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3953}

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3}{7f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*c*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.70759, size = 55, normalized size = 1.34

$$\frac{16a^3\cos^6\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx)}}{7f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (16*a^3*Cos[(e + f*x)/2]^6*Cot[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[c - c*Sec[e
 + f*x]])/(7*f)
```

Maple [A] time = 0.248, size = 55, normalized size = 1.3

$$\frac{2\,a^3\left(\sin\left(fx+e\right)\right)^7}{7\,f\left(\cos\left(fx+e\right)\right)^3\left(-1+\cos\left(fx+e\right)\right)^4}\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/7*a^3/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^7/cos(f*x+e)^3/(-
1+cos(f*x+e))^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.468582, size = 231, normalized size = 5.63

$$\frac{2\left(a^{3}\cos\left(fx+e\right)^{4}+4\,a^{3}\cos\left(fx+e\right)^{3}+6\,a^{3}\cos\left(fx+e\right)^{2}+4\,a^{3}\cos\left(fx+e\right)+a^{3}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{7\,f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] 2/7*(a^3*cos(f*x + e)^4 + 4*a^3*cos(f*x + e)^3 + 6*a^3*cos(f*x + e)^2 + 4*a
^3*cos(f*x + e) + a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x +
 e)^3*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{3}\left(\int\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)dx+\int 3\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)dx+\int 3\sqrt{-c\sec\left(e+fx\right)+c}\sec^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(3*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(3*sqrt(-c*sec(e + f*x)
 + c)*sec(e + f*x)**3, x) + Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)
**4, x))
```

Giac [A] time = 1.55148, size = 46, normalized size = 1.12

$$\frac{16\sqrt{2}a^3c^4}{7\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{7}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] 16/7*sqrt(2)*a^3*c^4/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(7/2)*f)
```

3.83 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=164

$$-\frac{8\sqrt{2}a^3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f}+\frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}+\frac{4\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3f\sqrt{c-c\sec(e+fx)}}+\frac{2a\tan(e+fx)(a\sec(e+}{5f\sqrt{c-c\sec(e+}$$

```
[Out] (-8*Sqrt[2]*a^3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f
*x]])])/(Sqrt[c]*f) + (8*a^3*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]]) + (
2*a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[c - c*Sec[e + f*x]]) + (
4*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.325402, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3956, 3795, 203}

$$-\frac{8\sqrt{2}a^3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f}+\frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}+\frac{4\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3f\sqrt{c-c\sec(e+fx)}}+\frac{2a\tan(e+fx)(a\sec(e+}{5f\sqrt{c-c\sec(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/Sqrt[c - c*Sec[e + f*x]],x]
```


```
[Out] (-8*Sqrt[2]*a^3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f
*x]])])/(Sqrt[c]*f) + (8*a^3*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]]) + (
2*a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[c - c*Sec[e + f*x]]) + (
4*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```


Rule 3956

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*d*Cot[e +
 f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]),
 x] + Dist[(2*c*(2*n - 1))/(2*n - 1), Int[(Csc[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}dx = \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + \frac{4\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} + (\cdots$$
$$= \frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + \frac{4\left(a^3+a^3\sec(e\cdots\right.}{3f\sqrt{c-c\cdots}}$$
$$= \frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + \frac{4\left(a^3+a^3\sec(e\cdots\right.}{3f\sqrt{c-c\cdots}}$$
$$= -\frac{8\sqrt{2}a^3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f} + \frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2a(a+a\sec(e+f\cdots}{5f\sqrt{c-c\sec\cdots}}$$

Mathematica [C] time = 1.53253, size = 185, normalized size = 1.13

$$\frac{4a^3e^{-\frac{1}{2}i(e+fx)}\sin\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)\left(3\sec^2(e+fx)+16\sec\cdots\right.\right.}{15f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (4*a^3*Sec[e + f*x]*((-30*Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1
+ E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/E^((I/2)*(e +
f*x)) + Cos[(e + f*x)/2]*(73 + 16*Sec[e + f*x] + 3*Sec[e + f*x]^2))*(Cos[(e
 + f*x)/2] + I*Sin[(e + f*x)/2])*Sin[(e + f*x)/2])/(15*E^((I/2)*(e + f*x))*
f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.273, size = 206, normalized size = 1.3

$$-\frac{2a^3\sin(fx+e)}{15f(\cos(fx+e))^3}\left(15(\cos(fx+e))^2\arctan\left(\frac{1}{\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}}\right)\left(-2\frac{\cos(fx+e)}{1+\cos(fx+e)}\right)^{5/2}+30\cos(fx+e)\text{arcta}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/15*a^3/f*(15*cos(f*x+e)^2*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))
*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+30*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)
/(1+cos(f*x+e)))^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+15*arctan(1/(-
2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)-73
*cos(f*x+e)^2-16*cos(f*x+e)-3)*sin(f*x+e)/cos(f*x+e)^3/(c*(-1+cos(f*x+e))/c
os(f*x+e))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.616898, size = 942, normalized size = 5.74

$$\left[\frac{2\left(30\sqrt{2}a^3c\sqrt{-\frac{1}{c}}\cos\left(fx+e\right)^2\log\left(-\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}-\left(3\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-\left(73\right.\right.}{15cf\cos\left(fx+e\right)^2\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [2/15*(30*sqrt(2)*a^3*c*sqrt(-1/c)*cos(f*x + e)^2*log(-(2*sqrt(2)*(cos(f*x
+ e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) -
 (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(
f*x + e) - (73*a^3*cos(f*x + e)^3 + 89*a^3*cos(f*x + e)^2 + 19*a^3*cos(f*x
+ e) + 3*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*cos(f*x + e)^2*
sin(f*x + e)), 2/15*(60*sqrt(2)*a^3*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x
+ e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*cos(f*x + e)^2
*sin(f*x + e) - (73*a^3*cos(f*x + e)^3 + 89*a^3*cos(f*x + e)^2 + 19*a^3*cos
(f*x + e) + 3*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*cos(f*x +
e)^2*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int\frac{\sec\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int\frac{3\sec^2\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int\frac{3\sec^3\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int\frac{\sec^4\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/sqrt(-c*sec(e + f*x) + c), x) + Integral(3*sec(
e + f*x)**2/sqrt(-c*sec(e + f*x) + c), x) + Integral(3*sec(e + f*x)**3/sqrt
(-c*sec(e + f*x) + c), x) + Integral(sec(e + f*x)**4/sqrt(-c*sec(e + f*x) +
 c), x))
```

Giac [C] time = 2.16939, size = 308, normalized size = 1.88

$$-\frac{2\left(4\,a^3c^3\left(\frac{15\,\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{7}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{\sqrt{2}\left(15\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2-5\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c+3\,c^2\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{5}{2}}c^3\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)+\frac{\left(60i\,\sqrt{2}a^{\cdots}\right.}{\cdots}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] -2/15*(4*a^3*c^3*(15*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt
(c))/(c^(7/2)*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e))) +
sqrt(2)*(15*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2 - 5*(c*tan(1/2*f*x + 1/2*e)^2
- c)*c + 3*c^2)/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*c^3*sgn(tan(1/2*f*x +
 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)))) + (60*I*sqrt(2)*a^3*sqrt(-c)*arc
tan(-I) - 92*sqrt(2)*a^3*sqrt(-c))*sgn(tan(1/2*f*x + 1/2*e))/c)/f
```

3.84 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=168

$$\frac{10\sqrt{2}a^3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}-\frac{5\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3cf\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)(a\sec(e+}{f(c-c\sec(e+fx)}$$

```
[Out] (10*Sqrt[2]*a^3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f
*x]])])/(c^(3/2)*f) - (a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec
[e + f*x])^(3/2)) - (10*a^3*Tan[e + f*x])/(c*f*Sqrt[c - c*Sec[e + f*x]]) -
(5*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(3*c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.336487, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3957, 3956, 3795, 203}

$$\frac{10\sqrt{2}a^3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}-\frac{5\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3cf\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)(a\sec(e+}{f(c-c\sec(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (10*Sqrt[2]*a^3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f
*x]])])/(c^(3/2)*f) - (a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec
[e + f*x])^(3/2)) - (10*a^3*Tan[e + f*x])/(c*f*Sqrt[c - c*Sec[e + f*x]]) -
(5*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(3*c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3956

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*d*Cot[e +
 f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]),
 x] + Dist[(2*c*(2*n - 1))/(2*n - 1), Int[(Csc[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{(5a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx}{2c}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{5\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{3cf\sqrt{c-c\sec(e+fx)}} - \ldots$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{5\left(a^3+a^3\sec(e\ldots\right.}{3cf\sqrt{c-\ldots}}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{5\left(a^3+a^3\sec(e\ldots\right.}{3cf\sqrt{c-\ldots}}$$

$$= \frac{10\sqrt{2}a^3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{10a^3\ldots}{cf\sqrt{c\ldots}}$$

Mathematica [C] time = 2.96077, size = 324, normalized size = 1.93

$$-\frac{a^3\csc\left(\frac{e}{2}\right)e^{-\frac{1}{2}i(e+fx)}\tan^3\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^3\left(-\frac{i\left(-1+e^{ie}\right)e^{\frac{ifx}{2}}\left(-24e^{i(e+fx)}+34e^{2i(e+fx)}-24e^{3i(e+fx)}+19\ldots\right)}{2\left(-1+e^{i(e+fx)}\right)^2\left(1+e^{2i(e+fx)}\right)}\ldots\right.}{3cf(\sec(e+fx)-1)\sec^{\frac{3}{2}}(e+fx)\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(3/2),
x]
```

```
[Out] -(a^3*Csc[e/2]*Sec[(e + f*x)/2]^2*(1 + Sec[e + f*x])^3*(((-I/2)*E^((I/2)*f*
x)*(-1 + E^(I*e))*(19 - 24*E^(I*(e + f*x)) + 34*E^((2*I)*(e + f*x)) - 24*E^
((3*I)*(e + f*x)) + 19*E^((4*I)*(e + f*x)))*Sqrt[Sec[e + f*x]])/((-1 + E^(I
*(e + f*x)))^2*(1 + E^((2*I)*(e + f*x)))) - 15*Sqrt[E^(I*(e + f*x))/(1 + E^
((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*
x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])]*Sec[(e + f*x)/2]*Sin[e/2])*Ta
n[(e + f*x)/2]^3)/(3*c*E^((I/2)*(e + f*x))*f*(-1 + Sec[e + f*x])*Sec[e + f*
x]^(3/2)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.232, size = 157, normalized size = 0.9

$$-\frac{a^3\sin\left(fx+e\right)}{3f\left(\cos\left(fx+e\right)\right)^3}\left(15\arctan\left(\frac{1}{\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}}\right)\left(-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{3/2}\left(\cos\left(fx+e\right)\right)^2-15\arctan\left(\frac{1}{\sqrt{-2\frac{\cos(\ldots}{1+\cos\ldots}}}\right)\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/3*a^3/f*(15*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(3/2)*cos(f*x+e)^2-15*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+38*cos(f*x+e)^2-24*cos(f*x
+e)-2)*sin(f*x+e)/cos(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.655823, size = 1067, normalized size = 6.35

$$\left[\frac{15\sqrt{2}\left(a^{3}c\cos\left(fx+e\right)^{2}-a^{3}c\cos\left(fx+e\right)\right)\sqrt{-\frac{1}{c}}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{c}}+\left(3\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin}{3\left(c^{2}f\cos\left(fx+e\right)^{2}-c^{2}f\cos\left(fx+e\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/3*(15*sqrt(2)*(a^3*c*cos(f*x + e)^2 - a^3*c*cos(f*x + e))*sqrt(-1/c)*log
((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*sqrt(-1/c) + (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1
)*sin(f*x + e)))*sin(f*x + e) + 2*(19*a^3*cos(f*x + e)^3 + 7*a^3*cos(f*x +
e)^2 - 13*a^3*cos(f*x + e) - a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/
((c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e))*sin(f*x + e)), -2/3*(15*sqrt(2
)*(a^3*c*cos(f*x + e)^2 - a^3*c*cos(f*x + e))*arctan(sqrt(2)*sqrt((c*cos(f*
x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e)
/sqrt(c) - (19*a^3*cos(f*x + e)^3 + 7*a^3*cos(f*x + e)^2 - 13*a^3*cos(f*x +
 e) - a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e)^2
- c^2*f*cos(f*x + e))*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int \frac{\sec\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*
sqrt(-c*sec(e + f*x) + c)), x) + Integral(3*sec(e + f*x)**2/(-c*sqrt(-c*sec
(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x) + Integral(3
*sec(e + f*x)**3/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec
(e + f*x) + c)), x) + Integral(sec(e + f*x)**4/(-c*sqrt(-c*sec(e + f*x) + c
)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x))
```

Giac [A] time = 1.82238, size = 300, normalized size = 1.79

$$\frac{2\,a^3c^2\left(\frac{15\,\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{7}{2}}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{2\,\sqrt{2}\left(6\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-7\,c\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}c^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{\ldots}{c^4\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,f\ldots\right.\right.}\right.}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] 2/3*a^3*c^2*(15*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/
(c^(7/2)*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e))) + 2*sqr
t(2)*(6*c*tan(1/2*f*x + 1/2*e)^2 - 7*c)/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/
2)*c^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e))) + 3*sqrt(
2)*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/(c^4*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*
sgn(tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e)^2))/f
```

3.85 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{15a^3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f}+\frac{15a^3\tan(e+fx)}{4c^2f\sqrt{c-c\sec(e+fx)}}+\frac{5\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{4cf(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)(a\sec(e+}{2f(c-c\sec(e+f}$$

```
[Out] (-15*a^3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])
/(2*Sqrt[2]*c^(5/2)*f) - (a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(2*f*(c -
c*Sec[e + f*x])^(5/2)) + (5*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(4*c*f*(
c - c*Sec[e + f*x])^(3/2)) + (15*a^3*Tan[e + f*x])/(4*c^2*f*Sqrt[c - c*Sec[
e + f*x]])
```

Rubi [A] time = 0.351981, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3957, 3956, 3795, 203}

$$-\frac{15a^3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f}+\frac{15a^3\tan(e+fx)}{4c^2f\sqrt{c-c\sec(e+fx)}}+\frac{5\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{4cf(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)(a\sec(e+}{2f(c-c\sec(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-15*a^3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])
/(2*Sqrt[2]*c^(5/2)*f) - (a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(2*f*(c -
c*Sec[e + f*x])^(5/2)) + (5*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(4*c*f*(
c - c*Sec[e + f*x])^(3/2)) + (15*a^3*Tan[e + f*x])/(4*c^2*f*Sqrt[c - c*Sec[
e + f*x]])
```

Rule 3957

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 3956

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*d*Cot[e +
 f*x]*(c + d*Csc[e + f*x])^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]),
 x] + Dist[(2*c*(2*n - 1))/(2*n - 1), Int[(Csc[e + f*x]*(c + d*Csc[e + f*x]
)^(n - 1))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
```

```
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}dx = -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} - \frac{(5a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}dx}{4c}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} + \cdots$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} + \cdots$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} + \cdots$$

$$= -\frac{15a^3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} - \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5\left(a^3+\cdots\right.}{4c\cdots}$$

Mathematica [C] time = 4.5866, size = 263, normalized size = 1.51

$$-\frac{a^3e^{-\frac{1}{2}i(2e+fx)}\tan^5\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^3\left(120e^{\frac{ie}{2}}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+f\cdots}}}\right.\right.}{32c^2f(\sec(e+fx)-1)^2\sqrt{\sec(e\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(5/2),
x]
```

```
[Out] -(a^3*Sec[(e + f*x)/2]*(1 + Sec[e + f*x])^3*(120*E^((I/2)*e)*Sqrt[E^(I*(e +
 f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1
+ E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])] + (25*Cos[(3*(e
 + f*x))/2] - 9*Cos[(5*(e + f*x))/2])*Csc[(e + f*x)/2]^4*Sqrt[Sec[e + f*x]]
*(Cos[e + (f*x)/2] + I*Sin[e + (f*x)/2]))*Tan[(e + f*x)/2]^5)/(32*c^2*E^((I
/2)*(2*e + f*x))*f*(-1 + Sec[e + f*x])^2*Sqrt[Sec[e + f*x]]*Sqrt[c - c*Sec[
e + f*x]])
```

Maple [A] time = 0.233, size = 206, normalized size = 1.2

$$-\frac{a^3\sin(fx+e)}{4f(\cos(fx+e))^3}\left(15(\cos(fx+e))^2\arctan\left(\frac{1}{\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}}\right)\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}-30\arctan\left(\frac{1}{\sqrt{-2\frac{\cos(f\cdots}{1+\cos(f\cdots}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/4*a^3/f*(15*cos(f*x+e)^2*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))*
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-30*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)-18*cos(f*x+e)^2
+15*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))*(-2*cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)+34*cos(f*x+e)-8)*sin(f*x+e)/cos(f*x+e)^3/(c*(-1+cos(f*x+e))/co
s(f*x+e))^(5/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.668452, size = 1107, normalized size = 6.36

$$\left[-\frac{15\sqrt{2}\left(a^3\cos\left(fx+e\right)^2-2\,a^3\cos\left(fx+e\right)+a^3\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3\,c\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right.}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right.}{8\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/8*(15*sqrt(2)*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*sqrt(-c)*
log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e)
- 1)*sin(f*x + e)))*sin(f*x + e) + 4*(9*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x
+ e)^2 - 13*a^3*cos(f*x + e) + 4*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e
)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1
/4*(15*sqrt(2)*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*sqrt(c)*arct
an(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*si
n(f*x + e)))*sin(f*x + e) - 2*(9*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x + e)^2
- 13*a^3*cos(f*x + e) + 4*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c
^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int\frac{\sec\left(e+fx\right)}{c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)-2c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c^2\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int\frac{}{c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2
 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x
) + c)), x) + Integral(3*sec(e + f*x)**2/(c**2*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-
c*sec(e + f*x) + c)), x) + Integral(3*sec(e + f*x)**3/(c**2*sqrt(-c*sec(e +
 f*x) + c)*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)
+ c**2*sqrt(-c*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**4/(c**2*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x) + c**2*sqrt(-c*sec(e + f*x) + c)), x))
```

Giac [A] time = 2.47044, size = 305, normalized size = 1.75

$$-\frac{a^3 c\left(\frac{15\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{7}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{8\sqrt{2}}{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^3\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(7\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}+9\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c\right)}{c^5\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/4*a^3*c*(15*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/(
c^(7/2)*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e))) + 8*sqrt
(2)/(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)
*sgn(tan(1/2*f*x + 1/2*e))) + sqrt(2)*(7*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/
2) + 9*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c)/(c^5*sgn(tan(1/2*f*x + 1/2*e)^
2 - 1)*sgn(tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e)^4))/f
```

3.86 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=142

$$\frac{128c^4\tan(e+fx)}{5af\sqrt{c-c\sec(e+fx)}}+\frac{32c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5af}+\frac{12c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5af}+\frac{2c\tan(e+fx)(c}{f(a\sec(}$$

```
[Out] (128*c^4*Tan[e + f*x])/(5*a*f*Sqrt[c - c*Sec[e + f*x]]) + (32*c^3*Sqrt[c -
c*Sec[e + f*x]]*Tan[e + f*x])/(5*a*f) + (12*c^2*(c - c*Sec[e + f*x])^(3/2)*
Tan[e + f*x])/(5*a*f) + (2*c*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(a
 + a*Sec[e + f*x]))
```

Rubi [A] time = 0.233667, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3954, 3793, 3792}

$$\frac{128c^4\tan(e+fx)}{5af\sqrt{c-c\sec(e+fx)}}+\frac{32c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5af}+\frac{12c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5af}+\frac{2c\tan(e+fx)(c}{f(a\sec(}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (128*c^4*Tan[e + f*x])/(5*a*f*Sqrt[c - c*Sec[e + f*x]]) + (32*c^3*Sqrt[c -
c*Sec[e + f*x]]*Tan[e + f*x])/(5*a*f) + (12*c^2*(c - c*Sec[e + f*x])^(3/2)*
Tan[e + f*x])/(5*a*f) + (2*c*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(a
 + a*Sec[e + f*x]))
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(6c)\int \sec(e+fx)(c-c\sec(e+fx))^{5/}}{a}$$
$$= \frac{12c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af} + \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{f(a+a\sec(e+fx))}$$
$$= \frac{32c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5af} + \frac{12c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af}$$
$$= \frac{128c^4\tan(e+fx)}{5af\sqrt{c-c\sec(e+fx)}} + \frac{32c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5af} + \frac{12c^2(c-c\,\mathrm{se}}{}$$

Mathematica [A] time = 0.756324, size = 86, normalized size = 0.61

$$-\frac{c^3(245\cos(e+fx)+86\cos(2(e+fx))+91\cos(3(e+fx))+90)\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx)}}{10af(\cos(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -(c^3*(90 + 245*Cos[e + f*x] + 86*Cos[2*(e + f*x)] + 91*Cos[3*(e + f*x)])*C
ot[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[c - c*Sec[e + f*x]])/(10*a*f*(1 + Cos[e
 + f*x]))
```

Maple [A] time = 0.187, size = 83, normalized size = 0.6

$$-\frac{\left(182\left(\cos\left(fx+e\right)\right)^3+86\left(\cos\left(fx+e\right)\right)^2-14\cos\left(fx+e\right)+2\right)\cos\left(fx+e\right)}{5fa\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^3}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] -2/5/a/f*(91*cos(f*x+e)^3+43*cos(f*x+e)^2-7*cos(f*x+e)+1)*cos(f*x+e)*(c*(-1
+cos(f*x+e))/cos(f*x+e))^(7/2)/sin(f*x+e)/(-1+cos(f*x+e))^3
```

Maxima [A] time = 1.56788, size = 220, normalized size = 1.55

$$\frac{8\left(16\sqrt{2}c^{\frac{7}{2}}-\frac{56\sqrt{2}c^{\frac{7}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{70\sqrt{2}c^{\frac{7}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{35\sqrt{2}c^{\frac{7}{2}}\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}+\frac{5\sqrt{2}c^{\frac{7}{2}}\sin\left(fx+e\right)^8}{\left(\cos\left(fx+e\right)+1\right)^8}\right)}{5af\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 8/5*(16*sqrt(2)*c^(7/2) - 56*sqrt(2)*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 70*sqrt(2)*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 35*sqrt(2)
*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5*sqrt(2)*c^(7/2)*sin(f*x +
e)^8/(cos(f*x + e) + 1)^8)/(a*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(7/2)
*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(7/2))
```

Fricas [A] time = 0.481039, size = 209, normalized size = 1.47

$$-\frac{2\left(91\,c^{3}\cos\left(fx+e\right)^{3}+43\,c^{3}\cos\left(fx+e\right)^{2}-7\,c^{3}\cos\left(fx+e\right)+c^{3}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{5\,af\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -2/5*(91*c^3*cos(f*x + e)^3 + 43*c^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) +
c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(a*f*cos(f*x + e)^2*sin(f*x +
e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.06639, size = 151, normalized size = 1.06

$$-\frac{8\sqrt{2}\left(5\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}\,c-\frac{15\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{2}c^{2}+5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)c^{3}+c^{4}}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{5}{2}}}\right)c^{2}}{5\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] -8/5*sqrt(2)*(5*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c - (15*(c*tan(1/2*f*x +
 1/2*e)^2 - c)^2*c^2 + 5*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^3 + c^4)/(c*tan(1
/2*f*x + 1/2*e)^2 - c)^(5/2))*c^2/(a*f)
```

3.87 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=108

$$\frac{32c^3\tan(e+fx)}{3af\sqrt{c-c\sec(e+fx)}}+\frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3af}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}$$

```
[Out] (32*c^3*Tan[e + f*x])/(3*a*f*Sqrt[c - c*Sec[e + f*x]]) + (8*c^2*Sqrt[c - c*
Sec[e + f*x]]*Tan[e + f*x])/(3*a*f) + (2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e
 + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rubi [A] time = 0.190516, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3954, 3793, 3792}

$$\frac{32c^3\tan(e+fx)}{3af\sqrt{c-c\sec(e+fx)}}+\frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3af}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (32*c^3*Tan[e + f*x])/(3*a*f*Sqrt[c - c*Sec[e + f*x]]) + (8*c^2*Sqrt[c - c*
Sec[e + f*x]]*Tan[e + f*x])/(3*a*f) + (2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e
 + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(4c)\int \sec(e+fx)(c-c\sec(e+fx))^{3/2}\,dx}{a}$$
$$= \frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3af} + \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(16\ldots}{}$$
$$= \frac{32c^3\tan(e+fx)}{3af\sqrt{c-c\sec(e+fx)}} + \frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3af} + \frac{2c(c-c\sec(e+\ldots}{f(a+a\,\mathrm{s}\ldots}$$

Mathematica [A] time = 0.429616, size = 74, normalized size = 0.69

$$-\frac{c^2(20\cos(e+fx)+23\cos(2(e+fx))+21)\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{3af(\cos(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -(c^2*(21 + 20*Cos[e + f*x] + 23*Cos[2*(e + f*x)])*Cot[(e + f*x)/2]*Sec[e +
 f*x]*Sqrt[c - c*Sec[e + f*x]])/(3*a*f*(1 + Cos[e + f*x]))
```

Maple [A] time = 0.177, size = 73, normalized size = 0.7

$$-\frac{\left(46\left(\cos\left(fx+e\right)\right)^2+20\cos\left(fx+e\right)-2\right)\cos\left(fx+e\right)}{3fa\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^2}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] -2/3/a/f*(23*cos(f*x+e)^2+10*cos(f*x+e)-1)*cos(f*x+e)*(c*(-1+cos(f*x+e))/co
s(f*x+e))^(5/2)/sin(f*x+e)/(-1+cos(f*x+e))^2
```

Maxima [A] time = 1.50242, size = 185, normalized size = 1.71

$$-\frac{4\left(8\sqrt{2}c^{\frac{5}{2}}-\frac{20\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{15\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{3\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}\right)}{3af\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -4/3*(8*sqrt(2)*c^(5/2) - 20*sqrt(2)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 15*sqrt(2)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3*sqrt(2)*
c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/(a*f*(sin(f*x + e)/(cos(f*x +
```

```
e) + 1) + 1)^(5/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

Fricas [A] time = 0.469432, size = 176, normalized size = 1.63

$$-\frac{2\left(23\,c^{2}\cos\left(fx+e\right)^{2}+10\,c^{2}\cos\left(fx+e\right)-c^{2}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\,af\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -2/3*(23*c^2*cos(f*x + e)^2 + 10*c^2*cos(f*x + e) - c^2)*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e))/(a*f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.63695, size = 115, normalized size = 1.06

$$-\frac{4\sqrt{2}\left(3\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}\,c-\frac{6\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c^{2}+c^{3}}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{3}{2}}}\right)c}{3\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] -4/3*sqrt(2)*(3*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c - (6*(c*tan(1/2*f*x +
1/2*e)^2 - c)*c^2 + c^3)/(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2))*c/(a*f)
```

3.88 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=72

$$\frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}}+\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)}$$

```
[Out] (4*c^2*Tan[e + f*x])/(a*f*Sqrt[c - c*Sec[e + f*x]]) + (2*c*Sqrt[c - c*Sec[e
 + f*x]]*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rubi [A] time = 0.152922, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3954, 3792}

$$\frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}}+\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (4*c^2*Tan[e + f*x])/(a*f*Sqrt[c - c*Sec[e + f*x]]) + (2*c*Sqrt[c - c*Sec[e
 + f*x]]*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx &= \frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{(2c)\int \sec(e+fx)\sqrt{c-c\sec(e+fx)}\,dx}{a}\\ &= \frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}}+\frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(a+a\sec(e+fx))}\end{aligned}$$

Mathematica [A] time = 0.241361, size = 54, normalized size = 0.75

$$-\frac{2c(3\cos(e+fx)+1)\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{af(\cos(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-2*c*(1 + 3*Cos[e + f*x])*Cot[(e + f*x)/2]*Sqrt[c - c*Sec[e + f*x]])/(a*f*
(1 + Cos[e + f*x]))
```

Maple [A] time = 0.174, size = 63, normalized size = 0.9

$$-2\,\frac{\left(3\,\cos\left(fx+e\right)+1\right)\cos\left(fx+e\right)}{fa\sin\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] -2/a/f*(3*cos(f*x+e)+1)*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin
(f*x+e)/(-1+cos(f*x+e))
```

Maxima [A] time = 1.52169, size = 149, normalized size = 2.07

$$\frac{2\left(2\,\sqrt{2}c^{\frac{3}{2}}-\frac{3\,\sqrt{2}c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{\sqrt{2}c^{\frac{3}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}\right)}{af\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 2*(2*sqrt(2)*c^(3/2) - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + sqrt(2)*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/(a*f*(sin(f*x + e)
/(cos(f*x + e) + 1) + 1)^(3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(3/2))
```

Fricas [A] time = 0.462894, size = 116, normalized size = 1.61

$$-\frac{2\left(3\,c\cos\left(fx+e\right)+c\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{af\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -2*(3*c*cos(f*x + e) + c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(a*f*sin(
f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)}{\sec(e+fx)+1}\,dx + \int -\frac{c\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x) + 1), x) +
 Integral(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2/(sec(e + f*x) + 1),
x))/a
```

Giac [A] time = 1.49003, size = 86, normalized size = 1.19

$$-\frac{2\sqrt{2}\left(\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^2-\frac{c^3}{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}\right)}{acf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] -2*sqrt(2)*(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2 - c^3/sqrt(c*tan(1/2*f*x
 + 1/2*e)^2 - c))/(a*c*f)
```

3.89 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=39

$$\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (2*c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.106548, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3953}

$$\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.131101, size = 29, normalized size = 0.74

$$-\frac{2\cot(e+fx)\sqrt{c-c\sec(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-2*Cot[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a*f)
```

Maple [A] time = 0.235, size = 43, normalized size = 1.1

$$-2\,\frac{\cos\left(fx+e\right)}{fa\sin\left(fx+e\right)}\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] -2/a/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)/sin(f*x+e)
```

Maxima [B] time = 1.52542, size = 113, normalized size = 2.9

$$-\frac{\sqrt{2}\sqrt{c}-\frac{\sqrt{2}\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}}{af\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1}\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -(sqrt(2)*sqrt(c) - sqrt(2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)/(a
*f*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e
) + 1) - 1))
```

Fricas [A] time = 0.456224, size = 103, normalized size = 2.64

$$-\frac{2\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{af\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -2*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(a*f*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x) + 1), x)/a
```

Giac [A] time = 1.42032, size = 85, normalized size = 2.18

$$-\frac{\sqrt{2}\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] -sqrt(2)*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*sgn(tan(1/2*f*x + 1/2*e)^3 + ta
n(1/2*f*x + 1/2*e))*sgn(cos(f*x + e))/(a*f)
```

3.90 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}$$

```
[Out] -(ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqrt[2
]*a*Sqrt[c]*f)) + Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f
*x]])
```

Rubi [A] time = 0.157163, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3960, 3795, 203}

$$\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}-\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -(ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqrt[2
]*a*Sqrt[c]*f)) + Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f
*x]])
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}dx = \frac{\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{2a}$$
$$= \frac{\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{2c+x^2}dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{af}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.495587, size = 155, normalized size = 1.74

$$-\frac{i\left(-1+e^{2i(e+fx)}\right)\left(2\left(1+e^{2i(e+fx)}\right)-\sqrt{2}\left(1+e^{i(e+fx)}\right)\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)\right)}{2af\left(1+e^{2i(e+fx)}\right)^2(\sec(e+fx)+1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] ((-I/2)*(-1 + E^((2*I)*(e + f*x)))*(2*(1 + E^((2*I)*(e + f*x))) - Sqrt[2]*(
1 + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f
*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])]))/(a*(1 + E^((2*I)*(e + f*x)
))^2*f*(1 + Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.194, size = 107, normalized size = 1.2

$$-\frac{-1+\cos(fx+e)}{fa\sin(fx+e)}\left(\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}+\arctan\left(\frac{1}{\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}}\right)\right)\frac{1}{\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}}\frac{1}{\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/a/f*(-1+cos(f*x+e))*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+arctan(1/(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(
f*x+e)/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)\sqrt{-c\sec(fx+e)+c}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)), x)
```

Fricas [A] time = 0.588019, size = 675, normalized size = 7.58

$$\left[\frac{\sqrt{2}c\sqrt{-\frac{1}{c}}\log\left(-\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{c}}-\left(3\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-4\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+\right.}{4acf\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*(sqrt(2)*c*sqrt(-1/c)*log(-(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) - (3*cos(f*x + e) + 1)*s
in(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 4*sqrt((c*co
s(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqrt
(2)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x
+ e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+\sqrt{-c\sec\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + sqrt(-c*sec
(e + f*x) + c)), x)/a
```

Giac [C] time = 1.40476, size = 184, normalized size = 2.07

$$-\frac{\frac{\sqrt{2}\left(i\sqrt{-c}\arctan\left(-i\right)-\sqrt{-c}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{ac}+\frac{\sqrt{2}\left(\frac{\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{c}\right)}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] -1/2*(sqrt(2)*(I*sqrt(-c)*arctan(-I) - sqrt(-c))*sgn(tan(1/2*f*x + 1/2*e))/
(a*c) + sqrt(2)*(arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt(c)
 - sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/c)/(a*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)
*sgn(tan(1/2*f*x + 1/2*e))))/f
```

3.91 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f}-\frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}$$

```
[Out] (-3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(4*S
qrt[2]*a*c^(3/2)*f) - (3*Tan[e + f*x])/(4*a*f*(c - c*Sec[e + f*x])^(3/2)) +
 Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.210511, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3960, 3796, 3795, 203}

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f}-\frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (-3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(4*S
qrt[2]*a*c^(3/2)*f) - (3*Tan[e + f*x])/(4*a*f*(c - c*Sec[e + f*x])^(3/2)) +
 Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2))
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}} + \frac{3\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}\,dx}{2a}$$

$$= -\frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/}}$$

$$= -\frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/}}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f} - \frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}} + \frac{}{f(a+a\sec}$$

Mathematica [C] time = 1.43111, size = 183, normalized size = 1.5

$$\frac{e^{-\frac{1}{2}i(e+fx)}\csc(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-8(\cos(e+fx)-3)+\frac{6\sqrt{2}e^{-i(e+fx)}\left(-1+e^{i(e+fx)}\right)^2\left(1+e^{i(e+fx)}\right)\tanh^{-}}{\sqrt{1+e^{2i(e+fx)}}}\right.}{32acf\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (((6*Sqrt[2]*(-1 + E^(I*(e + f*x)))^2*(1 + E^(I*(e + f*x)))*ArcTanh[(1 + E^
(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/(E^(I*(e + f*x))*S
qrt[1 + E^((2*I)*(e + f*x))]) - 8*(-3 + Cos[e + f*x]))*Csc[e + f*x]*(Cos[(e
 + f*x)/2] + I*Sin[(e + f*x)/2]))/(32*a*c*E^((I/2)*(e + f*x))*f*Sqrt[c - c*
Sec[e + f*x]])
```

Maple [B] time = 0.21, size = 266, normalized size = 2.2

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^2}{2fa\left(\sin\left(fx+e\right)\right)^3}\left(\cos\left(fx+e\right)\left(-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{5}{2}}+\left(-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{\frac{5}{2}}+\cos\left(fx+e\right)\left(-2\frac{\cos\left(fx+\right.}{1+\cos\left(fx\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/2/a/f*(-1+cos(f*x+e))^2*(cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+
(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e
)))^(3/2)-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-3*(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*cos(f*x+e)-3*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2))+3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+3*arctan(1/(-2*cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)^3/(-2*
```

```
cos(f*x+e)/(1+cos(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)),
x)
```

Fricas [A] time = 0.617447, size = 857, normalized size = 7.02

$$\left[-\frac{3\sqrt{2}\sqrt{-c}(\cos(fx+e)-1)\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{-c}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}+(3c\cos(fx+e)+c)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)+4\left(\cos(\right.}{16\left(ac^2f\cos(fx+e)-ac^2f\right)\sin(fx+e)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
fricas")
```

```
[Out] [-1/16*(3*sqrt(2)*sqrt(-c)*(cos(f*x + e) - 1)*log((2*sqrt(2)*(cos(f*x + e)^
2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*c
os(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x +
 e) + 4*(cos(f*x + e)^2 - 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e)))/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e)), 1/8*(3*sqrt(2)*sqr
t(c)*(cos(f*x + e) - 1)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x +
e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(cos(f*x + e)^2 -
 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a*c^2*f*cos(f*x
 + e) - a*c^2*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c*sqr
t(-c*sec(e + f*x) + c)), x)/a
```

Giac [A] time = 1.60222, size = 177, normalized size = 1.45

$$-\frac{\sqrt{2}\left(3\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)-2\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}-\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}\right)}{8ac^2f\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
giac")
```

```
[Out] -1/8*sqrt(2)*(3*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))
- 2*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)
/tan(1/2*f*x + 1/2*e)^2)/(a*c^2*f*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1
/2*f*x + 1/2*e)))
```

3.92 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{15\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f}-\frac{15\tan(e+fx)}{32acf(c-c\sec(e+fx))^{3/2}}-\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e}$$

```
[Out] (-15*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(32
*Sqrt[2]*a*c^(5/2)*f) - (5*Tan[e + f*x])/(8*a*f*(c - c*Sec[e + f*x])^(5/2))
 + Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)) - (15*T
an[e + f*x])/(32*a*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.255264, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3960, 3796, 3795, 203}

$$-\frac{15\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f}-\frac{15\tan(e+fx)}{32acf(c-c\sec(e+fx))^{3/2}}-\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)),x]
```


```
[Out] (-15*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(32
*Sqrt[2]*a*c^(5/2)*f) - (5*Tan[e + f*x])/(8*a*f*(c - c*Sec[e + f*x])^(5/2))
 + Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)) - (15*T
an[e + f*x])/(32*a*c*f*(c - c*Sec[e + f*x])^(3/2))
```


Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}} + \frac{5\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{5/2}}\,dx}{2a}$$

$$= -\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/}}$$

$$= -\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/}}$$

$$= -\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/}}$$

$$= -\frac{15\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f} - \frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{}{f(a+a\,\mathrm{se}}$$

Mathematica [C] time = 3.01863, size = 306, normalized size = 1.96

$$\frac{e^{-\frac{1}{2}i(e+fx)}\sin\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{7}{2}}(e+fx)\left(-\frac{1}{32}e^{-\frac{5}{2}i(e+fx)}\left(40e^{i(e+fx)}-51e^{2i(e+fx)}+80e^{3i(e+fx)}-51e^{4i(e+fx)}\right.\right.}{4ac^2f(\sec(e+fx)-1)^2(\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (Cos[(e + f*x)/2]*Sec[e + f*x]^(7/2)*Sin[(e + f*x)/2]*(-((3 + 40*E^(I*(e +
f*x)) - 51*E^((2*I)*(e + f*x)) + 80*E^((3*I)*(e + f*x)) - 51*E^((4*I)*(e +
f*x)) + 40*E^((5*I)*(e + f*x)) + 3*E^((6*I)*(e + f*x)))*Sqrt[Sec[e + f*x]])
/(32*E^(((5*I)/2)*(e + f*x))) - (15*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e +
 f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[
2]*Sqrt[1 + E^((2*I)*(e + f*x))])]*Sin[(e + f*x)/2]^3*Sin[e + f*x])/2))/(4*
a*c^2*E^((I/2)*(e + f*x))*f*(-1 + Sec[e + f*x])^2*(1 + Sec[e + f*x])*Sqrt[c
 - c*Sec[e + f*x]])
```

Maple [B] time = 0.22, size = 471, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/8/a/f*(-1+cos(f*x+e))^3*(5*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*cos(f*x+
e)^2+4*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*cos(f*x+e)-(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(7/2)+3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)*cos(f*x+e)^2-6*cos(
f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+3*(-2*cos(f*x+e)/(1+cos(f*x+e))
```

```
)^(5/2)-5*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+10*cos(f*x+e)*(
-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-5*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+
15*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+15*cos(f*x+e)^2*arctan
(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))-30*(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*cos(f*x+e)-30*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2
))+15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+15*arctan(1/(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)^5/(-2*co
s(f*x+e)/(1+cos(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\left(a\sec\left(fx+e\right)+a\right)\left(-c\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(5/2)),
x)
```

Fricas [A] time = 0.664488, size = 1049, normalized size = 6.72

$$\left[-\frac{15\sqrt{2}\left(\cos\left(fx+e\right)^{2}-2\cos\left(fx+e\right)+1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin}{128\left(ac^{3}f\cos\left(fx+e\right)^{2}-2ac^{3}f\cos\left(fx+e\right)+ac^{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
fricas")
```

```
[Out] [-1/128*(15*sqrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-c)*log((2*s
qrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin
(f*x + e)))*sin(f*x + e) - 4*(3*cos(f*x + e)^3 + 20*cos(f*x + e)^2 - 15*cos
(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a*c^3*f*cos(f*x + e)^
2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e)), 1/64*(15*sqrt(2)*(cos(
f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) + 2*
(3*cos(f*x + e)^3 + 20*cos(f*x + e)^2 - 15*cos(f*x + e))*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) +
a*c^3*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec\left(e+fx\right)}{c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^3\left(e+fx\right)-c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)-c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c^2\sqrt{-c\sec\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 - c**
2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - c**2*sqrt(-c*sec(e + f*x) + c
)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x) + c)), x)/a
```

Giac [A] time = 1.58549, size = 220, normalized size = 1.41

$$-\frac{\sqrt{2}\left(15\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)-8\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}-\frac{9\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c+7\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^2}{c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}\right)}{64\,ac^3f\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
giac")
```

```
[Out] -1/64*sqrt(2)*(15*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c)
) - 8*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c) - (9*(c*tan(1/2*f*x + 1/2*e)^2 - c
)^(3/2)*c + 7*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2)/(c^2*tan(1/2*f*x + 1/
2*e)^4))/(a*c^3*f*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e))
)
```

3.93 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=155

$$-\frac{64c^4\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}}-\frac{16c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3a^2f}-\frac{4c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2c\tan(e+fx)(}{3f(a\sec}$$

```
[Out] (-64*c^4*Tan[e + f*x])/(3*a^2*f*Sqrt[c - c*Sec[e + f*x]]) - (16*c^3*Sqrt[c
- c*Sec[e + f*x]]*Tan[e + f*x])/(3*a^2*f) - (4*c^2*(c - c*Sec[e + f*x])^(3/
2)*Tan[e + f*x])/(f*(a^2 + a^2*Sec[e + f*x])) + (2*c*(c - c*Sec[e + f*x])^(
5/2)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rubi [A] time = 0.317446, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3954, 3793, 3792}

$$-\frac{64c^4\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}}-\frac{16c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3a^2f}-\frac{4c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2c\tan(e+fx)(}{3f(a\sec}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^2,x]
```


```
[Out] (-64*c^4*Tan[e + f*x])/(3*a^2*f*Sqrt[c - c*Sec[e + f*x]]) - (16*c^3*Sqrt[c
- c*Sec[e + f*x]]*Tan[e + f*x])/(3*a^2*f) - (4*c^2*(c - c*Sec[e + f*x])^(3/
2)*Tan[e + f*x])/(f*(a^2 + a^2*Sec[e + f*x])) + (2*c*(c - c*Sec[e + f*x])^(
5/2)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```


Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(2c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx}{a}$$

$$= -\frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

$$= -\frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3a^2f} - \frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)}$$

$$= -\frac{64c^4\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}} - \frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3a^2f} - \frac{4c^2(c-c\,s}{f\left(a\right.}$$

Mathematica [A] time = 0.94085, size = 84, normalized size = 0.54

$$\frac{c^3(195\cos(e+fx)+138\cos(2(e+fx))+45\cos(3(e+fx))+134)\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{6a^2f(\cos(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^2,
x]
```

```
[Out] (c^3*(134 + 195*Cos[e + f*x] + 138*Cos[2*(e + f*x)] + 45*Cos[3*(e + f*x)])*
Cot[(e + f*x)/2]*Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(6*a^2*f*(1 + Cos[e
 + f*x])^2)
```

Maple [A] time = 0.193, size = 85, normalized size = 0.6

$$-\frac{\left(6\cos\left(fx+e\right)+2\right)\left(15\left(\cos\left(fx+e\right)\right)^2+18\cos\left(fx+e\right)-1\right)\left(\cos\left(fx+e\right)\right)^2}{3fa^2\left(\sin\left(fx+e\right)\right)^3\left(-1+\cos\left(fx+e\right)\right)^2}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -2/3/a^2/f*(3*cos(f*x+e)+1)*(15*cos(f*x+e)^2+18*cos(f*x+e)-1)*cos(f*x+e)^2*
(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)/sin(f*x+e)^3/(-1+cos(f*x+e))^2
```

Maxima [A] time = 1.72648, size = 254, normalized size = 1.64

$$-\frac{4\left(16\sqrt{2}c^{\frac{7}{2}} - \frac{56\sqrt{2}c^{\frac{7}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2} + \frac{70\sqrt{2}c^{\frac{7}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4} - \frac{35\sqrt{2}c^{\frac{7}{2}}\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6} + \frac{4\sqrt{2}c^{\frac{7}{2}}\sin\left(fx+e\right)^8}{\left(\cos\left(fx+e\right)+1\right)^8} + \frac{\sqrt{2}c^{\frac{7}{2}}\sin\left(fx+e\right)^{10}}{\left(\cos\left(fx+e\right)+1\right)^{10}}\right)}{3a^2f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -4/3*(16*sqrt(2)*c^(7/2) - 56*sqrt(2)*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 70*sqrt(2)*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 35*sqrt(2
)*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 4*sqrt(2)*c^(7/2)*sin(f*x +
 e)^8/(cos(f*x + e) + 1)^8 + sqrt(2)*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e)
+ 1)^10)/(a^2*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(7/2)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1)^(7/2))
```

Fricas [A] time = 0.487042, size = 243, normalized size = 1.57

$$\frac{2\left(45\,c^{3}\cos\left(fx+e\right)^{3}+69\,c^{3}\cos\left(fx+e\right)^{2}+15\,c^{3}\cos\left(fx+e\right)-c^{3}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\left(a^{2}f\cos\left(fx+e\right)^{2}+a^{2}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 2/3*(45*c^3*cos(f*x + e)^3 + 69*c^3*cos(f*x + e)^2 + 15*c^3*cos(f*x + e) -
c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^2*f*cos(f*x + e)^2 + a^2*f
*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 4.21786, size = 144, normalized size = 0.93

$$\frac{4\,\sqrt{2}\left(\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{3}{2}}+9\,\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}\,c-\frac{9\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c^{2}+c^{3}}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{3}{2}}}\right)c^{2}}{3\,a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 4/3*sqrt(2)*((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2) + 9*sqrt(c*tan(1/2*f*x +
1/2*e)^2 - c)*c - (9*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2 + c^3)/(c*tan(1/2*f
*x + 1/2*e)^2 - c)^(3/2))*c^2/(a^2*f)
```

3.94 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=123

$$-\frac{16c^3\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}}-\frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}$$

```
[Out] (-16*c^3*Tan[e + f*x])/(3*a^2*f*Sqrt[c - c*Sec[e + f*x]]) - (8*c^2*Sqrt[c -
 c*Sec[e + f*x]]*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x])) + (2*c*(c - c
*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rubi [A] time = 0.274039, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3954, 3792}

$$-\frac{16c^3\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}}-\frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-16*c^3*Tan[e + f*x])/(3*a^2*f*Sqrt[c - c*Sec[e + f*x]]) - (8*c^2*Sqrt[c -
 c*Sec[e + f*x]]*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x])) + (2*c*(c - c
*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx &= \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{(4c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx}{3a}\\ &= -\frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}+\frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\\ &= -\frac{16c^3\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}}-\frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}+\frac{2c(c-c\sec}{3f(a}\end{aligned}$$

Mathematica [A] time = 0.397296, size = 68, normalized size = 0.55

$$\frac{c^2(36\cos(e+fx)+11\cos(2(e+fx))+17)\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{3a^2f(\cos(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^2,
x]
```

```
[Out] (c^2*(17 + 36*Cos[e + f*x] + 11*Cos[2*(e + f*x)])*Cot[(e + f*x)/2]*Sqrt[c -
 c*Sec[e + f*x]])/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [A] time = 0.208, size = 75, normalized size = 0.6

$$-\frac{\left(22\left(\cos\left(fx+e\right)\right)^{2}+36\cos\left(fx+e\right)+6\right)\left(\cos\left(fx+e\right)\right)^{2}}{3fa^{2}\left(\sin\left(fx+e\right)\right)^{3}\left(-1+\cos\left(fx+e\right)\right)}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -2/3/a^2/f*(11*cos(f*x+e)^2+18*cos(f*x+e)+3)*cos(f*x+e)^2*(c*(-1+cos(f*x+e)
)/cos(f*x+e))^(5/2)/sin(f*x+e)^3/(-1+cos(f*x+e))
```

Maxima [A] time = 1.58908, size = 220, normalized size = 1.79

$$\frac{2\left(8\sqrt{2}c^{\frac{5}{2}}-\frac{20\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{15\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{2\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\frac{\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^{8}}{\left(\cos\left(fx+e\right)+1\right)^{8}}\right)}{3a^{2}f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 2/3*(8*sqrt(2)*c^(5/2) - 20*sqrt(2)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 15*sqrt(2)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2*sqrt(2)*c
^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - sqrt(2)*c^(5/2)*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8)/(a^2*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(5/2)*(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

Fricas [A] time = 0.470874, size = 193, normalized size = 1.57

$$\frac{2\left(11\,c^{2}\cos\left(fx+e\right)^{2}+18\,c^{2}\cos\left(fx+e\right)+3\,c^{2}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\left(a^{2}f\cos\left(fx+e\right)+a^{2}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 2/3*(11*c^2*cos(f*x + e)^2 + 18*c^2*cos(f*x + e) + 3*c^2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e))/((a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.9263, size = 109, normalized size = 0.89

$$\frac{2\sqrt{2}\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}+6\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c-\frac{3c^2}{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}\right)c}{3a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 2/3*sqrt(2)*((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2) + 6*sqrt(c*tan(1/2*f*x +
1/2*e)^2 - c)*c - 3*c^2/sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c))*c/(a^2*f)
```

3.95 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=89

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f(a\sec(e+fx)+a)^2}-\frac{4c^2\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (-4*c^2*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]
]) + (2*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^
2)
```

Rubi [A] time = 0.219272, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3954, 3953}

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f(a\sec(e+fx)+a)^2}-\frac{4c^2\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-4*c^2*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]
]) + (2*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^
2)
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx=\frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{(2c)\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx}{3a}$$
$$=-\frac{4c^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}}+\frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+f}{3f(a+a\sec(e+fx))^2}$$

Mathematica [A] time = 0.248986, size = 60, normalized size = 0.67

$$\frac{2c\cos(e+fx)(\cos(e+fx)+3)\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{3a^2f(\cos(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^2,
x]
```

```
[Out] (2*c*Cos[e + f*x]*(3 + Cos[e + f*x])*Cot[(e + f*x)/2]*Sqrt[c - c*Sec[e + f*
x]])/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [A] time = 0.207, size = 53, normalized size = 0.6

$$-\frac{\left(2\cos\left(fx+e\right)+6\right)\left(\cos\left(fx+e\right)\right)^{2}}{3fa^{2}\left(\sin\left(fx+e\right)\right)^{3}}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -2/3/a^2/f*(cos(f*x+e)+3)*cos(f*x+e)^2*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)
/sin(f*x+e)^3
```

Maxima [A] time = 1.53611, size = 149, normalized size = 1.67

$$-\frac{2\sqrt{2}c^{\frac{3}{2}}-\frac{3\sqrt{2}c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{\sqrt{2}c^{\frac{3}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}}{3a^{2}f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -1/3*(2*sqrt(2)*c^(3/2) - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + sqrt(2)*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/(a^2*f*(sin(f*x
 + e)/(cos(f*x + e) + 1) + 1)^(3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(
3/2))
```

Fricas [A] time = 0.464593, size = 171, normalized size = 1.92

$$\frac{2\left(c\cos\left(fx+e\right)^{2}+3c\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\left(a^{2}f\cos\left(fx+e\right)+a^{2}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 2/3*(c*cos(f*x + e)^2 + 3*c*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))/((a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int -\frac{c\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec
(e + f*x) + 1), x) + Integral(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2/
(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A] time = 2.22855, size = 84, normalized size = 0.94

$$\frac{\sqrt{2}\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c+3\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}c^2\right)}{3a^2cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/3*sqrt(2)*((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c + 3*sqrt(c*tan(1/2*f*x
+ 1/2*e)^2 - c)*c^2)/(a^2*c*f)
```

3.96 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=41

$$\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (2*c*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.100431, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3953}

$$\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*c*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx = \frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.150477, size = 55, normalized size = 1.34

$$-\frac{\cos^2(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{6a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -(Cos[e + f*x]^2*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^3*Sqrt[c - c*Sec[e + f*x
]])/(6*a^2*f)
```

Maple [A] time = 0.256, size = 53, normalized size = 1.3

$$\frac{\left(-2+2\cos\left(fx+e\right)\right)\left(\cos\left(fx+e\right)\right)^{2}}{3fa^{2}\left(\sin\left(fx+e\right)\right)^{3}}\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x)
```

```
[Out] 2/3/a^2/f*(-1+cos(f*x+e))*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)^2
/sin(f*x+e)^3
```

Maxima [B] time = 1.52617, size = 147, normalized size = 3.59

$$-\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt{2}\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{\sqrt{2}\sqrt{c}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}}{6a^{2}f\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1}\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -1/6*(sqrt(2)*sqrt(c) - 2*sqrt(2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + sqrt(2)*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/(a^2*f*sqrt(sin(f
*x + e)/(cos(f*x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) - 1))
```

Fricas [A] time = 0.459774, size = 142, normalized size = 3.46

$$-\frac{2\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}}{3\left(a^{2}f\cos\left(fx+e\right)+a^{2}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] -2/3*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2/((a^2*f*cos(f*x
 + e) + a^2*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e
+ f*x) + 1), x)/a**2
```

Giac [A] time = 1.39376, size = 89, normalized size = 2.17

$$\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{6\,a^{2}cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/6*sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*sgn(tan(1/2*f*x + 1/2*e)^3
 + tan(1/2*f*x + 1/2*e))*sgn(cos(f*x + e))/(a^2*c*f)
```

3.97 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=138

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}+\frac{\tan(e+fx)}{2f\left(a^2\sec(e+fx)+a^2\right)\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(2*Sqrt[
2]*a^2*Sqrt[c]*f) + Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec
[e + f*x]]) + Tan[e + f*x]/(2*f*(a^2 + a^2*Sec[e + f*x])*Sqrt[c - c*Sec[e +
 f*x]])
```

Rubi [A] time = 0.271098, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3960, 3795, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}+\frac{\tan(e+fx)}{2f\left(a^2\sec(e+fx)+a^2\right)\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(2*Sqrt[
2]*a^2*Sqrt[c]*f) + Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec
[e + f*x]]) + Tan[e + f*x]/(2*f*(a^2 + a^2*Sec[e + f*x])*Sqrt[c - c*Sec[e +
 f*x]])
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}dx = \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}}{2a}$$

$$= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+f}{2f\left(a^2+a^2\sec(e+fx)\right)\sqrt{}}$$

$$= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+f}{2f\left(a^2+a^2\sec(e+fx)\right)\sqrt{}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 2.14287, size = 259, normalized size = 1.88

$$\frac{2e^{-\frac{1}{2}i(e+fx)}\sin\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{5}{2}}(e+fx)\left(\frac{1}{8}e^{-\frac{3}{2}i(e+fx)}\left(6e^{i(e+fx)}+10e^{2i(e+fx)}+6e^{3i(e+fx)}+5e^{4i(e+fx)}+5\right)\right.}{3a^2f(\sec(e+fx)+1)^2\sqrt{c-c\sec(e+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(-3*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqr
t[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 +
E^((2*I)*(e + f*x))])]*Cos[(e + f*x)/2]^3 + ((5 + 6*E^(I*(e + f*x)) + 10*E^
((2*I)*(e + f*x)) + 6*E^((3*I)*(e + f*x)) + 5*E^((4*I)*(e + f*x)))*Sqrt[Sec
[e + f*x]])/(8*E^(((3*I)/2)*(e + f*x))))*Sec[e + f*x]^(5/2)*Sin[(e + f*x)/2
])/(3*a^2*E^((I/2)*(e + f*x))*f*(1 + Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x
]])
```

Maple [A] time = 0.219, size = 131, normalized size = 1.

$$\frac{-1+\cos(fx+e)}{6fa^2\sin(fx+e)}\left(\left(-2\frac{\cos(fx+e)}{1+\cos(fx+e)}\right)^{\frac{3}{2}} - 3\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}} - 3\arctan\left(\frac{1}{\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}}\right)\right)\frac{1}{\sqrt{\frac{c(-1+\cos(fx+e}{\cos(fx+e)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/6/a^2/f*(-1+cos(f*x+e))*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-3*(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)-3*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/(-2*cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\left(a\sec\left(fx+e\right)+a\right)^2\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^2*sqrt(-c*sec(f*x + e) + c)),
x)
```

Fricas [A] time = 0.617205, size = 864, normalized size = 6.26

$$\left[-\frac{3\sqrt{2}\sqrt{-c}\left(\cos\left(fx+e\right)+1\right)\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+4\left(5\cos\right.}{24\left(a^2cf\cos\left(fx+e\right)+a^2cf\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/24*(3*sqrt(2)*sqrt(-c)*(cos(f*x + e) + 1)*log((2*sqrt(2)*(cos(f*x + e)^
2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*c
os(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x +
 e) + 4*(5*cos(f*x + e)^2 + 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e)))/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e)), 1/12*(3*sqrt(2)*
sqrt(c)*(cos(f*x + e) + 1)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x + e
)^2 + 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c*f*co
s(f*x + e) + a^2*c*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)+2\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+\sqrt{-c\sec\left(e+fx\right)+c}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + 2*sqrt(-
c*sec(e + f*x) + c)*sec(e + f*x) + sqrt(-c*sec(e + f*x) + c)), x)/a**2
```

Giac [C] time = 1.57959, size = 224, normalized size = 1.62

$$-\frac{\frac{\sqrt{2}\left(3i\sqrt{-c}\arctan(-i)-4\sqrt{-c}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^2c}+\frac{\sqrt{2}\left(\frac{3\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c^4-3\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^5}{c^6}\right)}{a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/12*(sqrt(2)*(3*I*sqrt(-c)*arctan(-I) - 4*sqrt(-c))*sgn(tan(1/2*f*x + 1/2
*e))/(a^2*c) + sqrt(2)*(3*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c)
)/sqrt(c) + ((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^4 - 3*sqrt(c*tan(1/2*f*
x + 1/2*e)^2 - c)*c^5)/c^6)/(a^2*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/
2*f*x + 1/2*e))))/f
```

3.98 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{5\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f}-\frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}}+\frac{5\tan(e+fx)}{6f\left(a^2\sec(e+fx)+a^2\right)(c-c\sec(e+fx))^{3/2}}+\frac{}{3f(a\sec(e+}$$

```
[Out] (-5*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(8*S
qrt[2]*a^2*c^(3/2)*f) - (5*Tan[e + f*x])/(8*a^2*f*(c - c*Sec[e + f*x])^(3/2
)) + Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)) +
 (5*Tan[e + f*x])/(6*f*(a^2 + a^2*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.343459, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3960, 3796, 3795, 203}

$$-\frac{5\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f}-\frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}}+\frac{5\tan(e+fx)}{6f\left(a^2\sec(e+fx)+a^2\right)(c-c\sec(e+fx))^{3/2}}+\frac{}{3f(a\sec(e+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)),x]
```


```
[Out] (-5*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(8*S
qrt[2]*a^2*c^(3/2)*f) - (5*Tan[e + f*x])/(8*a^2*f*(c - c*Sec[e + f*x])^(3/2
)) + Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)) +
 (5*Tan[e + f*x])/(6*f*(a^2 + a^2*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2))
```


Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}dx = \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}} + \frac{5\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec}}{6a}$$

$$= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}} + \frac{5\tan}{6f\left(a^2+a^2\sec(e+f\right.}$$

$$= -\frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+f}$$

$$= -\frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+f}$$

$$= -\frac{5\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f} - \frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}} + \frac{}{3f(a+a}$$

Mathematica [C] time = 1.39913, size = 365, normalized size = 2.16

$$\frac{-\frac{15i\sqrt{2}\left(-1+e^{i(e+fx)}\right)^3\left(1+e^{i(e+fx)}\right)^4\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)}{\left(1+e^{2i(e+fx)}\right)^{7/2}} - 416\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sin^8(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\csc^4(2(e+fx)) - 40}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)),
x]
```

```
[Out] (((-15*I)*Sqrt[2]*(-1 + E^(I*(e + f*x)))^3*(1 + E^(I*(e + f*x)))^4*ArcTanh[
(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/(1 + E^((2*
I)*(e + f*x)))^(7/2) - 48*Cos[(e + f*x)/2]^4*Csc[e/2]*Sec[e + f*x]^4*Sin[(f
*x)/2]*Sin[(e + f*x)/2] + 48*Cos[(e + f*x)/2]^4*Cot[e/2]*Sec[e + f*x]^4*Sin
[(e + f*x)/2]^2 + 32*Cos[(e + f*x)/2]*Sec[e + f*x]^4*Sin[(e + f*x)/2]^3 + 4
16*Cos[e/2]*Cos[(f*x)/2]*Cos[(e + f*x)/2]^4*Sec[e + f*x]^4*Sin[(e + f*x)/2]
^3 - 416*Csc[(e + f*x)/2]*Csc[2*(e + f*x)]^4*Sin[e/2]*Sin[(f*x)/2]*Sin[e +
f*x]^8 - 40*Sec[e + f*x]*Tan[e + f*x]^3)/(48*a^2*c*f*(-1 + Sec[e + f*x])*(1
 + Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] time = 0.233, size = 320, normalized size = 1.9

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^2}{12fa^2\left(\sin\left(fx+e\right)\right)^3}\left(3\left(-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{7/2}\cos\left(fx+e\right)+3\left(-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{7/2}+3\cos\left(fx+e\right)\left(-2\frac{}{1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/12/a^2/f*(-1+cos(f*x+e))^2*(3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*cos(f
*x+e)+3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)+3*cos(f*x+e)*(-2*cos(f*x+e)/(1
+cos(f*x+e)))^(5/2)-3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)-5*cos(f*x+e)*(-2
*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+5*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+15
*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)+15*cos(f*x+e)*arctan(1/(-2
*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))-15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-
15*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f
*x+e))^(3/2)/sin(f*x+e)^3/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\left(a\sec\left(fx+e\right)+a\right)^{2}\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^(3/2))
, x)
```

Fricas [A] time = 0.660829, size = 948, normalized size = 5.61

$$\left[-\frac{15\sqrt{2}\left(\cos\left(fx+e\right)^{2}-1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+4\left(13\right.}{96\left(a^{2}c^{2}f\cos\left(fx+e\right)^{2}-a^{2}c^{2}f\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] [-1/96*(15*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(-c)*log((2*sqrt(2)*(cos(f*x +
e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*
c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*
x + e) + 4*(13*cos(f*x + e)^3 - 10*cos(f*x + e)^2 - 15*cos(f*x + e))*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*
sin(f*x + e)), 1/48*(15*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(c)*arctan(sqrt(2)
*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)
))*sin(f*x + e) - 2*(13*cos(f*x + e)^3 - 10*cos(f*x + e)^2 - 15*cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^
2*c^2*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec^{3}\left(e+fx\right)-c\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 - c*sqr
t(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c*sqrt(-c*sec(e + f*x) + c)*sec(e
+ f*x) + c*sqrt(-c*sec(e + f*x) + c)), x)/a**2
```

Giac [A] time = 1.51185, size = 221, normalized size = 1.31

$$-\frac{\sqrt{2}\left(15\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}}{\sqrt{c}}\right)-\frac{3\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}+\frac{2\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{3}{2}}c^{2}-6\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}\,c^{3}\right)}{c^{3}}\right)}{48\,a^{2}c^{2}f\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/48*sqrt(2)*(15*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c)
) - 3*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/tan(1/2*f*x + 1/2*e)^2 + 2*((c*tan
(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^2 - 6*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c
^3)/c^3)/(a^2*c^2*f*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e
)))
```

3.99 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{35\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f}-\frac{35\tan(e+fx)}{64a^2cf(c-c\sec(e+fx))^{3/2}}-\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}}+\frac{7\tan(e+fx}{6f\left(a^2\sec(e+fx)+a^2\right)(c-}$$

```
[Out] (-35*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(64
*Sqrt[2]*a^2*c^(5/2)*f) - (35*Tan[e + f*x])/(48*a^2*f*(c - c*Sec[e + f*x])^
(5/2)) + Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2
)) + (7*Tan[e + f*x])/(6*f*(a^2 + a^2*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5
/2)) - (35*Tan[e + f*x])/(64*a^2*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.388975, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3960, 3796, 3795, 203}

$$-\frac{35\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f}-\frac{35\tan(e+fx)}{64a^2cf(c-c\sec(e+fx))^{3/2}}-\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}}+\frac{7\tan(e+fx}{6f\left(a^2\sec(e+fx)+a^2\right)(c-}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (-35*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(64
*Sqrt[2]*a^2*c^(5/2)*f) - (35*Tan[e + f*x])/(48*a^2*f*(c - c*Sec[e + f*x])^
(5/2)) + Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2
)) + (7*Tan[e + f*x])/(6*f*(a^2 + a^2*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5
/2)) - (35*Tan[e + f*x])/(64*a^2*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}} + \frac{7\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\,\mathrm{se}}}{6a}$$

$$= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}} + \frac{7\tan}{6f\left(a^2+a^2\sec(e+f\right.}$$

$$= -\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+}$$

$$= -\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+}$$

$$= -\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+}$$

$$= -\frac{35\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f} - \frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}} + \frac{}{3f(a+}$$

Mathematica [C] time = 2.38116, size = 434, normalized size = 2.14

$$\cot^4(e+fx)\left(\frac{105i\sqrt{2}\left(-1+e^{i(e+fx)}\right)^5\left(1+e^{i(e+fx)}\right)^4\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)}{\left(1+e^{2i(e+fx)}\right)^{9/2}} - 3648\csc\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sin^9(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\csc^5(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)),
x]
```

```
[Out] (Cot[e + f*x]^4*(((105*I)*Sqrt[2]*(-1 + E^(I*(e + f*x)))^5*(1 + E^(I*(e + f
*x)))^4*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))
])])/(1 + E^((2*I)*(e + f*x)))^(9/2) + 192*Cos[(e + f*x)/2]^4*Csc[e/2]*Sec[
e + f*x]^5*Sin[(f*x)/2]*Sin[(e + f*x)/2] - 192*Cos[(e + f*x)/2]^4*Cot[e/2]*
Sec[e + f*x]^5*Sin[(e + f*x)/2]^2 + 256*Cos[(e + f*x)/2]*Sec[e + f*x]^5*Sin
[(e + f*x)/2]^5 + 2752*Cos[e/2]*Cos[(f*x)/2]*Cos[(e + f*x)/2]^4*Sec[e + f*x
]^5*Sin[(e + f*x)/2]^5 - 13312*Csc[2*(e + f*x)]^5*Sin[(e + f*x)/2]^2*Sin[e
+ f*x]^8 - 3648*Csc[e/2]*Csc[(e + f*x)/2]*Csc[2*(e + f*x)]^5*Sin[(f*x)/2]*S
in[e + f*x]^9 - 5504*Csc[2*(e + f*x)]^5*Sin[e/2]*Sin[(f*x)/2]*Sin[(e + f*x)
/2]*Sin[e + f*x]^9 + 114*Cot[e/2]*Sec[e + f*x]*Tan[e + f*x]^4))/(384*a^2*c^
2*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] time = 0.242, size = 551, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/48/a^2/f*(-1+cos(f*x+e))^3*(21*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+cos(f*x+e))
)^(9/2)+12*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)+15*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(7/2)*cos(f*x+e)^2-9*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)
-30*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*cos(f*x+e)-21*(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(5/2)*cos(f*x+e)^2+15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)+42*co
s(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+35*cos(f*x+e)^2*(-2*cos(f*x+e
)/(1+cos(f*x+e)))^(3/2)-21*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)-70*cos(f*x+
e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-105*cos(f*x+e)^2*(-2*cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)-105*cos(f*x+e)^2*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))
^(1/2))+35*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+210*(-2*cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*cos(f*x+e)+210*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+
e)))^(1/2))-105*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-105*arctan(1/(-2*cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+
e)^5/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.759568, size = 1239, normalized size = 6.1

$$\left[-\frac{105\sqrt{2}\left(\cos\left(fx+e\right)^{3}-\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)+1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)+\right.}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right.}{768\left(a^{2}c^{3}f\cos\left(fx+e\right)^{3}-a^{2}c^{3}f\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/768*(105*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*s
qrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f
*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) + 4*(43*cos(f*x + e)^4 - 161*cos(f
*x + e)^3 - 35*cos(f*x + e)^2 + 105*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2
*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e)), 1/384*(105*sqrt(2)*(cos(f*x
 + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c
```

```
*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f
*x + e) - 2*(43*cos(f*x + e)^4 - 161*cos(f*x + e)^3 - 35*cos(f*x + e)^2 + 1
05*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^3*f*cos(f
*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*
sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.64403, size = 265, normalized size = 1.31

$$-\frac{\sqrt{2}\left(105\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)+\frac{8\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c^2-9\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^3\right)}{c^3}-\frac{3\left(13\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c+11\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^2\right)}{c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}\right)}{384\,a^2c^3f\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/384*sqrt(2)*(105*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(
c)) + 8*((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^2 - 9*sqrt(c*tan(1/2*f*x +
1/2*e)^2 - c)*c^3)/c^3 - 3*(13*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c + 11*
sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4))/(a^2*
c^3*f*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)))
```

3.100 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=169

$$\frac{32c^4\tan(e+fx)}{5a^3f\sqrt{c-c\sec(e+fx)}}+\frac{16c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5f\left(a^3\sec(e+fx)+a^3\right)}-\frac{4c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c}{5f(a\sec(e}$$

```
[Out] (32*c^4*Tan[e + f*x])/(5*a^3*f*Sqrt[c - c*Sec[e + f*x]]) + (16*c^3*Sqrt[c -
 c*Sec[e + f*x]]*Tan[e + f*x])/(5*f*(a^3 + a^3*Sec[e + f*x])) - (4*c^2*(c -
 c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*a*f*(a + a*Sec[e + f*x])^2) + (2*c*
(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3)
```

Rubi [A] time = 0.402054, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3954, 3792}

$$\frac{32c^4\tan(e+fx)}{5a^3f\sqrt{c-c\sec(e+fx)}}+\frac{16c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5f\left(a^3\sec(e+fx)+a^3\right)}-\frac{4c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c}{5f(a\sec(e}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (32*c^4*Tan[e + f*x])/(5*a^3*f*Sqrt[c - c*Sec[e + f*x]]) + (16*c^3*Sqrt[c -
 c*Sec[e + f*x]]*Tan[e + f*x])/(5*f*(a^3 + a^3*Sec[e + f*x])) - (4*c^2*(c -
 c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*a*f*(a + a*Sec[e + f*x])^2) + (2*c*
(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3)
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(6c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}dx}{5a}$$

$$= -\frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

$$= \frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)} - \frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af(a+a\sec(e+fx))^2} -$$

$$= \frac{32c^4\tan(e+fx)}{5a^3f\sqrt{c-c\sec(e+fx)}} + \frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)} - \frac{4c^2(c-c\,\mathrm{se}}{5af(}$$

Mathematica [A] time = 0.626385, size = 78, normalized size = 0.46

$$-\frac{c^3(249\cos(e+fx)+110\cos(2(e+fx))+23\cos(3(e+fx))+130)\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{10a^3f(\cos(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^3,
x]
```

```
[Out] -(c^3*(130 + 249*Cos[e + f*x] + 110*Cos[2*(e + f*x)] + 23*Cos[3*(e + f*x)])
*Cot[(e + f*x)/2]*Sqrt[c - c*Sec[e + f*x]])/(10*a^3*f*(1 + Cos[e + f*x])^3)
```

Maple [A] time = 0.241, size = 85, normalized size = 0.5

$$-\frac{\left(46\left(\cos\left(fx+e\right)\right)^3+110\left(\cos\left(fx+e\right)\right)^2+90\cos\left(fx+e\right)+10\right)\left(\cos\left(fx+e\right)\right)^3}{5fa^3\left(\sin\left(fx+e\right)\right)^5\left(-1+\cos\left(fx+e\right)\right)}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -2/5/a^3/f*(23*cos(f*x+e)^3+55*cos(f*x+e)^2+45*cos(f*x+e)+5)*cos(f*x+e)^3*(
c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)/sin(f*x+e)^5/(-1+cos(f*x+e))
```

Maxima [A] time = 1.49678, size = 289, normalized size = 1.71

$$\frac{2\left(16\sqrt{2}c^{\frac{7}{2}} - \frac{56\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{70\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{35\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{5\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8} - \frac{\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}} + \frac{\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)}{(\cos(fx+e)+1)}\right.}{5a^3f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] 2/5*(16*sqrt(2)*c^(7/2) - 56*sqrt(2)*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 70*sqrt(2)*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 35*sqrt(2)
*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5*sqrt(2)*c^(7/2)*sin(f*x +
e)^8/(cos(f*x + e) + 1)^8 - sqrt(2)*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) +
 1)^10 + sqrt(2)*c^(7/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)/(a^3*f*(sin
(f*x + e)/(cos(f*x + e) + 1) + 1)^(7/2)*(sin(f*x + e)/(cos(f*x + e) + 1) -
1)^(7/2))
```

Fricas [A] time = 0.484222, size = 261, normalized size = 1.54

$$-\frac{2\left(23\,c^3\cos\left(fx+e\right)^3+55\,c^3\cos\left(fx+e\right)^2+45\,c^3\cos\left(fx+e\right)+5\,c^3\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{5\left(a^3f\cos\left(fx+e\right)^2+2\,a^3f\cos\left(fx+e\right)+a^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -2/5*(23*c^3*cos(f*x + e)^3 + 55*c^3*cos(f*x + e)^2 + 45*c^3*cos(f*x + e) +
 5*c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*
a^3*f*cos(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 6.36034, size = 143, normalized size = 0.85

$$-\frac{2\sqrt{2}\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}+5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c+15\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^2-\frac{5\,c^3}{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}\right)c}{5\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -2/5*sqrt(2)*((c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2) + 5*(c*tan(1/2*f*x + 1/2
*e)^2 - c)^(3/2)*c + 15*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2 - 5*c^3/sqrt
(c*tan(1/2*f*x + 1/2*e)^2 - c))*c/(a^3*f)
```

3.101 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=135

$$\frac{16c^3\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)\sqrt{c-c\sec(e+fx)}}-\frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{15af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5f(a\sec(e+fx)+a)^3}$$

```
[Out] (16*c^3*Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x
]]) - (8*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(15*a*f*(a + a*Sec[e +
f*x])^2) + (2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f*(a + a*Sec[e
+ f*x])^3)
```

Rubi [A] time = 0.348345, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3954, 3953}

$$\frac{16c^3\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)\sqrt{c-c\sec(e+fx)}}-\frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{15af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (16*c^3*Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x
]]) - (8*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(15*a*f*(a + a*Sec[e +
f*x])^2) + (2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f*(a + a*Sec[e
+ f*x])^3)
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(4c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx}{5a}$$

$$= -\frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(8}{}$$

$$= \frac{16c^3\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}} - \frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+f}{15af(a+a\sec(e+fx))^2}$$

Mathematica [A] time = 0.343672, size = 74, normalized size = 0.55

$$-\frac{c^2\cos(e+fx)(20\cos(e+fx)+7\cos(2(e+fx))+37)\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{15a^3f(\cos(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^3,
x]
```

```
[Out] -(c^2*Cos[e + f*x]*(37 + 20*Cos[e + f*x] + 7*Cos[2*(e + f*x)])*Cot[(e + f*x
)/2]*Sqrt[c - c*Sec[e + f*x]])/(15*a^3*f*(1 + Cos[e + f*x])^3)
```

Maple [A] time = 0.247, size = 65, normalized size = 0.5

$$-\frac{\left(14\left(\cos\left(fx+e\right)\right)^2+20\cos\left(fx+e\right)+30\right)\left(\cos\left(fx+e\right)\right)^3}{15\,fa^3\left(\sin\left(fx+e\right)\right)^5}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -2/15/a^3/f*(7*cos(f*x+e)^2+10*cos(f*x+e)+15)*cos(f*x+e)^3*(c*(-1+cos(f*x+e
))/cos(f*x+e))^(5/2)/sin(f*x+e)^5
```

Maxima [A] time = 1.50809, size = 255, normalized size = 1.89

$$-\frac{8\sqrt{2}c^{\frac{5}{2}} - \frac{20\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2} + \frac{15\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4} - \frac{5\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6} + \frac{5\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^8}{\left(\cos\left(fx+e\right)+1\right)^8} - \frac{3\sqrt{2}c^{\frac{5}{2}}\sin\left(fx+e\right)^{10}}{\left(\cos\left(fx+e\right)+1\right)^{10}}}{15\,a^3f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -1/15*(8*sqrt(2)*c^(5/2) - 20*sqrt(2)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 15*sqrt(2)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*sqrt(2)
```

```
*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5*sqrt(2)*c^(5/2)*sin(f*x +
e)^8/(cos(f*x + e) + 1)^8 - 3*sqrt(2)*c^(5/2)*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10)/(a^3*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(5/2)*(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)^(5/2))
```

Fricas [A] time = 0.471848, size = 250, normalized size = 1.85

$$-\frac{2\left(7\,c^2\cos\left(fx+e\right)^3+10\,c^2\cos\left(fx+e\right)^2+15\,c^2\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{15\left(a^3f\cos\left(fx+e\right)^2+2\,a^3f\cos\left(fx+e\right)+a^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -2/15*(7*c^2*cos(f*x + e)^3 + 10*c^2*cos(f*x + e)^2 + 15*c^2*cos(f*x + e))*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*a^3*f*co
s(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 4.376, size = 111, normalized size = 0.82

$$-\frac{\sqrt{2}\left(3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}+10\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c+15\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^2\right)}{15\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/15*sqrt(2)*(3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2) + 10*(c*tan(1/2*f*x +
 1/2*e)^2 - c)^(3/2)*c + 15*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2)/(a^3*f)
```

3.102 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=88

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5f(a\sec(e+fx)+a)^3}-\frac{4c^2\tan(e+fx)}{15af(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (-4*c^2*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x
]]) + (2*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])
^3)
```

Rubi [A] time = 0.222202, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3954, 3953}

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5f(a\sec(e+fx)+a)^3}-\frac{4c^2\tan(e+fx)}{15af(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-4*c^2*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x
]]) + (2*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])
^3)
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx &= \frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f(a+a\sec(e+fx))^3}-\frac{(2c)\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx}{5a}\\ &= -\frac{4c^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}+\frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+f}{5f(a+a\sec(e+fx))^3}\end{aligned}$$

Mathematica [A] time = 0.317465, size = 60, normalized size = 0.68

$$-\frac{2c(\cos(e+fx)-5)\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{15a^3f(\sec(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^3,
x]
```

```
[Out] (-2*c*(-5 + Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e + f*x]*Sqrt[c - c*Sec[e +
f*x]])/(15*a^3*f*(1 + Sec[e + f*x])^3)
```

Maple [A] time = 0.237, size = 63, normalized size = 0.7

$$-\frac{\left(2\left(\cos\left(fx+e\right)\right)^2+10-12\cos\left(fx+e\right)\right)\left(\cos\left(fx+e\right)\right)^3}{15fa^3\left(\sin\left(fx+e\right)\right)^5}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -2/15/a^3/f*(cos(f*x+e)^2+5-6*cos(f*x+e))*cos(f*x+e)^3*(c*(-1+cos(f*x+e))/c
os(f*x+e))^(3/2)/sin(f*x+e)^5
```

Maxima [B] time = 1.51664, size = 220, normalized size = 2.5

$$-\frac{2\sqrt{2}c^{\frac{3}{2}}-\frac{3\sqrt{2}c^{\frac{3}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{3\sqrt{2}c^{\frac{3}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{7\sqrt{2}c^{\frac{3}{2}}\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}-\frac{3\sqrt{2}c^{\frac{3}{2}}\sin\left(fx+e\right)^8}{\left(\cos\left(fx+e\right)+1\right)^8}}{30a^3f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -1/30*(2*sqrt(2)*c^(3/2) - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7*sqrt(2)*c
^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3*sqrt(2)*c^(3/2)*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8)/(a^3*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(3/2)
*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(3/2))
```

Fricas [A] time = 0.46934, size = 211, normalized size = 2.4

$$-\frac{2\left(c\cos\left(fx+e\right)^3-5c\cos\left(fx+e\right)^2\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{15\left(a^3f\cos\left(fx+e\right)^2+2a^3f\cos\left(fx+e\right)+a^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -2/15*(c*cos(f*x + e)^3 - 5*c*cos(f*x + e)^2)*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*a^3*f*cos(f*x + e) + a^3*f)*sin(f*x +
 e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int -\frac{c\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec
(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-c*sqrt(-c*sec(e + f*x) +
 c)*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) +
 1), x))/a**3
```

Giac [A] time = 2.85786, size = 81, normalized size = 0.92

$$-\frac{\sqrt{2}\left(3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}+5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c\right)}{30a^3cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/30*sqrt(2)*(3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2) + 5*(c*tan(1/2*f*x +
1/2*e)^2 - c)^(3/2)*c)/(a^3*c*f)
```

3.103 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=41

$$\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (2*c*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.101593, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3953}

$$\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*c*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.153059, size = 55, normalized size = 1.34

$$-\frac{\cos^3(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sec^5\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{20a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -(Cos[e + f*x]^3*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^5*Sqrt[c - c*Sec[e + f*x
]])/(20*a^3*f)
```

Maple [A] time = 0.29, size = 55, normalized size = 1.3

$$-\frac{2\left(-1+\cos\left(fx+e\right)\right)^{2}\left(\cos\left(fx+e\right)\right)^{3}}{5fa^{3}\left(\sin\left(fx+e\right)\right)^{5}}\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -2/5/a^3/f*(-1+cos(f*x+e))^2*cos(f*x+e)^3*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1
/2)/sin(f*x+e)^5
```

Maxima [B] time = 1.49168, size = 184, normalized size = 4.49

$$-\frac{\sqrt{2}\sqrt{c}-\frac{3\sqrt{2}\sqrt{c}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{3\sqrt{2}\sqrt{c}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{\sqrt{2}\sqrt{c}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}}{20\,a^{3}f\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1}\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -1/20*(sqrt(2)*sqrt(c) - 3*sqrt(2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 3*sqrt(2)*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - sqrt(2)*sqrt(
c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/(a^3*f*sqrt(sin(f*x + e)/(cos(f*x +
 e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) - 1))
```

Fricas [A] time = 0.467493, size = 176, normalized size = 4.29

$$-\frac{2\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{3}}{5\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -2/5*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^3/((a^3*f*cos(f*x
 + e)^2 + 2*a^3*f*cos(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x)/a**3
```

Giac [B] time = 1.64461, size = 194, normalized size = 4.73

$$\frac{\sqrt{2}\left(5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\frac{\left(3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}+5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)\right.}{c}\right.}{60\,a^3cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] 1/60*sqrt(2)*(5*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*sgn(tan(1/2*f*x + 1/2*
e)^3 + tan(1/2*f*x + 1/2*e)) - (3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2) + 5*
(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/
2*f*x + 1/2*e))/c)*sgn(cos(f*x + e))/(a^3*c*f)
```

3.104 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=181

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f}+\frac{\tan(e+fx)}{4f\left(a^3\sec(e+fx)+a^3\right)\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{6af(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}+$$

```
[Out] -ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(4*Sqrt[
2]*a^3*Sqrt[c]*f) + Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec
[e + f*x]]) + Tan[e + f*x]/(6*a*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e +
 f*x]]) + Tan[e + f*x]/(4*f*(a^3 + a^3*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x
]])
```

Rubi [A] time = 0.394111, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3960, 3795, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f}+\frac{\tan(e+fx)}{4f\left(a^3\sec(e+fx)+a^3\right)\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{6af(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(4*Sqrt[
2]*a^3*Sqrt[c]*f) + Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec
[e + f*x]]) + Tan[e + f*x]/(6*a*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e +
 f*x]]) + Tan[e + f*x]/(4*f*(a^3 + a^3*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x
]])
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+f}}{2a}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx}{6af(a+a\sec(e+fx))^2\sqrt{}}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx}{6af(a+a\sec(e+fx))^2\sqrt{}}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx}{6af(a+a\sec(e+fx))^2\sqrt{}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 1.58037, size = 225, normalized size = 1.24

$$\frac{2e^{-\frac{1}{2}i(e+fx)}\sin\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{7}{2}}(e+fx)\left(\frac{e^{\frac{1}{2}i(e+fx)}(80\cos(e+fx)+37\cos(2(e+fx))+67)}{8\sqrt{\sec(e+fx)}} - 15\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e-}}\right.}{15a^3f(\sec(e+fx)+1)^3\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(-15*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sq
rt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 +
 E^((2*I)*(e + f*x))])]*Cos[(e + f*x)/2]^5 + (E^((I/2)*(e + f*x))*(67 + 80*
Cos[e + f*x] + 37*Cos[2*(e + f*x)]))/(8*Sqrt[Sec[e + f*x]]))*Sec[e + f*x]^(
7/2)*Sin[(e + f*x)/2])/(15*a^3*E^((I/2)*(e + f*x))*f*(1 + Sec[e + f*x])^3*S
qrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.244, size = 155, normalized size = 0.9

$$-\frac{-1+\cos(fx+e)}{60\,fa^3\sin(fx+e)}\left(3\left(-2\frac{\cos(fx+e)}{1+\cos(fx+e)}\right)^{5/2} - 5\left(-2\frac{\cos(fx+e)}{1+\cos(fx+e)}\right)^{3/2} + 15\arctan\left(\frac{1}{\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}}\right) + 15\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/60/a^3/f*(-1+cos(f*x+e))*(3*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)-5*(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(3/2)+15*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(1/2))+15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))/(c*(-1+cos(f*x+e))/cos(f*x+
e))^(1/2)/sin(f*x+e)/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)^3\sqrt{-c\sec(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^3*sqrt(-c*sec(f*x + e) + c)),
x)
```

Fricas [A] time = 0.659831, size = 1053, normalized size = 5.82

$$\left[-\frac{15\sqrt{2}\left(\cos(fx+e)^2+2\cos(fx+e)+1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{-c}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}+(3c\cos(fx+e)+c)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin}{240\left(a^3cf\cos(fx+e)^2+2a^3cf\cos(fx+e)+a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/240*(15*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(-c)*log((2*s
qrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin
(f*x + e)))*sin(f*x + e) + 4*(37*cos(f*x + e)^3 + 40*cos(f*x + e)^2 + 15*co
s(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c*f*cos(f*x + e)
^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e)), 1/120*(15*sqrt(2)*(co
s(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) -
2*(37*cos(f*x + e)^3 + 40*cos(f*x + e)^2 + 15*cos(f*x + e))*sqrt((c*cos(f*x
 + e) - c)/cos(f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e)
 + a^3*c*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(e+fx)}{\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)+3\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)+3\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+\sqrt{-c\sec(e+fx)+c}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 3*sqrt(-
c*sec(e + f*x) + c)*sec(e + f*x)**2 + 3*sqrt(-c*sec(e + f*x) + c)*sec(e + f
```

```
*x) + sqrt(-c*sec(e + f*x) + c)), x)/a**3
```

Giac [C] time = 1.61584, size = 261, normalized size = 1.44

$$-\frac{\frac{\sqrt{2}\left(15 i \sqrt{-c}\arctan\left(-i\right)-23 \sqrt{-c}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)\right)}{a^{3} c}+\frac{\sqrt{2}\left(\frac{15 \arctan\left(\frac{\sqrt{c \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}-c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{3 \left(c \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}-c\right)^{\frac{5}{2}} c^{12}-5 \left(c \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}-c\right)^{\frac{3}{2}} c^{13}+15 \sqrt{c \tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}-c}\, c^{14}}{c^{15}}\right)}{a^{3}\operatorname{sgn}\left(\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)^{2}-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2} f x+\frac{1}{2} e\right)\right)}}{120 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/120*(sqrt(2)*(15*I*sqrt(-c)*arctan(-I) - 23*sqrt(-c))*sgn(tan(1/2*f*x +
1/2*e))/(a^3*c) + sqrt(2)*(15*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqr
t(c))/sqrt(c) - (3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*c^12 - 5*(c*tan(1/2
*f*x + 1/2*e)^2 - c)^(3/2)*c^13 + 15*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^1
4)/c^15)/(a^3*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e))))/f
```

3.105 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f}-\frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}}+\frac{7\tan(e+fx)}{12f\left(a^3\sec(e+fx)+a^3\right)(c-c\sec(e+fx))^{3/2}}+\frac{}{30af(a\sec(}$$

```
[Out] (-7*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(16*
Sqrt[2]*a^3*c^(3/2)*f) - (7*Tan[e + f*x])/(16*a^3*f*(c - c*Sec[e + f*x])^(3
/2)) + Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2))
 + (7*Tan[e + f*x])/(30*a*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/
2)) + (7*Tan[e + f*x])/(12*f*(a^3 + a^3*Sec[e + f*x])*(c - c*Sec[e + f*x])^
(3/2))
```

Rubi [A] time = 0.477317, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3960, 3796, 3795, 203}

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f}-\frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}}+\frac{7\tan(e+fx)}{12f\left(a^3\sec(e+fx)+a^3\right)(c-c\sec(e+fx))^{3/2}}+\frac{}{30af(a\sec(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)),x]
```


```
[Out] (-7*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(16*
Sqrt[2]*a^3*c^(3/2)*f) - (7*Tan[e + f*x])/(16*a^3*f*(c - c*Sec[e + f*x])^(3
/2)) + Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2))
 + (7*Tan[e + f*x])/(30*a*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/
2)) + (7*Tan[e + f*x])/(12*f*(a^3 + a^3*Sec[e + f*x])*(c - c*Sec[e + f*x])^
(3/2))
```


Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
```

```
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}} + \frac{7\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\,\mathrm{s}}}{10a}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}} + \frac{7\tan}{30af(a+a\sec(e+f}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}} + \frac{7\tan}{30af(a+a\sec(e+f}$$

$$= -\frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+}$$

$$= -\frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f} - \frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}} + \frac{}{5f(a+a}$$

Mathematica [C] time = 6.44551, size = 398, normalized size = 1.88

$$\frac{\sin^3\left(\frac{e}{2}+\frac{fx}{2}\right)\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^5(e+fx)\left(\frac{278\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)}{15f} - \frac{278\cos\left(\frac{e}{2}\right)\cos\left(\frac{fx}{2}\right)}{15f} + \frac{2\sec^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f} - \frac{56\sec^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{15f} + \frac{242\sec\left(\frac{e}{2}+\frac{f}{2}\right.}{15f}\right.}{(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)),
x]
```

```
[Out] (7*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f
*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])
]*Cos[e/2 + (f*x)/2]^6*Sec[e + f*x]^(9/2)*Sin[e/2 + (f*x)/2]^3)/(E^((I/2)*(
e + f*x))*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)) + (Cos[e/2 +
 (f*x)/2]^6*Sec[e + f*x]^5*((-278*Cos[e/2]*Cos[(f*x)/2])/(15*f) - (Cot[e/2]
*Csc[e/2 + (f*x)/2])/f + (242*Sec[e/2 + (f*x)/2])/(15*f) - (56*Sec[e/2 + (f
*x)/2]^3)/(15*f) + (2*Sec[e/2 + (f*x)/2]^5)/(5*f) + (Csc[e/2]*Csc[e/2 + (f*
x)/2]^2*Sin[(f*x)/2])/f + (278*Sin[e/2]*Sin[(f*x)/2])/(15*f))*Sin[e/2 + (f*
x)/2]^3)/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2))
```

Maple [A] time = 0.27, size = 370, normalized size = 1.8

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}}{120\,fa^{3}\left(\sin\left(fx+e\right)\right)^{3}}\left(15\,\cos\left(fx+e\right)\left(-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{9/2}+15\left(-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}\right)^{9/2}+15\left(-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/120/a^3/f*(-1+cos(f*x+e))^2*(15*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))
^(9/2)+15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)+15*(-2*cos(f*x+e)/(1+cos(f*x
+e)))^(7/2)*cos(f*x+e)-15*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)-21*cos(f*x+e
)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+21*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5
/2)+35*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-35*(-2*cos(f*x+e)/(1
+cos(f*x+e)))^(3/2)-105*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)-105
*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))+105*(-2*cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)+105*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2))
)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)^3/(-2*cos(f*x+e)/(1+cos(f
*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\left(a\sec\left(fx+e\right)+a\right)^{3}\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^(3/2))
, x)
```

Fricas [A] time = 0.6973, size = 1242, normalized size = 5.86

$$\left[-\frac{105\,\sqrt{2}\left(\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)-1\right)\sqrt{-c}\log\left(\frac{2\,\sqrt{2}\left(\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3\,c\cos\left(fx+e\right)+\right.}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right.}{960\left(a^{3}c^{2}f\cos\left(fx+e\right)^{3}+a^{3}c^{2}f\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] [-1/960*(105*sqrt(2)*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*s
qrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos
```

```
(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f
*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) + 4*(139*cos(f*x + e)^4 + 21*cos(f
*x + e)^3 - 175*cos(f*x + e)^2 - 105*cos(f*x + e))*sqrt((c*cos(f*x + e) - c
)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^
3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e)), 1/480*(105*sqrt(2)*(cos(f*
x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(c)*arctan(sqrt(2)*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(
f*x + e) - 2*(139*cos(f*x + e)^4 + 21*cos(f*x + e)^3 - 175*cos(f*x + e)^2 -
 105*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^2*f*cos
(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f
)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.75236, size = 257, normalized size = 1.21

$$-\frac{\sqrt{2}\left(105\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)-\frac{15\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}-\frac{2\left(3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}c^8-10\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c^9+45\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^{10}\right)}{c^{10}}\right)}{480\,a^3c^2f\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/480*sqrt(2)*(105*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(
c)) - 15*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/tan(1/2*f*x + 1/2*e)^2 - 2*(3*(
c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*c^8 - 10*(c*tan(1/2*f*x + 1/2*e)^2 - c)
^(3/2)*c^9 + 45*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^10)/c^10)/(a^3*c^2*f*s
gn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)))
```

3.106 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=246

$$-\frac{63\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f}-\frac{63\tan(e+fx)}{128a^3cf(c-c\sec(e+fx))^{3/2}}-\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}}+\frac{21\tan(e+f}{20f\left(a^3\sec(e+fx)+a^3\right)(c}$$

```
[Out] (-63*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(12
8*Sqrt[2]*a^3*c^(5/2)*f) - (21*Tan[e + f*x])/(32*a^3*f*(c - c*Sec[e + f*x])
^(5/2)) + Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/
2)) + (3*Tan[e + f*x])/(10*a*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^
(5/2)) + (21*Tan[e + f*x])/(20*f*(a^3 + a^3*Sec[e + f*x])*(c - c*Sec[e + f*
x])^(5/2)) - (63*Tan[e + f*x])/(128*a^3*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.532018, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3960, 3796, 3795, 203}

$$-\frac{63\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f}-\frac{63\tan(e+fx)}{128a^3cf(c-c\sec(e+fx))^{3/2}}-\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}}+\frac{21\tan(e+f}{20f\left(a^3\sec(e+fx)+a^3\right)(c}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)),x]
```


```
[Out] (-63*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(12
8*Sqrt[2]*a^3*c^(5/2)*f) - (21*Tan[e + f*x])/(32*a^3*f*(c - c*Sec[e + f*x])
^(5/2)) + Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/
2)) + (3*Tan[e + f*x])/(10*a*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^
(5/2)) + (21*Tan[e + f*x])/(20*f*(a^3 + a^3*Sec[e + f*x])*(c - c*Sec[e + f*
x])^(5/2)) - (63*Tan[e + f*x])/(128*a^3*c*f*(c - c*Sec[e + f*x])^(3/2))
```


Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
```

```
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}} + \frac{9\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\,\mathrm{s}}}{10a}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}} + \frac{3\tan}{10af(a+a\sec(e+f}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}} + \frac{3\tan}{10af(a+a\sec(e+f}$$

$$= -\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+}$$

$$= -\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+}$$

$$= -\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+}$$

$$= -\frac{63\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f} - \frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}} + \frac{}{5f(a+}$$

Mathematica [C] time = 6.63869, size = 468, normalized size = 1.9

$$\frac{\sin^5\left(\frac{e}{2}+\frac{fx}{2}\right)\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^6(e+fx)\left(-\frac{257\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)}{10f}+\frac{257\cos\left(\frac{e}{2}\right)\cos\left(\frac{fx}{2}\right)}{10f}-\frac{2\sec^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f}+\frac{22\sec^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f}-\frac{124\sec\left(\frac{e}{2}+\right.}{5f}\right.}{(a\sec(e+fx)+a)^3(c-c\sec(e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)),
x]
```

```
[Out] (-63*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e +
 f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))
])]*Cos[e/2 + (f*x)/2]^6*Sec[e + f*x]^(11/2)*Sin[e/2 + (f*x)/2]^5)/(4*E^((I
/2)*(e + f*x))*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)) + (Cos[
e/2 + (f*x)/2]^6*Sec[e + f*x]^6*((257*Cos[e/2]*Cos[(f*x)/2])/(10*f) + (23*C
ot[e/2]*Csc[e/2 + (f*x)/2])/(4*f) - (Cot[e/2]*Csc[e/2 + (f*x)/2]^3)/(2*f) -
 (124*Sec[e/2 + (f*x)/2])/(5*f) + (22*Sec[e/2 + (f*x)/2]^3)/(5*f) - (2*Sec[
e/2 + (f*x)/2]^5)/(5*f) - (23*Csc[e/2]*Csc[e/2 + (f*x)/2]^2*Sin[(f*x)/2])/(
4*f) + (Csc[e/2]*Csc[e/2 + (f*x)/2]^4*Sin[(f*x)/2])/(2*f) - (257*Sin[e/2]*S
in[(f*x)/2])/(10*f))*Sin[e/2 + (f*x)/2]^5)/((a + a*Sec[e + f*x])^3*(c - c*S
ec[e + f*x])^(5/2))
```

Maple [B] time = 0.278, size = 631, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/160/a^3/f*(-1+cos(f*x+e))^3*(45*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(11/2)*co
s(f*x+e)^2+20*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(11/2)*cos(f*x+e)+35*cos(f*x+e
)^2*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)-25*(-2*cos(f*x+e)/(1+cos(f*x+e)))^
(11/2)-70*cos(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)-45*(-2*cos(f*x+e)
/(1+cos(f*x+e)))^(7/2)*cos(f*x+e)^2+35*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(9/2)
+90*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)*cos(f*x+e)+63*(-2*cos(f*x+e)/(1+co
s(f*x+e)))^(5/2)*cos(f*x+e)^2-45*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(7/2)-126*c
os(f*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)-105*cos(f*x+e)^2*(-2*cos(f*x
+e)/(1+cos(f*x+e)))^(3/2)+63*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)+210*cos(f
*x+e)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)+315*cos(f*x+e)^2*(-2*cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)+315*cos(f*x+e)^2*arctan(1/(-2*cos(f*x+e)/(1+cos(f*x+e
)))^(1/2))-105*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(3/2)-630*(-2*cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*cos(f*x+e)-630*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(1+cos(
f*x+e)))^(1/2))+315*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+315*arctan(1/(-2*c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(
f*x+e)^5/(-2*cos(f*x+e)/(1+cos(f*x+e)))^(5/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0.835112, size = 1207, normalized size = 4.91

$$\left[-\frac{315\sqrt{2}\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)s}{2560\left(a^3c^3f\cos\left(fx+e\right)^4-2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/2560*(315*sqrt(2)*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-c)*log(
(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) -
c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)
*sin(f*x + e)))*sin(f*x + e) + 4*(257*cos(f*x + e)^5 - 354*cos(f*x + e)^4 -
 588*cos(f*x + e)^3 + 210*cos(f*x + e)^2 + 315*cos(f*x + e))*sqrt((c*cos(f*
x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x
 + e)^2 + a^3*c^3*f)*sin(f*x + e)), 1/1280*(315*sqrt(2)*(cos(f*x + e)^4 - 2
*cos(f*x + e)^2 + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(257*cos(f*x
 + e)^5 - 354*cos(f*x + e)^4 - 588*cos(f*x + e)^3 + 210*cos(f*x + e)^2 + 31
5*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*
x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.89017, size = 298, normalized size = 1.21

$$-\frac{\sqrt{2}\left(315\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)-\frac{5\left(17\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c+15\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^2\right)}{c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}-\frac{8\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}c^8-5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c^9+30\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^{10}\right)}{c^{10}}\right)}{1280\,a^3c^3f\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/1280*sqrt(2)*(315*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt
(c)) - 5*(17*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c + 15*sqrt(c*tan(1/2*f*x
 + 1/2*e)^2 - c)*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4) - 8*((c*tan(1/2*f*x + 1/
2*e)^2 - c)^(5/2)*c^8 - 5*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^9 + 30*sqr
t(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^10)/c^10)/(a^3*c^3*f*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1)*sgn(tan(1/2*f*x + 1/2*e)))
```

3.107 $\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=43

$$\frac{a\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (a*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.132595, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3953}

$$\frac{a\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx=\frac{a(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [B] time = 0.463943, size = 87, normalized size = 2.02

$$\frac{c^2(-6\cos(e+fx)+3\cos(2(e+fx))+5)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),
x]
```

```
[Out] (c^2*(5 - 6*Cos[e + f*x] + 3*Cos[2*(e + f*x)])*Csc[(e + f*x)/2]*Sec[(e + f*
x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(
12*f)
```

Maple [B] time = 0.3, size = 82, normalized size = 1.9

$$-\frac{\sin\left(fx+e\right)\left(7\left(\cos\left(fx+e\right)\right)^{2}-4\cos\left(fx+e\right)+1\right)}{3f\left(-1+\cos\left(fx+e\right)\right)^{3}}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*sin(f*x+e)*(7*cos(f*x+e)^2-4*cos(f*x+e)+1)*(c*(-1+cos(f*x+e))/cos(f*
x+e))^(5/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/(-1+cos(f*x+e))^3
```

Maxima [B] time = 1.87654, size = 861, normalized size = 20.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(30*c^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 9*c^2*cos(2*f*x + 2*e)*sin(
f*x + e) - 3*c^2*sin(f*x + e) - (3*c^2*sin(5*f*x + 5*e) - 6*c^2*sin(4*f*x +
 4*e) + 10*c^2*sin(3*f*x + 3*e) - 6*c^2*sin(2*f*x + 2*e) + 3*c^2*sin(f*x +
e))*cos(6*f*x + 6*e) + 9*(c^2*sin(4*f*x + 4*e) + c^2*sin(2*f*x + 2*e))*cos(
5*f*x + 5*e) - 3*(10*c^2*sin(3*f*x + 3*e) + 3*c^2*sin(f*x + e))*cos(4*f*x +
 4*e) + (3*c^2*cos(5*f*x + 5*e) - 6*c^2*cos(4*f*x + 4*e) + 10*c^2*cos(3*f*x
 + 3*e) - 6*c^2*cos(2*f*x + 2*e) + 3*c^2*cos(f*x + e))*sin(6*f*x + 6*e) - 3
*(3*c^2*cos(4*f*x + 4*e) + 3*c^2*cos(2*f*x + 2*e) + c^2)*sin(5*f*x + 5*e) +
 3*(10*c^2*cos(3*f*x + 3*e) + 3*c^2*cos(f*x + e) + 2*c^2)*sin(4*f*x + 4*e)
- 10*(3*c^2*cos(2*f*x + 2*e) + c^2)*sin(3*f*x + 3*e) + 3*(3*c^2*cos(f*x + e
) + 2*c^2)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*co
s(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*x
+ 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^2
+ 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*
e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*sin(
2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [B] time = 0.473063, size = 225, normalized size = 5.23

$$\frac{\left(3c^{2}\cos\left(fx+e\right)^{2}-3c^{2}\cos\left(fx+e\right)+c^{2}\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*c^2*cos(f*x + e)^2 - 3*c^2*cos(f*x + e) + c^2)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^
```

```
2*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.108 $\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=43

$$\frac{a\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (a*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.132618, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3953}

$$\frac{a\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\frac{a(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] time = 0.301323, size = 73, normalized size = 1.7

$$\frac{c(2\cos(e+fx)-1)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),
x]
```

```
[Out] (c*(-1 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]*Sqr
t[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(4*f)
```

Maple [A] time = 0.313, size = 72, normalized size = 1.7

$$-\frac{\sin\left(fx+e\right)\left(3\cos\left(fx+e\right)-1\right)}{2f\left(-1+\cos\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*sin(f*x+e)*(3*cos(f*x+e)-1)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(c
*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/(-1+cos(f*x+e))^2
```

Maxima [B] time = 1.86547, size = 402, normalized size = 9.35

$$\frac{2\left(2c\cos\left(3fx+3e\right)\sin\left(2fx+2e\right)-2c\cos\left(2fx+2e\right)\sin\left(fx+e\right)-\left(c\sin\left(3fx+3e\right)-c\sin\left(2fx+2e\right)+c\sin\left(f\right.\right.\right.}{\left(2\left(2\cos\left(2fx+2e\right)+1\right)\cos\left(4fx+4e\right)+\cos\left(4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*(2*c*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 2*c*cos(2*f*x + 2*e)*sin(f*x + e
) - (c*sin(3*f*x + 3*e) - c*sin(2*f*x + 2*e) + c*sin(f*x + e))*cos(4*f*x +
4*e) + (c*cos(3*f*x + 3*e) - c*cos(2*f*x + 2*e) + c*cos(f*x + e))*sin(4*f*x
 + 4*e) - (2*c*cos(2*f*x + 2*e) + c)*sin(3*f*x + 3*e) + (2*c*cos(f*x + e) +
 c)*sin(2*f*x + 2*e) - c*sin(f*x + e))*sqrt(a)*sqrt(c)/((2*(2*cos(2*f*x + 2
*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin
(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^
2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [B] time = 0.4648, size = 186, normalized size = 4.33

$$\frac{\left(2c\cos\left(fx+e\right)-c\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{2f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(2*c*cos(f*x + e) - c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.109 $\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=41

$$-\frac{c\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -((c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]]))
```

Rubi [A] time = 0.120371, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3953}

$$-\frac{c\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -((c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]]))
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx=-\frac{c\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.162722, size = 56, normalized size = 1.37

$$\frac{\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Se
c[e + f*x]])/(2*f)
```

Maple [A] time = 0.304, size = 62, normalized size = 1.5

$$-\frac{\sin(fx+e)}{f(-1+\cos(fx+e))}\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(
1/2)*sin(f*x+e)/(-1+cos(f*x+e))
```

Maxima [A] time = 1.55705, size = 74, normalized size = 1.8

$$\frac{2\sqrt{-a}\sqrt{c}}{f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*sqrt(-a)*sqrt(c)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1))
```

Fricas [A] time = 0.465663, size = 132, normalized size = 3.22

$$\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{f\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))/(f*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x
), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.110 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{a\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (a*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rubi [A] time = 0.125257, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3952}

$$\frac{a\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.852781, size = 99, normalized size = 1.94

$$-\frac{i\left(-1+e^{i(e+fx)}\right)\left(2\log\left(1-e^{i(e+fx)}\right)-\log\left(1+e^{2i(e+fx)}\right)\right)\sqrt{a(\sec(e+fx)+1)}}{f\left(1+e^{i(e+fx)}\right)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x]],
x]
```

```
[Out] ((-I)*(-1 + E^(I*(e + f*x)))*(2*Log[1 - E^(I*(e + f*x))] - Log[1 + E^((2*I)
*(e + f*x))])*Sqrt[a*(1 + Sec[e + f*x])])/((1 + E^(I*(e + f*x)))*f*Sqrt[c -
 c*Sec[e + f*x]])
```

Maple [B] time = 0.309, size = 136, normalized size = 2.7

$$\frac{\cos\left(fx+e\right)}{f\sin\left(fx+e\right)c}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(ln(-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))+ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*ln(-(-1+cos(f*x+e))
/sin(f*x+e)))*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/c
```

Maxima [A] time = 1.5226, size = 124, normalized size = 2.43

$$-\frac{\frac{\sqrt{-a}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{\sqrt{c}}+\frac{\sqrt{-a}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{\sqrt{c}}-\frac{2\sqrt{-a}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{\sqrt{c}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -(sqrt(-a)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(c) + sqrt(-a)*log(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - 2*sqrt(-a)*log(sin(f*x + e)/
(cos(f*x + e) + 1))/sqrt(c))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}\sec\left(fx+e\right)}{c\sec\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)*sec(f*x + e)/(
c*sec(f*x + e) - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/sqrt(-c*(sec(e + f*x) - 1)
), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.111 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{2f(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -(Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(2*f*(c - c*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.139167, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3950}

$$-\frac{\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{2f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(2*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 0.251898, size = 62, normalized size = 1.48

$$\frac{\tan\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}}{cf(\sec(e+fx)-1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(3/2
),x]
```

```
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c*f*(-1 + Sec[e
 + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.303, size = 60, normalized size = 1.4

$$-\frac{\sin\left(fx+e\right)}{2f\cos\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*sin(f*x+e)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/cos(f*x+e)/(c*(-1+c
os(f*x+e))/cos(f*x+e))^(3/2)
```

Maxima [B] time = 1.83427, size = 694, normalized size = 16.52

$$-\frac{}{\left(c^2\cos\left(4fx+4e\right)^2+4c^2\cos\left(3fx+3e\right)^2+4c^2\cos\left(2fx+2e\right)^2+4c^2\cos\left(fx+e\right)^2+c^2\sin\left(4fx+4e\right)^2+4c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] -2*((sin(3*f*x + 3*e) + sin(f*x + e))*cos(4*f*x + 4*e) - (cos(3*f*x + 3*e)
+ cos(f*x + e))*sin(4*f*x + 4*e) + (2*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e
) - 2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 2*cos(f*x + e)*sin(2*f*x + 2*e) +
 2*cos(2*f*x + 2*e)*sin(f*x + e) + sin(f*x + e))*sqrt(a)*sqrt(c)/((c^2*cos(
4*f*x + 4*e)^2 + 4*c^2*cos(3*f*x + 3*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + 4*c^
2*cos(f*x + e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(3*f*x + 3*e)^2 + 4*c^
2*sin(2*f*x + 2*e)^2 - 8*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*c^2*sin(f*x
+ e)^2 - 4*c^2*cos(f*x + e) + c^2 - 2*(2*c^2*cos(3*f*x + 3*e) - 2*c^2*cos(2
*f*x + 2*e) + 2*c^2*cos(f*x + e) - c^2)*cos(4*f*x + 4*e) - 4*(2*c^2*cos(2*f
*x + 2*e) - 2*c^2*cos(f*x + e) + c^2)*cos(3*f*x + 3*e) - 4*(2*c^2*cos(f*x +
 e) - c^2)*cos(2*f*x + 2*e) - 4*(c^2*sin(3*f*x + 3*e) - c^2*sin(2*f*x + 2*e
) + c^2*sin(f*x + e))*sin(4*f*x + 4*e) - 8*(c^2*sin(2*f*x + 2*e) - c^2*sin(
f*x + e))*sin(3*f*x + 3*e))*f)
```

Fricas [B] time = 0.461, size = 186, normalized size = 4.43

$$\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{\left(c^2f\cos\left(fx+e\right)-c^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))*cos(f*x + e)/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(-c*(sec(e + f*x) - 1))**(
3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.112 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

```
[Out] -(a*Tan[e + f*x])/(2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2))
```

Rubi [A] time = 0.136812, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3953}

$$-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(a*Tan[e + f*x])/(2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2))
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}$$

Mathematica [A] time = 0.38449, size = 69, normalized size = 1.6

$$-\frac{(2\cos(e+fx)-1)\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}}{2c^2f(\cos(e+fx)-1)^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(5/2
),x]
```

```
[Out] -((-1 + 2*Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(2*c^2
*f*(-1 + Cos[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.293, size = 70, normalized size = 1.6

$$-\frac{\left(3\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}{8f\left(\cos\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/8/f*(3*cos(f*x+e)-1)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*sin(f*x+e)/co
s(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)
```

Maxima [B] time = 2.09418, size = 1023, normalized size = 23.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2*((sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + (sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - cos(2*f*x + 2*e)*sin(4*f*x + 4*e
) + cos(4*f*x + 4*e)*sin(2*f*x + 2*e) - (cos(4*f*x + 4*e) + 2*cos(2*f*x + 2
*e) + 1)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(4*f*x
+ 4*e) + 2*cos(2*f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((c^3*cos(4*f*x + 4*e)^2 + 3
6*c^3*cos(2*f*x + 2*e)^2 + 16*c^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 16*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
^2 + c^3*sin(4*f*x + 4*e)^2 + 12*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36
*c^3*sin(2*f*x + 2*e)^2 + 16*c^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 16*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + 12*c^3*cos(2*f*x + 2*e) + c^3 + 2*(6*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*
f*x + 4*e) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*x + 2*e) - 4*c^3*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^3)*cos(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*
x + 2*e) + c^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c
^3*sin(4*f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e) - 4*c^3*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 8*(c^3*sin(4*f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e))*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [B] time = 0.471764, size = 254, normalized size = 5.91

$$\frac{\left(2\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{2\left(c^{3}f\cos\left(fx+e\right)^{2}-2c^{3}f\cos\left(fx+e\right)+c^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f
*cos(f*x + e) + c^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.113 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=89

$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{7/2}}{10f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}{5f}$$

```
[Out] (a^2*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(10*f*Sqrt[a + a*Sec[e + f*x]
]) + (a*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(
5*f)
```

Rubi [A] time = 0.275052, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3953}

$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{7/2}}{10f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a^2*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(10*f*Sqrt[a + a*Sec[e + f*x]
]) + (a*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(
5*f)
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx=\frac{a\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{5f}+\frac{1}{5}($$

$$=\frac{a^2(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{10f\sqrt{a+a\sec(e+fx)}}+\frac{a\sqrt{a+a\sec(e+fx)}}{}$$

Mathematica [A] time = 1.07061, size = 108, normalized size = 1.21

$$\frac{ac^3(-10\cos(e+fx)+20\cos(2(e+fx))-10\cos(3(e+fx))+5\cos(4(e+fx))+7)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\mathrm{se}}{80f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/2
),x]
```

```
[Out] (a*c^3*(7 - 10*Cos[e + f*x] + 20*Cos[2*(e + f*x)] - 10*Cos[3*(e + f*x)] + 5
*Cos[4*(e + f*x)])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[a*
(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(80*f)
```

Maple [A] time = 0.275, size = 103, normalized size = 1.2

$$\frac{a\left(13\left(\cos\left(fx+e\right)\right)^{3}-16\left(\cos\left(fx+e\right)\right)^{2}+9\cos\left(fx+e\right)-2\right)\left(\sin\left(fx+e\right)\right)^{3}}{10f\left(-1+\cos\left(fx+e\right)\right)^{5}\cos\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\frac{c\left(-1+\cos\left(\right.\right.}{\cos\left(fx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] 1/10/f*a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e
))^(7/2)*(13*cos(f*x+e)^3-16*cos(f*x+e)^2+9*cos(f*x+e)-2)*sin(f*x+e)^3/(-1+
cos(f*x+e))^5/cos(f*x+e)
```

Maxima [B] time = 1.89077, size = 2268, normalized size = 25.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/5*(100*a*c^3*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 25*a*c^3*cos(2*f*x + 2*e
)*sin(f*x + e) - 5*a*c^3*sin(f*x + e) - (5*a*c^3*sin(9*f*x + 9*e) - 10*a*c^
3*sin(8*f*x + 8*e) + 20*a*c^3*sin(7*f*x + 7*e) - 10*a*c^3*sin(6*f*x + 6*e)
+ 14*a*c^3*sin(5*f*x + 5*e) - 10*a*c^3*sin(4*f*x + 4*e) + 20*a*c^3*sin(3*f*
x + 3*e) - 10*a*c^3*sin(2*f*x + 2*e) + 5*a*c^3*sin(f*x + e))*cos(10*f*x + 1
0*e) + 25*(a*c^3*sin(8*f*x + 8*e) + 2*a*c^3*sin(6*f*x + 6*e) + 2*a*c^3*sin(
4*f*x + 4*e) + a*c^3*sin(2*f*x + 2*e))*cos(9*f*x + 9*e) - 5*(20*a*c^3*sin(7
*f*x + 7*e) + 10*a*c^3*sin(6*f*x + 6*e) + 14*a*c^3*sin(5*f*x + 5*e) + 10*a*
c^3*sin(4*f*x + 4*e) + 20*a*c^3*sin(3*f*x + 3*e) + 5*a*c^3*sin(f*x + e))*co
s(8*f*x + 8*e) + 100*(2*a*c^3*sin(6*f*x + 6*e) + 2*a*c^3*sin(4*f*x + 4*e) +
 a*c^3*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 10*(14*a*c^3*sin(5*f*x + 5*e) +
 20*a*c^3*sin(3*f*x + 3*e) - 5*a*c^3*sin(2*f*x + 2*e) + 5*a*c^3*sin(f*x + e
))*cos(6*f*x + 6*e) + 70*(2*a*c^3*sin(4*f*x + 4*e) + a*c^3*sin(2*f*x + 2*e)
)*cos(5*f*x + 5*e) - 50*(4*a*c^3*sin(3*f*x + 3*e) - a*c^3*sin(2*f*x + 2*e)
+ a*c^3*sin(f*x + e))*cos(4*f*x + 4*e) + (5*a*c^3*cos(9*f*x + 9*e) - 10*a*c
^3*cos(8*f*x + 8*e) + 20*a*c^3*cos(7*f*x + 7*e) - 10*a*c^3*cos(6*f*x + 6*e)
```

```
 + 14*a*c^3*cos(5*f*x + 5*e) - 10*a*c^3*cos(4*f*x + 4*e) + 20*a*c^3*cos(3*f
*x + 3*e) - 10*a*c^3*cos(2*f*x + 2*e) + 5*a*c^3*cos(f*x + e))*sin(10*f*x +
10*e) - 5*(5*a*c^3*cos(8*f*x + 8*e) + 10*a*c^3*cos(6*f*x + 6*e) + 10*a*c^3*
cos(4*f*x + 4*e) + 5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(9*f*x + 9*e) + 5*(
20*a*c^3*cos(7*f*x + 7*e) + 10*a*c^3*cos(6*f*x + 6*e) + 14*a*c^3*cos(5*f*x
+ 5*e) + 10*a*c^3*cos(4*f*x + 4*e) + 20*a*c^3*cos(3*f*x + 3*e) + 5*a*c^3*co
s(f*x + e) + 2*a*c^3)*sin(8*f*x + 8*e) - 20*(10*a*c^3*cos(6*f*x + 6*e) + 10
*a*c^3*cos(4*f*x + 4*e) + 5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(7*f*x + 7*e
) + 10*(14*a*c^3*cos(5*f*x + 5*e) + 20*a*c^3*cos(3*f*x + 3*e) - 5*a*c^3*cos
(2*f*x + 2*e) + 5*a*c^3*cos(f*x + e) + a*c^3)*sin(6*f*x + 6*e) - 14*(10*a*c
^3*cos(4*f*x + 4*e) + 5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(5*f*x + 5*e) +
10*(20*a*c^3*cos(3*f*x + 3*e) - 5*a*c^3*cos(2*f*x + 2*e) + 5*a*c^3*cos(f*x
+ e) + a*c^3)*sin(4*f*x + 4*e) - 20*(5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(
3*f*x + 3*e) + 5*(5*a*c^3*cos(f*x + e) + 2*a*c^3)*sin(2*f*x + 2*e))*sqrt(a)
*sqrt(c)/((2*(5*cos(8*f*x + 8*e) + 10*cos(6*f*x + 6*e) + 10*cos(4*f*x + 4*e
) + 5*cos(2*f*x + 2*e) + 1)*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 10*
(10*cos(6*f*x + 6*e) + 10*cos(4*f*x + 4*e) + 5*cos(2*f*x + 2*e) + 1)*cos(8*
f*x + 8*e) + 25*cos(8*f*x + 8*e)^2 + 20*(10*cos(4*f*x + 4*e) + 5*cos(2*f*x
+ 2*e) + 1)*cos(6*f*x + 6*e) + 100*cos(6*f*x + 6*e)^2 + 20*(5*cos(2*f*x + 2
*e) + 1)*cos(4*f*x + 4*e) + 100*cos(4*f*x + 4*e)^2 + 25*cos(2*f*x + 2*e)^2
+ 10*(sin(8*f*x + 8*e) + 2*sin(6*f*x + 6*e) + 2*sin(4*f*x + 4*e) + sin(2*f*
x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 50*(2*sin(6*f*x + 6*e
) + 2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 25*sin(8*f*x
+ 8*e)^2 + 100*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 1
00*sin(6*f*x + 6*e)^2 + 100*sin(4*f*x + 4*e)^2 + 100*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + 25*sin(2*f*x + 2*e)^2 + 10*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A] time = 0.489845, size = 274, normalized size = 3.08

$$\frac{\left(10\,ac^3\cos\left(fx+e\right)^4-10\,ac^3\cos\left(fx+e\right)^3+5\,ac^3\cos\left(fx+e\right)-2\,ac^3\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{10\,f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/10*(10*a*c^3*cos(f*x + e)^4 - 10*a*c^3*cos(f*x + e)^3 + 5*a*c^3*cos(f*x +
 e) - 2*a*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.114 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=89

$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{6f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}{4f}$$

```
[Out] (a^2*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(6*f*Sqrt[a + a*Sec[e + f*x]]
) + (a*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(4
*f)
```

Rubi [A] time = 0.275789, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3953}

$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{6f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(6*f*Sqrt[a + a*Sec[e + f*x]]
) + (a*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(4
*f)
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx &= \frac{a\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{4f}+\frac{1}{2}a\\ &= \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{6f\sqrt{a+a\sec(e+fx)}}+a\sqrt{a+a\sec(e+fx)}\end{aligned}$$

Mathematica [A] time = 0.591755, size = 97, normalized size = 1.09

$$\frac{ac^2(5\cos(e+fx)-3\cos(2(e+fx))+3\cos(3(e+fx)))\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(\sec(e+fx)+}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2
),x]
```

```
[Out] (a*c^2*(5*Cos[e + f*x] - 3*Cos[2*(e + f*x)] + 3*Cos[3*(e + f*x)])*Csc[(e +
f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c -
 c*Sec[e + f*x]])/(24*f)
```

Maple [A] time = 0.263, size = 93, normalized size = 1.

$$\frac{a\left(11\left(\cos\left(fx+e\right)\right)^2-10\cos\left(fx+e\right)+3\right)\left(\sin\left(fx+e\right)\right)^3}{12f\left(-1+\cos\left(fx+e\right)\right)^4\cos\left(fx+e\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/12/f*a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e
))^(5/2)*(11*cos(f*x+e)^2-10*cos(f*x+e)+3)*sin(f*x+e)^3/(-1+cos(f*x+e))^4/c
os(f*x+e)
```

Maxima [B] time = 1.85881, size = 1492, normalized size = 16.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(20*a*c^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 12*a*c^2*cos(2*f*x + 2*e)
*sin(f*x + e) - 3*a*c^2*sin(f*x + e) - (3*a*c^2*sin(7*f*x + 7*e) - 3*a*c^2*
sin(6*f*x + 6*e) + 5*a*c^2*sin(5*f*x + 5*e) + 5*a*c^2*sin(3*f*x + 3*e) - 3*
a*c^2*sin(2*f*x + 2*e) + 3*a*c^2*sin(f*x + e))*cos(8*f*x + 8*e) + 6*(2*a*c^
2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e) + 2*a*c^2*sin(2*f*x + 2*e))*c
os(7*f*x + 7*e) - 2*(10*a*c^2*sin(5*f*x + 5*e) + 9*a*c^2*sin(4*f*x + 4*e) +
 10*a*c^2*sin(3*f*x + 3*e) + 6*a*c^2*sin(f*x + e))*cos(6*f*x + 6*e) + 10*(3
*a*c^2*sin(4*f*x + 4*e) + 2*a*c^2*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 6*(5
*a*c^2*sin(3*f*x + 3*e) - 3*a*c^2*sin(2*f*x + 2*e) + 3*a*c^2*sin(f*x + e))*
cos(4*f*x + 4*e) + (3*a*c^2*cos(7*f*x + 7*e) - 3*a*c^2*cos(6*f*x + 6*e) + 5
*a*c^2*cos(5*f*x + 5*e) + 5*a*c^2*cos(3*f*x + 3*e) - 3*a*c^2*cos(2*f*x + 2*
e) + 3*a*c^2*cos(f*x + e))*sin(8*f*x + 8*e) - 3*(4*a*c^2*cos(6*f*x + 6*e) +
 6*a*c^2*cos(4*f*x + 4*e) + 4*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(7*f*x + 7
*e) + (20*a*c^2*cos(5*f*x + 5*e) + 18*a*c^2*cos(4*f*x + 4*e) + 20*a*c^2*cos
(3*f*x + 3*e) + 12*a*c^2*cos(f*x + e) + 3*a*c^2)*sin(6*f*x + 6*e) - 5*(6*a*
c^2*cos(4*f*x + 4*e) + 4*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(5*f*x + 5*e) +
```

```
6*(5*a*c^2*cos(3*f*x + 3*e) - 3*a*c^2*cos(2*f*x + 2*e) + 3*a*c^2*cos(f*x +
 e))*sin(4*f*x + 4*e) - 5*(4*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(3*f*x + 3*
e) + 3*(4*a*c^2*cos(f*x + e) + a*c^2)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2
*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*f
*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)
 + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) + 1
)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*s
in(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e)
 + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*
f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x +
 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A] time = 0.481349, size = 273, normalized size = 3.07

$$\frac{\left(12\,ac^2\cos\left(fx+e\right)^3-6\,ac^2\cos\left(fx+e\right)^2-4\,ac^2\cos\left(fx+e\right)+3\,ac^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{12\,f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*(12*a*c^2*cos(f*x + e)^3 - 6*a*c^2*cos(f*x + e)^2 - 4*a*c^2*cos(f*x +
e) + 3*a*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.115 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=89

$$-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{3f\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}{3f}$$

```
[Out] -(c^2*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]
]) - (c*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(
3*f)
```

Rubi [A] time = 0.274389, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3953}

$$-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{3f\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(c^2*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]
]) - (c*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(
3*f)
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx=-\frac{c(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f}$$

$$=-\frac{c^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}-\frac{c(a+a\sec(e+}{}$$

Mathematica [A] time = 0.430983, size = 78, normalized size = 0.88

$$\frac{ac(3\cos(2(e+fx))+1)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2
),x]
```

```
[Out] (a*c*(1 + 3*Cos[2*(e + f*x)])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x
]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(12*f)
```

Maple [A] time = 0.27, size = 83, normalized size = 0.9

$$\frac{a\left(\sin\left(fx+e\right)\right)^3\left(2\cos\left(fx+e\right)-1\right)}{3f\left(-1+\cos\left(fx+e\right)\right)^3\cos\left(fx+e\right)}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/3/f*a*sin(f*x+e)^3*(2*cos(f*x+e)-1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)*
(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/(-1+cos(f*x+e))^3/cos(f*x+e)
```

Maxima [B] time = 1.84834, size = 743, normalized size = 8.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(6*a*c*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 9*a*c*cos(f*x + e)*sin(2*f*x
 + 2*e) - 9*a*c*cos(2*f*x + 2*e)*sin(f*x + e) - 3*a*c*sin(f*x + e) - (3*a*c
*sin(5*f*x + 5*e) + 2*a*c*sin(3*f*x + 3*e) + 3*a*c*sin(f*x + e))*cos(6*f*x
+ 6*e) + 9*(a*c*sin(4*f*x + 4*e) + a*c*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) -
 3*(2*a*c*sin(3*f*x + 3*e) + 3*a*c*sin(f*x + e))*cos(4*f*x + 4*e) + (3*a*c*
cos(5*f*x + 5*e) + 2*a*c*cos(3*f*x + 3*e) + 3*a*c*cos(f*x + e))*sin(6*f*x +
 6*e) - 3*(3*a*c*cos(4*f*x + 4*e) + 3*a*c*cos(2*f*x + 2*e) + a*c)*sin(5*f*x
 + 5*e) + 3*(2*a*c*cos(3*f*x + 3*e) + 3*a*c*cos(f*x + e))*sin(4*f*x + 4*e)
- 2*(3*a*c*cos(2*f*x + 2*e) + a*c)*sin(3*f*x + 3*e))*sqrt(a)*sqrt(c)/((2*(3
*cos(4*f*x + 4*e) + 3*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x +
6*e)^2 + 6*(3*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2
 + 9*cos(2*f*x + 2*e)^2 + 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x
 + 6*e) + sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*s
in(2*f*x + 2*e) + 9*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A] time = 0.471238, size = 197, normalized size = 2.21

$$\frac{\left(3\,ac\cos\left(fx+e\right)^{2}-ac\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\,f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*a*c*cos(f*x + e)^2 - a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^2*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.116 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=43

$$-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -(c*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.129904, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3953}

$$-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -(c*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=-\frac{c(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.275162, size = 73, normalized size = 1.7

$$\frac{a(2\cos(e+fx)+1)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],
x]
```

```
[Out] (a*(1 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]*Sqrt
[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(4*f)
```

Maple [A] time = 0.322, size = 73, normalized size = 1.7

$$\frac{a\left(\sin\left(fx+e\right)\right)^{3}}{2f\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*sin(f*x+e)^3*(c*(-1+cos(f*x+e
))/cos(f*x+e))^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))^2
```

Maxima [A] time = 1.53053, size = 76, normalized size = 1.77

$$-\frac{2\sqrt{-a}a\sqrt{c}}{f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{2}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*sqrt(-a)*a*sqrt(c)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^2*(sin(f*x +
 e)/(cos(f*x + e) + 1) - 1)^2)
```

Fricas [B] time = 0.467276, size = 186, normalized size = 4.33

$$\frac{\left(2a\cos\left(fx+e\right)+a\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{2f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(2*a*cos(f*x + e) + a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.117 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2a^2\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (2*a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]) + (a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c
 - c*Sec[e + f*x]])
```

Rubi [A] time = 0.263783, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3952}

$$\frac{2a^2\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (2*a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]) + (a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c
 - c*Sec[e + f*x]])
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx &= \frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}+(2a)\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,d\\ &= \frac{2a^2\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [C] time = 1.44814, size = 174, normalized size = 1.83

$$\frac{\sqrt{2}a\sin\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{3}{2}}(e+fx)\sqrt{a(\sec(e+fx)+1)}\left(1+\left(4\log\left(1-e^{i(e+fx)}\right)-2\log\left(1+e^{2i(e+fx)}\right)\right)\cos(e+fx)\right)\left(\cos\left(\frac{1}{2}\right.\right.}{f\left(1+e^{i(e+fx)}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/Sqrt[c - c*Sec[e + f*x]
],x]
```

```
[Out] (Sqrt[2]*a*(1 + Cos[e + f*x]*(4*Log[1 - E^(I*(e + f*x))] - 2*Log[1 + E^((2*
I)*(e + f*x))]))*Sec[e + f*x]^(3/2)*Sqrt[a*(1 + Sec[e + f*x])]*(Cos[(e + f*
x)/2] + I*Sin[(e + f*x)/2])*Sin[(e + f*x)/2])/((1 + E^(I*(e + f*x)))*Sqrt[E
^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.338, size = 162, normalized size = 1.7

$$\frac{a}{f\sin(fx+e)c}\left(2\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)+2\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)+\sin(f}{\sin(fx+e)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*a*(2*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)
*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*cos(f*x+e)*ln(-(-1+cos(f*x+e)
)/sin(f*x+e))-cos(f*x+e)-1)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(c*(-1+co
s(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/c
```

Maxima [B] time = 1.90335, size = 371, normalized size = 3.91

$$-\frac{2\left(a\cos\left(\frac{1}{2}\arctan\left(\sin(2fx+2e),\cos(2fx+2e)\right)\right)\sin(2fx+2e)+\left(a\cos(2fx+2e)^{2}+a\sin(2fx+2e)^{2}+2a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*(a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e)
 + (a*cos(2*f*x + 2*e)^2 + a*sin(2*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*(a*cos(2*f*x + 2*e)^2
+ a*sin(2*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*arctan2(sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - 1) - (a*cos(2*f*x + 2*e) + a)*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c*cos(2*f*x + 2*e)^2 + c*sin(2
*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a\sec\left(fx+e\right)^{2}+a\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{c\sec\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(a*sec(f*x + e)^2 + a*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqrt
(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.118 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^2\tan(e+fx)\log(1-\sec(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -((a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2)))
 - (a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.275484, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3954, 3952}

$$-\frac{a^2\tan(e+fx)\log(1-\sec(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2)))
 - (a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx &= -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}-\frac{a\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c}\\ &= -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}-\frac{a^2\log(1-\sec(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.655738, size = 134, normalized size = 1.35

$$-\frac{a\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(-2\log\left(1-e^{i(e+fx)}\right)+\log\left(1+e^{2i(e+fx)}\right)+\left(2\log\left(1-e^{i(e+fx)}\right)-\log\left(1+e^{2i(e+\cdots}\right.\right.\right.}{cf(\cos(e+fx)-1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(3
/2),x]
```

```
[Out] -((a*(2 - 2*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(2*Log[1 - E^(I*(e + f*
x))] - Log[1 + E^((2*I)*(e + f*x))]) + Log[1 + E^((2*I)*(e + f*x))])*Sqrt[a
*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c*f*(-1 + Cos[e + f*x])*Sqrt[c - c*
Sec[e + f*x]]))
```

Maple [B] time = 0.308, size = 251, normalized size = 2.5

$$-\frac{a\left(-1+\cos\left(fx+e\right)\right)}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}\left(\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+\cdots\right.}{\sin\left(\cdots\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f*a*(-1+cos(f*x+e))*(cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))+cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*cos(f*x+e)*ln(-(
-1+cos(f*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-ln(-(
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-cos
(f*x+e)-1)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/cos(f*x+e)/(c*(-1+cos(f*x+
e))/cos(f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [A] time = 1.59513, size = 165, normalized size = 1.67

$$\frac{\frac{\sqrt{-a}a\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{-a}a\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{c^{\frac{3}{2}}}-\frac{2\sqrt{-a}a\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{-a}a\left(\cos\left(fx+e\right)+1\right)^{2}}{c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] (sqrt(-a)*a*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^(3/2) + sqrt(-a)*a*l
og(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - 2*sqrt(-a)*a*log(sin(f*x
+ e)/(cos(f*x + e) + 1))/c^(3/2) + sqrt(-a)*a*(cos(f*x + e) + 1)^2/(c^(3/2)
*sin(f*x + e)^2))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a\sec\left(fx+e\right)^{2}+a\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{c^{2}\sec\left(fx+e\right)^{2}-2\,c^{2}\sec\left(fx+e\right)+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((a*sec(f*x + e)^2 + a*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqrt(
-c*sec(f*x + e) + c)/(c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.119 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{4f(c-c\sec(e+fx))^{5/2}}$$

```
[Out] -((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(4*f*(c - c*Sec[e + f*x])^(5/2))
```

Rubi [A] time = 0.149108, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{4f(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(4*f*(c - c*Sec[e + f*x])^(5/2))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}$$

Mathematica [A] time = 0.492289, size = 63, normalized size = 1.5

$$\frac{a\tan\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}{c^3f(\sec(e+fx)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(5
/2),x]
```

```
[Out] (a*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]*Tan[(e
+ f*x)/2])/(c^3*f*(-1 + Sec[e + f*x])^3)
```

Maple [A] time = 0.267, size = 73, normalized size = 1.7

$$\frac{a\left(\sin\left(fx+e\right)\right)^{3}}{4f\left(-1+\cos\left(fx+e\right)\right)\left(\cos\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/4/f*a*sin(f*x+e)^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/(-1+cos(f*x+e))/
cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)
```

Maxima [B] time = 1.82011, size = 720, normalized size = 17.14

$$\left(c^{3}\cos\left(4fx+4e\right)^{2}+16c^{3}\cos\left(3fx+3e\right)^{2}+36c^{3}\cos\left(2fx+2e\right)^{2}+16c^{3}\cos\left(fx+e\right)^{2}+c^{3}\sin\left(4fx+4e\right)^{2}+16c^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2*(6*a*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 6*a*cos(f*x + e)*sin(2*f*x + 2*e
) - 6*a*cos(2*f*x + 2*e)*sin(f*x + e) - (a*sin(3*f*x + 3*e) + a*sin(f*x + e
))*cos(4*f*x + 4*e) + (a*cos(3*f*x + 3*e) + a*cos(f*x + e))*sin(4*f*x + 4*e
) - (6*a*cos(2*f*x + 2*e) + a)*sin(3*f*x + 3*e) - a*sin(f*x + e))*sqrt(a)*s
qrt(c)/((c^3*cos(4*f*x + 4*e)^2 + 16*c^3*cos(3*f*x + 3*e)^2 + 36*c^3*cos(2*
f*x + 2*e)^2 + 16*c^3*cos(f*x + e)^2 + c^3*sin(4*f*x + 4*e)^2 + 16*c^3*sin(
3*f*x + 3*e)^2 + 36*c^3*sin(2*f*x + 2*e)^2 - 48*c^3*sin(2*f*x + 2*e)*sin(f*
x + e) + 16*c^3*sin(f*x + e)^2 - 8*c^3*cos(f*x + e) + c^3 - 2*(4*c^3*cos(3*
f*x + 3*e) - 6*c^3*cos(2*f*x + 2*e) + 4*c^3*cos(f*x + e) - c^3)*cos(4*f*x +
 4*e) - 8*(6*c^3*cos(2*f*x + 2*e) - 4*c^3*cos(f*x + e) + c^3)*cos(3*f*x + 3
*e) - 12*(4*c^3*cos(f*x + e) - c^3)*cos(2*f*x + 2*e) - 4*(2*c^3*sin(3*f*x +
 3*e) - 3*c^3*sin(2*f*x + 2*e) + 2*c^3*sin(f*x + e))*sin(4*f*x + 4*e) - 16*
(3*c^3*sin(2*f*x + 2*e) - 2*c^3*sin(f*x + e))*sin(3*f*x + 3*e))*f)
```

Fricas [B] time = 0.470585, size = 225, normalized size = 5.36

$$\frac{a\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}}{\left(c^{3}f\cos\left(fx+e\right)^{2}-2c^{3}f\cos\left(fx+e\right)+c^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] a*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f
)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.120 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{24cf(c-c\sec(e+fx))^{5/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

```
[Out] -((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(6*f*(c - c*Sec[e + f*x])^(7/2))
 - ((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(24*c*f*(c - c*Sec[e + f*x])^(
5/2))
```

Rubi [A] time = 0.300941, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3951, 3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{24cf(c-c\sec(e+fx))^{5/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] -((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(6*f*(c - c*Sec[e + f*x])^(7/2))
 - ((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(24*c*f*(c - c*Sec[e + f*x])^(
5/2))
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx &= -\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}}+\frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx}{6c}\\ &= -\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}}-\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{24cf(c-c\sec(e+fx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.553457, size = 80, normalized size = 0.91

$$\frac{a(3\cos(e+fx)-3\cos(2(e+fx))-4)\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}}{6c^3f(\cos(e+fx)-1)^3\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(7
/2),x]
```

```
[Out] (a*(-4 + 3*Cos[e + f*x] - 3*Cos[2*(e + f*x)])*Sqrt[a*(1 + Sec[e + f*x])]*Ta
n[(e + f*x)/2])/(6*c^3*f*(-1 + Cos[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.283, size = 83, normalized size = 0.9

$$\frac{a\left(5\cos\left(fx+e\right)-1\right)\left(\sin\left(fx+e\right)\right)^{3}}{24f\left(-1+\cos\left(fx+e\right)\right)\left(\cos\left(fx+e\right)\right)^{3}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] 1/24/f*a*(5*cos(f*x+e)-1)*sin(f*x+e)^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2
)/(-1+cos(f*x+e))/cos(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)
```

Maxima [B] time = 4.4234, size = 2105, normalized size = 23.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*(a*sin(4*f*x + 4*e) + a*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 3*(a*si
n(6*f*x + 6*e) + 9*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e) - 4*a*sin(3/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*(2*a*sin(6*f*x + 6*e) + 15*a*sin(4*f*x + 4*e) +
 15*a*sin(2*f*x + 2*e) + 3*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(a*sin(6*f
*x + 6*e) + 9*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e))*cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*(a*cos(4*f*x + 4*e) + a*cos(2*f*x +
2*e))*sin(6*f*x + 6*e) + 3*a*sin(4*f*x + 4*e) + 3*a*sin(2*f*x + 2*e) - 3*(a
*cos(6*f*x + 6*e) + 9*a*cos(4*f*x + 4*e) + 9*a*cos(2*f*x + 2*e) - 4*a*cos(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*sin(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) - 4*(2*a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x +
 4*e) + 15*a*cos(2*f*x + 2*e) + 3*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 2*a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 3*(a*cos(6*f*x + 6*e) + 9*a*cos(4*f*x + 4*e) + 9*a*cos(2*f*x + 2*e) + a)*s
in(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^4*
cos(6*f*x + 6*e)^2 + 225*c^4*cos(4*f*x + 4*e)^2 + 225*c^4*cos(2*f*x + 2*e)^
2 + 36*c^4*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*c^4
```

```
*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*c^4*cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^4*sin(6*f*x + 6*e)^2 + 225
*c^4*sin(4*f*x + 4*e)^2 + 450*c^4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 225*c
^4*sin(2*f*x + 2*e)^2 + 36*c^4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 400*c^4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 36*c^4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 30*c^4*c
os(2*f*x + 2*e) + c^4 + 2*(15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e
) + c^4)*cos(6*f*x + 6*e) + 30*(15*c^4*cos(2*f*x + 2*e) + c^4)*cos(4*f*x +
4*e) - 12*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*
x + 2*e) - 20*c^4*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*
c^4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^4)*cos(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(c^4*cos(6*f*x + 6*e) + 15*c
^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + c^4)*cos(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) - 12*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*
c^4*cos(2*f*x + 2*e) + c^4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 30*(c^4*sin(4*f*x + 4*e) + c^4*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) -
 12*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*
e) - 20*c^4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*c^4*si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) - 40*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x
 + 4*e) + 15*c^4*sin(2*f*x + 2*e) - 6*c^4*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 12*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*
e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [A] time = 0.482839, size = 321, normalized size = 3.65

$$\frac{\left(6\,a\cos\left(fx+e\right)^3-3\,a\cos\left(fx+e\right)^2+a\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{6\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f\cos\left(fx+e\right)-c^4f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(6*a*cos(f*x + e)^3 - 3*a*cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^4*f
*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*si
n(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.121 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=92

$$\frac{a^2\tan(e+fx)}{12cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{4f(c-c\sec(e+fx))^{9/2}}$$

```
[Out] -(a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(4*f*(c - c*Sec[e + f*x])^(9/2))
 + (a^2*Tan[e + f*x])/(12*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])
^(7/2))
```

Rubi [A] time = 0.284766, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3954, 3953}

$$\frac{a^2\tan(e+fx)}{12cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{4f(c-c\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(9/2),x]
```

```
[Out] -(a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(4*f*(c - c*Sec[e + f*x])^(9/2))
 + (a^2*Tan[e + f*x])/(12*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])
^(7/2))
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx &= -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{4f(c-c\sec(e+fx))^{9/2}}-\frac{a\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx}{4c}\\ &= -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{4f(c-c\sec(e+fx))^{9/2}}+\frac{a^2\tan(e+fx)}{12cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.846981, size = 90, normalized size = 0.98

$$-\frac{a(17\cos(e+fx)-6\cos(2(e+fx))+3\cos(3(e+fx))-8)\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}}{12c^4f(\cos(e+fx)-1)^4\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(9
/2),x]
```

```
[Out] -(a*(-8 + 17*Cos[e + f*x] - 6*Cos[2*(e + f*x)] + 3*Cos[3*(e + f*x)])*Sqrt[a
*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(12*c^4*f*(-1 + Cos[e + f*x])^4*Sqrt
[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.283, size = 93, normalized size = 1.

$$\frac{a\left(17\left(\cos\left(fx+e\right)\right)^{2}-6\cos\left(fx+e\right)+1\right)\left(\sin\left(fx+e\right)\right)^{3}}{96f\left(-1+\cos\left(fx+e\right)\right)\left(\cos\left(fx+e\right)\right)^{4}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x)
```

```
[Out] 1/96/f*a*(17*cos(f*x+e)^2-6*cos(f*x+e)+1)*sin(f*x+e)^3*(1/cos(f*x+e)*a*(1+c
os(f*x+e)))^(1/2)/(-1+cos(f*x+e))/cos(f*x+e)^4/(c*(-1+cos(f*x+e))/cos(f*x+e
))^(9/2)
```

Maxima [B] time = 19.9984, size = 3521, normalized size = 38.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(28*a*cos(6*f*x + 6*e)*sin(4*f*x + 4*e) - 28*a*cos(4*f*x + 4*e)*sin(2*f
*x + 2*e) + 2*(3*a*sin(6*f*x + 6*e) + 8*a*sin(4*f*x + 4*e) + 3*a*sin(2*f*x
+ 2*e))*cos(8*f*x + 8*e) + (3*a*sin(8*f*x + 8*e) + 36*a*sin(6*f*x + 6*e) +
82*a*sin(4*f*x + 4*e) + 36*a*sin(2*f*x + 2*e) - 32*a*sin(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) - 32*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17
*a*sin(8*f*x + 8*e) + 140*a*sin(6*f*x + 6*e) + 294*a*sin(4*f*x + 4*e) + 140
*a*sin(2*f*x + 2*e) + 32*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17*a*sin(8*f*
x + 8*e) + 140*a*sin(6*f*x + 6*e) + 294*a*sin(4*f*x + 4*e) + 140*a*sin(2*f*
x + 2*e) + 32*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (3*a*sin(8*f*x + 8*e) + 3
6*a*sin(6*f*x + 6*e) + 82*a*sin(4*f*x + 4*e) + 36*a*sin(2*f*x + 2*e))*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(3*a*cos(6*f*x + 6*e) +
 8*a*cos(4*f*x + 4*e) + 3*a*cos(2*f*x + 2*e))*sin(8*f*x + 8*e) - 2*(14*a*co
s(4*f*x + 4*e) - 3*a)*sin(6*f*x + 6*e) + 4*(7*a*cos(2*f*x + 2*e) + 4*a)*sin
```

```
(4*f*x + 4*e) + 6*a*sin(2*f*x + 2*e) - (3*a*cos(8*f*x + 8*e) + 36*a*cos(6*f
*x + 6*e) + 82*a*cos(4*f*x + 4*e) + 36*a*cos(2*f*x + 2*e) - 32*a*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*a*cos(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 3*a)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - (17*a*cos(8*f*x + 8*e) + 140*a*cos(6*f*x + 6*e) + 294*a*cos(
4*f*x + 4*e) + 140*a*cos(2*f*x + 2*e) + 32*a*cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 17*a)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - (17*a*cos(8*f*x + 8*e) + 140*a*cos(6*f*x + 6*e) + 294*a*cos(4*f*
x + 4*e) + 140*a*cos(2*f*x + 2*e) + 32*a*cos(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 17*a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - (3*a*cos(8*f*x + 8*e) + 36*a*cos(6*f*x + 6*e) + 82*a*cos(4*f*x + 4*e
) + 36*a*cos(2*f*x + 2*e) + 3*a)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))))*sqrt(a)*sqrt(c)/((c^5*cos(8*f*x + 8*e)^2 + 784*c^5*cos(6*f*x +
6*e)^2 + 4900*c^5*cos(4*f*x + 4*e)^2 + 784*c^5*cos(2*f*x + 2*e)^2 + 64*c^5*
cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*cos(5/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*cos(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*c^5*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 + c^5*sin(8*f*x + 8*e)^2 + 784*c^5*sin(6*f*x + 6*
e)^2 + 4900*c^5*sin(4*f*x + 4*e)^2 + 3920*c^5*sin(4*f*x + 4*e)*sin(2*f*x +
2*e) + 784*c^5*sin(2*f*x + 2*e)^2 + 64*c^5*sin(7/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e)))^2 + 3136*c^5*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 3136*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 64*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 56*c
^5*cos(2*f*x + 2*e) + c^5 + 2*(28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x +
 4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5)*cos(8*f*x + 8*e) + 56*(70*c^5*cos(4*
f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5)*cos(6*f*x + 6*e) + 140*(28*c^5*
cos(2*f*x + 2*e) + c^5)*cos(4*f*x + 4*e) - 16*(c^5*cos(8*f*x + 8*e) + 28*c^
5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 56
*c^5*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*c^5*cos(3/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + c^5)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - 112*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*
c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 56*c^5*cos(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + c^5)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - 112*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f
*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + c^5)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) - 16*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*
f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5)*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 28*(2*c^5*sin(6*f*x + 6*e) + 5*c^5*sin(4*f*x + 4*e)
 + 2*c^5*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 784*(5*c^5*sin(4*f*x + 4*e) +
 2*c^5*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 16*(c^5*sin(8*f*x + 8*e) + 28*c
^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 5
6*c^5*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*c^5*sin(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 112*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*s
in(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 56*c^5*sin(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*
(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) +
 28*c^5*sin(2*f*x + 2*e) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*
sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c
^5*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*
f)
```

Fricas [A] time = 0.497793, size = 383, normalized size = 4.16

$$\frac{\left(6\,a\cos\left(fx+e\right)^4-6\,a\cos\left(fx+e\right)^3+4\,a\cos\left(fx+e\right)^2-a\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{6\left(c^5f\cos\left(fx+e\right)^4-4\,c^5f\cos\left(fx+e\right)^3+6\,c^5f\cos\left(fx+e\right)^2-4\,c^5f\cos\left(fx+e\right)+c^5f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(6*a*cos(f*x + e)^4 - 6*a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 - a*cos(f
*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/
cos(f*x + e))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos
(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.122 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=92

$$\frac{a^2\tan(e+fx)}{20cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{5f(c-c\sec(e+fx))^{11/2}}$$

```
[Out] -(a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(5*f*(c - c*Sec[e + f*x])^(11/2)
) + (a^2*Tan[e + f*x])/(20*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]
)^(9/2))
```

Rubi [A] time = 0.286713, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3954, 3953}

$$\frac{a^2\tan(e+fx)}{20cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{5f(c-c\sec(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(11/2),x
]
```

```
[Out] -(a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(5*f*(c - c*Sec[e + f*x])^(11/2)
) + (a^2*Tan[e + f*x])/(20*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]
)^(9/2))
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx &= -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{5f(c-c\sec(e+fx))^{11/2}}-\frac{a\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{9/2}}\,dx}{5c}\\ &= -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{5f(c-c\sec(e+fx))^{11/2}}+\frac{a^2\tan(e+fx)}{20cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx)}\end{aligned}$$

Mathematica [A] time = 1.23778, size = 100, normalized size = 1.09

$$\frac{a(75\cos(e+fx)-50\cos(2(e+fx))+15\cos(3(e+fx))-5\cos(4(e+fx))-51)\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}}{40c^5f(\cos(e+fx)-1)^5\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(1
1/2),x]
```

```
[Out] (a*(-51 + 75*Cos[e + f*x] - 50*Cos[2*(e + f*x)] + 15*Cos[3*(e + f*x)] - 5*C
os[4*(e + f*x)])*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(40*c^5*f*(-1
 + Cos[e + f*x])^5*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.277, size = 103, normalized size = 1.1

$$\frac{a\left(49\left(\cos\left(fx+e\right)\right)^{3}-23\left(\cos\left(fx+e\right)\right)^{2}+7\cos\left(fx+e\right)-1\right)\left(\sin\left(fx+e\right)\right)^{3}}{320f\left(-1+\cos\left(fx+e\right)\right)\left(\cos\left(fx+e\right)\right)^{5}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x)
```

```
[Out] 1/320/f*a*(49*cos(f*x+e)^3-23*cos(f*x+e)^2+7*cos(f*x+e)-1)*sin(f*x+e)^3*(1/
cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/(-1+cos(f*x+e))/cos(f*x+e)^5/(c*(-1+cos(
f*x+e))/cos(f*x+e))^(11/2)
```

Maxima [B] time = 109.892, size = 5273, normalized size = 57.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="maxima")
```

```
[Out] -2/5*(225*a*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) + 225*a*cos(4*f*x + 4*e)*sin(
2*f*x + 2*e) - 15*(a*sin(8*f*x + 8*e) + 5*a*sin(6*f*x + 6*e) + 5*a*sin(4*f*
x + 4*e) + a*sin(2*f*x + 2*e))*cos(10*f*x + 10*e) - 225*(a*sin(6*f*x + 6*e)
 + a*sin(4*f*x + 4*e))*cos(8*f*x + 8*e) - 5*(a*sin(10*f*x + 10*e) + 15*a*si
n(8*f*x + 8*e) + 60*a*sin(6*f*x + 6*e) + 60*a*sin(4*f*x + 4*e) + 15*a*sin(2
*f*x + 2*e) - 20*a*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4
8*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*a*sin(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(9/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 10*(5*a*sin(10*f*x + 10*e) + 45*a*sin(8*f*x + 8*e) +
 150*a*sin(6*f*x + 6*e) + 150*a*sin(4*f*x + 4*e) + 45*a*sin(2*f*x + 2*e) -
36*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 10*a*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 6*(17*a*sin(10*f*x + 10*e) + 135*a*sin(8*f*x + 8*e)
 + 420*a*sin(6*f*x + 6*e) + 420*a*sin(4*f*x + 4*e) + 135*a*sin(2*f*x + 2*e)
 + 60*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 40*a*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x +
```

```
2*e), cos(2*f*x + 2*e))) - 50*(a*sin(10*f*x + 10*e) + 9*a*sin(8*f*x + 8*e)
+ 30*a*sin(6*f*x + 6*e) + 30*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e) + 2*
a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*(a*sin(10*f*x + 10*e) + 15*a*sin(8*f*
x + 8*e) + 60*a*sin(6*f*x + 6*e) + 60*a*sin(4*f*x + 4*e) + 15*a*sin(2*f*x +
 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 15*(a*cos(8*f
*x + 8*e) + 5*a*cos(6*f*x + 6*e) + 5*a*cos(4*f*x + 4*e) + a*cos(2*f*x + 2*e
))*sin(10*f*x + 10*e) + 15*(15*a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x + 4*e) -
 a)*sin(8*f*x + 8*e) - 75*(3*a*cos(2*f*x + 2*e) + a)*sin(6*f*x + 6*e) - 75*
(3*a*cos(2*f*x + 2*e) + a)*sin(4*f*x + 4*e) - 15*a*sin(2*f*x + 2*e) + 5*(a*
cos(10*f*x + 10*e) + 15*a*cos(8*f*x + 8*e) + 60*a*cos(6*f*x + 6*e) + 60*a*c
os(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) - 20*a*cos(7/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 48*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 20*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*
sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 10*(5*a*cos(10*f*x +
 10*e) + 45*a*cos(8*f*x + 8*e) + 150*a*cos(6*f*x + 6*e) + 150*a*cos(4*f*x +
 4*e) + 45*a*cos(2*f*x + 2*e) - 36*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 10*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ 5*a)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(17*a*cos(1
0*f*x + 10*e) + 135*a*cos(8*f*x + 8*e) + 420*a*cos(6*f*x + 6*e) + 420*a*cos
(4*f*x + 4*e) + 135*a*cos(2*f*x + 2*e) + 60*a*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 40*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 17*a)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 50*
(a*cos(10*f*x + 10*e) + 9*a*cos(8*f*x + 8*e) + 30*a*cos(6*f*x + 6*e) + 30*a
*cos(4*f*x + 4*e) + 9*a*cos(2*f*x + 2*e) + 2*a*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + 5*(a*cos(10*f*x + 10*e) + 15*a*cos(8*f*x + 8*e) + 60*a*cos(6*f*x
+ 6*e) + 60*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) + a)*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^6*cos(10*f*x + 1
0*e)^2 + 2025*c^6*cos(8*f*x + 8*e)^2 + 44100*c^6*cos(6*f*x + 6*e)^2 + 44100
*c^6*cos(4*f*x + 4*e)^2 + 2025*c^6*cos(2*f*x + 2*e)^2 + 100*c^6*cos(9/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*cos(7/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*c^6*cos(5/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))^2 + 100*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + c^6*sin(10*f*x + 10*e)^2 + 2025*c^6*sin(8*f*x + 8*e)^2 + 44100*
c^6*sin(6*f*x + 6*e)^2 + 44100*c^6*sin(4*f*x + 4*e)^2 + 18900*c^6*sin(4*f*x
 + 4*e)*sin(2*f*x + 2*e) + 2025*c^6*sin(2*f*x + 2*e)^2 + 100*c^6*sin(9/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*sin(7/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*c^6*sin(5/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e)))^2 + 100*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 90*c^6*cos(2*f*x + 2*e) + c^6 + 2*(45*c^6*cos(8*f*x + 8*e) + 2
10*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e
) + c^6)*cos(10*f*x + 10*e) + 90*(210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*
f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(8*f*x + 8*e) + 420*(210*c^6
*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(6*f*x + 6*e) + 420*(
45*c^6*cos(2*f*x + 2*e) + c^6)*cos(4*f*x + 4*e) - 20*(c^6*cos(10*f*x + 10*e
) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x
+ 4*e) + 45*c^6*cos(2*f*x + 2*e) - 120*c^6*cos(7/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 252*c^6*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - 120*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1
0*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^6)*cos(9/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*cos(10*f*x + 10*e) +
 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4
*e) + 45*c^6*cos(2*f*x + 2*e) - 252*c^6*cos(5/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) - 120*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^6)*
cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 504*(c^6*cos(10*f*x
```

```
+ 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(
4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 120*c^6*cos(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + c^6)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 240*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x
+ 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 10*c^6*cos(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^6)*cos(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8
*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*
cos(2*f*x + 2*e) + c^6)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + 30*(3*c^6*sin(8*f*x + 8*e) + 14*c^6*sin(6*f*x + 6*e) + 14*c^6*sin(4*f*x
 + 4*e) + 3*c^6*sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + 1350*(14*c^6*sin(6*f
*x + 6*e) + 14*c^6*sin(4*f*x + 4*e) + 3*c^6*sin(2*f*x + 2*e))*sin(8*f*x + 8
*e) + 6300*(14*c^6*sin(4*f*x + 4*e) + 3*c^6*sin(2*f*x + 2*e))*sin(6*f*x + 6
*e) - 20*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*
f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e) - 120*c^6*s
in(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 252*c^6*sin(5/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*sin(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))))*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*
(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e
) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e) - 252*c^6*sin(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*sin(3/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5
04*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x +
6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e) - 120*c^6*sin(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) - 240*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c
^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e) -
10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(c^6*sin(10*f*x + 10*e) + 45*c^
6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) +
45*c^6*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)))*f)
```

Fricas [B] time = 0.505016, size = 458, normalized size = 4.98

$$\frac{\left(20\, a \cos\left(fx+e\right)^{5} - 30\, a \cos\left(fx+e\right)^{4} + 30\, a \cos\left(fx+e\right)^{3} - 15\, a \cos\left(fx+e\right)^{2} + 3\, a \cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c}{}}}{20\left(c^{6} f \cos\left(fx+e\right)^{5} - 5\, c^{6} f \cos\left(fx+e\right)^{4} + 10\, c^{6} f \cos\left(fx+e\right)^{3} - 10\, c^{6} f \cos\left(fx+e\right)^{2} + 5\, c^{6} f \cos\left(fx+e\right) - c^{6} f\right) \mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="fricas")
```

```
[Out] 1/20*(20*a*cos(f*x + e)^5 - 30*a*cos(f*x + e)^4 + 30*a*cos(f*x + e)^3 - 15*
a*cos(f*x + e)^2 + 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^6*f*cos(f*x + e)^5 - 5*c^6*f*
cos(f*x + e)^4 + 10*c^6*f*cos(f*x + e)^3 - 10*c^6*f*cos(f*x + e)^2 + 5*c^6*
f*cos(f*x + e) - c^6*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

3.123 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=134

$$\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}{15f}+\frac{a^3\tan(e+fx)(c-c\sec(e+fx))^{7/2}}{15f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{7/2}}{6f}$$

```
[Out] (a^3*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(15*f*Sqrt[a + a*Sec[e + f*x]
]) + (2*a^2*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x
])/(15*f) + (a*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/2)*Tan[e
+ f*x])/(6*f)
```

Rubi [A] time = 0.421419, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3953}

$$\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}{15f}+\frac{a^3\tan(e+fx)(c-c\sec(e+fx))^{7/2}}{15f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{7/2}}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a^3*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(15*f*Sqrt[a + a*Sec[e + f*x]
]) + (2*a^2*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x
])/(15*f) + (a*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/2)*Tan[e
+ f*x])/(6*f)
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{a(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{6f} +$$
$$= \frac{2a^2\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{15f} +$$
$$= \frac{a^3(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sqrt{a+a\sec(e+f}}{}$$

Mathematica [A] time = 1.34259, size = 113, normalized size = 0.84

$$\frac{a^2c^3(78\cos(e+fx)+5(7\cos(3(e+fx))-3\cos(4(e+fx))+3\cos(5(e+fx))-5))\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^5(}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(7/2
),x]
```

```
[Out] (a^2*c^3*(78*Cos[e + f*x] + 5*(-5 + 7*Cos[3*(e + f*x)] - 3*Cos[4*(e + f*x)]
 + 3*Cos[5*(e + f*x)]))*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]^5*Sq
rt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(480*f)
```

Maple [A] time = 0.265, size = 105, normalized size = 0.8

$$-\frac{a^2\left(21\left(\cos\left(fx+e\right)\right)^3-33\left(\cos\left(fx+e\right)\right)^2+21\cos\left(fx+e\right)-5\right)\left(\sin\left(fx+e\right)\right)^5}{30f\left(-1+\cos\left(fx+e\right)\right)^6\left(\cos\left(fx+e\right)\right)^2}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\frac{c\left(-1+\cos(\right.}{\cos\left(fx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] -1/30/f*a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*
x+e))^(7/2)*(21*cos(f*x+e)^3-33*cos(f*x+e)^2+21*cos(f*x+e)-5)*sin(f*x+e)^5/
(-1+cos(f*x+e))^6/cos(f*x+e)^2
```

Maxima [B] time = 1.96033, size = 3313, normalized size = 24.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/15*(210*a^2*c^3*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 90*a^2*c^3*cos(2*f*x
+ 2*e)*sin(f*x + e) - 15*a^2*c^3*sin(f*x + e) - (15*a^2*c^3*sin(11*f*x + 11
*e) - 15*a^2*c^3*sin(10*f*x + 10*e) + 35*a^2*c^3*sin(9*f*x + 9*e) + 78*a^2*
c^3*sin(7*f*x + 7*e) - 50*a^2*c^3*sin(6*f*x + 6*e) + 78*a^2*c^3*sin(5*f*x +
 5*e) + 35*a^2*c^3*sin(3*f*x + 3*e) - 15*a^2*c^3*sin(2*f*x + 2*e) + 15*a^2*
```

```
c^3*sin(f*x + e))*cos(12*f*x + 12*e) + 15*(6*a^2*c^3*sin(10*f*x + 10*e) + 1
5*a^2*c^3*sin(8*f*x + 8*e) + 20*a^2*c^3*sin(6*f*x + 6*e) + 15*a^2*c^3*sin(4
*f*x + 4*e) + 6*a^2*c^3*sin(2*f*x + 2*e))*cos(11*f*x + 11*e) - 3*(70*a^2*c^
3*sin(9*f*x + 9*e) + 75*a^2*c^3*sin(8*f*x + 8*e) + 156*a^2*c^3*sin(7*f*x +
7*e) + 156*a^2*c^3*sin(5*f*x + 5*e) + 75*a^2*c^3*sin(4*f*x + 4*e) + 70*a^2*
c^3*sin(3*f*x + 3*e) + 30*a^2*c^3*sin(f*x + e))*cos(10*f*x + 10*e) + 35*(15
*a^2*c^3*sin(8*f*x + 8*e) + 20*a^2*c^3*sin(6*f*x + 6*e) + 15*a^2*c^3*sin(4*
f*x + 4*e) + 6*a^2*c^3*sin(2*f*x + 2*e))*cos(9*f*x + 9*e) - 15*(78*a^2*c^3*
sin(7*f*x + 7*e) - 50*a^2*c^3*sin(6*f*x + 6*e) + 78*a^2*c^3*sin(5*f*x + 5*e
) + 35*a^2*c^3*sin(3*f*x + 3*e) - 15*a^2*c^3*sin(2*f*x + 2*e) + 15*a^2*c^3*
sin(f*x + e))*cos(8*f*x + 8*e) + 78*(20*a^2*c^3*sin(6*f*x + 6*e) + 15*a^2*c
^3*sin(4*f*x + 4*e) + 6*a^2*c^3*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 10*(15
6*a^2*c^3*sin(5*f*x + 5*e) + 75*a^2*c^3*sin(4*f*x + 4*e) + 70*a^2*c^3*sin(3
*f*x + 3*e) + 30*a^2*c^3*sin(f*x + e))*cos(6*f*x + 6*e) + 234*(5*a^2*c^3*si
n(4*f*x + 4*e) + 2*a^2*c^3*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 75*(7*a^2*c
^3*sin(3*f*x + 3*e) - 3*a^2*c^3*sin(2*f*x + 2*e) + 3*a^2*c^3*sin(f*x + e))*
cos(4*f*x + 4*e) + (15*a^2*c^3*cos(11*f*x + 11*e) - 15*a^2*c^3*cos(10*f*x +
 10*e) + 35*a^2*c^3*cos(9*f*x + 9*e) + 78*a^2*c^3*cos(7*f*x + 7*e) - 50*a^2
*c^3*cos(6*f*x + 6*e) + 78*a^2*c^3*cos(5*f*x + 5*e) + 35*a^2*c^3*cos(3*f*x
+ 3*e) - 15*a^2*c^3*cos(2*f*x + 2*e) + 15*a^2*c^3*cos(f*x + e))*sin(12*f*x
+ 12*e) - 15*(6*a^2*c^3*cos(10*f*x + 10*e) + 15*a^2*c^3*cos(8*f*x + 8*e) +
20*a^2*c^3*cos(6*f*x + 6*e) + 15*a^2*c^3*cos(4*f*x + 4*e) + 6*a^2*c^3*cos(2
*f*x + 2*e) + a^2*c^3)*sin(11*f*x + 11*e) + 3*(70*a^2*c^3*cos(9*f*x + 9*e)
+ 75*a^2*c^3*cos(8*f*x + 8*e) + 156*a^2*c^3*cos(7*f*x + 7*e) + 156*a^2*c^3*
cos(5*f*x + 5*e) + 75*a^2*c^3*cos(4*f*x + 4*e) + 70*a^2*c^3*cos(3*f*x + 3*e
) + 30*a^2*c^3*cos(f*x + e) + 5*a^2*c^3)*sin(10*f*x + 10*e) - 35*(15*a^2*c^
3*cos(8*f*x + 8*e) + 20*a^2*c^3*cos(6*f*x + 6*e) + 15*a^2*c^3*cos(4*f*x + 4
*e) + 6*a^2*c^3*cos(2*f*x + 2*e) + a^2*c^3)*sin(9*f*x + 9*e) + 15*(78*a^2*c
^3*cos(7*f*x + 7*e) - 50*a^2*c^3*cos(6*f*x + 6*e) + 78*a^2*c^3*cos(5*f*x +
5*e) + 35*a^2*c^3*cos(3*f*x + 3*e) - 15*a^2*c^3*cos(2*f*x + 2*e) + 15*a^2*c
^3*cos(f*x + e))*sin(8*f*x + 8*e) - 78*(20*a^2*c^3*cos(6*f*x + 6*e) + 15*a^
2*c^3*cos(4*f*x + 4*e) + 6*a^2*c^3*cos(2*f*x + 2*e) + a^2*c^3)*sin(7*f*x +
7*e) + 10*(156*a^2*c^3*cos(5*f*x + 5*e) + 75*a^2*c^3*cos(4*f*x + 4*e) + 70*
a^2*c^3*cos(3*f*x + 3*e) + 30*a^2*c^3*cos(f*x + e) + 5*a^2*c^3)*sin(6*f*x +
 6*e) - 78*(15*a^2*c^3*cos(4*f*x + 4*e) + 6*a^2*c^3*cos(2*f*x + 2*e) + a^2*
c^3)*sin(5*f*x + 5*e) + 75*(7*a^2*c^3*cos(3*f*x + 3*e) - 3*a^2*c^3*cos(2*f*
x + 2*e) + 3*a^2*c^3*cos(f*x + e))*sin(4*f*x + 4*e) - 35*(6*a^2*c^3*cos(2*f
*x + 2*e) + a^2*c^3)*sin(3*f*x + 3*e) + 15*(6*a^2*c^3*cos(f*x + e) + a^2*c^
3)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2*(6*cos(10*f*x + 10*e) + 15*cos(8*f
*x + 8*e) + 20*cos(6*f*x + 6*e) + 15*cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e)
+ 1)*cos(12*f*x + 12*e) + cos(12*f*x + 12*e)^2 + 12*(15*cos(8*f*x + 8*e) +
20*cos(6*f*x + 6*e) + 15*cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(10*
f*x + 10*e) + 36*cos(10*f*x + 10*e)^2 + 30*(20*cos(6*f*x + 6*e) + 15*cos(4*
f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e) + 225*cos(8*f*x + 8*e
)^2 + 40*(15*cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) +
400*cos(6*f*x + 6*e)^2 + 30*(6*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 225
*cos(4*f*x + 4*e)^2 + 36*cos(2*f*x + 2*e)^2 + 2*(6*sin(10*f*x + 10*e) + 15*
sin(8*f*x + 8*e) + 20*sin(6*f*x + 6*e) + 15*sin(4*f*x + 4*e) + 6*sin(2*f*x
+ 2*e))*sin(12*f*x + 12*e) + sin(12*f*x + 12*e)^2 + 12*(15*sin(8*f*x + 8*e)
 + 20*sin(6*f*x + 6*e) + 15*sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(10*f
*x + 10*e) + 36*sin(10*f*x + 10*e)^2 + 30*(20*sin(6*f*x + 6*e) + 15*sin(4*f
*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 225*sin(8*f*x + 8*e)^2 +
 120*(5*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 400*sin(6
*f*x + 6*e)^2 + 225*sin(4*f*x + 4*e)^2 + 180*sin(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 36*sin(2*f*x + 2*e)^2 + 12*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A] time = 0.503169, size = 360, normalized size = 2.69

$$\frac{\left(30\,a^2c^3\cos\left(fx+e\right)^5-15\,a^2c^3\cos\left(fx+e\right)^4-20\,a^2c^3\cos\left(fx+e\right)^3+15\,a^2c^3\cos\left(fx+e\right)^2+6\,a^2c^3\cos\left(fx+e\right)-5\,a^2c\right.}{30\,f\cos\left(fx+e\right)^5\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/30*(30*a^2*c^3*cos(f*x + e)^5 - 15*a^2*c^3*cos(f*x + e)^4 - 20*a^2*c^3*co
s(f*x + e)^3 + 15*a^2*c^3*cos(f*x + e)^2 + 6*a^2*c^3*cos(f*x + e) - 5*a^2*c
^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))/(f*cos(f*x + e)^5*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.124 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=134

$$-\frac{2c^3\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{15f\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{5f}-\frac{c\tan(e+fx)(a\sec$$

```
[Out] (-2*c^3*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(15*f*Sqrt[c - c*Sec[e + f
*x]]) - (c^2*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*
x])/(5*f) - (c*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)*Tan[e
+ f*x])/(5*f)
```

Rubi [A] time = 0.424053, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3953}

$$-\frac{2c^3\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{15f\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{5f}-\frac{c\tan(e+fx)(a\sec$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*c^3*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(15*f*Sqrt[c - c*Sec[e + f
*x]]) - (c^2*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*
x])/(5*f) - (c*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)*Tan[e
+ f*x])/(5*f)
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{c(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5f} +$$
$$= -\frac{c^2(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f} -$$
$$= -\frac{2c^3(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}} - \frac{c^2(a+a\sec(e+f}{}$$

Mathematica [A] time = 0.778556, size = 92, normalized size = 0.69

$$\frac{a^2c^2(20\cos(2(e+fx))+15\cos(4(e+fx))+29)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec}}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2
),x]
```

```
[Out] (a^2*c^2*(29 + 20*Cos[2*(e + f*x)] + 15*Cos[4*(e + f*x)])*Csc[(e + f*x)/2]*
Sec[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e
 + f*x]])/(240*f)
```

Maple [A] time = 0.26, size = 95, normalized size = 0.7

$$-\frac{a^2\left(\sin\left(fx+e\right)\right)^5\left(8\left(\cos\left(fx+e\right)\right)^2-9\cos\left(fx+e\right)+3\right)}{15f\left(-1+\cos\left(fx+e\right)\right)^5\left(\cos\left(fx+e\right)\right)^2}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/15/f*a^2*sin(f*x+e)^5*(8*cos(f*x+e)^2-9*cos(f*x+e)+3)*(c*(-1+cos(f*x+e))
/cos(f*x+e))^(5/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/(-1+cos(f*x+e))^5/
cos(f*x+e)^2
```

Maxima [B] time = 1.90967, size = 2060, normalized size = 15.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2/15*(100*a^2*c^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 75*a^2*c^2*cos(f*x +
e)*sin(2*f*x + 2*e) - 75*a^2*c^2*cos(2*f*x + 2*e)*sin(f*x + e) - 15*a^2*c^2
*sin(f*x + e) - (15*a^2*c^2*sin(9*f*x + 9*e) + 20*a^2*c^2*sin(7*f*x + 7*e)
+ 58*a^2*c^2*sin(5*f*x + 5*e) + 20*a^2*c^2*sin(3*f*x + 3*e) + 15*a^2*c^2*si
n(f*x + e))*cos(10*f*x + 10*e) + 75*(a^2*c^2*sin(8*f*x + 8*e) + 2*a^2*c^2*s
```

```
in(6*f*x + 6*e) + 2*a^2*c^2*sin(4*f*x + 4*e) + a^2*c^2*sin(2*f*x + 2*e))*co
s(9*f*x + 9*e) - 5*(20*a^2*c^2*sin(7*f*x + 7*e) + 58*a^2*c^2*sin(5*f*x + 5*
e) + 20*a^2*c^2*sin(3*f*x + 3*e) + 15*a^2*c^2*sin(f*x + e))*cos(8*f*x + 8*e
) + 100*(2*a^2*c^2*sin(6*f*x + 6*e) + 2*a^2*c^2*sin(4*f*x + 4*e) + a^2*c^2*
sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 10*(58*a^2*c^2*sin(5*f*x + 5*e) + 20*a
^2*c^2*sin(3*f*x + 3*e) + 15*a^2*c^2*sin(f*x + e))*cos(6*f*x + 6*e) + 290*(
2*a^2*c^2*sin(4*f*x + 4*e) + a^2*c^2*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 5
0*(4*a^2*c^2*sin(3*f*x + 3*e) + 3*a^2*c^2*sin(f*x + e))*cos(4*f*x + 4*e) +
(15*a^2*c^2*cos(9*f*x + 9*e) + 20*a^2*c^2*cos(7*f*x + 7*e) + 58*a^2*c^2*cos
(5*f*x + 5*e) + 20*a^2*c^2*cos(3*f*x + 3*e) + 15*a^2*c^2*cos(f*x + e))*sin(
10*f*x + 10*e) - 15*(5*a^2*c^2*cos(8*f*x + 8*e) + 10*a^2*c^2*cos(6*f*x + 6*
e) + 10*a^2*c^2*cos(4*f*x + 4*e) + 5*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*si
n(9*f*x + 9*e) + 5*(20*a^2*c^2*cos(7*f*x + 7*e) + 58*a^2*c^2*cos(5*f*x + 5*
e) + 20*a^2*c^2*cos(3*f*x + 3*e) + 15*a^2*c^2*cos(f*x + e))*sin(8*f*x + 8*e
) - 20*(10*a^2*c^2*cos(6*f*x + 6*e) + 10*a^2*c^2*cos(4*f*x + 4*e) + 5*a^2*c
^2*cos(2*f*x + 2*e) + a^2*c^2)*sin(7*f*x + 7*e) + 10*(58*a^2*c^2*cos(5*f*x
+ 5*e) + 20*a^2*c^2*cos(3*f*x + 3*e) + 15*a^2*c^2*cos(f*x + e))*sin(6*f*x +
 6*e) - 58*(10*a^2*c^2*cos(4*f*x + 4*e) + 5*a^2*c^2*cos(2*f*x + 2*e) + a^2*
c^2)*sin(5*f*x + 5*e) + 50*(4*a^2*c^2*cos(3*f*x + 3*e) + 3*a^2*c^2*cos(f*x
+ e))*sin(4*f*x + 4*e) - 20*(5*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*sin(3*f*
x + 3*e))*sqrt(a)*sqrt(c)/((2*(5*cos(8*f*x + 8*e) + 10*cos(6*f*x + 6*e) + 1
0*cos(4*f*x + 4*e) + 5*cos(2*f*x + 2*e) + 1)*cos(10*f*x + 10*e) + cos(10*f*
x + 10*e)^2 + 10*(10*cos(6*f*x + 6*e) + 10*cos(4*f*x + 4*e) + 5*cos(2*f*x +
 2*e) + 1)*cos(8*f*x + 8*e) + 25*cos(8*f*x + 8*e)^2 + 20*(10*cos(4*f*x + 4*
e) + 5*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 100*cos(6*f*x + 6*e)^2 + 20
*(5*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 100*cos(4*f*x + 4*e)^2 + 25*co
s(2*f*x + 2*e)^2 + 10*(sin(8*f*x + 8*e) + 2*sin(6*f*x + 6*e) + 2*sin(4*f*x
+ 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 50*(
2*sin(6*f*x + 6*e) + 2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e
) + 25*sin(8*f*x + 8*e)^2 + 100*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin
(6*f*x + 6*e) + 100*sin(6*f*x + 6*e)^2 + 100*sin(4*f*x + 4*e)^2 + 100*sin(4
*f*x + 4*e)*sin(2*f*x + 2*e) + 25*sin(2*f*x + 2*e)^2 + 10*cos(2*f*x + 2*e)
+ 1)*f)
```

Fricas [A] time = 0.483366, size = 251, normalized size = 1.87

$$\frac{\left(15\,a^2c^2\cos\left(fx+e\right)^4-10\,a^2c^2\cos\left(fx+e\right)^2+3\,a^2c^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{15\,f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(15*a^2*c^2*cos(f*x + e)^4 - 10*a^2*c^2*cos(f*x + e)^2 + 3*a^2*c^2)*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e
))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.125 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=89

$$-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{6f\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{4f}$$

```
[Out] -(c^2*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(6*f*Sqrt[c - c*Sec[e + f*x]
]) - (c*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(
4*f)
```

Rubi [A] time = 0.27494, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3953}

$$-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{6f\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(c^2*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(6*f*Sqrt[c - c*Sec[e + f*x]
]) - (c*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(
4*f)
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=-\frac{c(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{4f}$$

$$=-\frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{6f\sqrt{c-c\sec(e+fx)}}-\frac{c(a+a\sec(e+$$

Mathematica [A] time = 0.599256, size = 96, normalized size = 1.08

$$\frac{a^2c(5\cos(e+fx)+3(\cos(2(e+fx))+\cos(3(e+fx))))\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(\sec(e+fx)+1)}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2
),x]
```

```
[Out] (a^2*c*(5*Cos[e + f*x] + 3*(Cos[2*(e + f*x)] + Cos[3*(e + f*x)]))*Csc[(e +
f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c -
 c*Sec[e + f*x]])/(24*f)
```

Maple [A] time = 0.261, size = 85, normalized size = 1.

$$-\frac{a^2\left(\sin\left(fx+e\right)\right)^5\left(5\cos\left(fx+e\right)-3\right)}{12f\left(-1+\cos\left(fx+e\right)\right)^4\left(\cos\left(fx+e\right)\right)^2}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/12/f*a^2*sin(f*x+e)^5*(5*cos(f*x+e)-3)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3
/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/(-1+cos(f*x+e))^4/cos(f*x+e)^2
```

Maxima [B] time = 1.885, size = 1493, normalized size = 16.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(20*a^2*c*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 12*a^2*c*cos(2*f*x + 2*e)
*sin(f*x + e) - 3*a^2*c*sin(f*x + e) - (3*a^2*c*sin(7*f*x + 7*e) + 3*a^2*c*
sin(6*f*x + 6*e) + 5*a^2*c*sin(5*f*x + 5*e) + 5*a^2*c*sin(3*f*x + 3*e) + 3*
a^2*c*sin(2*f*x + 2*e) + 3*a^2*c*sin(f*x + e))*cos(8*f*x + 8*e) + 6*(2*a^2*
c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e) + 2*a^2*c*sin(2*f*x + 2*e))*c
os(7*f*x + 7*e) - 2*(10*a^2*c*sin(5*f*x + 5*e) - 9*a^2*c*sin(4*f*x + 4*e) +
 10*a^2*c*sin(3*f*x + 3*e) + 6*a^2*c*sin(f*x + e))*cos(6*f*x + 6*e) + 10*(3
*a^2*c*sin(4*f*x + 4*e) + 2*a^2*c*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 6*(5
*a^2*c*sin(3*f*x + 3*e) + 3*a^2*c*sin(2*f*x + 2*e) + 3*a^2*c*sin(f*x + e))*
cos(4*f*x + 4*e) + (3*a^2*c*cos(7*f*x + 7*e) + 3*a^2*c*cos(6*f*x + 6*e) + 5
*a^2*c*cos(5*f*x + 5*e) + 5*a^2*c*cos(3*f*x + 3*e) + 3*a^2*c*cos(2*f*x + 2*
e) + 3*a^2*c*cos(f*x + e))*sin(8*f*x + 8*e) - 3*(4*a^2*c*cos(6*f*x + 6*e) +
 6*a^2*c*cos(4*f*x + 4*e) + 4*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(7*f*x + 7
*e) + (20*a^2*c*cos(5*f*x + 5*e) - 18*a^2*c*cos(4*f*x + 4*e) + 20*a^2*c*cos
(3*f*x + 3*e) + 12*a^2*c*cos(f*x + e) - 3*a^2*c)*sin(6*f*x + 6*e) - 5*(6*a^
2*c*cos(4*f*x + 4*e) + 4*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(5*f*x + 5*e) +
 6*(5*a^2*c*cos(3*f*x + 3*e) + 3*a^2*c*cos(2*f*x + 2*e) + 3*a^2*c*cos(f*x +
```

```
e))*sin(4*f*x + 4*e) - 5*(4*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(3*f*x + 3*
e) + 3*(4*a^2*c*cos(f*x + e) - a^2*c)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2
*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*f
*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)
 + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) + 1
)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*s
in(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e)
 + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*
f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x +
 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A] time = 0.479723, size = 273, normalized size = 3.07

$$\frac{\left(12\,a^2c\cos\left(fx+e\right)^3+6\,a^2c\cos\left(fx+e\right)^2-4\,a^2c\cos\left(fx+e\right)-3\,a^2c\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{12\,f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*(12*a^2*c*cos(f*x + e)^3 + 6*a^2*c*cos(f*x + e)^2 - 4*a^2*c*cos(f*x +
e) - 3*a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.126 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=43

$$-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -(c*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.128307, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3953}

$$-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -(c*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx=-\frac{c(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [B] time = 0.462833, size = 88, normalized size = 2.05

$$\frac{a^2\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}\left(4\cos(e+fx)+\cos^2(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)+2\right.}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],
x]
```

```
[Out] (a^2*Cot[(e + f*x)/2]*(2 + 4*Cos[e + f*x] + Cos[e + f*x]^2*Sec[(e + f*x)/2]
^2)*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(6*
f)
```

Maple [A] time = 0.28, size = 75, normalized size = 1.7

$$-\frac{a^{2} \left(\sin \left(f x +e \right)\right)^{5}}{3 f \left(\cos \left(f x +e \right)\right)^{2} \left(-1+\cos \left(f x +e \right)\right)^{3}}\sqrt{\frac{a \left(1+\cos \left(f x +e \right)\right)}{\cos \left(f x +e \right)}}\, \sqrt{\frac{c \left(-1+\cos \left(f x +e \right)\right)}{\cos \left(f x +e \right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*sin(f*x+e)^5*(c*(-1+cos(f*
x+e))/cos(f*x+e))^(1/2)/cos(f*x+e)^2/(-1+cos(f*x+e))^3
```

Maxima [A] time = 1.55089, size = 78, normalized size = 1.81

$$\frac{8 \sqrt{-a} a^{2} \sqrt{c}}{3 f \left(\frac{\sin\left(f x+e\right)}{\cos\left(f x+e\right)+1}+1\right)^{3} \left(\frac{\sin\left(f x+e\right)}{\cos\left(f x+e\right)+1}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 8/3*sqrt(-a)*a^2*sqrt(c)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^3*(sin(f*
x + e)/(cos(f*x + e) + 1) - 1)^3)
```

Fricas [B] time = 0.469005, size = 225, normalized size = 5.23

$$\frac{\left(3 a^{2} \cos\left(f x+e\right)^{2}+3 a^{2} \cos\left(f x+e\right)+a^{2}\right) \sqrt{\frac{a \cos\left(f x+e\right)+a}{\cos\left(f x+e\right)}} \sqrt{\frac{c \cos\left(f x+e\right)-c}{\cos\left(f x+e\right)}}}{3 f \cos\left(f x+e\right)^{2} \sin\left(f x+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) + a^2)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^
2*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(1/2),x)
```

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.127 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}+\frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (4*a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]) + (2*a^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[c - c*Sec[e + f*x]]) + (a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*
Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.404526, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3952}

$$\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}+\frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (4*a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]) + (2*a^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[c - c*Sec[e + f*x]]) + (a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*
Sqrt[c - c*Sec[e + f*x]])
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{2a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} + (4a$$
$$= \frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} +$$

Mathematica [C] time = 6.62073, size = 328, normalized size = 2.33

$$\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\sec(e+fx)(a(\sec(e+fx)+1))^{5/2}\sqrt{(\cos(e+fx)+1)\sec(e+fx)}\left(\frac{5\sec\left(\frac{e}{2}+\frac{fx}{2}\right)}{2f}+\frac{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\sec(e+fx)}{f}\right)}{(\sec(e+fx)+1)^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{4\sqrt{2}e^{\frac{1}{2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/Sqrt[c - c*Sec[e + f*x]
],x]
```

```
[Out] (4*Sqrt[2]*E^((I/2)*(e + f*x))*Sqrt[(1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(
e + f*x)))]*(2*Log[1 - E^(I*(e + f*x))] - Log[1 + E^((2*I)*(e + f*x))])*Sqr
t[Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2)*Sin[e/2 + (f*x)/2])/((1 + E^(I
*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*f*(1 + Sec[e +
 f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]) + (Sec[e + f*x]*Sqrt[(1 + Cos[e + f*
x])*Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2)*((5*Sec[e/2 + (f*x)/2])/(2*f
) + (Cos[e/2 + (f*x)/2]*Sec[e + f*x])/f)*Sin[e/2 + (f*x)/2])/((1 + Sec[e +
f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.319, size = 189, normalized size = 1.3

$$-\frac{a^2}{2f\sin(fx+e)\cos(fx+e)c}\left(16\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)(\cos(fx+e))^2-8(\cos(fx+e))^2\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*a^2*(16*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-8*cos(f*x+e)^2*
ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-8*ln(-(-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))*cos(f*x+e)^2+5*cos(f*x+e)^2+6*cos(f*x+e)+1)*(1/cos(f*x+e)*a*(1
+cos(f*x+e)))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/cos(f*x
+e)/c
```

Maxima [B] time = 1.99269, size = 995, normalized size = 7.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*(a^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - a^2*cos(4*f*x + 4*e)*sin(2*f*x
+ 2*e) - a^2*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x
 + 2*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e
) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(
2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1) - 4*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + a^2*sin
(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x
 + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2*f*x + 2*e) + a^2)
*cos(4*f*x + 4*e))*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + 3*(a^2*si
n(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 3*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*(a^2*cos(4*f*x + 4*e)
+ 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 3*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c*cos(4*f*
x + 4*e)^2 + 4*c*cos(2*f*x + 2*e)^2 + c*sin(4*f*x + 4*e)^2 + 4*c*sin(4*f*x
+ 4*e)*sin(2*f*x + 2*e) + 4*c*sin(2*f*x + 2*e)^2 + 2*(2*c*cos(2*f*x + 2*e)
+ c)*cos(4*f*x + 4*e) + 4*c*cos(2*f*x + 2*e) + c)*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2\sec\left(fx+e\right)^3+2\,a^2\sec\left(fx+e\right)^2+a^2\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{c\sec\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(a^2*sec(f*x + e)^3 + 2*a^2*sec(f*x + e)^2 + a^2*sec(f*x + e))*sq
rt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.128 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{cf\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{f(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -((a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2)
)) - (4*a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]]) - (2*a^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x]
)/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.419942, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3954, 3955, 3952}

$$-\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{cf\sqrt{c-c\sec(e+fx)}}-\frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2)
)) - (4*a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]]) - (2*a^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x]
)/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{(2a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c}$$

$$= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{2a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{(4$$

$$= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 1.4307, size = 188, normalized size = 1.3

$$\frac{a^2\tan\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(\sec(e+fx)+1)}\left(-4\log\left(1-e^{i(e+fx)}\right)+2\log\left(1+e^{2i(e+fx)}\right)+\left(8\log\left(1-e^{i(e+fx)}\right)-4\log\right.\right.}{cf(\cos(e+fx)-1)\sqrt{c-c\sec(e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(3
/2),x]
```

```
[Out] (a^2*(1 - 4*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(-5 + 8*Log[1 - E^(I*(e
 + f*x))] - 4*Log[1 + E^((2*I)*(e + f*x))]) + 2*Log[1 + E^((2*I)*(e + f*x))
] + Cos[2*(e + f*x)]*(-4*Log[1 - E^(I*(e + f*x))] + 2*Log[1 + E^((2*I)*(e +
 f*x))]))*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c*f*(-
1 + Cos[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] time = 0.277, size = 288, normalized size = 2.

$$\frac{a^2\left(-1+\cos\left(fx+e\right)\right)}{f\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)}\left(8\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos\left(fx+e\right)\right)^2-4\left(\cos\left(fx+e\right)\right)^2\ln\left(-\frac{-1+\cos\left(fx+e\right)-s}{\sin\left(fx+e\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/f*a^2*(-1+cos(f*x+e))*(8*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-4*c
os(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-4*ln(-(-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+3*cos(f*x+e)^2-8*cos(f*x+e)*ln(-(-1+c
os(f*x+e))/sin(f*x+e))+4*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))+4*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)-1)
*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(
f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [B] time = 2.55438, size = 2747, normalized size = 18.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 2*(8*a^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*cos(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*a^2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
8*a^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*a^2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*a
^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) + 2*a^2*cos(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 2*a^2*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x
+ 2*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2
*f*x + 2*e) + a^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1) - 4*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*co
s(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*s
in(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*sin(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a
^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e) - 4*(a^2*cos(4*f*x + 4*e) + 2*a
^2*cos(2*f*x + 2*e) - 2*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a^2*
cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*
e) - 2*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^
2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*a
rctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + (16*a^2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 5*a^2*sin(4*f*x + 4*e) + 6*a^2*sin(2*f*x + 2*e) - 8*(a^2*cos(4*f*x +
 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (5*a^2*s
in(4*f*x + 4*e) + 6*a^2*sin(2*f*x + 2*e) - 8*(a^2*cos(4*f*x + 4*e) + 2*a^2*
cos(2*f*x + 2*e) + a^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (16*a^2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) - 5*a^2*cos(4*f*x + 4*e) - 6*a^2*cos(2*f*x + 2*e) - 5*a^2 - 8*(
a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 (5*a^2*cos(4*f*x + 4*e) + 6*a^2*cos(2*f*x + 2*e) + 5*a^2 + 8*(a^2*sin(4*f*
x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt
(c)/((c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + 4*c^2*cos(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x
+ 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2*
f*x + 2*e) + c^2)*cos(4*f*x + 4*e) - 4*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*
f*x + 2*e) - 2*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c
^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(c^2*cos(4*f*x
 + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) - 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e) - 2*c^
2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f
*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2\sec\left(fx+e\right)^3+2\,a^2\sec\left(fx+e\right)^2+a^2\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{c^2\sec\left(fx+e\right)^2-2\,c^2\sec\left(fx+e\right)+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^3 + 2*a^2*sec(f*x + e)^2 + a^2*sec(f*x + e))*sqr
t(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x + e)^2 - 2*c^2
*sec(f*x + e) + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.129 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$\frac{a^3\tan(e+fx)\log(1-\sec(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{cf(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f(c-c\sec(e+fx))^{5/2}}$$

```
[Out] -(a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*(c - c*Sec[e + f*x])^(5/2
)) + (a^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(c*f*(c - c*Sec[e + f*x])^
(3/2)) + (a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.431088, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3954, 3952}

$$\frac{a^3\tan(e+fx)\log(1-\sec(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{cf(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*(c - c*Sec[e + f*x])^(5/2
)) + (a^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(c*f*(c - c*Sec[e + f*x])^
(3/2)) + (a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} - \frac{a\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx}{c}$$

$$= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{cf(c-c\sec(e+fx))^{3/2}} + \frac{a^2}{}$$

$$= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{cf(c-c\sec(e+fx))^{3/2}} + \frac{}{c^2}$$

Mathematica [C] time = 1.32972, size = 182, normalized size = 1.26

$$-\frac{a^2\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(-6\log\left(1-e^{i(e+fx)}\right)+3\log\left(1+e^{2i(e+fx)}\right)+\left(8\log\left(1-e^{i(e+fx)}\right)-4\log\left(1+e^{2i(}\right.\right.\right.}{2c^2f(\cos(e+fx)-1)^2\sqrt{c-c\sec(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(5
/2),x]
```

```
[Out] -(a^2*(4 - 6*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(8*Log[1 - E^(I*(e + f
*x))] - 4*Log[1 + E^((2*I)*(e + f*x))]) + 3*Log[1 + E^((2*I)*(e + f*x))] +
Cos[2*(e + f*x)]*(-2*Log[1 - E^(I*(e + f*x))] + Log[1 + E^((2*I)*(e + f*x))
]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(2*c^2*f*(-1 + Cos[e + f*x
])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] time = 0.276, size = 366, normalized size = 2.5

$$\frac{a^2\left(-1+\cos\left(fx+e\right)\right)}{2f\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)}\left(2\left(\cos\left(fx+e\right)\right)^2\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+2\ln\left(-\frac{-1+\cos\left(fx+e\right)+}{\sin\left(fx+\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/2/f*a^2*(-1+cos(f*x+e))*(2*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))+2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-4*ln(-(
-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-4*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))-4*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)+8*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-cos(f*x+e)^2+2*ln(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))+2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-
4*ln(-(-1+cos(f*x+e))/sin(f*x+e))+2*cos(f*x+e)+3)*(1/cos(f*x+e)*a*(1+cos(f*
x+e)))^(1/2)/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)
```

Maxima [A] time = 1.54619, size = 228, normalized size = 1.57

$$-\frac{\frac{2\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^{\frac{5}{2}}} + \frac{2\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{5}{2}}} - \frac{4\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{5}{2}}} + \frac{\left(\sqrt{-a}a^2\sqrt{c}+\frac{2\sqrt{-a}a^2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^4}{c^3\sin(fx+e)^4}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*(2*sqrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^(5/2) + 2*s
qrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) - 4*sqrt(-a)*a
^2*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(5/2) + (sqrt(-a)*a^2*sqrt(c) + 2
*sqrt(-a)*a^2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) +
1)^4/(c^3*sin(f*x + e)^4))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2\sec\left(fx+e\right)^3+2\,a^2\sec\left(fx+e\right)^2+a^2\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{c^3\sec\left(fx+e\right)^3-3\,c^3\sec\left(fx+e\right)^2+3\,c^3\sec\left(fx+e\right)-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(a^2*sec(f*x + e)^3 + 2*a^2*sec(f*x + e)^2 + a^2*sec(f*x + e))*sq
rt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x + e)^3 - 3*c^
3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.130 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

```
[Out] -((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(6*f*(c - c*Sec[e + f*x])^(7/2))
```

Rubi [A] time = 0.145583, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] -((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(6*f*(c - c*Sec[e + f*x])^(7/2))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx=-\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}}$$

Mathematica [A] time = 0.550775, size = 76, normalized size = 1.81

$$\frac{a^2(3\cos(2(e+fx))+5)\csc^5\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}}{48c^3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(7
/2),x]
```

```
[Out] (a^2*(5 + 3*Cos[2*(e + f*x)])*Csc[(e + f*x)/2]^5*Sec[(e + f*x)/2]*Sqrt[a*(1
 + Sec[e + f*x])])/(48*c^3*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] time = 0.248, size = 75, normalized size = 1.8

$$-\frac{a^{2} \left(\sin \left(f x +e \right)\right)^{5}}{6 f \left(-1+\cos \left(f x +e \right)\right)^{2} \left(\cos \left(f x +e \right)\right)^{3}}\sqrt{\frac{a \left(1+\cos \left(f x +e \right)\right)}{\cos \left(f x +e \right)}}\left(\frac{c \left(-1+\cos \left(f x +e \right)\right)}{\cos \left(f x +e \right)}\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] -1/6/f*a^2*sin(f*x+e)^5*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/(-1+cos(f*x+e
))^2/cos(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)
```

Maxima [B] time = 1.92277, size = 2450, normalized size = 58.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(208*a^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 48*a^2*cos(f*x + e)*sin(2*
f*x + 2*e) - 48*a^2*cos(2*f*x + 2*e)*sin(f*x + e) - 3*a^2*sin(f*x + e) - (3
*a^2*sin(7*f*x + 7*e) + 13*a^2*sin(5*f*x + 5*e) + 13*a^2*sin(3*f*x + 3*e) +
 3*a^2*sin(f*x + e))*cos(8*f*x + 8*e) + 6*(8*a^2*sin(6*f*x + 6*e) + 15*a^2*
sin(4*f*x + 4*e) + 8*a^2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 16*(13*a^2*si
n(5*f*x + 5*e) + 13*a^2*sin(3*f*x + 3*e) + 3*a^2*sin(f*x + e))*cos(6*f*x +
6*e) + 26*(15*a^2*sin(4*f*x + 4*e) + 8*a^2*sin(2*f*x + 2*e))*cos(5*f*x + 5*
e) - 30*(13*a^2*sin(3*f*x + 3*e) + 3*a^2*sin(f*x + e))*cos(4*f*x + 4*e) + (
3*a^2*cos(7*f*x + 7*e) + 13*a^2*cos(5*f*x + 5*e) + 13*a^2*cos(3*f*x + 3*e)
+ 3*a^2*cos(f*x + e))*sin(8*f*x + 8*e) - 3*(16*a^2*cos(6*f*x + 6*e) + 30*a^
2*cos(4*f*x + 4*e) + 16*a^2*cos(2*f*x + 2*e) + a^2)*sin(7*f*x + 7*e) + 16*(
13*a^2*cos(5*f*x + 5*e) + 13*a^2*cos(3*f*x + 3*e) + 3*a^2*cos(f*x + e))*sin
(6*f*x + 6*e) - 13*(30*a^2*cos(4*f*x + 4*e) + 16*a^2*cos(2*f*x + 2*e) + a^2
)*sin(5*f*x + 5*e) + 30*(13*a^2*cos(3*f*x + 3*e) + 3*a^2*cos(f*x + e))*sin(
4*f*x + 4*e) - 13*(16*a^2*cos(2*f*x + 2*e) + a^2)*sin(3*f*x + 3*e))*sqrt(a)
*sqrt(c)/((c^4*cos(8*f*x + 8*e)^2 + 36*c^4*cos(7*f*x + 7*e)^2 + 256*c^4*cos
(6*f*x + 6*e)^2 + 676*c^4*cos(5*f*x + 5*e)^2 + 900*c^4*cos(4*f*x + 4*e)^2 +
 676*c^4*cos(3*f*x + 3*e)^2 + 256*c^4*cos(2*f*x + 2*e)^2 + 36*c^4*cos(f*x +
 e)^2 + c^4*sin(8*f*x + 8*e)^2 + 36*c^4*sin(7*f*x + 7*e)^2 + 256*c^4*sin(6*
f*x + 6*e)^2 + 676*c^4*sin(5*f*x + 5*e)^2 + 900*c^4*sin(4*f*x + 4*e)^2 + 67
6*c^4*sin(3*f*x + 3*e)^2 + 256*c^4*sin(2*f*x + 2*e)^2 - 192*c^4*sin(2*f*x +
 2*e)*sin(f*x + e) + 36*c^4*sin(f*x + e)^2 - 12*c^4*cos(f*x + e) + c^4 - 2*
(6*c^4*cos(7*f*x + 7*e) - 16*c^4*cos(6*f*x + 6*e) + 26*c^4*cos(5*f*x + 5*e)
 - 30*c^4*cos(4*f*x + 4*e) + 26*c^4*cos(3*f*x + 3*e) - 16*c^4*cos(2*f*x + 2
*e) + 6*c^4*cos(f*x + e) - c^4)*cos(8*f*x + 8*e) - 12*(16*c^4*cos(6*f*x + 6
*e) - 26*c^4*cos(5*f*x + 5*e) + 30*c^4*cos(4*f*x + 4*e) - 26*c^4*cos(3*f*x
+ 3*e) + 16*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(7*f*x + 7*
e) - 32*(26*c^4*cos(5*f*x + 5*e) - 30*c^4*cos(4*f*x + 4*e) + 26*c^4*cos(3*f
*x + 3*e) - 16*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(6*f*x +
 6*e) - 52*(30*c^4*cos(4*f*x + 4*e) - 26*c^4*cos(3*f*x + 3*e) + 16*c^4*cos(
2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(5*f*x + 5*e) - 60*(26*c^4*cos(
3*f*x + 3*e) - 16*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(4*f*
x + 4*e) - 52*(16*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(3*f*
x + 3*e) - 32*(6*c^4*cos(f*x + e) - c^4)*cos(2*f*x + 2*e) - 4*(3*c^4*sin(7*
```

```
f*x + 7*e) - 8*c^4*sin(6*f*x + 6*e) + 13*c^4*sin(5*f*x + 5*e) - 15*c^4*sin(
4*f*x + 4*e) + 13*c^4*sin(3*f*x + 3*e) - 8*c^4*sin(2*f*x + 2*e) + 3*c^4*sin
(f*x + e))*sin(8*f*x + 8*e) - 24*(8*c^4*sin(6*f*x + 6*e) - 13*c^4*sin(5*f*x
 + 5*e) + 15*c^4*sin(4*f*x + 4*e) - 13*c^4*sin(3*f*x + 3*e) + 8*c^4*sin(2*f
*x + 2*e) - 3*c^4*sin(f*x + e))*sin(7*f*x + 7*e) - 64*(13*c^4*sin(5*f*x + 5
*e) - 15*c^4*sin(4*f*x + 4*e) + 13*c^4*sin(3*f*x + 3*e) - 8*c^4*sin(2*f*x +
 2*e) + 3*c^4*sin(f*x + e))*sin(6*f*x + 6*e) - 104*(15*c^4*sin(4*f*x + 4*e)
 - 13*c^4*sin(3*f*x + 3*e) + 8*c^4*sin(2*f*x + 2*e) - 3*c^4*sin(f*x + e))*s
in(5*f*x + 5*e) - 120*(13*c^4*sin(3*f*x + 3*e) - 8*c^4*sin(2*f*x + 2*e) + 3
*c^4*sin(f*x + e))*sin(4*f*x + 4*e) - 104*(8*c^4*sin(2*f*x + 2*e) - 3*c^4*s
in(f*x + e))*sin(3*f*x + 3*e))*f)
```

Fricas [B] time = 0.481269, size = 298, normalized size = 7.1

$$\frac{\left(3 a^{2} \cos \left(f x+e\right)^{3}+a^{2} \cos \left(f x+e\right)\right) \sqrt{\frac{a \cos \left(f x+e\right)+a}{\cos \left(f x+e\right)}} \sqrt{\frac{c \cos \left(f x+e\right)-c}{\cos \left(f x+e\right)}}}{3 \left(c^{4} f \cos \left(f x+e\right)^{3}-3 c^{4} f \cos \left(f x+e\right)^{2}+3 c^{4} f \cos \left(f x+e\right)-c^{4} f\right) \sin \left(f x+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*a^2*cos(f*x + e)^3 + a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^4*f*cos(f*x + e)^3 -
 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.131 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{48cf(c-c\sec(e+fx))^{7/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{8f(c-c\sec(e+fx))^{9/2}}$$

```
[Out] -((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(8*f*(c - c*Sec[e + f*x])^(9/2))
 - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(48*c*f*(c - c*Sec[e + f*x])^(
7/2))
```

Rubi [A] time = 0.297633, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3951, 3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{48cf(c-c\sec(e+fx))^{7/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{8f(c-c\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(9/2),x]
```

```
[Out] -((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(8*f*(c - c*Sec[e + f*x])^(9/2))
 - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(48*c*f*(c - c*Sec[e + f*x])^(
7/2))
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx &= -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{8f(c-c\sec(e+fx))^{9/2}}+\frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx}{8c}\\ &= -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{8f(c-c\sec(e+fx))^{9/2}}-\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{48cf(c-c\sec(e+fx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.827386, size = 92, normalized size = 1.05

$$-\frac{a^2(17\cos(e+fx)-3\cos(2(e+fx))+3\cos(3(e+fx))-5)\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}}{12c^4f(\cos(e+fx)-1)^4\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(9
/2),x]
```

```
[Out] -(a^2*(-5 + 17*Cos[e + f*x] - 3*Cos[2*(e + f*x)] + 3*Cos[3*(e + f*x)])*Sqrt
[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(12*c^4*f*(-1 + Cos[e + f*x])^4*Sq
rt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.257, size = 85, normalized size = 1.

$$-\frac{a^2\left(7\cos\left(fx+e\right)-1\right)\left(\sin\left(fx+e\right)\right)^5}{48f\left(-1+\cos\left(fx+e\right)\right)^2\left(\cos\left(fx+e\right)\right)^4}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x)
```

```
[Out] -1/48/f*a^2*(7*cos(f*x+e)-1)*sin(f*x+e)^5*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(
1/2)/(-1+cos(f*x+e))^2/cos(f*x+e)^4/(c*(-1+cos(f*x+e))/cos(f*x+e))^(9/2)
```

Maxima [B] time = 19.6824, size = 3671, normalized size = 41.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(70*a^2*cos(6*f*x + 6*e)*sin(4*f*x + 4*e) - 70*a^2*cos(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 3*a^2*sin(2*f*x + 2*e) + (3*a^2*sin(6*f*x + 6*e) + 10*a^2*s
in(4*f*x + 4*e) + 3*a^2*sin(2*f*x + 2*e))*cos(8*f*x + 8*e) + (3*a^2*sin(8*f
*x + 8*e) + 60*a^2*sin(6*f*x + 6*e) + 130*a^2*sin(4*f*x + 4*e) + 60*a^2*sin
(2*f*x + 2*e) - 32*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 32*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17*a^2*sin(8*f*x + 8*e) + 308*
a^2*sin(6*f*x + 6*e) + 630*a^2*sin(4*f*x + 4*e) + 308*a^2*sin(2*f*x + 2*e)
+ 32*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17*a^2*sin(8*f*x + 8*e) + 308*a
^2*sin(6*f*x + 6*e) + 630*a^2*sin(4*f*x + 4*e) + 308*a^2*sin(2*f*x + 2*e) +
 32*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (3*a^2*sin(8*f*x + 8*e) + 60*a^2*
sin(6*f*x + 6*e) + 130*a^2*sin(4*f*x + 4*e) + 60*a^2*sin(2*f*x + 2*e))*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (3*a^2*cos(6*f*x + 6*e)
+ 10*a^2*cos(4*f*x + 4*e) + 3*a^2*cos(2*f*x + 2*e))*sin(8*f*x + 8*e) - (70*
a^2*cos(4*f*x + 4*e) - 3*a^2)*sin(6*f*x + 6*e) + 10*(7*a^2*cos(2*f*x + 2*e)
```

```
 + a^2)*sin(4*f*x + 4*e) - (3*a^2*cos(8*f*x + 8*e) + 60*a^2*cos(6*f*x + 6*e
) + 130*a^2*cos(4*f*x + 4*e) + 60*a^2*cos(2*f*x + 2*e) - 32*a^2*cos(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*a^2*cos(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) + 3*a^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - (17*a^2*cos(8*f*x + 8*e) + 308*a^2*cos(6*f*x + 6*e) + 630
*a^2*cos(4*f*x + 4*e) + 308*a^2*cos(2*f*x + 2*e) + 32*a^2*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + 17*a^2)*sin(5/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - (17*a^2*cos(8*f*x + 8*e) + 308*a^2*cos(6*f*x + 6*
e) + 630*a^2*cos(4*f*x + 4*e) + 308*a^2*cos(2*f*x + 2*e) + 32*a^2*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 17*a^2)*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - (3*a^2*cos(8*f*x + 8*e) + 60*a^2*cos(6*f*
x + 6*e) + 130*a^2*cos(4*f*x + 4*e) + 60*a^2*cos(2*f*x + 2*e) + 3*a^2)*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^5*cos
(8*f*x + 8*e)^2 + 784*c^5*cos(6*f*x + 6*e)^2 + 4900*c^5*cos(4*f*x + 4*e)^2
+ 784*c^5*cos(2*f*x + 2*e)^2 + 64*c^5*cos(7/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 3136*c^5*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + 3136*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
+ 64*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^5*sin(8
*f*x + 8*e)^2 + 784*c^5*sin(6*f*x + 6*e)^2 + 4900*c^5*sin(4*f*x + 4*e)^2 +
3920*c^5*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 784*c^5*sin(2*f*x + 2*e)^2 + 6
4*c^5*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*sin
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*sin(3/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*c^5*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 56*c^5*cos(2*f*x + 2*e) + c^5 + 2*(28*c^5
*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5
)*cos(8*f*x + 8*e) + 56*(70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e)
+ c^5)*cos(6*f*x + 6*e) + 140*(28*c^5*cos(2*f*x + 2*e) + c^5)*cos(4*f*x + 4
*e) - 16*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x
 + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 56*c^5*cos(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 56*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^5)
*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*cos(8*f*x
+ 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f
*x + 2*e) - 56*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8
*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^5)*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*cos(8*f*x + 8*e) + 28
*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) -
 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^5)*cos(3/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*cos(8*f*x + 8*e) + 2
8*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e)
+ c^5)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 28*(2*c^5*sin
(6*f*x + 6*e) + 5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x + 2*e))*sin(8*f*x
+ 8*e) + 784*(5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x + 2*e))*sin(6*f*x +
6*e) - 16*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*
x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 56*c^5*sin(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 56*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(
7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*sin(8*f*x + 8*e
) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x +
2*e) - 56*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*
f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 8*c^5*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x +
 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e))*sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [B] time = 0.492618, size = 394, normalized size = 4.48

$$\frac{\left(6 a^{2} \cos \left(f x+e\right)^{4}-3 a^{2} \cos \left(f x+e\right)^{3}+4 a^{2} \cos \left(f x+e\right)^{2}-a^{2} \cos \left(f x+e\right)\right) \sqrt{\frac{a \cos \left(f x+e\right)+a}{\cos \left(f x+e\right)}} \sqrt{\frac{c \cos \left(f x+e\right)-c}{\cos \left(f x+e\right)}}}{6 \left(c^{5} f \cos \left(f x+e\right)^{4}-4 c^{5} f \cos \left(f x+e\right)^{3}+6 c^{5} f \cos \left(f x+e\right)^{2}-4 c^{5} f \cos \left(f x+e\right)+c^{5} f\right) \sin \left(f x+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(6*a^2*cos(f*x + e)^4 - 3*a^2*cos(f*x + e)^3 + 4*a^2*cos(f*x + e)^2 - a
^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c
^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.132 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{240c^2f(c-c\sec(e+fx))^{7/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{40cf(c-c\sec(e+fx))^{9/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{10f(c-c\sec(e+fx))^{11/2}}$$

```
[Out] -((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(10*f*(c - c*Sec[e + f*x])^(11/2
)) - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(40*c*f*(c - c*Sec[e + f*x])
^(9/2)) - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(240*c^2*f*(c - c*Sec[e
 + f*x])^(7/2))
```

Rubi [A] time = 0.455579, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3951, 3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{240c^2f(c-c\sec(e+fx))^{7/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{40cf(c-c\sec(e+fx))^{9/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{10f(c-c\sec(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(11/2),x
]
```

```
[Out] -((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(10*f*(c - c*Sec[e + f*x])^(11/2
)) - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(40*c*f*(c - c*Sec[e + f*x])
^(9/2)) - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(240*c^2*f*(c - c*Sec[e
 + f*x])^(7/2))
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{10f(c-c\sec(e+fx))^{11/2}} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}dx}{5c}$$

$$= -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{10f(c-c\sec(e+fx))^{11/2}} - \frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{40cf(c-c\sec(e+fx))^{9/2}} + \frac{\int \mathrm{se}}{}$$

$$= -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{10f(c-c\sec(e+fx))^{11/2}} - \frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{40cf(c-c\sec(e+fx))^{9/2}} - \frac{(a+}{2}$$

Mathematica [A] time = 1.15805, size = 102, normalized size = 0.77

$$\frac{a^2(170\cos(e+fx)-140\cos(2(e+fx))+30\cos(3(e+fx))-15\cos(4(e+fx))-141)\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+}}{120c^5f(\cos(e+fx)-1)^5\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(1
1/2),x]
```

```
[Out] (a^2*(-141 + 170*Cos[e + f*x] - 140*Cos[2*(e + f*x)] + 30*Cos[3*(e + f*x)]
- 15*Cos[4*(e + f*x)])*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(120*c^
5*f*(-1 + Cos[e + f*x])^5*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.264, size = 95, normalized size = 0.7

$$-\frac{a^2\left(31\left(\cos\left(fx+e\right)\right)^2-8\cos\left(fx+e\right)+1\right)\left(\sin\left(fx+e\right)\right)^5}{240f\left(-1+\cos\left(fx+e\right)\right)^2\left(\cos\left(fx+e\right)\right)^5}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x)
```

```
[Out] -1/240/f*a^2*(31*cos(f*x+e)^2-8*cos(f*x+e)+1)*sin(f*x+e)^5*(1/cos(f*x+e)*a*
(1+cos(f*x+e)))^(1/2)/(-1+cos(f*x+e))^2/cos(f*x+e)^5/(c*(-1+cos(f*x+e))/cos
(f*x+e))^(11/2)
```

Maxima [B] time = 109.035, size = 5546, normalized size = 41.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="maxima")
```

```
[Out] -2/15*(1350*a^2*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) + 1350*a^2*cos(4*f*x + 4*
e)*sin(2*f*x + 2*e) - 30*a^2*sin(2*f*x + 2*e) - 10*(3*a^2*sin(8*f*x + 8*e)
+ 17*a^2*sin(6*f*x + 6*e) + 17*a^2*sin(4*f*x + 4*e) + 3*a^2*sin(2*f*x + 2*e
))*cos(10*f*x + 10*e) - 1350*(a^2*sin(6*f*x + 6*e) + a^2*sin(4*f*x + 4*e))*
```

cos(8*f*x + 8*e) - 5*(3*a^2*sin(10*f*x + 10*e) + 75*a^2*sin(8*f*x + 8*e) + 290*a^2*sin(6*f*x + 6*e) + 290*a^2*sin(4*f*x + 4*e) + 75*a^2*sin(2*f*x + 2*e) - 80*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 192*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 80*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(7*a^2*sin(10*f*x + 10*e) + 135*a^2*sin(8*f*x + 8*e) + 450*a^2*sin(6*f*x + 6*e) + 450*a^2*sin(4*f*x + 4*e) + 135*a^2*sin(2*f*x + 2*e) - 72*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 20*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*(47*a^2*sin(10*f*x + 10*e) + 855*a^2*sin(8*f*x + 8*e) + 2730*a^2*sin(6*f*x + 6*e) + 2730*a^2*sin(4*f*x + 4*e) + 855*a^2*sin(2*f*x + 2*e) + 240*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 160*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(7*a^2*sin(10*f*x + 10*e) + 135*a^2*sin(8*f*x + 8*e) + 450*a^2*sin(6*f*x + 6*e) + 450*a^2*sin(4*f*x + 4*e) + 135*a^2*sin(2*f*x + 2*e) + 20*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*(3*a^2*sin(10*f*x + 10*e) + 75*a^2*sin(8*f*x + 8*e) + 290*a^2*sin(6*f*x + 6*e) + 290*a^2*sin(4*f*x + 4*e) + 75*a^2*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 10*(3*a^2*cos(8*f*x + 8*e) + 17*a^2*cos(6*f*x + 6*e) + 17*a^2*cos(4*f*x + 4*e) + 3*a^2*cos(2*f*x + 2*e))*sin(10*f*x + 10*e) + 30*(45*a^2*cos(6*f*x + 6*e) + 45*a^2*cos(4*f*x + 4*e) - a^2)*sin(8*f*x + 8*e) - 10*(135*a^2*cos(2*f*x + 2*e) + 17*a^2)*sin(6*f*x + 6*e) - 10*(135*a^2*cos(2*f*x + 2*e) + 17*a^2)*sin(4*f*x + 4*e) + 5*(3*a^2*cos(10*f*x + 10*e) + 75*a^2*cos(8*f*x + 8*e) + 290*a^2*cos(6*f*x + 6*e) + 290*a^2*cos(4*f*x + 4*e) + 75*a^2*cos(2*f*x + 2*e) - 80*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 192*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 80*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*a^2)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 20*(7*a^2*cos(10*f*x + 10*e) + 135*a^2*cos(8*f*x + 8*e) + 450*a^2*cos(6*f*x + 6*e) + 450*a^2*cos(4*f*x + 4*e) + 135*a^2*cos(2*f*x + 2*e) - 72*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 20*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 7*a^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(47*a^2*cos(10*f*x + 10*e) + 855*a^2*cos(8*f*x + 8*e) + 2730*a^2*cos(6*f*x + 6*e) + 2730*a^2*cos(4*f*x + 4*e) + 855*a^2*cos(2*f*x + 2*e) + 240*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 160*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 47*a^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 20*(7*a^2*cos(10*f*x + 10*e) + 135*a^2*cos(8*f*x + 8*e) + 450*a^2*cos(6*f*x + 6*e) + 450*a^2*cos(4*f*x + 4*e) + 135*a^2*cos(2*f*x + 2*e) + 20*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 7*a^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 5*(3*a^2*cos(10*f*x + 10*e) + 75*a^2*cos(8*f*x + 8*e) + 290*a^2*cos(6*f*x + 6*e) + 290*a^2*cos(4*f*x + 4*e) + 75*a^2*cos(2*f*x + 2*e) + 3*a^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^6*cos(10*f*x + 10*e)^2 + 2025*c^6*cos(8*f*x + 8*e)^2 + 44100*c^6*cos(6*f*x + 6*e)^2 + 44100*c^6*cos(4*f*x + 4*e)^2 + 2025*c^6*cos(2*f*x + 2*e)^2 + 100*c^6*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*c^6*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^6*sin(10*f*x + 10*e)^2 + 2025*c^6*sin(8*f*x + 8*e)^2 + 44100*c^6*sin(6*f*x + 6*e)^2 + 44100*c^6*sin(4*f*x + 4*e)^2 + 18900*c^6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 2025*c^6*sin(2*f*x + 2*e)^2 + 100*c^6*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 90*c^6*cos(2*f*x + 2*e) + c^6 + 2*(45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(

```
4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(10*f*x + 10*e) + 90*(210*
c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) +
 c^6)*cos(8*f*x + 8*e) + 420*(210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x +
 2*e) + c^6)*cos(6*f*x + 6*e) + 420*(45*c^6*cos(2*f*x + 2*e) + c^6)*cos(4*f
*x + 4*e) - 20*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*
cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 120
*c^6*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 252*c^6*cos(5/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*cos(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + c^6)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 240*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos
(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 252*c^
6*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*cos(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + c^6)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 504*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 21
0*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e)
 - 120*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^6)*cos(5/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*cos(10*f*x + 10*e) + 45*c^6*
cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45
*c^6*cos(2*f*x + 2*e) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + c^6)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(
c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e)
 + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(3*c^6*sin(8*f*x + 8*e) + 14*c^
6*sin(6*f*x + 6*e) + 14*c^6*sin(4*f*x + 4*e) + 3*c^6*sin(2*f*x + 2*e))*sin(
10*f*x + 10*e) + 1350*(14*c^6*sin(6*f*x + 6*e) + 14*c^6*sin(4*f*x + 4*e) +
3*c^6*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6300*(14*c^6*sin(4*f*x + 4*e) +
3*c^6*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 20*(c^6*sin(10*f*x + 10*e) + 45*
c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e)
+ 45*c^6*sin(2*f*x + 2*e) - 120*c^6*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) - 252*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - 120*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*s
in(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(9/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8
*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*
sin(2*f*x + 2*e) - 252*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - 120*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^
6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 504*(c^6*sin(10*f*x + 10*e) + 45*c^6*si
n(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c
^6*sin(2*f*x + 2*e) - 120*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(
5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*sin(10*f*x + 10
*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*
x + 4*e) + 45*c^6*sin(2*f*x + 2*e) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 20*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x
 + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e))*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [A] time = 0.505749, size = 471, normalized size = 3.54

$$\frac{\left(15\,a^{2}\cos\left(fx+e\right)^{5}-15\,a^{2}\cos\left(fx+e\right)^{4}+20\,a^{2}\cos\left(fx+e\right)^{3}-10\,a^{2}\cos\left(fx+e\right)^{2}+2\,a^{2}\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{}}{15\left(c^{6}f\cos\left(fx+e\right)^{5}-5\,c^{6}f\cos\left(fx+e\right)^{4}+10\,c^{6}f\cos\left(fx+e\right)^{3}-10\,c^{6}f\cos\left(fx+e\right)^{2}+5\,c^{6}f\cos\left(fx+e\right)-c^{6}f\right)\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="fricas")
```

```
[Out] 1/15*(15*a^2*cos(f*x + e)^5 - 15*a^2*cos(f*x + e)^4 + 20*a^2*cos(f*x + e)^3
 - 10*a^2*cos(f*x + e)^2 + 2*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^6*f*cos(f*x + e)^5
- 5*c^6*f*cos(f*x + e)^4 + 10*c^6*f*cos(f*x + e)^3 - 10*c^6*f*cos(f*x + e)^
2 + 5*c^6*f*cos(f*x + e) - c^6*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

3.133 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=139

$$-\frac{2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}-\frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (-4*c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) - (2*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*S
qrt[a + a*Sec[e + f*x]]) - (c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f
*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.409218, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3952}

$$-\frac{2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}-\frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/Sqrt[a + a*Sec[e + f*x]],x]
```


```
[Out] (-4*c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) - (2*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*S
qrt[a + a*Sec[e + f*x]]) - (c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f
*Sqrt[a + a*Sec[e + f*x]])
```


Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} + (2c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3}}{\sqrt{a+a\sec(e+fx)}}$$
$$= -\frac{2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} + ($$
$$= -\frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx}{f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] time = 1.49376, size = 141, normalized size = 1.01

$$\frac{c^2\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx)}\left(8\log\left(1+e^{i(e+fx)}\right)-4\log\left(1+e^{2i(e+fx)}\right)-6\cos(e+fx)+\left(8\log\left(1\right.\right.\right.}{2f\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/Sqrt[a + a*Sec[e + f*x]
],x]
```

```
[Out] (c^2*Cot[(e + f*x)/2]*(1 - 6*Cos[e + f*x] + 8*Log[1 + E^(I*(e + f*x))] + Co
s[2*(e + f*x)]*(8*Log[1 + E^(I*(e + f*x))] - 4*Log[1 + E^((2*I)*(e + f*x))]
) - 4*Log[1 + E^((2*I)*(e + f*x))])*Sec[e + f*x]^2*Sqrt[c - c*Sec[e + f*x]]
)/(2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] time = 0.315, size = 165, normalized size = 1.2

$$-\frac{\cos(fx+e)}{2\,af\sin(fx+e)\left(-1+\cos(fx+e)\right)^2}\left(8\left(\cos(fx+e)\right)^2\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)+8\ln\left(-\frac{-1+\cos}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f/a*(8*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+8*ln(-(
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+7*cos(f*x+e)^2+6*cos(f*x
+e)-1)*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*(1/cos(f*x+e)*a*(1+c
os(f*x+e)))^(1/2)/sin(f*x+e)/(-1+cos(f*x+e))^2
```

Maxima [B] time = 1.92371, size = 995, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*(c^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - c^2*cos(4*f*x + 4*e)*sin(2*f*x +
 2*e) - c^2*sin(2*f*x + 2*e) + 2*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x
```

```
+ 2*e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2
*f*x + 2*e) + c^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1) - 4*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + c^2*sin(
4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x
+ 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2*f*x + 2*e) + c^2)*
cos(4*f*x + 4*e))*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 3*(c^2*sin
(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) - 3*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(c^2*cos(4*f*x + 4*e) +
 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 3*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((a*cos(4*f*x
 + 4*e)^2 + 4*a*cos(2*f*x + 2*e)^2 + a*sin(4*f*x + 4*e)^2 + 4*a*sin(4*f*x +
 4*e)*sin(2*f*x + 2*e) + 4*a*sin(2*f*x + 2*e)^2 + 2*(2*a*cos(2*f*x + 2*e) +
 a)*cos(4*f*x + 4*e) + 4*a*cos(2*f*x + 2*e) + a)*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c^2\sec\left(fx+e\right)^3-2\,c^2\sec\left(fx+e\right)^2+c^2\sec\left(fx+e\right)\right)\sqrt{-c\sec\left(fx+e\right)+c}}{\sqrt{a\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*sqr
t(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 3.66539, size = 188, normalized size = 1.35

$$\frac{2\left(2\,c^3\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)-\frac{3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^3+4\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^4+c^5}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2}\right)\mathrm{csgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx\right.\right.}{\sqrt{-ac}f|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 2*(2*c^3*log(c*tan(1/2*f*x + 1/2*e)^2 - c) - (3*(c*tan(1/2*f*x + 1/2*e)^2 -
 c)^2*c^3 + 4*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^4 + c^5)/(c*tan(1/2*f*x + 1/
2*e)^2 - c)^2)*c*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/(sqrt(-
a*c)*f*abs(c))
```

3.134 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=94

$$-\frac{2c^2\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (-2*c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) - (c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[
a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.265812, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3955, 3952}

$$-\frac{2c^2\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (-2*c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) - (c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[
a + a*Sec[e + f*x]])
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx &= -\frac{c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+(2c)\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx\\ &= -\frac{2c^2\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

Mathematica [C] time = 1.84673, size = 173, normalized size = 1.84

$$\frac{ce^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2\cos\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx)}\left(-1+\left(4\log\left(1+e^{i(e+fx)}\right)-2\right.\right.}{2f\left(1+e^{i(e+fx)}\right)\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/Sqrt[a + a*Sec[e + f*x]
],x]
```

```
[Out] (c*(1 + E^((2*I)*(e + f*x)))^2*Cos[(e + f*x)/2]*Cot[(e + f*x)/2]*(-1 + Cos[
e + f*x]*(4*Log[1 + E^(I*(e + f*x))] - 2*Log[1 + E^((2*I)*(e + f*x))]))*Sec
[e + f*x]^3*Sqrt[c - c*Sec[e + f*x]]*(Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2]
))/(2*E^((2*I)*(e + f*x))*(1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[e + f*x])
])
```

Maple [A] time = 0.303, size = 149, normalized size = 1.6

$$-\frac{\cos(fx+e)}{af\sin(fx+e)(-1+\cos(fx+e))}\left(2\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)+2\cos(fx+e)\ln\left(-\frac{-1}{\,}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f/a*(2*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e
)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)+1)*cos(f*x+e)*(c*(-
1+cos(f*x+e))/cos(f*x+e))^(3/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f
*x+e)/(-1+cos(f*x+e))
```

Maxima [B] time = 1.88113, size = 373, normalized size = 3.97

$$-\frac{2\left(c\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)\sin\left(2fx+2e\right)-\left(c\cos\left(2fx+2e\right)^2+c\sin\left(2fx+2e\right)^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*(c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e)
 - (c*cos(2*f*x + 2*e)^2 + c*sin(2*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(c*cos(2*f*x + 2*e)^2
+ c*sin(2*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*arctan2(sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + 1) - (c*cos(2*f*x + 2*e) + c)*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((a*cos(2*f*x + 2*e)^2 + a*sin(2
*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c\sec\left(fx+e\right)^{2}-c\sec\left(fx+e\right)\right)\sqrt{-c\sec\left(fx+e\right)+c}}{\sqrt{a\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(c*sec(f*x + e)^2 - c*sec(f*x + e))*sqrt(-c*sec(f*x + e) + c)/sqr
t(a*sec(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.17438, size = 154, normalized size = 1.64

$$\frac{2\left(c^{4}\log\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)-\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)c^{4}+c^{5}}{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}\right)\text{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\sqrt{-ac}cf|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 2*(c^4*log(c*tan(1/2*f*x + 1/2*e)^2 - c) - ((c*tan(1/2*f*x + 1/2*e)^2 - c)*
c^4 + c^5)/(c*tan(1/2*f*x + 1/2*e)^2 - c))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan
(1/2*f*x + 1/2*e))/(sqrt(-a*c)*c*f*abs(c))
```

3.135 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=50

$$-\frac{c\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -((c*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]))
```

Rubi [A] time = 0.122232, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3952}

$$-\frac{c\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -((c*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]))
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{c\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 0.37146, size = 140, normalized size = 2.8

$$\frac{i\left(1+e^{i(e+fx)}\right)\sqrt{\frac{c\left(-1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}}\left(2\log\left(1+e^{i(e+fx)}\right)-\log\left(1+e^{2i(e+fx)}\right)\right)}{f\left(-1+e^{i(e+fx)}\right)\sqrt{\frac{a\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],
x]
```

```
[Out] (I*(1 + E^(I*(e + f*x)))*Sqrt[(c*(-1 + E^(I*(e + f*x)))^2)/(1 + E^((2*I)*(e
 + f*x)))]*(2*Log[1 + E^(I*(e + f*x))] - Log[1 + E^((2*I)*(e + f*x))]))/((-
```

```
1 + E^(I*(e + f*x)))*Sqrt[(a*(1 + E^(I*(e + f*x)))^2)/(1 + E^((2*I)*(e + f*
x)))]*f)
```

Maple [B] time = 0.303, size = 116, normalized size = 2.3

$$-\frac{\cos\left(fx+e\right)}{af\sin\left(fx+e\right)}\left(\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\right)\sqrt{\frac{c\left(-1+\cos\left(fx+\right.\right.}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f/a*(ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+ln(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e)))*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(1/cos(
f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)
```

Maxima [A] time = 1.52388, size = 86, normalized size = 1.72

$$-\frac{\frac{\sqrt{c}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{\sqrt{-a}}+\frac{\sqrt{c}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{\sqrt{-a}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -(sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(-a) + sqrt(c)*log(s
in(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(-a))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c\sec\left(fx+e\right)+c}\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(-c*sec(f*x + e) + c)*sec(f*x + e)/sqrt(a*sec(f*x + e) + a), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)/sqrt(a*(sec(e + f*x) + 1)
), x)
```

Giac [A] time = 1.63163, size = 116, normalized size = 2.32

$$\frac{c^2 \log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{\sqrt{-ac}f|c|\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] c^2*log(abs(c*tan(1/2*f*x + 1/2*e)^2 - c))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan
(1/2*f*x + 1/2*e))*sgn(cos(f*x + e))/(sqrt(-a*c)*f*abs(c)*sgn(tan(1/2*f*x +
 1/2*e)^2 - 1))
```

3.136 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -((ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]))
```

Rubi [A] time = 0.13497, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3959, 3770}

$$-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -((ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]))
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx &= \frac{\tan(e+fx)\int \csc(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= -\frac{\tanh^{-1}(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.792874, size = 94, normalized size = 2.

$$\frac{4i\left(-1+e^{i(e+fx)}\right)\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\tanh^{-1}\left(e^{i(e+fx)}\right)}{f\left(1+e^{i(e+fx)}\right)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),
x]
```

```
[Out] ((4*I)*(-1 + E^(I*(e + f*x)))*ArcTanh[E^(I*(e + f*x))]*Cos[(e + f*x)/2]^2*S
ec[e + f*x])/((1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c
*Sec[e + f*x]])
```

Maple [A] time = 0.267, size = 85, normalized size = 1.8

$$-\frac{\cos\left(fx+e\right)}{af\sin\left(fx+e\right)c}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f/a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*ln(-(-1+cos(f*x+e))/sin(f*x+e)
)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)/sin(f*x+e)/c
```

Maxima [A] time = 1.83463, size = 59, normalized size = 1.26

$$-\frac{\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)+1\right)-\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)-1\right)}{\sqrt{a}\sqrt{c}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -(arctan2(sin(f*x + e), cos(f*x + e) + 1) - arctan2(sin(f*x + e), cos(f*x +
 e) - 1))/(sqrt(a)*sqrt(c)*f)
```

Fricas [A] time = 0.607107, size = 487, normalized size = 10.36

$$\left[-\frac{\sqrt{-ac}\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^2+\left(ac\cos\left(fx+e\right)^2+ac\right)\sin\left(fx+e\right)\right)}{\left(\cos\left(fx+e\right)^2-1\right)\sin\left(fx+e\right)}\right)}{2\,acf},\ \frac{\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos(\ldots}{\cos(\ldots}}}{ac\sin\left(fx+e\right)}\right)}{acf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x +
```

```
e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))/(a*c*f), sqr
t(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e))/(a*c*sin(f*x + e)))/(a*c*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1
))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.137 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -Tan[e + f*x]/(2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)) - (
ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
- c*Sec[e + f*x]])
```

Rubi [A] time = 0.280026, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3960, 3959, 3770}

$$-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] -Tan[e + f*x]/(2*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)) - (
ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
- c*Sec[e + f*x]])
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+f}}}{2c}$$
$$= -\frac{\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)\int \csc(e}{2c\sqrt{a+a\sec(e+fx)}\sqrt{c}}$$
$$= -\frac{\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} - \frac{\tanh^{-1}(\cos(e+fx))}{2cf\sqrt{a+a\sec(e+fx)}\sqrt{c}}$$

Mathematica [C] time = 0.880798, size = 79, normalized size = 0.83

$$-\frac{\tan(e+fx)\left(1+2(\cos(e+fx)-1)\tanh^{-1}\left(e^{i(e+fx)}\right)\right)}{2cf(\cos(e+fx)-1)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)
),x]
```

```
[Out] -((1 + 2*ArcTanh[E^(I*(e + f*x))]*(-1 + Cos[e + f*x]))*Tan[e + f*x])/(2*c*f
*(-1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.299, size = 131, normalized size = 1.4

$$-\frac{-1+\cos(fx+e)}{4af\cos(fx+e)\sin(fx+e)}\left(2\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-\cos(fx+e)-2\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/4/f/a*(-1+cos(f*x+e))*(2*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-cos(
f*x+e)-2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-1)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))
^(1/2)/cos(f*x+e)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [B] time = 1.92862, size = 548, normalized size = 5.77

$$\frac{\left(\left(2\left(2\cos(fx+e)-1\right)\cos(2fx+2e)-\cos(2fx+2e)^2-4\cos(fx+e)^2-\sin(2fx+2e)^2+4\sin(2fx+2e)\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*((2*(2*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos(
f*x + e)^2 - sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f
*x + e)^2 + 4*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) - (
```

```
2*(2*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos(f*x +
e)^2 - sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e
)^2 + 4*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*cos(f
*x + e)*sin(2*f*x + 2*e) - 2*cos(2*f*x + 2*e)*sin(f*x + e) - 2*sin(f*x + e)
)*sqrt(a)*sqrt(c)/((a*c^2*cos(2*f*x + 2*e)^2 + 4*a*c^2*cos(f*x + e)^2 + a*c
^2*sin(2*f*x + 2*e)^2 - 4*a*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*a*c^2*sin
(f*x + e)^2 - 4*a*c^2*cos(f*x + e) + a*c^2 - 2*(2*a*c^2*cos(f*x + e) - a*c^
2)*cos(2*f*x + 2*e))*f)
```

Fricas [B] time = 0.667688, size = 952, normalized size = 10.02

$$\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)-1\right)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}+\left(ac\cos\left(fx+e\right)^{2}+ac\right)\sin\left(fx+e\right)\right)}{\left(\cos\left(fx+e\right)^{2}-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-2\ \cdots}{4\left(ac^{2}f\cos\left(fx+e\right)-ac^{2}f\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*c)*(cos(f*x + e) - 1)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)
^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x
 + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*co
s(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a*c^2*f*cos(f*x + e) - a*c^2*
f)*sin(f*x + e)), 1/2*(sqrt(a*c)*(cos(f*x + e) - 1)*arctan(sqrt(a*c)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(
a*c*sin(f*x + e)))*sin(f*x + e) + sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a*c^2*f*cos(f*x + e)
 - a*c^2*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**
(3/2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.138 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=140

$$-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{4c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}-\frac{\tan(e+ f}{4f\sqrt{a\sec(e+fx)+a}(c -}$$

```
[Out] -Tan[e + f*x]/(4*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)) - T
an[e + f*x]/(4*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)) - (
ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(4*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[
c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.429864, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3960, 3959, 3770}

$$-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{4c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}-\frac{\tan(e+ f}{4f\sqrt{a\sec(e+fx)+a}(c -}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] -Tan[e + f*x]/(4*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)) - T
an[e + f*x]/(4*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)) - (
ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(4*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[
c - c*Sec[e + f*x]])
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} + \frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+f}}{2c}$$

$$= -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{\tan(e+f}{4cf\sqrt{a+a\sec(e+fx)}(c}$$

$$= -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{\tan(e+f}{4cf\sqrt{a+a\sec(e+fx)}(c}$$

$$= -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{\tan(e+f}{4cf\sqrt{a+a\sec(e+fx)}(c}$$

Mathematica [C] time = 0.839713, size = 91, normalized size = 0.65

$$-\frac{\tan(e+fx)\left(3\cos(e+fx)+8\sin^4\left(\frac{1}{2}(e+fx)\right)\tanh^{-1}\left(e^{i(e+fx)}\right)-2\right)}{4c^2f(\cos(e+fx)-1)^2\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)
),x]
```

```
[Out] -((-2 + 3*Cos[e + f*x] + 8*ArcTanh[E^(I*(e + f*x))]*Sin[(e + f*x)/2]^4)*Tan
[e + f*x])/(4*c^2*f*(-1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
 - c*Sec[e + f*x]])
```

Maple [A] time = 0.301, size = 170, normalized size = 1.2

$$-\frac{-1+\cos(fx+e)}{16af(\cos(fx+e))^2\sin(fx+e)}\left(4\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)(\cos(fx+e))^2-8\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/16/f/a*(-1+cos(f*x+e))*(4*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-8
*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-5*cos(f*x+e)^2+4*ln(-(-1+cos(f*
x+e))/sin(f*x+e))-2*cos(f*x+e)+3)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/cos
(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)
```

Maxima [B] time = 2.16308, size = 1621, normalized size = 11.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((2*(4*cos(3*f*x + 3*e) - 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) - 1)*cos(
4*f*x + 4*e) - cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) - 4*cos(f*x + e)
+ 1)*cos(3*f*x + 3*e) - 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) - 1)*cos
(2*f*x + 2*e) - 36*cos(2*f*x + 2*e)^2 - 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x
+ 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) - sin(4*f*x
+ 4*e)^2 + 16*(3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*sin(3*f*x + 3*e) - 16*s
in(3*f*x + 3*e)^2 - 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x + e
) - 16*sin(f*x + e)^2 + 8*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x +
 e) + 1) - (2*(4*cos(3*f*x + 3*e) - 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) - 1
)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) - 4*cos(f*x
 + e) + 1)*cos(3*f*x + 3*e) - 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) -
1)*cos(2*f*x + 2*e) - 36*cos(2*f*x + 2*e)^2 - 16*cos(f*x + e)^2 + 4*(2*sin(
3*f*x + 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) - sin(
4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*sin(3*f*x + 3*e)
- 16*sin(3*f*x + 3*e)^2 - 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f
*x + e) - 16*sin(f*x + e)^2 + 8*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos
(f*x + e) - 1) - 2*(3*sin(3*f*x + 3*e) - 4*sin(2*f*x + 2*e) + 3*sin(f*x + e
))*cos(4*f*x + 4*e) + 2*(3*cos(3*f*x + 3*e) - 4*cos(2*f*x + 2*e) + 3*cos(f*
x + e))*sin(4*f*x + 4*e) - 2*(2*cos(2*f*x + 2*e) + 3)*sin(3*f*x + 3*e) + 4*
(cos(f*x + e) + 2)*sin(2*f*x + 2*e) + 4*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) -
 4*cos(2*f*x + 2*e)*sin(f*x + e) - 6*sin(f*x + e))*sqrt(a)*sqrt(c)/((a*c^3*
cos(4*f*x + 4*e)^2 + 16*a*c^3*cos(3*f*x + 3*e)^2 + 36*a*c^3*cos(2*f*x + 2*e
)^2 + 16*a*c^3*cos(f*x + e)^2 + a*c^3*sin(4*f*x + 4*e)^2 + 16*a*c^3*sin(3*f
*x + 3*e)^2 + 36*a*c^3*sin(2*f*x + 2*e)^2 - 48*a*c^3*sin(2*f*x + 2*e)*sin(f
*x + e) + 16*a*c^3*sin(f*x + e)^2 - 8*a*c^3*cos(f*x + e) + a*c^3 - 2*(4*a*c
^3*cos(3*f*x + 3*e) - 6*a*c^3*cos(2*f*x + 2*e) + 4*a*c^3*cos(f*x + e) - a*c
^3)*cos(4*f*x + 4*e) - 8*(6*a*c^3*cos(2*f*x + 2*e) - 4*a*c^3*cos(f*x + e) +
 a*c^3)*cos(3*f*x + 3*e) - 12*(4*a*c^3*cos(f*x + e) - a*c^3)*cos(2*f*x + 2*
e) - 4*(2*a*c^3*sin(3*f*x + 3*e) - 3*a*c^3*sin(2*f*x + 2*e) + 2*a*c^3*sin(f
*x + e))*sin(4*f*x + 4*e) - 16*(3*a*c^3*sin(2*f*x + 2*e) - 2*a*c^3*sin(f*x
+ e))*sin(3*f*x + 3*e))*f)
```

Fricas [A] time = 0.69242, size = 1138, normalized size = 8.13

$$\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)^{2}-2\,\cos\left(fx+e\right)+1\right)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)^{2}+\left(ac\cos(fx+e)^{2}+ac\right)\sin(fx+e)\right)}{\left(\cos(fx+e)^{2}-1\right)\sin(fx+e)}\right)}{8\left(ac^{3}f\cos\left(fx+e\right)^{2}-2\,ac^{3}f\cos\left(fx+e\right)+ac^{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a*c)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(-4*(2*sqrt(-a*c
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x
+ e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(3*cos(f*x + e)^2 - 2*cos(f*x +
 e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(
f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin
(f*x + e)), 1/4*(sqrt(a*c)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*arctan(sqr
t(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (3*cos(f*x + e)^2 - 2*cos(f*
```

```
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*
sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.139 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{af\sqrt{a\sec(e+fx)+a}}+\frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)^{3/2}}$$

```
[Out] (4*c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sq
rt[c - c*Sec[e + f*x]]) + (2*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(a*
f*Sqrt[a + a*Sec[e + f*x]]) + (c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(
f*(a + a*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.423644, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3954, 3955, 3952}

$$\frac{2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{af\sqrt{a\sec(e+fx)+a}}+\frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sq
rt[c - c*Sec[e + f*x]]) + (2*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(a*
f*Sqrt[a + a*Sec[e + f*x]]) + (c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(
f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}} - \frac{(2c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a}$$

$$= \frac{2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}} - \frac{(4c^2)}{}$$

$$= \frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} +$$

Mathematica [C] time = 1.04371, size = 183, normalized size = 1.29

$$-\frac{c^2\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}\left(4\log\left(1+e^{i(e+fx)}\right)-2\log\left(1+e^{2i(e+fx)}\right)+\left(8\log\left(1+e^{i(e+fx)}\right)-4\log\right.\right.}{af(\cos(e+fx)+1)\sqrt{a(\sec(e+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(3
/2),x]
```

```
[Out] -((c^2*Cot[(e + f*x)/2]*(-1 + 4*Log[1 + E^(I*(e + f*x))] + Cos[e + f*x]*(-5
 + 8*Log[1 + E^(I*(e + f*x))] - 4*Log[1 + E^((2*I)*(e + f*x))]) + Cos[2*(e
+ f*x)]*(4*Log[1 + E^(I*(e + f*x))] - 2*Log[1 + E^((2*I)*(e + f*x))]) - 2*L
og[1 + E^((2*I)*(e + f*x))])*Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a*f*(1
 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]))
```

Maple [A] time = 0.263, size = 235, normalized size = 1.7

$$-\frac{\left(\cos\left(fx+e\right)\right)^2}{fa^2\left(\sin\left(fx+e\right)\right)^3\left(-1+\cos\left(fx+e\right)\right)}\left(4\left(\cos\left(fx+e\right)\right)^2\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+4\ln\left(-\frac{-1+\cos\left(\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f/a^2*(4*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+4*ln(-(
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+4*cos(f*x+e)*ln(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))+4*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))-cos(f*x+e)^2+4*cos(f*x+e)+1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5
/2)*cos(f*x+e)^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+e)^3/(-1+cos
(f*x+e))
```

Maxima [B] time = 2.51524, size = 2747, normalized size = 19.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] -2*(8*c^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*cos(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*c^2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 8*c^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*c^2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*
c^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) + 2*c^2*cos(4*f*x + 4*e)*sin(2*f*x +
2*e) + 2*c^2*sin(2*f*x + 2*e) + 2*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x
 + 2*e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e
) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(
2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1) - 4*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + 4*c^2*c
os(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*sin(3/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*
c^2*cos(2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e) + 4*(c^2*cos(4*f*x + 4*e) + 2*
c^2*cos(2*f*x + 2*e) + 2*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + c^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^2
*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*cos(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2
*e) + 2*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^2*sin(4*f*x + 4*e) + 2*c
^2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*
arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + (16*c^2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 5*c^2*sin(4*f*x + 4*e) - 6*c^2*sin(2*f*x + 2*e) + 8*(c^2*cos(4*f*x
+ 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (5*c^2*
sin(4*f*x + 4*e) + 6*c^2*sin(2*f*x + 2*e) - 8*(c^2*cos(4*f*x + 4*e) + 2*c^2
*cos(2*f*x + 2*e) + c^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (16*c^2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 5*c^2*cos(4*f*x + 4*e) + 6*c^2*cos(2*f*x + 2*e) + 5*c^2 + 8*
(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ (5*c^2*cos(4*f*x + 4*e) + 6*c^2*cos(2*f*x + 2*e) + 5*c^2 + 8*(c^2*sin(4*f
*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqr
t(c)/((a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*cos(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x
 + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*sin(3/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2
*f*x + 2*e) + a^2)*cos(4*f*x + 4*e) + 4*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2
*f*x + 2*e) + 2*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a^2*cos(4*f*
x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e) + 2*a
^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*
f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2\sec\left(fx+e\right)^3-2\,c^2\sec\left(fx+e\right)^2+c^2\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*sqr
t(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e)^2 + 2*a^2
*sec(f*x + e) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.75211, size = 188, normalized size = 1.32

$$-\frac{2\left(2\,c^3\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)+\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^2-\frac{2\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^3+c^4}{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}\right)c\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\mathrm{t}\right.}{\sqrt{-ac}af|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] -2*(2*c^3*log(c*tan(1/2*f*x + 1/2*e)^2 - c) + (c*tan(1/2*f*x + 1/2*e)^2 - c
)*c^2 - (2*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^3 + c^4)/(c*tan(1/2*f*x + 1/2*e
)^2 - c))*c*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/(sqrt(-a*c)*
a*f*abs(c))
```

3.140 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{c^2\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)^{3/2}}$$

```
[Out] (c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]) + (c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*(a + a
*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.278487, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3954, 3952}

$$\frac{c^2\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]) + (c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*(a + a
*Sec[e + f*x])^(3/2))
```

Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx &= \frac{c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}-\frac{c\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a}\\ &= \frac{c^2\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [C] time = 1.05326, size = 132, normalized size = 1.39

$$-\frac{c\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}\left(2\log\left(1+e^{i(e+fx)}\right)-\log\left(1+e^{2i(e+fx)}\right)+\left(2\log\left(1+e^{i(e+fx)}\right)-\log\left(1+e^{2i(e+fx)}\right)\right)c}{af(\cos(e+fx)+1)\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(3
/2),x]
```

```
[Out] -((c*Cot[(e + f*x)/2]*(-2 + 2*Log[1 + E^(I*(e + f*x))] + Cos[e + f*x]*(2*Lo
g[1 + E^(I*(e + f*x))] - Log[1 + E^((2*I)*(e + f*x))]) - Log[1 + E^((2*I)*(
e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(a*f*(1 + Cos[e + f*x])*Sqrt[a*(1 + S
ec[e + f*x])]))
```

Maple [B] time = 0.253, size = 193, normalized size = 2.

$$-\frac{\left(\cos\left(fx+e\right)\right)^{2}}{fa^{2}\left(\sin\left(fx+e\right)\right)^{3}}\left(\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)-\sin\left(\right.}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f/a^2*(cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)*
ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+ln(-(-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))+ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-cos(f*x+e)+1)*(c*(-1+
cos(f*x+e))/cos(f*x+e))^(3/2)*cos(f*x+e)^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^
(1/2)/sin(f*x+e)^3
```

Maxima [A] time = 1.53183, size = 134, normalized size = 1.41

$$\frac{\frac{c^{\frac{3}{2}}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{\sqrt{-a}a}+\frac{c^{\frac{3}{2}}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{\sqrt{-a}a}+\frac{c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\sqrt{-a}a\left(\cos\left(fx+e\right)+1\right)^{2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] (c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(sqrt(-a)*a) + c^(3/2)*lo
g(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(sqrt(-a)*a) + c^(3/2)*sin(f*x + e)^
2/(sqrt(-a)*a*(cos(f*x + e) + 1)^2))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c\sec\left(fx+e\right)^{2}-c\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(c*sec(f*x + e)^2 - c*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqrt
(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 3.56969, size = 123, normalized size = 1.29

$$-\frac{\left(c^4\log\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)+\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^3\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\sqrt{-ac}acf|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] -(c^4*log(c*tan(1/2*f*x + 1/2*e)^2 - c) + (c*tan(1/2*f*x + 1/2*e)^2 - c)*c^
3)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/(sqrt(-a*c)*a*c*f*abs
(c))
```

3.141 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{2f(a\sec(e+fx)+a)^{3/2}}$$

```
[Out] (Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.134305, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3950}

$$\frac{\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{2f(a\sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 0.183704, size = 42, normalized size = 1.

$$\frac{\csc(e+fx)\sqrt{c-c\sec(e+fx)}}{af\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(3/2
),x]
```

```
[Out] (Csc[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] time = 0.284, size = 73, normalized size = 1.7

$$\frac{\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^{2}}{2\,fa^{2}\left(\sin\left(fx+e\right)\right)^{3}}\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f/a^2*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e
)))^(1/2)*cos(f*x+e)*(-1+cos(f*x+e))^2/sin(f*x+e)^3
```

Maxima [A] time = 1.52174, size = 73, normalized size = 1.74

$$\frac{\sqrt{c}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{2\sqrt{-a}af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*sqrt(c)*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*(sin(f*x + e)/(cos(f*x +
e) + 1) - 1)/(sqrt(-a)*a*f)
```

Fricas [B] time = 0.466875, size = 186, normalized size = 4.43

$$\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{\left(a^{2}f\cos\left(fx+e\right)+a^{2}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))*cos(f*x + e)/((a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)/(a*(sec(e + f*x) + 1))**(
3/2), x)
```

Giac [B] time = 2.67175, size = 116, normalized size = 2.76

$$-\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)c\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}{2\sqrt{-ac}af|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] -1/2*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*
f*x + 1/2*e))*sgn(cos(f*x + e))/(sqrt(-a*c)*a*f*abs(c)*sgn(tan(1/2*f*x + 1/
2*e)^2 - 1))
```

3.142 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=95

$$\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{2af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] Tan[e + f*x]/(2*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]) - (A
rcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rubi [A] time = 0.278977, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3960, 3959, 3770}

$$\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{2af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] Tan[e + f*x]/(2*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]) - (A
rcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx}}}{2a}$$

$$= \frac{\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)\int \csc(e}{2a\sqrt{a+a\sec(e+fx)}\sqrt{c-}}$$

$$= \frac{\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} - \frac{\tanh^{-1}(\cos(e+fx))\,\mathrm{t}}{2af\sqrt{a+a\sec(e+fx)}\sqrt{c}}$$

Mathematica [C] time = 1.40295, size = 157, normalized size = 1.65

$$-\frac{\sin\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(1+2(\cos(e+fx)+1)\tanh^{-1}\left(e^{i(e+fx)}\right)\right)}{\sqrt{2}af\left(1+e^{i(e+fx)}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]
),x]
```

```
[Out] -(((1 + 2*ArcTanh[E^(I*(e + f*x))]*(1 + Cos[e + f*x]))*Sec[e + f*x]^(3/2)*(
Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2])*Sin[(e + f*x)/2])/(Sqrt[2]*a*(1 + E^
(I*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*f*Sqrt[a*(1
+ Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [A] time = 0.3, size = 123, normalized size = 1.3

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}}{4\,fa^{2}\left(\sin\left(fx+e\right)\right)^{3}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(2\,\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+2\,\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/4/f/a^2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-1+cos(f*x+e))^2*(2*cos(f*
x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))+2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+cos(
f*x+e)-1)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^3
```

Maxima [B] time = 1.86656, size = 536, normalized size = 5.64

$$-\frac{\left(\left(2\left(2\,\cos\left(fx+e\right)+1\right)\cos\left(2\,fx+2\,e\right)+\cos\left(2\,fx+2\,e\right)^{2}+4\,\cos\left(fx+e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+4\,\sin\left(2\,fx+2\,e\right)\mathrm{si}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*((2*(2*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*cos
(f*x + e)^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*sin(
f*x + e)^2 + 4*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) -
(2*(2*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*cos(f*x +
 e)^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*sin(f*x +
e)^2 + 4*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) - 2*cos(
f*x + e)*sin(2*f*x + 2*e) + 2*cos(2*f*x + 2*e)*sin(f*x + e) + 2*sin(f*x + e
))*sqrt(a)*sqrt(c)/((a^2*c*cos(2*f*x + 2*e)^2 + 4*a^2*c*cos(f*x + e)^2 + a^
2*c*sin(2*f*x + 2*e)^2 + 4*a^2*c*sin(2*f*x + 2*e)*sin(f*x + e) + 4*a^2*c*si
n(f*x + e)^2 + 4*a^2*c*cos(f*x + e) + a^2*c + 2*(2*a^2*c*cos(f*x + e) + a^2
*c)*cos(2*f*x + 2*e))*f)
```

Fricas [B] time = 0.665104, size = 952, normalized size = 10.02

$$\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)+1\right)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}+\left(ac\cos\left(fx+e\right)^{2}+ac\right)\sin\left(fx+e\right)\right)}{\left(\cos\left(fx+e\right)^{2}-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-2\,\sqrt{}}{4\left(a^{2}cf\cos\left(fx+e\right)+a^{2}cf\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*c)*(cos(f*x + e) + 1)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)
^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x
 + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*co
s(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a^2*c*f*cos(f*x + e) + a^2*c*
f)*sin(f*x + e)), 1/2*(sqrt(a*c)*(cos(f*x + e) + 1)*arctan(sqrt(a*c)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(
a*c*sin(f*x + e)))*sin(f*x + e) + sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a^2*c*f*cos(f*x + e)
 + a^2*c*f)*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x)
- 1))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.143 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=104

$$\frac{\csc(e+fx)}{2acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{2acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] Csc[e + f*x]/(2*a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) -
(ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.181877, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3959, 2611, 3770}

$$\frac{\csc(e+fx)}{2acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{2acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] Csc[e + f*x]/(2*a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) -
(ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]])
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\tan(e+fx)\int \cot^2(e+fx)\csc(e+fx)\,dx}{ac\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{\csc(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)\int \csc}{2ac\sqrt{a+a\sec(e+fx)}\sqrt{}}$$
$$= \frac{\csc(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tanh^{-1}(\cos(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] time = 0.772458, size = 69, normalized size = 0.66

$$\frac{\csc(e+fx)-2\tan(e+fx)\tanh^{-1}\left(e^{i(e+fx)}\right)}{2acf\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/
2)),x]
```

```
[Out] (Csc[e + f*x] - 2*ArcTanh[E^(I*(e + f*x))]*Tan[e + f*x])/(2*a*c*f*Sqrt[a*(1
 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.261, size = 133, normalized size = 1.3

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^2}{2\,fa^2\left(\sin\left(fx+e\right)\right)^3\cos\left(fx+e\right)}\left(\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos\left(fx+e\right)\right)^2-\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-\cos\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f/a^2*(-1+cos(f*x+e))^2*(ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-l
n(-(-1+cos(f*x+e))/sin(f*x+e))-cos(f*x+e))*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^
(1/2)/sin(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/cos(f*x+e)
```

Maxima [B] time = 1.97262, size = 765, normalized size = 7.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*((2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - 4*
cos(2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*
e) - 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arctan2(sin(f*x + e), c
os(f*x + e) + 1) - (2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x
 + 4*e)^2 - 4*cos(2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) - 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arctan2(s
```

```
in(f*x + e), cos(f*x + e) - 1) - 2*(sin(3*f*x + 3*e) + sin(f*x + e))*cos(4*
f*x + 4*e) + 2*(cos(3*f*x + 3*e) + cos(f*x + e))*sin(4*f*x + 4*e) + 2*(2*co
s(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e) - 4*cos(3*f*x + 3*e)*sin(2*f*x + 2*e)
- 4*cos(f*x + e)*sin(2*f*x + 2*e) + 4*cos(2*f*x + 2*e)*sin(f*x + e) - 2*sin
(f*x + e))*sqrt(a)*sqrt(c)/((a^2*c^2*cos(4*f*x + 4*e)^2 + 4*a^2*c^2*cos(2*f
*x + 2*e)^2 + a^2*c^2*sin(4*f*x + 4*e)^2 - 4*a^2*c^2*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + 4*a^2*c^2*sin(2*f*x + 2*e)^2 - 4*a^2*c^2*cos(2*f*x + 2*e) + a
^2*c^2 - 2*(2*a^2*c^2*cos(2*f*x + 2*e) - a^2*c^2)*cos(4*f*x + 4*e))*f)
```

Fricas [A] time = 0.695179, size = 979, normalized size = 9.41

$$\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)^2-1\right)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^2+\left(ac\cos\left(fx+e\right)^2+ac\right)\sin\left(fx+e\right)\right)}{\left(\cos\left(fx+e\right)^2-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-2}{4\left(a^2c^2f\cos\left(fx+e\right)^2-a^2c^2f\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*c)*(cos(f*x + e)^2 - 1)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x +
e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f
*x + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2)/((a^2*c^2*f*cos(f*x + e)^2
- a^2*c^2*f)*sin(f*x + e)), 1/2*(sqrt(a*c)*(cos(f*x + e)^2 - 1)*arctan(sqrt
(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2)/((a^2*c^
2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.144 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=146

$$\frac{3\csc(e+fx)}{8ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{8ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+}{4f(a\sec(e+fx)+a)^{3/2}(}$$

```
[Out] (3*Csc[e + f*x])/(8*a*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x
]]) - Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/
2)) - (3*ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(8*a*c^2*f*Sqrt[a + a*Sec[e +
f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.340645, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3960, 3959, 2611, 3770}

$$\frac{3\csc(e+fx)}{8ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{8ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+}{4f(a\sec(e+fx)+a)^{3/2}(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (3*Csc[e + f*x])/(8*a*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x
]]) - Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/
2)) - (3*ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(8*a*c^2*f*Sqrt[a + a*Sec[e +
f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}} + \frac{3\int \frac{\sec(e+f}{(a+a\sec(e+fx))^{3/2}(c-}}{4c}$$

$$= -\frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}} - \frac{(3\tan(e+fx))\int \mathrm{co}}{4ac^2\sqrt{a+a\sec(e+}}$$

$$= \frac{3\csc(e+fx)}{8ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e-}{4f(a+a\sec(e+fx))^{3/}}$$

$$= \frac{3\csc(e+fx)}{8ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e-}{4f(a+a\sec(e+fx))^{3/}}$$

Mathematica [C] time = 1.50492, size = 122, normalized size = 0.84

$$-\frac{\tan(e+fx)\left(-2\cos(e+fx)+5\cos(2(e+fx))+24\sin^2\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\tanh^{-1}\left(e^{i(e+fx)}\right)+1\right)}{16ac^2f(\cos(e+fx)-1)^2(\cos(e+fx)+1)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/
2)),x]
```

```
[Out] -((1 - 2*Cos[e + f*x] + 5*Cos[2*(e + f*x)] + 24*ArcTanh[E^(I*(e + f*x))]*Si
n[(e + f*x)/2]^2*Sin[e + f*x]^2)*Tan[e + f*x])/(16*a*c^2*f*(-1 + Cos[e + f*
x])^2*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]
])
```

Maple [A] time = 0.269, size = 211, normalized size = 1.5

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^2}{32\,fa^2\left(\sin\left(fx+e\right)\right)^3\left(\cos\left(fx+e\right)\right)^2}\left(12\left(\cos\left(fx+e\right)\right)^3\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-5\left(\cos\left(fx+e\right)\right)^3-12\ln\left(-\frac{-1+\mathrm{c}}{\mathrm{sin}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/32/f/a^2*(-1+cos(f*x+e))^2*(12*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e
))-5*cos(f*x+e)^3-12*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-15*cos(f*
x+e)^2-12*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))+9*cos(f*x+e)+12*ln(-(-
1+cos(f*x+e))/sin(f*x+e))+3)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin(f*x+
e)^3/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.744877, size = 1315, normalized size = 9.01

$$\left[-\frac{3\left(\cos\left(fx+e\right)^{3}-\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)+1\right)\sqrt{-ac}\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)^{2}+\left(ac\cos(fx+\cdots\right.\right.}{\left(\cos(fx+e)^{2}-1\right)\sin(fx+e)}\cdots\right.}{16\left(a^{2}c^{3}f\cos\left(fx+e\right)^{3}-a^{2}c^{3}f\cos\left(fx+e\right)^{2}\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(3*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(-a*c)*l
og(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
 + e) - c)/cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*
x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x +
e)^3 - cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 -
a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e)
), 1/8*(3*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(a*c)*ar
ctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (5*cos(f*x + e)^3 -
cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3
*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.145 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{c^3\tan(e+fx)\log(\sec(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{af(a\sec(e+fx)+a)^{3/2}}+\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f(a\sec(e+fx)+a)^{5/2}}$$

```
[Out] -((c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c - c*Sec[e + f*x]])) - (c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(a
*f*(a + a*Sec[e + f*x])^(3/2)) + (c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x]
)/(2*f*(a + a*Sec[e + f*x])^(5/2))
```

Rubi [A] time = 0.435758, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3954, 3952}

$$-\frac{c^3\tan(e+fx)\log(\sec(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{af(a\sec(e+fx)+a)^{3/2}}+\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f(a\sec(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(5/2),x]
```


```
[Out] -((c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c - c*Sec[e + f*x]])) - (c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(a
*f*(a + a*Sec[e + f*x])^(3/2)) + (c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x]
)/(2*f*(a + a*Sec[e + f*x])^(5/2))
```


Rule 3954

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(2*a*c*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(b*f*(2*m + 1)),
 x] - Dist[(d*(2*n - 1))/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 3952

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(a*c*Log[1 + (b
*Csc[e + f*x])/a]*Cot[e + f*x])/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Cs
c[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}} - \frac{c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}dx}{a}$$

$$= -\frac{c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}} + \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}} + \frac{c^2\int$$

$$= -\frac{c^3\log(1+\sec(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}}$$

Mathematica [C] time = 1.42275, size = 178, normalized size = 1.23

$$\frac{c^2\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}\left(6\log\left(1+e^{i(e+fx)}\right)-3\log\left(1+e^{2i(e+fx)}\right)+\left(8\log\left(1+e^{i(e+fx)}\right)-4\log\left(1+e^{2i(e+fx)}\right)\right.\right.}{2a^2f(\cos(e+fx)+1)^2\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(5
/2),x]
```

```
[Out] (c^2*Cot[(e + f*x)/2]*(-4 + 6*Log[1 + E^(I*(e + f*x))] + Cos[e + f*x]*(8*Lo
g[1 + E^(I*(e + f*x))] - 4*Log[1 + E^((2*I)*(e + f*x))]) + Cos[2*(e + f*x)]
*(2*Log[1 + E^(I*(e + f*x))] - Log[1 + E^((2*I)*(e + f*x))]) - 3*Log[1 + E^
((2*I)*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(2*a^2*f*(1 + Cos[e + f*x])^2
*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.261, size = 281, normalized size = 1.9

$$-\frac{(\cos(fx+e))^3}{2fa^3(\sin(fx+e))^5}\left(2\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)(\cos(fx+e))^2+2(\cos(fx+e))^2\ln\left(-\frac{-1+\cos(fx}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f/a^3*(2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*cos
(f*x+e)^2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+4*cos(f*x+e)*ln(-(-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))-cos(f*x+e)^2+2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*
ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*cos(f*x+e)+3)*(c*(-1+cos(f*x+e
))/cos(f*x+e))^(5/2)*cos(f*x+e)^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/sin
(f*x+e)^5
```

Maxima [A] time = 1.50696, size = 180, normalized size = 1.24

$$-\frac{\frac{2c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{-a}a^2}+\frac{2c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{-a}a^2}-\frac{\frac{2\sqrt{-a}c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{\sqrt{-a}c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}{a^3}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*(2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(sqrt(-a)*a^2) + 2
*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(sqrt(-a)*a^2) - (2*sqrt(
-a)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(-a)*c^(5/2)*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4)/a^3)/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c^2\sec\left(fx+e\right)^3-2\,c^2\sec\left(fx+e\right)^2+c^2\sec\left(fx+e\right)\right)\sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}}{a^3\sec\left(fx+e\right)^3+3\,a^3\sec\left(fx+e\right)^2+3\,a^3\sec\left(fx+e\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*sqr
t(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^3 + 3*a^3
*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 7.3788, size = 153, normalized size = 1.06

$$\frac{\left(2\,c^3\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)+\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c+4\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^2\right)c\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.\right.}{2\,\sqrt{-ac}a^2f|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] 1/2*(2*c^3*log(c*tan(1/2*f*x + 1/2*e)^2 - c) + (c*tan(1/2*f*x + 1/2*e)^2 -
c)^2*c + 4*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2)*c*sgn(tan(1/2*f*x + 1/2*e)^3
 + tan(1/2*f*x + 1/2*e))/(sqrt(-a*c)*a^2*f*abs(c))
```

3.146 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}} dx$

Optimal. Leaf size=42

$$\frac{\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{4f(a\sec(e+fx)+a)^{5/2}}$$

```
[Out] ((c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(4*f*(a + a*Sec[e + f*x])^(5/2))
```

Rubi [A] time = 0.146948, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3950}

$$\frac{\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{4f(a\sec(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(4*f*(a + a*Sec[e + f*x])^(5/2))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}} dx = \frac{(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}$$

Mathematica [A] time = 0.284028, size = 68, normalized size = 1.62

$$\frac{c\cos(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{4a^2f\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(5
/2),x]
```

```
[Out] (c*Cos[e + f*x]*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^3*Sqrt[c - c*Sec[e + f*x]
])/(4*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.251, size = 75, normalized size = 1.8

$$-\frac{\left(\cos\left(fx+e\right)\right)^{2}\left(-1+\cos\left(fx+e\right)\right)^{3}}{4fa^{3}\left(\sin\left(fx+e\right)\right)^{5}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/4/f/a^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^2*(-1+cos(f*x+e
))^3*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)^5
```

Maxima [B] time = 1.54388, size = 132, normalized size = 3.14

$$-\frac{\sqrt{-a}c^{\frac{3}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)\sin\left(fx+e\right)^{4}}{4\left(a^{3}-\frac{a^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)f\left(\cos\left(fx+e\right)+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*sqrt(-a)*c^(3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1)*sin(f*x + e)^4/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2)*f*(cos(f*x + e) + 1)^4)
```

Fricas [B] time = 0.475209, size = 225, normalized size = 5.36

$$\frac{c\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}}{\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2/((a^3*f*cos(f*x + e)^2 + 2*a^3*f*cos(f*x + e) + a^3*f
)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 4.04385, size = 115, normalized size = 2.74

$$\frac{\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2+2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c\right)c\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{4\sqrt{-ac}a^2f|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] 1/4*((c*tan(1/2*f*x + 1/2*e)^2 - c)^2 + 2*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c)
*c*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/(sqrt(-a*c)*a^2*f*abs
(c))
```

3.147 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (c*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.134325, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3953}

$$\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (c*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] time = 0.246441, size = 71, normalized size = 1.65

$$\frac{(2\cos(e+fx)+1)\csc\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{8a^2f\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(5/2
),x]
```

```
[Out] ((1 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^3*Sqrt[c - c*Sec[e
+ f*x]])/(8*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.29, size = 83, normalized size = 1.9

$$-\frac{\cos\left(fx+e\right)\left(3\cos\left(fx+e\right)+1\right)\left(-1+\cos\left(fx+e\right)\right)^{3}}{8fa^{3}\left(\sin\left(fx+e\right)\right)^{5}}\sqrt{\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/8/f/a^3*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(1/cos(f*x+e)*a*(1+cos(f*x+
e)))^(1/2)*cos(f*x+e)*(3*cos(f*x+e)+1)*(-1+cos(f*x+e))^3/sin(f*x+e)^5
```

Maxima [A] time = 1.53636, size = 78, normalized size = 1.81

$$-\frac{\sqrt{c}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{2}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{2}}{8\sqrt{-a}a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/8*sqrt(c)*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^2*(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)^2/(sqrt(-a)*a^2*f)
```

Fricas [B] time = 0.472106, size = 254, normalized size = 5.91

$$\frac{\left(2\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{2\left(a^{3}f\cos\left(fx+e\right)^{2}+2a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(2*cos(f*x + e)^2 + cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*a^3*f
*cos(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.91911, size = 117, normalized size = 2.72

$$\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{2}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}{8\sqrt{-ac}a^{2}f|c|\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] 1/8*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f
*x + 1/2*e))*sgn(cos(f*x + e))/(sqrt(-a*c)*a^2*f*abs(c)*sgn(tan(1/2*f*x + 1
/2*e)^2 - 1))
```

3.148 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=140

$$-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{4a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{4af(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}\sqrt{c}}$$

```
[Out] Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]) + Ta
n[e + f*x]/(4*a*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]) - (A
rcTanh[Cos[e + f*x]]*Tan[e + f*x])/(4*a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]])
```

Rubi [A] time = 0.424006, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3960, 3959, 3770}

$$-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{4a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{4af(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]) + Ta
n[e + f*x]/(4*a*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]) - (A
rcTanh[Cos[e + f*x]]*Tan[e + f*x])/(4*a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]])
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}dx = \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec}}}{2a}$$

$$= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+}{4af(a+a\sec(e+fx))^{3}}$$

$$= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+}{4af(a+a\sec(e+fx))^{3}}$$

$$= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+}{4af(a+a\sec(e+fx))^{3}}$$

Mathematica [C] time = 1.05336, size = 91, normalized size = 0.65

$$-\frac{\tan(e+fx)\left(3\cos(e+fx)+8\cos^4\left(\frac{1}{2}(e+fx)\right)\tanh^{-1}\left(e^{i(e+fx)}\right)+2\right)}{4a^2f(\cos(e+fx)+1)^2\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]
),x]
```

```
[Out] -((2 + 8*ArcTanh[E^(I*(e + f*x))]*Cos[(e + f*x)/2]^4 + 3*Cos[e + f*x])*Tan[
e + f*x])/(4*a^2*f*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c -
 c*Sec[e + f*x]])
```

Maple [A] time = 0.3, size = 164, normalized size = 1.2

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^3}{16\,fa^3\left(\sin\left(fx+e\right)\right)^5}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(4\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos\left(fx+e\right)\right)^2+5\left(\cos\left(fx+e\right)\right)^2+8\,\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/16/f/a^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-1+cos(f*x+e))^3*(4*ln(-
(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+5*cos(f*x+e)^2+8*cos(f*x+e)*ln(-(-
1+cos(f*x+e))/sin(f*x+e))-2*cos(f*x+e)+4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-3)
/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^5
```

Maxima [B] time = 2.14867, size = 1608, normalized size = 11.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*((2*(4*cos(3*f*x + 3*e) + 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos
(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) + 4*cos(f*x + e)
 + 1)*cos(3*f*x + 3*e) + 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) + 1)*co
s(2*f*x + 2*e) + 36*cos(2*f*x + 2*e)^2 + 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x
 + 3*e) + 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) + sin(4*f*x
 + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(3*f*x + 3*e) + 16*
sin(3*f*x + 3*e)^2 + 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x +
e) + 16*sin(f*x + e)^2 + 8*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x
+ e) + 1) - (2*(4*cos(3*f*x + 3*e) + 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) +
1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) + 4*cos(f*
x + e) + 1)*cos(3*f*x + 3*e) + 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) +
 1)*cos(2*f*x + 2*e) + 36*cos(2*f*x + 2*e)^2 + 16*cos(f*x + e)^2 + 4*(2*sin
(3*f*x + 3*e) + 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) + sin
(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(3*f*x + 3*e)
 + 16*sin(3*f*x + 3*e)^2 + 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(
f*x + e) + 16*sin(f*x + e)^2 + 8*cos(f*x + e) + 1)*arctan2(sin(f*x + e), co
s(f*x + e) - 1) + 2*(3*sin(3*f*x + 3*e) + 4*sin(2*f*x + 2*e) + 3*sin(f*x +
e))*cos(4*f*x + 4*e) - 2*(3*cos(3*f*x + 3*e) + 4*cos(2*f*x + 2*e) + 3*cos(f
*x + e))*sin(4*f*x + 4*e) + 2*(2*cos(2*f*x + 2*e) + 3)*sin(3*f*x + 3*e) - 4
*(cos(f*x + e) - 2)*sin(2*f*x + 2*e) - 4*cos(3*f*x + 3*e)*sin(2*f*x + 2*e)
+ 4*cos(2*f*x + 2*e)*sin(f*x + e) + 6*sin(f*x + e))*sqrt(a)*sqrt(c)/((a^3*c
*cos(4*f*x + 4*e)^2 + 16*a^3*c*cos(3*f*x + 3*e)^2 + 36*a^3*c*cos(2*f*x + 2*
e)^2 + 16*a^3*c*cos(f*x + e)^2 + a^3*c*sin(4*f*x + 4*e)^2 + 16*a^3*c*sin(3*
f*x + 3*e)^2 + 36*a^3*c*sin(2*f*x + 2*e)^2 + 48*a^3*c*sin(2*f*x + 2*e)*sin(
f*x + e) + 16*a^3*c*sin(f*x + e)^2 + 8*a^3*c*cos(f*x + e) + a^3*c + 2*(4*a^
3*c*cos(3*f*x + 3*e) + 6*a^3*c*cos(2*f*x + 2*e) + 4*a^3*c*cos(f*x + e) + a^
3*c)*cos(4*f*x + 4*e) + 8*(6*a^3*c*cos(2*f*x + 2*e) + 4*a^3*c*cos(f*x + e)
+ a^3*c)*cos(3*f*x + 3*e) + 12*(4*a^3*c*cos(f*x + e) + a^3*c)*cos(2*f*x + 2
*e) + 4*(2*a^3*c*sin(3*f*x + 3*e) + 3*a^3*c*sin(2*f*x + 2*e) + 2*a^3*c*sin(
f*x + e))*sin(4*f*x + 4*e) + 16*(3*a^3*c*sin(2*f*x + 2*e) + 2*a^3*c*sin(f*x
 + e))*sin(3*f*x + 3*e))*f)
```

Fricas [A] time = 0.693415, size = 1138, normalized size = 8.13

$$\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)^2+2\cos\left(fx+e\right)+1\right)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)^2+\left(ac\cos(fx+e)^2+ac\right)\sin(fx+e)\right)}{\left(\cos(fx+e)^2-1\right)\sin(fx+e)}\right)\sin}{8\left(a^3cf\cos\left(fx+e\right)^2+2a^3cf\cos\left(fx+e\right)+a^3cf\right)s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a*c)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(-4*(2*sqrt(-a*c
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x
+ e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(3*cos(f*x + e)^2 + 2*cos(f*x +
 e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(
f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin
(f*x + e)), 1/4*(sqrt(a*c)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*arctan(sqr
t(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (3*cos(f*x + e)^2 + 2*cos(f*
```

```
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*
sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.149 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=146

$$\frac{3\csc(e+fx)}{8a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{8a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}(c-$$

```
[Out] (3*Csc[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x
]]) + Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/
2)) - (3*ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sec[e +
f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.344092, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3960, 3959, 2611, 3770}

$$\frac{3\csc(e+fx)}{8a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{8a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}(c-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (3*Csc[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x
]]) + Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/
2)) - (3*ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sec[e +
f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3960

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}} + \frac{3\int \frac{\sec(e\text{-}}{(a+a\sec(e+fx))^{3/2}(}}{4a}$$
$$= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}} - \frac{(3\tan(e+fx))\int c}{4a^2c\sqrt{a+a\sec(e}}$$
$$= \frac{3\csc(e+fx)}{8a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\tan}{4f(a+a\sec(e+fx}$$
$$= \frac{3\csc(e+fx)}{8a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\tan}{4f(a+a\sec(e+fx}$$

Mathematica [C] time = 1.39315, size = 130, normalized size = 0.89

$$-\frac{\tan(e+fx)\left(2\cos(e+fx)+5\cos(2(e+fx))-3(\cos(e+fx)-2\cos(2(e+fx))-\cos(3(e+fx))+2)\tanh^{-1}\left(e^{i(e+}\right.\right.}{16a^2cf(\cos(e+fx)-1)(\cos(e+fx)+1)^2\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/
2)),x]
```

```
[Out] -((1 + 2*Cos[e + f*x] + 5*Cos[2*(e + f*x)] - 3*ArcTanh[E^(I*(e + f*x))]*(2
+ Cos[e + f*x] - 2*Cos[2*(e + f*x)] - Cos[3*(e + f*x)]))*Tan[e + f*x])/(16*
a^2*c*f*(-1 + Cos[e + f*x])*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])]
*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] time = 0.265, size = 204, normalized size = 1.4

$$-\frac{\left(\cos\left(fx+e\right)\right)^2}{32\,fa^3c^3\left(\sin\left(fx+e\right)\right)^5}\left(12\left(\cos\left(fx+e\right)\right)^3\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+5\left(\cos\left(fx+e\right)\right)^3+12\ln\left(-\frac{-1+\cos\left(fx+\right.}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/32/f/a^3*(12*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e))+5*cos(f*x+e)^3
+12*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-15*cos(f*x+e)^2-12*cos(f*x
+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-9*cos(f*x+e)-12*ln(-(-1+cos(f*x+e))/sin
(f*x+e))+3)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^2*(c*(-1+cos(f
*x+e))/cos(f*x+e))^(3/2)/c^3/sin(f*x+e)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.739696, size = 1315, normalized size = 9.01

$$\left[-\frac{3\left(\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)-1\right)\sqrt{-ac}\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)^{2}+\left(ac\cos(fx+e)^{2}+\cdots\right.\right.}{\left(\cos(fx+e)^{2}-1\right)\sin(fx+e)}\cdots\right.}{16\left(a^{3}c^{2}f\cos\left(fx+e\right)^{3}+a^{3}c^{2}f\cos\left(fx+e\right)^{2}-a\cdots\right.}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(3*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(-a*c)*l
og(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
 + e) - c)/cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*
x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x +
e)^3 + cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 +
a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e)
), 1/8*(3*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(a*c)*ar
ctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (5*cos(f*x + e)^3 +
cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2
*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.150 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=160

$$\frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\cot^2(e+fx)\csc(e+fx)}{4a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+fx)\tanh^{-1}}{8a^2c^2f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (3*Csc[e + f*x])/(8*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f
*x]]) - (Cot[e + f*x]^2*Csc[e + f*x])/(4*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]]) - (3*ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(8*a^2*
c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.201946, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3959, 2611, 3770}

$$\frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\cot^2(e+fx)\csc(e+fx)}{4a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+fx)\tanh^{-1}}{8a^2c^2f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```



```
[Out] (3*Csc[e + f*x])/(8*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f
*x]]) - (Cot[e + f*x]^2*Csc[e + f*x])/(4*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]]) - (3*ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(8*a^2*
c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```


Rule 3959

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-(a*c))^(m +
1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)\int \cot^4(e+fx)\csc(e+fx)\,dx}{a^2c^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{\cot^2(e+fx)\csc(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{(3\tan(e+fx))\int}{4a^2c^2\sqrt{a+a\sec}}$$
$$= \frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\cot^2(e+}{4a^2c^2f\sqrt{a+a\sec(}}$$
$$= \frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\cot^2(e+}{4a^2c^2f\sqrt{a+a\sec(}}$$

Mathematica [C] time = 1.39122, size = 84, normalized size = 0.52

$$\frac{(1-5\cos(2(e+fx)))\csc^3(e+fx)-12\tan(e+fx)\tanh^{-1}\left(e^{i(e+fx)}\right)}{16a^2c^2f\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/
2)),x]
```

```
[Out] ((1 - 5*Cos[2*(e + f*x)])*Csc[e + f*x]^3 - 12*ArcTanh[E^(I*(e + f*x))]*Tan[
e + f*x])/(16*a^2*c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]
)
```

Maple [A] time = 0.277, size = 173, normalized size = 1.1

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^3}{8fa^3\left(\sin\left(fx+e\right)\right)^5\left(\cos\left(fx+e\right)\right)^2}\left(3\left(\cos\left(fx+e\right)\right)^4\ln\left(-\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-5\left(\cos\left(fx+e\right)\right)^3-6\ln\left(-\frac{-1+}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/8/f/a^3*(-1+cos(f*x+e))^3*(3*cos(f*x+e)^4*ln(-(-1+cos(f*x+e))/sin(f*x+e)
)-5*cos(f*x+e)^3-6*ln(-(-1+cos(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+3*cos(f*x+e
)+3*ln(-(-1+cos(f*x+e))/sin(f*x+e)))*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)/
sin(f*x+e)^5/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)
```

Maxima [B] time = 2.93168, size = 2240, normalized size = 14.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/8*(3*(2*(4*cos(6*f*x + 6*e) - 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1
)*cos(8*f*x + 8*e) - cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2*f
```

```
*x + 2*e) + 1)*cos(6*f*x + 6*e) - 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x +
 2*e) - 1)*cos(4*f*x + 4*e) - 36*cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*e)^2
 + 4*(2*sin(6*f*x + 6*e) - 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f
*x + 8*e) - sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e
))*sin(6*f*x + 6*e) - 16*sin(6*f*x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2 + 48*si
n(4*f*x + 4*e)*sin(2*f*x + 2*e) - 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e
) - 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) - 3*(2*(4*cos(6*f*x + 6*e) -
 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1)*cos(8*f*x + 8*e) - cos(8*f*x
+ 8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)
 - 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - 3
6*cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) - 3*si
n(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - sin(8*f*x + 8*e)^2
+ 16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 16*sin(6*
f*x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 - 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) - 1)*arctan2(sin(f*x + e), co
s(f*x + e) - 1) - 2*(5*sin(7*f*x + 7*e) + 3*sin(5*f*x + 5*e) + 3*sin(3*f*x
+ 3*e) + 5*sin(f*x + e))*cos(8*f*x + 8*e) - 20*(2*sin(6*f*x + 6*e) - 3*sin(
4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) + 8*(3*sin(5*f*x + 5*e)
 + 3*sin(3*f*x + 3*e) + 5*sin(f*x + e))*cos(6*f*x + 6*e) + 12*(3*sin(4*f*x
+ 4*e) - 2*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 12*(3*sin(3*f*x + 3*e) + 5*
sin(f*x + e))*cos(4*f*x + 4*e) + 2*(5*cos(7*f*x + 7*e) + 3*cos(5*f*x + 5*e)
 + 3*cos(3*f*x + 3*e) + 5*cos(f*x + e))*sin(8*f*x + 8*e) + 10*(4*cos(6*f*x
+ 6*e) - 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1)*sin(7*f*x + 7*e) - 8*
(3*cos(5*f*x + 5*e) + 3*cos(3*f*x + 3*e) + 5*cos(f*x + e))*sin(6*f*x + 6*e)
 - 6*(6*cos(4*f*x + 4*e) - 4*cos(2*f*x + 2*e) + 1)*sin(5*f*x + 5*e) + 12*(3
*cos(3*f*x + 3*e) + 5*cos(f*x + e))*sin(4*f*x + 4*e) + 6*(4*cos(2*f*x + 2*e
) - 1)*sin(3*f*x + 3*e) - 24*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 40*cos(f*x
 + e)*sin(2*f*x + 2*e) + 40*cos(2*f*x + 2*e)*sin(f*x + e) - 10*sin(f*x + e)
)*sqrt(a)*sqrt(c)/((a^3*c^3*cos(8*f*x + 8*e)^2 + 16*a^3*c^3*cos(6*f*x + 6*e
)^2 + 36*a^3*c^3*cos(4*f*x + 4*e)^2 + 16*a^3*c^3*cos(2*f*x + 2*e)^2 + a^3*c
^3*sin(8*f*x + 8*e)^2 + 16*a^3*c^3*sin(6*f*x + 6*e)^2 + 36*a^3*c^3*sin(4*f*
x + 4*e)^2 - 48*a^3*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*a^3*c^3*sin(
2*f*x + 2*e)^2 - 8*a^3*c^3*cos(2*f*x + 2*e) + a^3*c^3 - 2*(4*a^3*c^3*cos(6*
f*x + 6*e) - 6*a^3*c^3*cos(4*f*x + 4*e) + 4*a^3*c^3*cos(2*f*x + 2*e) - a^3*
c^3)*cos(8*f*x + 8*e) - 8*(6*a^3*c^3*cos(4*f*x + 4*e) - 4*a^3*c^3*cos(2*f*x
 + 2*e) + a^3*c^3)*cos(6*f*x + 6*e) - 12*(4*a^3*c^3*cos(2*f*x + 2*e) - a^3*
c^3)*cos(4*f*x + 4*e) - 4*(2*a^3*c^3*sin(6*f*x + 6*e) - 3*a^3*c^3*sin(4*f*x
 + 4*e) + 2*a^3*c^3*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - 16*(3*a^3*c^3*sin(
4*f*x + 4*e) - 2*a^3*c^3*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*f)
```

Fricas [A] time = 0.771649, size = 1177, normalized size = 7.36

$$\left[-\frac{3\left(\cos\left(fx+e\right)^4 - 2\cos\left(fx+e\right)^2 + 1\right)\sqrt{-ac}\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)^2+\left(ac\cos(fx+e)^2+ac\right)\sin(fx+e)\right)}{\left(\cos(fx+e)^2-1\right)\sin(fx+e)}\right)}{16\left(a^3c^3f\cos\left(fx+e\right)^4 - 2a^3c^3f\cos\left(fx+e\right)^2 + a^3c^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-a*c)*log(-4*(2*sqrt
(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
```

```
s(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos
(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x + e)^4 - 3*cos(
f*x + e)^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) -
c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 +
 a^3*c^3*f)*sin(f*x + e)), 1/8*(3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*s
qrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*c
os(f*x + e) - c)/cos(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (5*cos(f*
x + e)^4 - 3*cos(f*x + e)^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f
*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")

[Out] Timed out
```

3.151 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$

Optimal. Leaf size=101

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{n-1}\text{Hypergeometric2F1}\left(m+\frac{1}{2},\frac{1}{2}-n,m\right.}{f(2m+1)}$$

```
[Out] -((2^(1/2 + n)*c*Hypergeometric2F1[1/2 + m, 1/2 - n, 3/2 + m, (1 + Sec[e +
f*x])/2]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^m*(c - c*Sec[e +
 f*x])^(-1 + n)*Tan[e + f*x])/(f*(1 + 2*m)))
```

Rubi [A] time = 0.120212, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3961, 70, 69}

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{n-1}\,{}_2F_1\left(m+\frac{1}{2},\frac{1}{2}-n;m+\frac{3}{2};\frac{1}{2}(\sec(e+fx\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*c*Hypergeometric2F1[1/2 + m, 1/2 - n, 3/2 + m, (1 + Sec[e +
f*x])/2]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^m*(c - c*Sec[e +
 f*x])^(-1 + n)*Tan[e + f*x])/(f*(1 + 2*m)))
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = -\frac{(ac\tan(e+fx))\operatorname{Subst}\left(\int(a+ax)^{-\frac{1}{2}+m}(c-cx)^{-\frac{1}{2}+n}\,dx, x\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e\right.}{f\sqrt{a+}}$$

$$= -\frac{2^{\frac{1}{2}+n}c\,{}_2F_1\left(\frac{1}{2}+m,\frac{1}{2}-n;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(1-}{}$$

Mathematica [F] time = 1.27041, size = 0, normalized size = 0.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n, x]
```

Maple [F] time = 0.523, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^m\left(c-c\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^m\left(-c\sec\left(fx+e\right)+c\right)^n\sec\left(fx+e\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n*sec(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^m\left(-c\sec\left(fx+e\right)+c\right)^n\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n*sec(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sec \left(fx+e\right)+a\right)^{m} \left(-c \sec \left(fx+e\right)+c\right)^{n} \sec \left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n*sec(f*x + e), x)
```

3.152 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=92

$$\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))^2(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-m,\frac{7}{2},\right.}{5f}$$

```
[Out] (2^(1/2 + m)*a*Hypergeometric2F1[5/2, 1/2 - m, 7/2, (1 - Sec[e + f*x])/2]*(
1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*(c - c*Sec[e + f*
x])^2*Tan[e + f*x])/(5*f)
```

Rubi [A] time = 0.108356, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3961, 70, 69}

$$\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))^2(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*a*Hypergeometric2F1[5/2, 1/2 - m, 7/2, (1 - Sec[e + f*x])/2]*(
1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*(c - c*Sec[e + f*
x])^2*Tan[e + f*x])/(5*f)
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int (a+ax)^{-\frac{1}{2}+m}(c-cx)^{3/2}\,dx, x, \sec(\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+m}ac(a+a\sec(e+fx))^{-1+m}\left(\frac{a+a\sec(e+fx)}{a}\right)^{\frac{1}{2}-m}\tan(e+\right.}{f\sqrt{c-c\sec}}$$

$$= \frac{2^{\frac{1}{2}+m}a\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))}{5f}$$

Mathematica [A] time = 0.24753, size = 89, normalized size = 0.97

$$\frac{c^2 2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)-1)^2(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a(\sec(e+fx)+1))^m\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-m,\frac{7}{2},\frac{1}{2}(1\right.}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*c^2*Hypergeometric2F1[5/2, 1/2 - m, 7/2, (1 - Sec[e + f*x])/2]
*(-1 + Sec[e + f*x])^2*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))
^m*Tan[e + f*x])/(5*f)
```

Maple [F] time = 0.433, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^m\left(c-c\sec\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c\sec\left(fx+e\right)-c\right)^2\left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] integrate((c*sec(f*x + e) - c)^2*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c^2\sec\left(fx+e\right)^3-2\,c^2\sec\left(fx+e\right)^2+c^2\sec\left(fx+e\right)\right)\left(a\sec\left(fx+e\right)+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*(a*
sec(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c \sec \left(fx+e\right)-c\right)^{2} \left(a \sec \left(fx+e\right)+a\right)^{m} \sec \left(fx+e\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((c*sec(f*x + e) - c)^2*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

3.153 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=90

$$\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-m,\frac{5}{2},\frac{1}{2}(1-\right.}{3f}$$

```
[Out] (2^(1/2 + m)*a*Hypergeometric2F1[3/2, 1/2 - m, 5/2, (1 - Sec[e + f*x])/2]*(
1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*(c - c*Sec[e + f*
x])*Tan[e + f*x])/(3*f)
```

Rubi [A] time = 0.0781782, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3961, 70, 69}

$$\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x]),x]
```

```
[Out] (2^(1/2 + m)*a*Hypergeometric2F1[3/2, 1/2 - m, 5/2, (1 - Sec[e + f*x])/2]*(
1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*(c - c*Sec[e + f*
x])*Tan[e + f*x])/(3*f)
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int (a+ax)^{-\frac{1}{2}+m}\sqrt{c-cx}\,dx,x,\sec(\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+m}ac(a+a\sec(e+fx))^{-1+m}\left(\frac{a+a\sec(e+fx)}{a}\right)^{\frac{1}{2}-m}\tan(e\right.}{f\sqrt{c-c\,\text{s}}}$$

$$= \frac{2^{\frac{1}{2}+m}a\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx}{3}$$

Mathematica [A] time = 0.162535, size = 85, normalized size = 0.94

$$-\frac{c2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)-1)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a(\sec(e+fx)+1))^m\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-m,\frac{5}{2},\frac{1}{2}\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x]),x]
```

```
[Out] -(2^(1/2 + m)*c*Hypergeometric2F1[3/2, 1/2 - m, 5/2, (1 - Sec[e + f*x])/2]*
(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*
Tan[e + f*x])/(3*f)
```

Maple [F] time = 0.366, size = 0, normalized size = 0.

$$\int \sec(fx+e)(a+a\sec(fx+e))^m(c-c\sec(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int (c\sec(fx+e)-c)(a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -integrate((c*sec(f*x + e) - c)*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(c\sec(fx+e)^2-c\sec(fx+e)\right)(a\sec(fx+e)+a)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] integral(-(c*sec(f*x + e)^2 - c*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int -\left(a\sec\left(e+fx\right)+a\right)^{m}\sec\left(e+fx\right)dx+\int\left(a\sec\left(e+fx\right)+a\right)^{m}\sec^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-(a*sec(e + f*x) + a)**m*sec(e + f*x), x) + Integral((a*sec(e
+ f*x) + a)**m*sec(e + f*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(c\sec\left(fx+e\right)-c\right)\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] integrate(-(c*sec(f*x + e) - c)*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

3.154 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=90

$$-\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\frac{1}{2}(1-\sec(e+fx\right.}{f(c-c\sec(e+fx))}$$

```
[Out] -((2^(1/2 + m)*a*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 - Sec[e + f*x])/2
]*(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*Tan[e + f*x])/
(f*(c - c*Sec[e + f*x])))
```

Rubi [A] time = 0.111511, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3961, 70, 69}

$$-\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x]),x]
```

```
[Out] -((2^(1/2 + m)*a*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 - Sec[e + f*x])/2
]*(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*Tan[e + f*x])/
(f*(c - c*Sec[e + f*x])))
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx = -\frac{(ac\tan(e+fx))\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+m}ac(a+a\sec(e+fx))^{-1+m}\left(\frac{a+a\sec(e+fx)}{a}\right)^{\frac{1}{2}-m}\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(\frac{1}{2}+\right.}{(c}\right.}{f\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e}{f(c-c\sec(e+fx))}$$

Mathematica [F] time = 0.555419, size = 0, normalized size = 0.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x]),x]
```

```
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x]), x]
```

Maple [F] time = 0.305, size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^m}{c-c\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)}{c\sec\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{c\sec\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] integral(-(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{\left(a\sec\left(e+fx\right)+a\right)^{m}\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral((a*sec(e + f*x) + a)**m*sec(e + f*x)/(sec(e + f*x) - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{c\sec\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] integrate(-(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c), x)
```

3.155 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=92

$$-\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}-m,-\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)}{3f(c-c\sec(e+fx))^2}$$

```
[Out] -(2^(1/2 + m)*a*Hypergeometric2F1[-3/2, 1/2 - m, -1/2, (1 - Sec[e + f*x])/2
]*(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*Tan[e + f*x])/
(3*f*(c - c*Sec[e + f*x])^2)
```

Rubi [A] time = 0.11493, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3961, 70, 69}

$$-\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{3f(c-c\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] -(2^(1/2 + m)*a*Hypergeometric2F1[-3/2, 1/2 - m, -1/2, (1 - Sec[e + f*x])/2
]*(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*Tan[e + f*x])/
(3*f*(c - c*Sec[e + f*x])^2)
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx = -\frac{(ac\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+m}ac(a+a\sec(e+fx))^{-1+m}\left(\frac{a+a\sec(e+fx)}{a}\right)^{\frac{1}{2}-m}\tan(e+fx)\right)\mathrm{Subst}\left(\int\right.}{f\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}a\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\,\mathrm{s}}{3f(c-c\sec(e+fx))^2}$$

Mathematica [F] time = 0.995004, size = 0, normalized size = 0.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^2, x]
```

Maple [F] time = 0.389, size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^m}{\left(c-c\sec\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)}{\left(c\sec\left(fx+e\right)-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{c^{2}\sec\left(fx+e\right)^{2}-2\,c^{2}\sec\left(fx+e\right)+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c^2*sec(f*x + e)^2 - 2*c^2*se
c(f*x + e) + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\left(a\sec\left(e+fx\right)+a\right)^{m}\sec\left(e+fx\right)}{\sec^{2}\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral((a*sec(e + f*x) + a)**m*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e +
f*x) + 1), x)/c**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\left(c\sec\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c)^2, x)
```

3.156 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=160

$$-\frac{16c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}(a\sec(e+fx)+a)^m}{f\left(4m^2+16m+15\right)}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^m}{f(2m+5)\left(4m^2+8m+3\right)\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e$$

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(5 + 2*m)*(3 + 8*m + 4*m^2
)*Sqrt[c - c*Sec[e + f*x]]) - (16*c^2*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec
[e + f*x]]*Tan[e + f*x])/(f*(15 + 16*m + 4*m^2)) - (2*c*(a + a*Sec[e + f*x]
)^m*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(5 + 2*m))
```

Rubi [A] time = 0.380242, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{16c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}(a\sec(e+fx)+a)^m}{f\left(4m^2+16m+15\right)}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^m}{f(2m+5)\left(4m^2+8m+3\right)\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(5 + 2*m)*(3 + 8*m + 4*m^2
)*Sqrt[c - c*Sec[e + f*x]]) - (16*c^2*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec
[e + f*x]]*Tan[e + f*x])/(f*(15 + 16*m + 4*m^2)) - (2*c*(a + a*Sec[e + f*x]
)^m*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(5 + 2*m))
```

Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2c(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(5+2m)} +$$
$$= -\frac{16c^2(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\left(15+16m+4m^2\right)} -$$
$$= -\frac{64c^3(a+a\sec(e+fx))^m\tan(e+fx)}{f\left(15+46m+36m^2+8m^3\right)\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a$$

Mathematica [F] time = 15.7782, size = 0, normalized size = 0.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(5/2), x
]
```

Maple [F] time = 0.265, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^m\left(c-c\sec\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x)
```

Maxima [A] time = 1.56329, size = 308, normalized size = 1.92

$$-\frac{2\left(\frac{\sqrt{2}\left(2^{m+5}m+5\cdot 2^{m+4}\right)(-a)^m c^{\frac{5}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2} - \frac{\sqrt{2}\left(2^{m+4}m^2+2^{m+6}m+15\cdot 2^{m+2}\right)(-a)^m c^{\frac{5}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4} - 2^{m+\frac{11}{2}}\left(-a\right)^m c^{\frac{5}{2}}\right)e^{\left(-m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-m\log}}{\left(8\,m^3+36\,m^2+46\,m+15\right)f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] -2*(sqrt(2)*(2^(m + 5)*m + 5*2^(m + 4))*(-a)^m*c^(5/2)*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - sqrt(2)*(2^(m + 4)*m^2 + 2^(m + 6)*m + 15*2^(m + 2))*(-a)
^m*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2^(m + 11/2)*(-a)^m*c^(5/2
))*e^(-m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1))/((8*m^3 + 36*m^2 + 46*m + 15)*f*(sin(f*x + e)/(cos(f*x
 + e) + 1) + 1)^(5/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

Fricas [A] time = 0.52022, size = 435, normalized size = 2.72

$$\frac{2\left(4c^2m^2 + \left(4c^2m^2 + 24c^2m + 43c^2\right)\cos\left(fx+e\right)^3 + 8c^2m - \left(4c^2m^2 + 8c^2m - 29c^2\right)\cos\left(fx+e\right)^2 + 3c^2 - \left(4c^2m^2\right.\right.}{\left(8fm^3 + 36fm^2 + 46fm + 15f\right)\cos\left(fx+e\right)^2\sin\left(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] 2*(4*c^2*m^2 + (4*c^2*m^2 + 24*c^2*m + 43*c^2)*cos(f*x + e)^3 + 8*c^2*m - (
4*c^2*m^2 + 8*c^2*m - 29*c^2)*cos(f*x + e)^2 + 3*c^2 - (4*c^2*m^2 + 24*c^2*
m + 11*c^2)*cos(f*x + e))*((a*cos(f*x + e) + a)/cos(f*x + e))^m*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e))/((8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x
+ e)^2*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-c\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^(5/2)*(a*sec(f*x + e) + a)^m*sec(f*x + e),
x)
```

3.157 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=100

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^m}{f\left(4m^2+8m+3\right)\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}(a\sec(e+fx)+a)^m}{f(2m+3)}$$

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(3 + 8*m + 4*m^2)*Sqrt[c -
c*Sec[e + f*x]]) - (2*c*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]]*Tan
[e + f*x])/(f*(3 + 2*m))
```

Rubi [A] time = 0.221745, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3955, 3953}

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^m}{f\left(4m^2+8m+3\right)\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}(a\sec(e+fx)+a)^m}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(3/2),x]
```


```
[Out] (-8*c^2*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(3 + 8*m + 4*m^2)*Sqrt[c -
c*Sec[e + f*x]]) - (2*c*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]]*Tan
[e + f*x])/(f*(3 + 2*m))
```


Rule 3955

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> -Simp[(d*Cot[e + f*
x]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x] + D
ist[(c*(2*n - 1))/(m + n), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d*C
sc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && !LtQ[m, -2^(-1)] &&
 !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx=-\frac{2c(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(3+2m)}+\frac{(4}{}$$

$$=-\frac{8c^2(a+a\sec(e+fx))^m\tan(e+fx)}{f\left(3+8m+4m^2\right)\sqrt{c-c\sec(e+fx)}}-\frac{2c(a+a\sec(e+f}{}$$

Mathematica [F] time = 38.3734, size = 0, normalized size = 0.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(3/2), x
]
```

Maple [F] time = 0.261, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x)
```

Maxima [A] time = 1.56104, size = 231, normalized size = 2.31

$$-\frac{2\left(\sqrt{2}2^{m+2}\left(-a\right)^{m}c^{\frac{3}{2}}-\frac{\sqrt{2}\left(2^{m+2}m+3\cdot 2^{m+1}\right)\left(-a\right)^{m}c^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)e^{\left(-m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)\right)}}{\left(4\,m^{2}+8\,m+3\right)f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] -2*(sqrt(2)*2^(m + 2)*(-a)^m*c^(3/2) - sqrt(2)*(2^(m + 2)*m + 3*2^(m + 1))*
(-a)^m*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*e^(-m*log(sin(f*x + e)/
(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1))/((4*m
^2 + 8*m + 3)*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(3/2)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1)^(3/2))
```

Fricas [A] time = 0.498297, size = 263, normalized size = 2.63

$$\frac{2\left(\left(2\,cm+5\,c\right)\cos\left(fx+e\right)^{2}-2\,cm+4\,c\cos\left(fx+e\right)-c\right)\left(\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}\right)^{m}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{\left(4\,fm^{2}+8\,fm+3\,f\right)\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2*((2*c*m + 5*c)*cos(f*x + e)^2 - 2*c*m + 4*c*cos(f*x + e) - c)*((a*cos(f*x
 + e) + a)/cos(f*x + e))^m*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((4*f*m^
2 + 8*f*m + 3*f)*cos(f*x + e)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^(3/2)*(a*sec(f*x + e) + a)^m*sec(f*x + e),
x)
```

3.158 $\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=46

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (-2*c*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.101803, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3953}

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*c*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3953

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(2*a*c*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] time = 18.7722, size = 163, normalized size = 3.54

$$\frac{\sqrt{2}e^{-\frac{1}{2}i(e+fx)}\left(1+e^{i(e+fx)}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^m\csc\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}(\sec(e+fx)+1)^{-m}(a(\sec(e+f}{(2fm+f)\sqrt{\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[2]*(1 + E^(I*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x))))^m*Csc[(e + f*x)/2]*(a*(1 + Sec[e + f*x]))^m*Sqrt[c - c*Sec[e + f*x]])/(E^((I/2)*(e + f*x))*(f + 2*f*m)*Sqrt[Sec[e + f*x]]*(1 + Sec[e + f*x])^m)
```

Maple [F] time = 0.313, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\sqrt{c-c\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x)
```

Maxima [B] time = 1.55983, size = 154, normalized size = 3.35

$$\frac{2^{m+\frac{3}{2}}\left(-a\right)^{m}\sqrt{c}e^{\left(-m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)\right)}}{f(2\,m+1)\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1}\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] 2^(m + 3/2)*(-a)^m*sqrt(c)*e^(-m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) -
 m*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1))/(f*(2*m + 1)*sqrt(sin(f*x + e)
/(cos(f*x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) - 1))
```

Fricas [A] time = 0.474301, size = 171, normalized size = 3.72

$$\frac{2\left(\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}\right)^{m}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\left(\cos\left(fx+e\right)+1\right)}{\left(2\,fm+f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] 2*((a*cos(f*x + e) + a)/cos(f*x + e))^m*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))*(cos(f*x + e) + 1)/((2*f*m + f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c\sec\left(fx+e\right)+c}\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

3.159 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=69

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\text{Hypergeometric2F1}\left(1,m+\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -((Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*Sec[
e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]]))
```

Rubi [A] time = 0.129482, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3961, 68}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -((Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*Sec[
e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]]))
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx &= -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{c-cx}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= -\frac{{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [F] time = 0.642719, size = 0, normalized size = 0.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/Sqrt[c - c*Sec[e + f*x]], x
]
```

Maple [F] time = 0.305, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\frac{1}{\sqrt{c-c\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-c\sec\left(fx+e\right)+c}\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{c\sec\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*
sec(f*x + e) - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\sec\left(e+fx\right)}{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)/sqrt(-c*(sec(e + f*x) - 1))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

3.160 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m \text{Hypergeometric2F1}\left(2,m+\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(\sec(e+fx)+1)\right)}{2cf(2m+1)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -(Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*Sec[e
 + f*x])^m*Tan[e + f*x])/(2*c*f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.13863, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3961, 68}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\, {}_2F_1\left(2,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{2cf(2m+1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*Sec[e
 + f*x])^m*Tan[e + f*x])/(2*c*f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx &= -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^2}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= -\frac{{}_2F_1\left(2,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{2cf(1+2m)\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [F] time = 0.693296, size = 0, normalized size = 0.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(3/2),
x]
```

```
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(3/2),
 x]
```

Maple [F] time = 0.257, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-c\sec\left(fx+e\right)+c}\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{c^{2}\sec\left(fx+e\right)^{2}-2\,c^{2}\sec\left(fx+e\right)+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c^2
*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
x)
```

3.161 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m \text{Hypergeometric2F1}\left(3,m+\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(\sec(e+fx)+1)\right)}{4c^2f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -(Hypergeometric2F1[3, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*Sec[e
 + f*x])^m*Tan[e + f*x])/(4*c^2*f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Rubi [A] time = 0.137036, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3961, 68}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\, {}_2F_1\left(3,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{4c^2f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(Hypergeometric2F1[3, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*Sec[e
 + f*x])^m*Tan[e + f*x])/(4*c^2*f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx &= -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^3}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= -\frac{{}_2F_1\left(3,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{4c^2f(1+2m)\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [F] time = 2.03071, size = 0, normalized size = 0.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(5/2),
x]
```

```
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(5/2),
 x]
```

Maple [F] time = 0.255, size = 0, normalized size = 0.

$$\int \sec \left(fx+e\right) \left(a+a\sec \left(fx+e\right)\right)^{m} \left(c-c\sec \left(fx+e\right)\right)^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c\sec \left(fx+e\right)+c}\left(a\sec \left(fx+e\right)+a\right)^{m}\sec \left(fx+e\right)}{c^{3}\sec \left(fx+e\right)^{3}-3\,c^{3}\sec \left(fx+e\right)^{2}+3\,c^{3}\sec \left(fx+e\right)-c^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral(-sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c^
3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(5/2),
x)
```

3.162 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx$

Optimal. Leaf size=169

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^{m+2}(c-c\sec(e+fx))^{-m-3}}{a^2f(2m+1)\left(4m^2+16m+15\right)}+\frac{2\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+fx))}{af\left(4m^2+8m+3\right)}$$

```
[Out] -(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m)*Tan[e + f*x])/(f*(1
 + 2*m))) + (2*(a + a*Sec[e + f*x])^(1 + m)*(c - c*Sec[e + f*x])^(-3 - m)*T
an[e + f*x])/(a*f*(3 + 8*m + 4*m^2)) - (2*(a + a*Sec[e + f*x])^(2 + m)*(c -
 c*Sec[e + f*x])^(-3 - m)*Tan[e + f*x])/(a^2*f*(1 + 2*m)*(15 + 16*m + 4*m^2
))
```

Rubi [A] time = 0.373941, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3951, 3950}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^{m+2}(c-c\sec(e+fx))^{-m-3}}{a^2f(2m+1)\left(4m^2+16m+15\right)}+\frac{2\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+fx))}{af\left(4m^2+8m+3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m),x]
```

```
[Out] -(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m)*Tan[e + f*x])/(f*(1
 + 2*m))) + (2*(a + a*Sec[e + f*x])^(1 + m)*(c - c*Sec[e + f*x])^(-3 - m)*T
an[e + f*x])/(a*f*(3 + 8*m + 4*m^2)) - (2*(a + a*Sec[e + f*x])^(2 + m)*(c -
 c*Sec[e + f*x])^(-3 - m)*Tan[e + f*x])/(a^2*f*(1 + 2*m)*(15 + 16*m + 4*m^2
))
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx = -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{f(1+2m)}$$
$$= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{f(1+2m)}$$
$$= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{f(1+2m)}$$

Mathematica [C] time = 9.02579, size = 321, normalized size = 1.9

$$-\frac{i2^{m+3}\left(1+e^{i(e+fx)}\right)\left(-ie^{-\frac{1}{2}i(e+fx)}\left(-1+e^{i(e+fx)}\right)\right)^{-2m}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-m}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^m\left(\left(4m^2+12m+7\right)e^{4i(e+fx)}+\left(8m^2+24m\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m)
,x]
```

```
[Out] ((-I)*2^(3 + m)*(1 + E^(I*(e + f*x)))*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I
)*(e + f*x))))^m*(7 + 12*m + 4*m^2 - 4*E^(I*(e + f*x))*(3 + 2*m) - 4*E^((3*
I)*(e + f*x))*(3 + 2*m) + E^((4*I)*(e + f*x))*(7 + 12*m + 4*m^2) + E^((2*I)
*(e + f*x))*(22 + 24*m + 8*m^2))*Sec[e + f*x]^(3 + m)*(a*(1 + Sec[e + f*x])
)^m*(c - c*Sec[e + f*x])^(-3 - m))/((-1 + E^(I*(e + f*x)))^5*(((-I)*(-1 + E
^(I*(e + f*x))))/E^((I/2)*(e + f*x)))^(2*m)*(E^(I*(e + f*x))/(1 + E^((2*I)*
(e + f*x))))^m*f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(1 + Sec[e + f*x])^m*Sin[e/2
 + (f*x)/2]^(2*(-3 - m)))
```

Maple [F] time = 0.658, size = 0, normalized size = 0.

$$\int \sec(fx+e)\left(a+a\sec(fx+e)\right)^m\left(c-c\sec(fx+e)\right)^{-3-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x)
```

Maxima [A] time = 1.53933, size = 211, normalized size = 1.25

$$\frac{\left(\left(4m^2+8m+3\right)(-a)^m-\frac{2\left(4m^2+12m+5\right)(-a)^m\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{\left(4m^2+16m+15\right)(-a)^m\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}\right)c^{-m-3}\left(\cos\left(fx+e\right)+1\right)^5}{4\left(8m^3+36m^2+46m+15\right)f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)^{2m}\sin\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x, algorith
m="maxima")
```

```
[Out] 1/4*((4*m^2 + 8*m + 3)*(-a)^m - 2*(4*m^2 + 12*m + 5)*(-a)^m*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + (4*m^2 + 16*m + 15)*(-a)^m*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4)*c^(-m - 3)*(cos(f*x + e) + 1)^5/((8*m^3 + 36*m^2 + 46*m + 15)*f
*(sin(f*x + e)/(cos(f*x + e) + 1))^(2*m)*sin(f*x + e)^5)
```

Fricas [A] time = 0.512713, size = 296, normalized size = 1.75

$$-\frac{\left(\left(4\,m^{2}+12\,m+7\right)\cos\left(fx+e\right)^{2}-2\,(2\,m+3)\cos\left(fx+e\right)+2\right)\left(\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}\right)^{m}\left(\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}\right)^{-m-3}\sin\left(fx+e\right)}{\left(8\,fm^{3}+36\,fm^{2}+46\,fm+15\,f\right)\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x, algorith
m="fricas")
```

```
[Out] -((4*m^2 + 12*m + 7)*cos(f*x + e)^2 - 2*(2*m + 3)*cos(f*x + e) + 2)*((a*cos
(f*x + e) + a)/cos(f*x + e))^m*((c*cos(f*x + e) - c)/cos(f*x + e))^(-m - 3)
*sin(f*x + e)/((8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(-3-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\left(-c\sec\left(fx+e\right)+c\right)^{-m-3}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x, algorith
m="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m - 3)*sec(f*x + e
), x)
```

3.163 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx$

Optimal. Leaf size=104

$$\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+fx))^{-m-2}}{af\left(4m^2+8m+3\right)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-2}}{f(2m+1)}$$

```
[Out] -(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m)*Tan[e + f*x])/(f*(1
 + 2*m))) + ((a + a*Sec[e + f*x])^(1 + m)*(c - c*Sec[e + f*x])^(-2 - m)*Tan
[e + f*x])/(a*f*(3 + 8*m + 4*m^2))
```

Rubi [A] time = 0.224782, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3951, 3950}

$$\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+fx))^{-m-2}}{af\left(4m^2+8m+3\right)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-2}}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m),x]
```

```
[Out] -(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m)*Tan[e + f*x])/(f*(1
 + 2*m))) + ((a + a*Sec[e + f*x])^(1 + m)*(c - c*Sec[e + f*x])^(-2 - m)*Tan
[e + f*x])/(a*f*(3 + 8*m + 4*m^2))
```

Rule 3951

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx &= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\tan(e+fx)}{f(1+2m)}\\ &= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\tan(e+fx)}{f(1+2m)}\end{aligned}$$

Mathematica [C] time = 3.14231, size = 250, normalized size = 2.4

$$\frac{i2^{m+3}\left(1+e^{i(e+fx)}\right)\left(-ie^{-\frac{1}{2}i(e+fx)}\left(-1+e^{i(e+fx)}\right)\right)^{-2m}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-m}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^m\left((m+1)e^{2i(e+fx)}-e^{i(e+fx)}+m+1\right)\sin}{f(2m+1)(2m+3)\left(-1+e^{i}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m)
,x]
```

```
[Out] (I*2^(3 + m)*(1 + E^(I*(e + f*x)))*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(
e + f*x))))^m*(1 - E^(I*(e + f*x)) + m + E^((2*I)*(e + f*x))*(1 + m))*Sec[e
 + f*x]^(2 + m)*(a*(1 + Sec[e + f*x]))^m*(c - c*Sec[e + f*x])^(-2 - m)*Sin[
(e + f*x)/2]^(2*(2 + m)))/((-1 + E^(I*(e + f*x)))^3*(((-I)*(-1 + E^(I*(e +
f*x))))/E^((I/2)*(e + f*x)))^(2*m)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)
)))^m*f*(1 + 2*m)*(3 + 2*m)*(1 + Sec[e + f*x])^m)
```

Maple [F] time = 0.668, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{-2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x)
```

Maxima [A] time = 1.5165, size = 144, normalized size = 1.38

$$-\frac{\left((-a)^m\,(2\,m+1)-\frac{(-a)^m(2\,m+3)\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)c^{-m-2}\left(\cos\left(fx+e\right)+1\right)^3}{2\left(4\,m^2+8\,m+3\right)f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)^{2\,m}\sin\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x, algorith
m="maxima")
```

```
[Out] -1/2*((-a)^m*(2*m + 1) - (-a)^m*(2*m + 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2)*c^(-m - 2)*(cos(f*x + e) + 1)^3/((4*m^2 + 8*m + 3)*f*(sin(f*x + e)/(cos
(f*x + e) + 1))^(2*m)*sin(f*x + e)^3)
```

Fricas [A] time = 0.50293, size = 227, normalized size = 2.18

$$-\frac{\left(2\,(m+1)\cos\left(fx+e\right)-1\right)\left(\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}\right)^m\left(\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}\right)^{-m-2}\sin\left(fx+e\right)}{\left(4\,fm^2+8\,fm+3\,f\right)\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x, algorith
m="fricas")
```

```
[Out] -(2*(m + 1)*cos(f*x + e) - 1)*((a*cos(f*x + e) + a)/cos(f*x + e))^m*((c*cos
(f*x + e) - c)/cos(f*x + e))^(-m - 2)*sin(f*x + e)/((4*f*m^2 + 8*f*m + 3*f)
*cos(f*x + e)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(-2-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\left(-c\sec\left(fx+e\right)+c\right)^{-m-2}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x, algorith
m="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m - 2)*sec(f*x + e
), x)
```

3.164 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-1}}{f(2m+1)}$$

```
[Out] -(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m)*Tan[e + f*x])/(f*(1
 + 2*m)))
```

Rubi [A] time = 0.101124, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {3950}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-1}}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m),x]
```

```
[Out] -(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m)*Tan[e + f*x])/(f*(1
 + 2*m)))
```

Rule 3950

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[(b*Cot[e + f*x
]*(a + b*Csc[e + f*x])^m*(c + d*Csc[e + f*x])^n)/(a*f*(2*m + 1)), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx = -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\tan(e+f}{f(1+2m)}$$

Mathematica [C] time = 1.14629, size = 208, normalized size = 4.43

$$-\frac{2^{m+1}e^{-\frac{1}{2}i(e+fx)}\left(1+e^{i(e+fx)}\right)\left(-ie^{-\frac{1}{2}i(e+fx)}\left(-1+e^{i(e+fx)}\right)\right)^{-2m-1}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-m}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^m\sin^{2(m+1)}\left(\frac{1}{2}(e+fx)\right)\sec^{m}}{2fm+f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m)
,x]
```

```
[Out] -((2^(1 + m)*(((-I)*(-1 + E^(I*(e + f*x))))/E^((I/2)*(e + f*x)))^(-1 - 2*m)
*(1 + E^(I*(e + f*x)))*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x))))^
m*Sec[e + f*x]^(1 + m)*(a*(1 + Sec[e + f*x]))^m*(c - c*Sec[e + f*x])^(-1 -
```

```
m)*Sin[(e + f*x)/2]^(2*(1 + m)))/(E^((I/2)*(e + f*x))*(E^(I*(e + f*x))/(1 +
 E^((2*I)*(e + f*x))))^m*(f + 2*f*m)*(1 + Sec[e + f*x])^m))
```

Maple [F] time = 0.689, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{-1-m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x)
```

Maxima [A] time = 1.51068, size = 84, normalized size = 1.79

$$\frac{(-a)^{m}c^{-m-1}\left(\cos\left(fx+e\right)+1\right)}{f(2m+1)\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)^{2m}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x, algorith
m="maxima")
```

```
[Out] (-a)^m*c^(-m - 1)*(cos(f*x + e) + 1)/(f*(2*m + 1)*(sin(f*x + e)/(cos(f*x +
e) + 1))^(2*m)*sin(f*x + e))
```

Fricas [A] time = 0.494884, size = 169, normalized size = 3.6

$$-\frac{\left(\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}\right)^{m}\left(\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}\right)^{-m-1}\sin\left(fx+e\right)}{\left(2\,fm+f\right)\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x, algorith
m="fricas")
```

```
[Out] -((a*cos(f*x + e) + a)/cos(f*x + e))^m*((c*cos(f*x + e) - c)/cos(f*x + e))^
(-m - 1)*sin(f*x + e)/((2*f*m + f)*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\left(-c\sec\left(fx+e\right)+c\right)^{-m-1}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x, algorith
m="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m - 1)*sec(f*x + e
), x)
```

3.165 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx$

Optimal. Leaf size=101

$$-\frac{c2^{\frac{1}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m+\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-1}\text{Hypergeometric2F1}\left(m+\frac{1}{2},m+\frac{1}{2}\right.}{f(2m+1)}$$

```
[Out] -((2^(1/2 - m)*c*Hypergeometric2F1[1/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e +
f*x])/2]*(1 - Sec[e + f*x])^(1/2 + m)*(a + a*Sec[e + f*x])^m*(c - c*Sec[e +
 f*x])^(-1 - m)*Tan[e + f*x])/(f*(1 + 2*m)))
```

Rubi [A] time = 0.129243, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3961, 70, 69}

$$-\frac{c2^{\frac{1}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m+\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-1}\,{}_2F_1\left(m+\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 - m)*c*Hypergeometric2F1[1/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e +
f*x])/2]*(1 - Sec[e + f*x])^(1/2 + m)*(a + a*Sec[e + f*x])^m*(c - c*Sec[e +
 f*x])^(-1 - m)*Tan[e + f*x])/(f*(1 + 2*m)))
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx = -\frac{(ac\tan(e+fx))\,\mathrm{Subst}\left(\int(a+ax)^{-\frac{1}{2}+m}(c-cx)^{-\frac{1}{2}-m}\,dx\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}-m}ac(c-c\sec(e+fx))^{-1-m}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}+m}\tan(\right.}{f\sqrt{a}}$$

$$= -\frac{2^{\frac{1}{2}-m}c\,{}_2F_1\left(\frac{1}{2}+m,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(1}{}$$

Mathematica [C] time = 1.44974, size = 257, normalized size = 2.54

$$\frac{2^{m-1}\left(-ie^{-\frac{1}{2}i(e+fx)}\left(-1+e^{i(e+fx)}\right)\right)^{-2m}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-m}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^m\sin^{2m}\left(\frac{1}{2}(e+fx)\right)\left(\frac{\sec(e+fx)}{\sec(e+fx)+1}\right)^m\left(\text{Hypergeometric}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^m,x]
```

```
[Out] (2^(-1 + m)*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x))))^m*(-Hyperge
ometric2F1[1, -2*m, 1 - 2*m, ((-I)*(-1 + E^(I*(e + f*x))))/(1 + E^(I*(e + f
*x)))] + Hypergeometric2F1[1, -2*m, 1 - 2*m, (I*(-1 + E^(I*(e + f*x))))/(1
+ E^(I*(e + f*x)))])*(Sec[e + f*x]/(1 + Sec[e + f*x]))^m*(a*(1 + Sec[e + f*
x]))^m*Sin[(e + f*x)/2]^(2*m))/((((-I)*(-1 + E^(I*(e + f*x))))/E^((I/2)*(e
+ f*x)))^(2*m)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^m*f*m*(c - c*Sec
[e + f*x])^m)
```

Maple [F] time = 0.52, size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^m}{\left(c-c\sec\left(fx+e\right)\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^m\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x, algorithm="
maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{m}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x, algorithm="
fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/((c-c*sec(f*x+e))**m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x, algorithm="
giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^m, x)
```

3.166 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$

Optimal. Leaf size=99

$$-\frac{c2^{\frac{3}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m}\text{Hypergeometric2F1}\left(m-\frac{1}{2},m+\right.}{f(2m+1)}$$

```
[Out] -((2^(3/2 - m)*c*Hypergeometric2F1[-1/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e +
 f*x])/2]*(1 - Sec[e + f*x])^(-1/2 + m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x]
)/(f*(1 + 2*m)*(c - c*Sec[e + f*x])^m))
```

Rubi [A] time = 0.143151, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3961, 70, 69}

$$-\frac{c2^{\frac{3}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m}\,{}_2F_1\left(m-\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(1 - m),x]
```

```
[Out] -((2^(3/2 - m)*c*Hypergeometric2F1[-1/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e +
 f*x])/2]*(1 - Sec[e + f*x])^(-1/2 + m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x]
)/(f*(1 + 2*m)*(c - c*Sec[e + f*x])^m))
```

Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx = -\frac{(ac\tan(e+fx))\,\mathrm{Subst}\left(\int(a+ax)^{-\frac{1}{2}+m}(c-cx)^{\frac{1}{2}-m}\,dx,x,\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{\frac{1}{2}-m}ac(c-c\sec(e+fx))^{-m}\left(\frac{c-c\sec(e+fx)}{c}\right)^{-\frac{1}{2}+m}\tan(e+\right.}{f\sqrt{a+a\,}}$$

$$= -\frac{2^{\frac{3}{2}-m}c\,{}_2F_1\left(-\frac{1}{2}+m,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(1}{}$$

Mathematica [F] time = 2.2868, size = 0, normalized size = 0.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(1 - m),
x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(1 - m),
 x]
```

Maple [F] time = 0.647, size = 0, normalized size = 0.

$$\int \sec(fx+e)\left(a+a\sec(fx+e)\right)^m\left(c-c\sec(fx+e)\right)^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec(fx+e)+a\right)^m\left(-c\sec(fx+e)+c\right)^{-m+1}\sec(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 1)*sec(f*x + e
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sec(fx+e)+a\right)^m\left(-c\sec(fx+e)+c\right)^{-m+1}\sec(fx+e),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x, algorithm
="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 1)*sec(f*x + e)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(1-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sec\left(fx+e\right)+a\right)^{m} \left(-c \sec\left(fx+e\right)+c\right)^{-m+1} \sec\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x, algorithm
="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 1)*sec(f*x + e
), x)
```

3.167 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$

Optimal. Leaf size=101

$$-\frac{c^2 2^{\frac{5}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m}\text{Hypergeometric2F1}\left(m-\frac{3}{2},m+\frac{1}{2},\right.}{f(2m+1)}$$

```
[Out] -((2^(5/2 - m)*c^2*Hypergeometric2F1[-3/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e
 + f*x])/2]*(1 - Sec[e + f*x])^(-1/2 + m)*(a + a*Sec[e + f*x])^m*Tan[e + f*
x])/(f*(1 + 2*m)*(c - c*Sec[e + f*x])^m))
```

Rubi [A] time = 0.154416, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3961, 70, 69}

$$-\frac{c^2 2^{\frac{5}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m}\,{}_2F_1\left(m-\frac{3}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+f\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(2 - m),x]
```


```
[Out] -((2^(5/2 - m)*c^2*Hypergeometric2F1[-3/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e
 + f*x])/2]*(1 - Sec[e + f*x])^(-1/2 + m)*(a + a*Sec[e + f*x])^m*Tan[e + f*
x])/(f*(1 + 2*m)*(c - c*Sec[e + f*x])^m))
```


Rule 3961

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(a*c*Cot[e + f
*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int (a+ax)^{-\frac{1}{2}+m}(c-cx)^{\frac{3}{2}-m}\,dx\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{\frac{3}{2}-m}ac^2(c-c\sec(e+fx))^{-m}\left(\frac{c-c\sec(e+fx)}{c}\right)^{-\frac{1}{2}+m}\tan(\right.}{f\sqrt{a}}$$

$$= -\frac{2^{\frac{5}{2}-m}c^2\,{}_2F_1\left(-\frac{3}{2}+m,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right.}{}$$

Mathematica [F] time = 2.77688, size = 0, normalized size = 0.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(2 - m),
x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(2 - m),
 x]
```

Maple [F] time = 0.665, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^m\left(c-c\sec\left(fx+e\right)\right)^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^m\left(-c\sec\left(fx+e\right)+c\right)^{-m+2}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 2)*sec(f*x + e
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^m\left(-c\sec\left(fx+e\right)+c\right)^{-m+2}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x, algorithm
="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 2)*sec(f*x + e)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(2-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sec\left(fx + e\right) + a\right)^{m} \left(-c \sec\left(fx + e\right) + c\right)^{-m+2} \sec\left(fx + e\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x, algorithm
="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 2)*sec(f*x + e
), x)
```

3.168 $\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=105

$$-\frac{a^3c\tan^5(e+fx)}{5f}-\frac{2a^3c\tan^3(e+fx)}{3f}+\frac{a^3c\tanh^{-1}(\sin(e+fx))}{4f}-\frac{a^3c\tan(e+fx)\sec^3(e+fx)}{2f}+\frac{a^3c\tan(e+fx)\,\mathrm{s}}{4f}$$

```
[Out] (a^3*c*ArcTanh[Sin[e + f*x]])/(4*f) + (a^3*c*Sec[e + f*x]*Tan[e + f*x])/(4*
f) - (a^3*c*Sec[e + f*x]^3*Tan[e + f*x])/(2*f) - (2*a^3*c*Tan[e + f*x]^3)/(
3*f) - (a^3*c*Tan[e + f*x]^5)/(5*f)
```

Rubi [A] time = 0.193214, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3962, 2607, 30, 2611, 3768, 3770, 14}

$$-\frac{a^3c\tan^5(e+fx)}{5f}-\frac{2a^3c\tan^3(e+fx)}{3f}+\frac{a^3c\tanh^{-1}(\sin(e+fx))}{4f}-\frac{a^3c\tan(e+fx)\sec^3(e+fx)}{2f}+\frac{a^3c\tan(e+fx)\,\mathrm{s}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```


```
[Out] (a^3*c*ArcTanh[Sin[e + f*x]])/(4*f) + (a^3*c*Sec[e + f*x]*Tan[e + f*x])/(4*
f) - (a^3*c*Sec[e + f*x]^3*Tan[e + f*x])/(2*f) - (2*a^3*c*Tan[e + f*x]^3)/(
3*f) - (a^3*c*Tan[e + f*x]^5)/(5*f)
```


Rule 3962

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[(-(a*c))^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = -\left((ac)\int \left(a^2\sec^2(e+fx)\tan^2(e+fx)+2a^2\sec^3(e+fx)\tan\right.\right.$$
$$= -\left(\left(a^3c\right)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\right)-\left(a^3c\right)\int \sec^4(e+$$
$$= -\frac{a^3c\sec^3(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}\left(a^3c\right)\int \sec^3(e+fx)\,dx-$$
$$= \frac{a^3c\sec(e+fx)\tan(e+fx)}{4f}-\frac{a^3c\sec^3(e+fx)\tan(e+fx)}{2f}-$$
$$= \frac{a^3c\tanh^{-1}(\sin(e+fx))}{4f}+\frac{a^3c\sec(e+fx)\tan(e+fx)}{4f}-\frac{a^3c}{}$$

Mathematica [A] time = 0.309133, size = 68, normalized size = 0.65

$$\frac{a^3c\left(15\tanh^{-1}(\sin(e+fx))-\tan(e+fx)\left(12\tan^4(e+fx)+40\tan^2(e+fx)+30\sec^3(e+fx)-15\sec(e+fx)\right)\right)}{60f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*c*(15*ArcTanh[Sin[e + f*x]] - Tan[e + f*x]*(-15*Sec[e + f*x] + 30*Sec[
e + f*x]^3 + 40*Tan[e + f*x]^2 + 12*Tan[e + f*x]^4)))/(60*f)
```

Maple [A] time = 0.027, size = 130, normalized size = 1.2

$$\frac{a^3c\sec(fx+e)\tan(fx+e)}{4f}+\frac{a^3c\ln(\sec(fx+e)+\tan(fx+e))}{4f}+\frac{7a^3c\tan(fx+e)}{15f}-\frac{a^3c(\sec(fx+e))^3\tan(fx}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x)
```

```
[Out] 1/4*a^3*c*sec(f*x+e)*tan(f*x+e)/f+1/4/f*a^3*c*ln(sec(f*x+e)+tan(f*x+e))+7/1
5/f*a^3*c*tan(f*x+e)-1/2*a^3*c*sec(f*x+e)^3*tan(f*x+e)/f-1/5/f*a^3*c*tan(f*
x+e)*sec(f*x+e)^4-4/15/f*a^3*c*tan(f*x+e)*sec(f*x+e)^2
```

Maxima [A] time = 1.03722, size = 232, normalized size = 2.21

$$-\frac{8\left(3\tan\left(fx+e\right)^{5}+10\tan\left(fx+e\right)^{3}+15\tan\left(fx+e\right)\right)a^{3}c-15a^{3}c\left(\frac{2\left(3\sin\left(fx+e\right)^{3}-5\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^{4}-2\sin\left(fx+e\right)^{2}+1}-3\log\left(\sin\left(fx+e\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -1/120*(8*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c -
15*a^3*c*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x
 + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 60*a^3*
c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*
x + e) - 1)) - 120*a^3*c*tan(f*x + e))/f
```

Fricas [A] time = 0.493847, size = 342, normalized size = 3.26

$$\frac{15a^{3}c\cos\left(fx+e\right)^{5}\log\left(\sin\left(fx+e\right)+1\right)-15a^{3}c\cos\left(fx+e\right)^{5}\log\left(-\sin\left(fx+e\right)+1\right)+2\left(28a^{3}c\cos\left(fx+e\right)^{4}+1\right.}{120f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*a^3*c*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 15*a^3*c*cos(f*x + e
)^5*log(-sin(f*x + e) + 1) + 2*(28*a^3*c*cos(f*x + e)^4 + 15*a^3*c*cos(f*x
+ e)^3 - 16*a^3*c*cos(f*x + e)^2 - 30*a^3*c*cos(f*x + e) - 12*a^3*c)*sin(f*
x + e))/(f*cos(f*x + e)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^{3}c\left(\int -\sec^{2}\left(e+fx\right)dx+\int -2\sec^{3}\left(e+fx\right)dx+\int 2\sec^{5}\left(e+fx\right)dx+\int \sec^{6}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**3*c*(Integral(-sec(e + f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) +
Integral(2*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Giac [A] time = 1.22543, size = 207, normalized size = 1.97

$$\frac{15\,a^{3}c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,a^{3}c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(15\,a^{3}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}-70\,a^{3}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+128\,a^{3}c\tan}{\left(\tan\left(\frac{1}{2}\,fx-\right.\right.}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] 1/60*(15*a^3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^3*c*log(abs(tan(1/
2*f*x + 1/2*e) - 1)) - 2*(15*a^3*c*tan(1/2*f*x + 1/2*e)^9 - 70*a^3*c*tan(1/
2*f*x + 1/2*e)^7 + 128*a^3*c*tan(1/2*f*x + 1/2*e)^5 - 250*a^3*c*tan(1/2*f*x
 + 1/2*e)^3 - 15*a^3*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^5
)/f
```

3.169 $\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=86

$$-\frac{a^2c\tan^3(e+fx)}{3f}+\frac{a^2c\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^2c\tan(e+fx)\sec^3(e+fx)}{4f}+\frac{a^2c\tan(e+fx)\sec(e+fx)}{8f}$$

```
[Out] (a^2*c*ArcTanh[Sin[e + f*x]])/(8*f) + (a^2*c*Sec[e + f*x]*Tan[e + f*x])/(8*
f) - (a^2*c*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) - (a^2*c*Tan[e + f*x]^3)/(3*
f)
```

Rubi [A] time = 0.15607, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3962, 2607, 30, 2611, 3768, 3770}

$$-\frac{a^2c\tan^3(e+fx)}{3f}+\frac{a^2c\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^2c\tan(e+fx)\sec^3(e+fx)}{4f}+\frac{a^2c\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*c*ArcTanh[Sin[e + f*x]])/(8*f) + (a^2*c*Sec[e + f*x]*Tan[e + f*x])/(8*
f) - (a^2*c*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) - (a^2*c*Tan[e + f*x]^3)/(3*
f)
```

Rule 3962

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[(-(a*c))^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = -\left((ac)\int \left(a\sec^2(e+fx)\tan^2(e+fx)+a\sec^3(e+fx)\tan^2(\right.\right.$$
$$= -\left(\left(a^2c\right)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\right)-\left(a^2c\right)\int \sec^3(e+$$
$$= -\frac{a^2c\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{1}{4}\left(a^2c\right)\int \sec^3(e+fx)\,dx-$$
$$= \frac{a^2c\sec(e+fx)\tan(e+fx)}{8f}-\frac{a^2c\sec^3(e+fx)\tan(e+fx)}{4f}-$$
$$= \frac{a^2c\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2c\sec(e+fx)\tan(e+fx)}{8f}-\frac{a^2c}{}$$

Mathematica [A] time = 0.182562, size = 57, normalized size = 0.66

$$\frac{a^2c\left(3\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(-8\tan^2(e+fx)-6\sec^3(e+fx)+3\sec(e+fx)\right)\right)}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*c*(3*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(3*Sec[e + f*x] - 6*Sec[e +
f*x]^3 - 8*Tan[e + f*x]^2)))/(24*f)
```

Maple [A] time = 0.023, size = 107, normalized size = 1.2

$$\frac{a^2c\sec(fx+e)\tan(fx+e)}{8f}+\frac{a^2c\ln(\sec(fx+e)+\tan(fx+e))}{8f}+\frac{a^2c\tan(fx+e)}{3f}-\frac{a^2c\tan(fx+e)(\sec(fx+e)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

```
[Out] 1/8*a^2*c*sec(f*x+e)*tan(f*x+e)/f+1/8/f*a^2*c*ln(sec(f*x+e)+tan(f*x+e))+1/3
/f*a^2*c*tan(f*x+e)-1/3/f*a^2*c*tan(f*x+e)*sec(f*x+e)^2-1/4*a^2*c*sec(f*x+e
)^3*tan(f*x+e)/f
```

Maxima [B] time = 0.991333, size = 216, normalized size = 2.51

$$-\frac{16\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^2c-3a^2c\left(\frac{2\left(3\sin\left(fx+e\right)^3-5\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^4-2\sin\left(fx+e\right)^2+1}-3\log\left(\sin\left(fx+e\right)+1\right)+3\log\left(\sin\left(fx\right.\right.\right.}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c - 3*a^2*c*(2*(3*sin(f*x +
 e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin
(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 12*a^2*c*(2*sin(f*x + e)/(sin(f
*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 48*a^2*c*
tan(f*x + e))/f
```

Fricas [A] time = 0.487301, size = 297, normalized size = 3.45

$$\frac{3a^2c\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-3a^2c\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)+2\left(8a^2c\cos\left(fx+e\right)^3+3a^2c\right.}{48f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*a^2*c*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 3*a^2*c*cos(f*x + e)^4
*log(-sin(f*x + e) + 1) + 2*(8*a^2*c*cos(f*x + e)^3 + 3*a^2*c*cos(f*x + e)^
2 - 8*a^2*c*cos(f*x + e) - 6*a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^2c\left(\int -\sec^2\left(e+fx\right)dx+\int -\sec^3\left(e+fx\right)dx+\int \sec^4\left(e+fx\right)dx+\int \sec^5\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*c*(Integral(-sec(e + f*x)**2, x) + Integral(-sec(e + f*x)**3, x) + In
tegral(sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))
```

Giac [A] time = 1.29682, size = 182, normalized size = 2.12

$$\frac{3a^2c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-3a^2c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{2\left(3a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-11a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+53a^2c\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^4}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*a^2*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a^2*c*log(abs(tan(1/2*
f*x + 1/2*e) - 1)) - 2*(3*a^2*c*tan(1/2*f*x + 1/2*e)^7 - 11*a^2*c*tan(1/2*f
*x + 1/2*e)^5 + 53*a^2*c*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*c*tan(1/2*f*x + 1/2
*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

3.170 $\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=17

$$-\frac{ac\tan^3(e+fx)}{3f}$$

```
[Out] -(a*c*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.0710234, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3962, 2607, 30}

$$-\frac{ac\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] -(a*c*Tan[e + f*x]^3)/(3*f)
```

Rule 3962

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[(-(a*c))^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx &= -\left((ac)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\right)\\ &= -\frac{(ac)\operatorname{Subst}\left(\int x^2\,dx,x,\tan(e+fx)\right)}{f}\\ &= -\frac{ac\tan^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.0186051, size = 17, normalized size = 1.

$$-\frac{ac\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] -(a*c*Tan[e + f*x]^3)/(3*f)
```

Maple [B] time = 0.02, size = 36, normalized size = 2.1

$$\frac{1}{f}\left(ac\tan\left(fx+e\right)+ac\left(-\frac{2}{3}-\frac{\left(\sec\left(fx+e\right)\right)^{2}}{3}\right)\tan\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] 1/f*(a*c*tan(f*x+e)+a*c*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [B] time = 0.97744, size = 49, normalized size = 2.88

$$-\frac{\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)ac-3\,ac\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*a*c - 3*a*c*tan(f*x + e))/f
```

Fricas [B] time = 0.438633, size = 86, normalized size = 5.06

$$\frac{\left(ac\cos\left(fx+e\right)^{2}-ac\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/3*(a*c*cos(f*x + e)^2 - a*c)*sin(f*x + e)/(f*cos(f*x + e)^3)
```

Sympy [A] time = 6.85426, size = 53, normalized size = 3.12

$$\begin{cases} -\frac{ac\left(\frac{\tan^3(e+fx)}{3}+\tan(e+fx)\right)-ac\tan(e+fx)}{f} & \text{for } f \neq 0 \\ x\,(a\sec(e)+a)\,(-c\sec(e)+c)\sec^2(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] Piecewise((-(a*c*(tan(e + f*x)**3/3 + tan(e + f*x)) - a*c*tan(e + f*x))/f,
Ne(f, 0)), (x*(a*sec(e) + a)*(-c*sec(e) + c)*sec(e)**2, True))
```

Giac [A] time = 1.26771, size = 22, normalized size = 1.29

$$-\frac{ac\tan\left(fx+e\right)^3}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/3*a*c*tan(f*x + e)^3/f
```

3.171 $\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=56

$$-\frac{c\tan(e+fx)}{af}+\frac{2c\tanh^{-1}(\sin(e+fx))}{af}-\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

```
[Out] (2*c*ArcTanh[Sin[e + f*x]])/(a*f) - (c*Tan[e + f*x])/(a*f) - (2*c*Tan[e + f
*x])/(f*(a + a*Sec[e + f*x]))
```

Rubi [A] time = 0.108156, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4008, 3787, 3770, 3767, 8}

$$-\frac{c\tan(e+fx)}{af}+\frac{2c\tanh^{-1}(\sin(e+fx))}{af}-\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```


```
[Out] (2*c*ArcTanh[Sin[e + f*x]])/(a*f) - (c*Tan[e + f*x])/(a*f) - (2*c*Tan[e + f
*x])/(f*(a + a*Sec[e + f*x]))
```


Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{\int \sec(e+fx)(-2ac+ac\sec(e+fx))\,dx}{a^2}$$
$$= -\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{c\int \sec^2(e+fx)\,dx}{a} + \frac{(2c)\int \sec(e+fx)\,dx}{a}$$
$$= \frac{2c\tanh^{-1}(\sin(e+fx))}{af} - \frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{c\,\mathrm{Subst}(\int 1\,dx, x, -\tan(e+$$
$$= \frac{2c\tanh^{-1}(\sin(e+fx))}{af} - \frac{c\tan(e+fx)}{af} - \frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}$$

Mathematica [B] time = 0.607151, size = 154, normalized size = 2.75

$$-\frac{c\left(\frac{2\tan\left(\frac{1}{2}(e+fx)\right)}{f} + \frac{\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)} + \frac{\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)} + \frac{2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} - \frac{2\log\left(\sin\left(\frac{1}{2}(e+\right.\right.}{}\right.}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*((2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])/f - (2*Log[Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]])/f + Sin[(e + f*x)/2]/(f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])) + Sin[(e + f*x)/2]/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) +
 (2*Tan[(e + f*x)/2])/f))/a)
```

Maple [A] time = 0.058, size = 104, normalized size = 1.9

$$-2\frac{c\tan\left(1/2\,fx+e/2\right)}{fa} + 2\frac{c\ln\left(\tan\left(1/2\,fx+e/2\right)+1\right)}{fa} + \frac{c}{fa}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1} + \frac{c}{fa}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{-1} - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] -2/f*c/a*tan(1/2*f*x+1/2*e)+2/f*c/a*ln(tan(1/2*f*x+1/2*e)+1)+1/f*c/a/(tan(1
/2*f*x+1/2*e)+1)+1/f*c/a/(tan(1/2*f*x+1/2*e)-1)-2/f*c/a*ln(tan(1/2*f*x+1/2*
e)-1)
```

Maxima [B] time = 1.02407, size = 262, normalized size = 4.68

$$\frac{c\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a} - \frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)} - \frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right) + c\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+\right.}{a}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

[Out] (c*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + c*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))))/f

---

Fricas [A] time = 0.476078, size = 270, normalized size = 4.82

$$\frac{\left(c\cos\left(fx+e\right)^{2}+c\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-\left(c\cos\left(fx+e\right)^{2}+c\cos\left(fx+e\right)\right)\log\left(-\sin\left(fx+e\right)+1\right)-(3}{af\cos\left(fx+e\right)^{2}+af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="fricas")

[Out] ((c*cos(f*x + e)^2 + c*cos(f*x + e))*log(sin(f*x + e) + 1) - (c*cos(f*x + e)^2 + c*cos(f*x + e))*log(-sin(f*x + e) + 1) - (3*c*cos(f*x + e) + c)*sin(f*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c\left(\int -\frac{\sec^{2}\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^{3}\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)**2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)

[Out] -c*(Integral(-sec(e + f*x)**2/(sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) + 1), x))/a

---

Giac [A] time = 1.20791, size = 124, normalized size = 2.21

$$\frac{2\left(\frac{c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a}-\frac{c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a}-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a}+\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)a}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac")

[Out] 2*(c*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - c*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a - c*tan(1/2*f*x + 1/2*e)/a + c*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a))/f

3.172 $\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=70

$$-\frac{c\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{7c\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

```
[Out] -((c*ArcTanh[Sin[e + f*x]])/(a^2*f)) + (7*c*Tan[e + f*x])/(3*a^2*f*(1 + Sec
[e + f*x])) - (2*c*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rubi [A] time = 0.161477, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4008, 3998, 3770, 3794}

$$-\frac{c\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{7c\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -((c*ArcTanh[Sin[e + f*x]])/(a^2*f)) + (7*c*Tan[e + f*x])/(3*a^2*f*(1 + Sec
[e + f*x])) - (2*c*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\int \frac{\sec(e+fx)(-4ac+3ac\sec(e+fx))}{a+a\sec(e+fx)}\,dx}{3a^2}$$
$$= -\frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{c\int \sec(e+fx)\,dx}{a^2} + \frac{(7c)\int \frac{\sec(e+fx)}{a+a\sec(e+fx)}\,dx}{3a}$$
$$= -\frac{c\tanh^{-1}(\sin(e+fx))}{a^2 f} - \frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{7c\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}$$

Mathematica [B] time = 0.451307, size = 335, normalized size = 4.79

$$\frac{c\sec\left(\frac{e}{2}\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left(-6\sin\left(e+\frac{fx}{2}\right)+10\sin\left(e+\frac{3fx}{2}\right)+3\cos\left(e+\frac{3fx}{2}\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c*Cos[(e + f*x)/2]*Sec[e/2]*Sec[e + f*x]^2*(3*Cos[e + (3*f*x)/2]*Log[Cos[(
e + f*x)/2] - Sin[(e + f*x)/2]] + 3*Cos[2*e + (3*f*x)/2]*Log[Cos[(e + f*x)/
2] - Sin[(e + f*x)/2]] + 9*Cos[(f*x)/2]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*
x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 9*Cos[e + (f*x)/2]*(Lo
g[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]]) - 3*Cos[e + (3*f*x)/2]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 3
*Cos[2*e + (3*f*x)/2]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 24*Sin[(f*
x)/2] - 6*Sin[e + (f*x)/2] + 10*Sin[e + (3*f*x)/2]))/(6*a^2*f*(1 + Sec[e +
f*x])^2)
```

Maple [A] time = 0.062, size = 81, normalized size = 1.2

$$\frac{c}{3fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+2\,\frac{c\tan\left(1/2\,fx+e/2\right)}{fa^2}+\frac{c}{fa^2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\frac{c}{fa^2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

```
[Out] 1/3/f*c/a^2*tan(1/2*f*x+1/2*e)^3+2/f*c/a^2*tan(1/2*f*x+1/2*e)+1/f*c/a^2*ln(
tan(1/2*f*x+1/2*e)-1)-1/f*c/a^2*ln(tan(1/2*f*x+1/2*e)+1)
```

Maxima [B] time = 0.977587, size = 194, normalized size = 2.77

$$\frac{c\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(c*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x
 + e)/(cos(f*x + e) + 1) - 1)/a^2) + c*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
 sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [A] time = 0.477495, size = 323, normalized size = 4.61

$$-\frac{3\left(c\cos\left(fx+e\right)^{2}+2\,c\cos\left(fx+e\right)+c\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(c\cos\left(fx+e\right)^{2}+2\,c\cos\left(fx+e\right)+c\right)\log\left(-\sin \cdots\right.}{6\left(a^{2}f\cos\left(fx+e\right)^{2}+2\,a^{2}f\cos\left(fx+e\right)+a^{2}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(3*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*log(sin(f*x + e) + 1) - 3
*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*log(-sin(f*x + e) + 1) - 2*(5*c*
cos(f*x + e) + 7*c)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x +
 e) + a^2*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c\left(\int -\frac{\sec^{2}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^{3}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c*(Integral(-sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A] time = 1.19779, size = 115, normalized size = 1.64

$$-\frac{\frac{3\,c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^{2}}-\frac{3\,c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^{2}}-\frac{a^{4}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+6\,a^{4}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{6}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 3*c*log(abs(tan(1/2*f*x
+ 1/2*e) - 1))/a^2 - (a^4*c*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c*tan(1/2*f*x +
1/2*e))/a^6)/f
```

3.173 $\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=86

$$-\frac{4c\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{11c\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}-\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

```
[Out] (-2*c*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (11*c*Tan[e + f*x])/(15*
a*f*(a + a*Sec[e + f*x])^2) - (4*c*Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f
*x]))
```

Rubi [A] time = 0.163336, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {4008, 4000, 3794}

$$-\frac{4c\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{11c\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}-\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-2*c*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (11*c*Tan[e + f*x])/(15*
a*f*(a + a*Sec[e + f*x])^2) - (4*c*Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f
*x]))
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{\int \frac{\sec(e+fx)(-6ac+5ac\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx}{5a^2}$$
$$= -\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{11c\tan(e+fx)}{15af(a+a\sec(e+fx))^2} - \frac{(4c)\int \frac{\sec(e+fx)}{a+a\sec(e+fx)}\,dx}{15a^2}$$
$$= -\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{11c\tan(e+fx)}{15af(a+a\sec(e+fx))^2} - \frac{4c\tan(e+fx)}{15f\left(a^3+a^3\sec(e+f\right.}$$

Mathematica [A] time = 0.17772, size = 43, normalized size = 0.5

$$-\frac{c(\cos(e+fx)+4)\tan^3\left(\frac{1}{2}(e+fx)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)}{30a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -(c*(4 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^3)/(30*a^3*f)
```

Maple [A] time = 0.071, size = 37, normalized size = 0.4

$$\frac{c}{2fa^3}\left(-\frac{1}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5 - \frac{1}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 1/2/f*c/a^3*(-1/5*tan(1/2*f*x+1/2*e)^5-1/3*tan(1/2*f*x+1/2*e)^3)
```

Maxima [A] time = 1.00829, size = 155, normalized size = 1.8

$$-\frac{\frac{c\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3} - \frac{3c\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(c*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 3*c*(5*sin(f*x +
e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A] time = 0.43997, size = 192, normalized size = 2.23

$$\frac{\left(c\cos\left(fx+e\right)^{2}+3\,c\cos\left(fx+e\right)-4\,c\right)\sin\left(fx+e\right)}{15\left(a^{3}f\cos\left(fx+e\right)^{3}+3\,a^{3}f\cos\left(fx+e\right)^{2}+3\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(c*cos(f*x + e)^2 + 3*c*cos(f*x + e) - 4*c)*sin(f*x + e)/(a^3*f*cos(f*
x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c\left(\int -\frac{\sec^{2}\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^{3}\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c*(Integral(-sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(e + f
*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A] time = 1.25364, size = 53, normalized size = 0.62

$$-\frac{3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+5\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}}{30\,a^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/30*(3*c*tan(1/2*f*x + 1/2*e)^5 + 5*c*tan(1/2*f*x + 1/2*e)^3)/(a^3*f)
```

3.174 $\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=140

$$-\frac{a^2c\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\sec(e+fx))^p\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{p+3}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3f}-\frac{a^2c\tan^3(e+fx)\,c}{}$$

```
[Out] -(a^2*c*(Cos[e + f*x]^2)^((3 + p)/2)*Hypergeometric2F1[3/2, (3 + p)/2, 5/2,
 Sin[e + f*x]^2]*(g*Sec[e + f*x])^p*Tan[e + f*x]^3)/(3*f) - (a^2*c*(Cos[e +
 f*x]^2)^((4 + p)/2)*Hypergeometric2F1[3/2, (4 + p)/2, 5/2, Sin[e + f*x]^2]
*(g*Sec[e + f*x])^(1 + p)*Tan[e + f*x]^3)/(3*f*g)
```

Rubi [A] time = 0.198732, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3962, 2617, 16}

$$-\frac{a^2c\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\sec(e+fx))^p\,{}_2F_1\left(\frac{3}{2},\frac{p+3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f}-\frac{a^2c\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+4}{2}}(g\,s}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```


```
[Out] -(a^2*c*(Cos[e + f*x]^2)^((3 + p)/2)*Hypergeometric2F1[3/2, (3 + p)/2, 5/2,
 Sin[e + f*x]^2]*(g*Sec[e + f*x])^p*Tan[e + f*x]^3)/(3*f) - (a^2*c*(Cos[e +
 f*x]^2)^((4 + p)/2)*Hypergeometric2F1[3/2, (4 + p)/2, 5/2, Sin[e + f*x]^2]
*(g*Sec[e + f*x])^(1 + p)*Tan[e + f*x]^3)/(3*f*g)
```


Rule 3962

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[(-(a*c))^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rubi steps

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = -\left((ac)\int \left(a(g\sec(e+fx))^p\tan^2(e+fx)+a\sec(e+fx)(g\right.\right.$$
$$= -\left(\left(a^2c\right)\int (g\sec(e+fx))^p\tan^2(e+fx)\,dx\right)-\left(a^2c\right)\int \sec$$
$$= -\frac{a^2c\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{3}{2},\frac{3+p}{2};\frac{5}{2};\sin^2(e+fx)\right)(g\sec(e+}{3f}$$
$$= -\frac{a^2c\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{3}{2},\frac{3+p}{2};\frac{5}{2};\sin^2(e+fx)\right)(g\sec(e+}{3f}$$

Mathematica [C] time = 55.6888, size = 13374, normalized size = 95.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.695, size = 0, normalized size = 0.

$$\int \left(g\sec\left(fx+e\right)\right)^p\left(a+a\sec\left(fx+e\right)\right)^2\left(c-c\sec\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

```
[Out] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(a\sec\left(fx+e\right)+a\right)^2\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -integrate((a*sec(f*x + e) + a)^2*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p,
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^2c\sec\left(fx+e\right)^3+a^2c\sec\left(fx+e\right)^2-a^2c\sec\left(fx+e\right)-a^2c\right)\left(g\sec\left(fx+e\right)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral(-(a^2*c*sec(f*x + e)^3 + a^2*c*sec(f*x + e)^2 - a^2*c*sec(f*x + e)
 - a^2*c)*(g*sec(f*x + e))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-a^2c\left(\int -\left(g\sec\left(e+fx\right)\right)^p\,dx + \int -\left(g\sec\left(e+fx\right)\right)^p\sec\left(e+fx\right)dx + \int \left(g\sec\left(e+fx\right)\right)^p\sec^2\left(e+fx\right)dx + \int \left(g\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*c*(Integral(-(g*sec(e + f*x))**p, x) + Integral(-(g*sec(e + f*x))**p*
sec(e + f*x), x) + Integral((g*sec(e + f*x))**p*sec(e + f*x)**2, x) + Integ
ral((g*sec(e + f*x))**p*sec(e + f*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(a\sec\left(fx+e\right)+a\right)^2\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate(-(a*sec(f*x + e) + a)^2*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p,
x)
```

3.175 $\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=65

$$-\frac{ac\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\sec(e+fx))^p\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{p+3}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3f}$$

```
[Out] -(a*c*(Cos[e + f*x]^2)^((3 + p)/2)*Hypergeometric2F1[3/2, (3 + p)/2, 5/2, S
in[e + f*x]^2]*(g*Sec[e + f*x])^p*Tan[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.089211, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3962, 2617}

$$-\frac{ac\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\sec(e+fx))^p\,{}_2F_1\left(\frac{3}{2},\frac{p+3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] -(a*c*(Cos[e + f*x]^2)^((3 + p)/2)*Hypergeometric2F1[3/2, (3 + p)/2, 5/2, S
in[e + f*x]^2]*(g*Sec[e + f*x])^p*Tan[e + f*x]^3)/(3*f)
```

Rule 3962

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[(-(a*c))^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\left((ac)\int (g\sec(e+fx))^p\tan^2(e+fx)\,dx\right)$$

$$= -\frac{ac\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{3}{2},\frac{3+p}{2};\frac{5}{2};\sin^2(e+fx)\right)(g\sec(e+}{3f}$$

Mathematica [A] time = 0.340417, size = 72, normalized size = 1.11

$$-\frac{ac\tan(e+fx)(g\sec(e+fx))^p\left(\frac{\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p}{2},\frac{p+2}{2},\sec^2(e+fx)\right)}{\sqrt{-\tan^2(e+fx)}}+p\right)}{fp(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] -((a*c*(g*Sec[e + f*x])^p*Tan[e + f*x]*(p + Hypergeometric2F1[1/2, p/2, (2
+ p)/2, Sec[e + f*x]^2]/Sqrt[-Tan[e + f*x]^2]))/(f*p*(1 + p)))
```

Maple [F] time = 0.62, size = 0, normalized size = 0.

$$\int \left(g\sec\left(fx+e\right)\right)^{p}\left(a+a\sec\left(fx+e\right)\right)\left(c-c\sec\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(a\sec\left(fx+e\right)+a\right)\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -integrate((a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(ac\sec\left(fx+e\right)^{2}-ac\right)\left(g\sec\left(fx+e\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-(a*c*sec(f*x + e)^2 - a*c)*(g*sec(f*x + e))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-ac\left(\int -\left(g\sec\left(e+fx\right)\right)^{p}dx+\int \left(g\sec\left(e+fx\right)\right)^{p}\sec^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] -a*c*(Integral(-(g*sec(e + f*x))**p, x) + Integral((g*sec(e + f*x))**p*sec(
e + f*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(a\sec\left(fx+e\right)+a\right)\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(-(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p, x)
```

3.176 $\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=180

$$-\frac{cg(1-2p)\sin(e+fx)(g\sec(e+fx))^{p-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-p}{2},\frac{3-p}{2},\cos^2(e+fx)\right)}{af(1-p)\sqrt{\sin^2(e+fx)}}+\frac{2c\sin(e+fx)(g\sec(e+$$

```
[Out] -((c*g*(1 - 2*p)*Hypergeometric2F1[1/2, (1 - p)/2, (3 - p)/2, Cos[e + f*x]^
2]*(g*Sec[e + f*x])^(-1 + p)*Sin[e + f*x])/(a*f*(1 - p)*Sqrt[Sin[e + f*x]^2
])) + (2*c*Hypergeometric2F1[1/2, -p/2, (2 - p)/2, Cos[e + f*x]^2]*(g*Sec[e
 + f*x])^p*Sin[e + f*x])/(a*f*Sqrt[Sin[e + f*x]^2]) - (2*c*(g*Sec[e + f*x])
^p*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rubi [A] time = 0.231625, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4020, 3787, 3772, 2643}

$$-\frac{cg(1-2p)\sin(e+fx)(g\sec(e+fx))^{p-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)}{af(1-p)\sqrt{\sin^2(e+fx)}}+\frac{2c\sin(e+fx)(g\sec(e+fx))^p\,{}_2F_1\left(\frac{1}{2},-\frac{p}{2};\frac{2}{}\right.}{af\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*g*(1 - 2*p)*Hypergeometric2F1[1/2, (1 - p)/2, (3 - p)/2, Cos[e + f*x]^
2]*(g*Sec[e + f*x])^(-1 + p)*Sin[e + f*x])/(a*f*(1 - p)*Sqrt[Sin[e + f*x]^2
])) + (2*c*Hypergeometric2F1[1/2, -p/2, (2 - p)/2, Cos[e + f*x]^2]*(g*Sec[e
 + f*x])^p*Sin[e + f*x])/(a*f*Sqrt[Sin[e + f*x]^2]) - (2*c*(g*Sec[e + f*x])
^p*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{\int(g\sec(e+fx))^p(ac(1-2p)+2acp\sec(e$$

$$= -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{(c(1-2p))\int(g\sec(e+fx))^p\,dx}{a} + \frac{(2cp)\int}{}$$

$$= -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{\left(c(1-2p)\left(\frac{\cos(e+fx)}{g}\right)^p(g\sec(e+fx))^p\right)\int(}{a}$$

$$= -\frac{c(1-2p)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)(g\sec(e+fx))^p\sin(e+}{af(1-p)\sqrt{\sin^2(e+fx)}}$$

Mathematica [C] time = 17.3653, size = 3396, normalized size = 18.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-6*c*Sec[e + f*x]^p*(g*Sec[e + f*x])^p*Tan[(e + f*x)/2]^3*(-((AppellF1[1/2
, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^
2)/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + 2*p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))/(a*f*(3*Sec[(e + f*x)/2]^2*S
ec[e + f*x]^p*(-((AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Cos[(e + f*x)/2]^2)/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + AppellF1
[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2,
 p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*p*(AppellF1[3/2,
p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 +
 p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
 + 6*p*Sec[e + f*x]^(1 + p)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-((AppellF1[1/2,
 p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2
)/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + 2*p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + 6*Sec[e + f*x]^p*Tan[(e + f*
```

```
x)/2]*((AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Cos[(e + f*x)/2]*Sin[(e + f*x)/2])/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (Cos
[(e + f*x)/2]^2*(-((1 - p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (p*AppellF1[
3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f
*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + ((p*AppellF
1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x
)/2]^2*Tan[(e + f*x)/2])/3 + (p*AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)/(3*Appe
llF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*p*(Appel
lF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1
[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2) - (AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*(2*p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*((p*AppellF1[3/2, p, 1 - p, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2])/3 + (p*AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*p*Tan[(e + f*x)/2]^2*((
-3*(1 - p)*AppellF1[5/2, p, 2 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (6*p*AppellF1[5/2, 1 + p, 1
- p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(
e + f*x)/2])/5 + (3*(1 + p)*AppellF1[5/2, 2 + p, -p, 7/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)))/(3*Appell
F1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*p*(AppellF
1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3
/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)
/2]^2)^2 + (AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Cos[(e + f*x)/2]^2*(2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] +
 3*(-((1 - p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (p*AppellF1[3/2, 1 + p, 1
 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[
(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + p)*((-3*(2 - p)*AppellF1[5/2
, p, 3 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/5 + (3*p*AppellF1[5/2, 1 + p, 2 - p, 7/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + p*((-
3*(1 - p)*AppellF1[5/2, 1 + p, 2 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + p)*AppellF1[5/2,
2 + p, 1 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2
]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2)))
```

Maple [F] time = 0.897, size = 0, normalized size = 0.

$$\int \frac{\left(g \sec \left(fx+e\right)\right)^{p} \left(c-c \sec \left(fx+e\right)\right)}{a+a \sec \left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^{p}}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -integrate((c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^{p}}{a\sec\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c\left(\int -\frac{\left(g\sec\left(e+fx\right)\right)^{p}}{\sec\left(e+fx\right)+1}\,dx+\int \frac{\left(g\sec\left(e+fx\right)\right)^{p}\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-(g*sec(e + f*x))**p/(sec(e + f*x) + 1), x) + Integral((g*sec(
e + f*x))**p*sec(e + f*x)/(sec(e + f*x) + 1), x))/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^{p}}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(-(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a), x)
```

3.177 $\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=226

$$-\frac{cg(3-4p)\sin(e+fx)(g\sec(e+fx))^{p-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-p}{2},\frac{3-p}{2},\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}+\frac{c(5-4p)\sin(e+fx)(g\sec}{}$$

```
[Out] -(c*g*(3 - 4*p)*Hypergeometric2F1[1/2, (1 - p)/2, (3 - p)/2, Cos[e + f*x]^2
]*(g*Sec[e + f*x])^(-1 + p)*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) +
(c*(5 - 4*p)*Hypergeometric2F1[1/2, -p/2, (2 - p)/2, Cos[e + f*x]^2]*(g*Sec
[e + f*x])^p*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - (c*(5 - 4*p)*(g
*Sec[e + f*x])^p*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])) - (2*c*(g*Sec[e
 + f*x])^p*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rubi [A] time = 0.412885, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4020, 3787, 3772, 2643}

$$-\frac{cg(3-4p)\sin(e+fx)(g\sec(e+fx))^{p-1}{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}+\frac{c(5-4p)\sin(e+fx)(g\sec(e+fx))^p{}_2F_1\left(\frac{1}{2},-\right.}{3a^2f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```


```
[Out] -(c*g*(3 - 4*p)*Hypergeometric2F1[1/2, (1 - p)/2, (3 - p)/2, Cos[e + f*x]^2
]*(g*Sec[e + f*x])^(-1 + p)*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) +
(c*(5 - 4*p)*Hypergeometric2F1[1/2, -p/2, (2 - p)/2, Cos[e + f*x]^2]*(g*Sec
[e + f*x])^p*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - (c*(5 - 4*p)*(g
*Sec[e + f*x])^p*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])) - (2*c*(g*Sec[e
 + f*x])^p*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```


Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{\int \frac{(g\sec(e+fx))^p(ac(3-2p)-2ac(1-p)\sec(e+fx))}{a+a\sec(e+fx)}\,d}{3a^2}$$

$$= -\frac{c(5-4p)(g\sec(e+fx))^p\tan(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+$$

$$= -\frac{c(5-4p)(g\sec(e+fx))^p\tan(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+$$

$$= -\frac{c(5-4p)(g\sec(e+fx))^p\tan(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+$$

$$= -\frac{c(3-4p)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)(g\sec(e+fx))^p\sin(}{3a^2f\sqrt{\sin^2(e+fx)}}$$

Mathematica [F] time = 3.11801, size = 0, normalized size = 0.

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,
x]
```

```
[Out] Integrate[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,
 x]
```

Maple [F] time = 0.504, size = 0, normalized size = 0.

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^p\left(c-c\sec\left(fx+e\right)\right)}{\left(a+a\sec\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

```
[Out] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^p}{\left(a\sec\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -integrate((c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a)^2,
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^{p}}{a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] integral(-(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a^2*sec(f*x + e)^2 + 2*a
^2*sec(f*x + e) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c\left(\int -\frac{\left(g\sec\left(e+fx\right)\right)^{p}}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{\left(g\sec\left(e+fx\right)\right)^{p}\sec\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c*(Integral(-(g*sec(e + f*x))**p/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x
) + Integral((g*sec(e + f*x))**p*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e +
f*x) + 1), x))/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(c\sec\left(fx+e\right)-c\right)\left(g\sec\left(fx+e\right)\right)^{p}}{\left(a\sec\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] integrate(-(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a)^2,
x)
```

3.178 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=104

$$\frac{2g\cot(e+fx)\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}{cf}-\frac{2\sqrt{a}g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{cf}$$

```
[Out] (-2*Sqrt[a]*g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[g*Sec[e +
f*x]]*Sqrt[a + a*Sec[e + f*x]])])/(c*f) + (2*g*Cot[e + f*x]*Sqrt[g*Sec[e +
f*x]]*Sqrt[a + a*Sec[e + f*x]])/(c*f)
```

Rubi [A] time = 0.278111, antiderivative size = 143, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3964, 47, 63, 217, 203}

$$\frac{2ag^{3/2}\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2ag\tan(e+fx)\sqrt{g\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x]),
x]
```

```
[Out] (-2*a*g*Sqrt[g*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c -
 c*Sec[e + f*x])) + (2*a*g^(3/2)*ArcTan[(Sqrt[c]*Sqrt[g*Sec[e + f*x]])/(Sqr
t[g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c]*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3964

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a*c*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = -\frac{(acg\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{\sqrt{gx}}{(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{\left(ag^2\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{gx}\ldots}\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c}\ldots}$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{(2ag\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-}\ldots}\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{}\ldots}$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{(2ag\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{c\ldots}{g}}\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-}\ldots}$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{2ag^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)\mathrm{ta}\ldots}{\sqrt{c}f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\,\mathrm{se}}\ldots}$$

Mathematica [A] time = 1.53321, size = 162, normalized size = 1.56

$$\frac{2\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}(g\sec(e+fx))^{3/2}\left(\sqrt{\sec(e+fx)}\sqrt{\sec(e+fx)+1}+\sqrt{\tan^2(e+fx)}\left(\log(\sec(e+f\ldots\right.\right.}{cf\sec^{\frac{3}{2}}(e+fx)(\sec(e+fx)+1)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e +
f*x]),x]
```

```
[Out] (2*Cot[(e + f*x)/2]*(g*Sec[e + f*x])^(3/2)*Sqrt[a*(1 + Sec[e + f*x])]*(Sqrt
[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]] + (Log[1 + Sec[e + f*x]] - Log[Sqrt[S
ec[e + f*x]] + Sec[e + f*x]^(3/2) + Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x
]^2]])*Sqrt[Tan[e + f*x]^2]))/(c*f*Sec[e + f*x]^(3/2)*(1 + Sec[e + f*x])^(3
/2))
```

Maple [B] time = 0.342, size = 234, normalized size = 2.3

$$\frac{(-1+\cos(fx+e))(\cos(fx+e))^2}{fc(\sin(fx+e))^4}\left(\cos(fx+e)\,\mathrm{Artanh}\left(\frac{\cos(fx+e)+1-\sin(fx+e)}{2}\sqrt{(1+\cos(fx+e))^{-1}}\right)-\cos\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] 1/c/f*(cos(f*x+e)*arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1-sin(f*
x+e)))-cos(f*x+e)*arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1+sin(f*
x+e)))-2*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)-arctanh(1/2*(1/(1+cos(f*x+e)))
^(1/2)*(cos(f*x+e)+1-sin(f*x+e)))+arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos
(f*x+e)+1+sin(f*x+e))))*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(-1+cos(f*x+e
))*cos(f*x+e)^2*(g/cos(f*x+e))^(3/2)/(1/(1+cos(f*x+e)))^(3/2)/sin(f*x+e)^4
```

Maxima [B] time = 2.06821, size = 1322, normalized size = 12.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] 1/2*(4*sqrt(2)*g*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sqrt(2)*g*cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 4*sqrt(2)*g*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - (g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*s
in(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + g)*log(2*cos(1/4*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) +
(g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + g)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(
2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (g*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + g)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)
*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) + (g*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + g)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin
(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + 2))*sqrt(a)*sqrt(g)/((c*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 - 2*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + c)*f)
```

Fricas [A] time = 0.80428, size = 855, normalized size = 8.22

$$\left[\frac{\sqrt{ag}\,g\log\left(\frac{ag\cos\left(fx+e\right)^{3}-7\,ag\cos\left(fx+e\right)^{2}+4\,\sqrt{ag}\left(\cos\left(fx+e\right)^{2}-2\,\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{g}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)+8\,ag}{\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}}\right)\sin\left(fx+e\right)+4\,g\sqrt{\frac{a\,\mathrm{co}}{\mathrm{c}}}}{2\,cf\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] [1/2*(sqrt(a*g)*g*log((a*g*cos(f*x + e)^3 - 7*a*g*cos(f*x + e)^2 + 4*sqrt(a
*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt(g/cos(f*x + e))*sin(f*x + e) + 8*a*g)/(cos(f*x + e)^3 + cos(f*x + e
)^2))*sin(f*x + e) + 4*g*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos
(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e)), -(sqrt(-a*g)*g*arctan(2*sqrt(-
a*g)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x +
 e)*sin(f*x + e)/(a*g*cos(f*x + e)^2 - a*g*cos(f*x + e) - 2*a*g))*sin(f*x +
 e) - 2*g*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(
f*x + e))/(c*f*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] Timed out
```

3.179 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=81

$$\frac{\cot(e+fx)\sqrt{a\sec(e+fx)+a}}{acf}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}cf}$$

```
[Out] -(ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])]/(Sqrt[2
]*Sqrt[a]*c*f)) + (Cot[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(a*c*f)
```

Rubi [A] time = 0.231719, antiderivative size = 116, normalized size of antiderivative = 1.43, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3964, 78, 63, 208}

$$-\frac{\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))}-\frac{\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\sqrt{c}f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] -(Tan[e + f*x]/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]))) - (ArcTan
h[Sqrt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c
]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3964

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a*c*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{x}{(a+ax)(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{(a\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{(a+ax)\cdots}\right.}{2f\sqrt{a+a\sec(e+fx)}\sqrt{\cdots}}$$

$$= -\frac{\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} - \frac{(a\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\cdots\right.}{cf\sqrt{a+a\sec(e+fx)}\cdots}$$

$$= -\frac{\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} - \frac{\tanh^{-1}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\tan\cdots}{\sqrt{2}\sqrt{c}f\sqrt{a+a\sec(e+fx)}\sqrt{c-\cdots}}$$

Mathematica [A] time = 0.473784, size = 73, normalized size = 0.9

$$-\frac{\cot\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{2}\sqrt{\sec(e+fx)-1}\tan^{-1}\left(\frac{\sqrt{\sec(e+fx)-1}}{\sqrt{2}}\right)-2\right)}{2cf\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] -(Cot[(e + f*x)/2]*(-2 + Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]]*Sq
rt[-1 + Sec[e + f*x]]))/(2*c*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.238, size = 204, normalized size = 2.5

$$-\frac{1}{2fca\left(\left(\cos\left(fx+e\right)\right)^2-1\right)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\left(\cos\left(fx+e\right)\right)^2\ln\left(-\frac{1}{\sin\left(fx+e\right)}\left(-\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sin\left(fx\cdots\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/c/f/a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)^2*ln(-(-(-2*co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)+2*cos(f*x+e)*sin(f*x+e)-ln(-(-(-2*cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e))*(-2*cos(f*x+e)/(
1+cos(f*x+e)))^(1/2))/(cos(f*x+e)^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sec\left(fx+e\right)^2}{\sqrt{a\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] -integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)),
x)
```

Fricas [A] time = 0.549708, size = 676, normalized size = 8.35

$$\left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e)+4\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx\ldots}{4acf\sin(fx+e)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/4*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x
+ e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqr
t(2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 2*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sec^2(e+fx)}{\sqrt{a\sec(e+fx)+a}\sec(e+fx)-\sqrt{a\sec(e+fx)+a}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -Integral(sec(e + f*x)**2/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(a*s
ec(e + f*x) + a)), x)/c
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.180 $\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=140

$$\frac{\csc(e+fx)\sqrt{a\sec(e+fx)+a}}{acf\sqrt{\sec(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(e+fx)\sqrt{\sec(e+fx)}}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{2}\sqrt{a}cf}-\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}cf}$$

```
[Out] (-2*ArcSinh[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/(Sqrt[a]*c*f)
 + ArcTanh[(Sqrt[a]*Sqrt[Sec[e + f*x]]*Sin[e + f*x])/(Sqrt[2]*Sqrt[a + a*Se
c[e + f*x]])]/(Sqrt[2]*Sqrt[a]*c*f) + (Csc[e + f*x]*Sqrt[a + a*Sec[e + f*x]
])/(a*c*f*Sqrt[Sec[e + f*x]])
```

Rubi [A] time = 0.281189, antiderivative size = 213, normalized size of antiderivative = 1.52, number of steps used = 8, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3964, 98, 157, 63, 217, 203, 93, 205}

$$-\frac{\sin(e+fx)\sec^{\frac{3}{2}}(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))}+\frac{2\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{c-c\cdots}}\right)}{\sqrt{2}\sqrt{c}f\sqrt{a\sec(e+fx)+a}\sqrt{c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] -((Sec[e + f*x]^(3/2)*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[
e + f*x]))) + (2*ArcTan[(Sqrt[c]*Sqrt[Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x
]]]*Tan[e + f*x])/(Sqrt[c]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*
x]]) - (ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x]
]]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]])
```

Rule 3964

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a*c*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{(ac\tan(e+fx))\,\text{Subst}\left(\int \frac{x^{3/2}}{(a+ax)(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{\frac{ac}{2}}{\sqrt{x}(a+}\right.}{cf\sqrt{a+a\sec(e+fx}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{c-}}\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{(2\tan(e+fx))\,\text{Subst}\left(\int \right.}{f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{c}f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)\text{t}}{\sqrt{c}f\sqrt{a+a\sec(e+fx)}\sqrt{c-}}$$

Mathematica [B] time = 6.45511, size = 724, normalized size = 5.17

$$\frac{\sin(e+fx)\sin^2\left(\frac{e}{2}+\frac{fx}{2}\right)\cos(e+fx)(\sec(e+fx)+1)^{3/2}\sqrt{\sec^2(e+fx)-1}\left(\log\left(-3\sec^2(e+fx)-2\sqrt{2}\sqrt{\sec(e+fx)}\right.\right.}{2f(\cos(e+fx)+1)\sqrt{2-2\cos}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])
),x]
```

```
[Out] (Sec[e + f*x]^(3/2)*Sqrt[(1 + Cos[e + f*x])*Sec[e + f*x]]*Sqrt[1 + Sec[e +
f*x]]*((-2*Cot[e])/f + (Csc[e/2]*Csc[e/2 + (f*x)/2]*Sin[(f*x)/2])/f + (Sec[
e/2]*Sec[e/2 + (f*x)/2]*Sin[(f*x)/2])/f)*Sin[e/2 + (f*x)/2]^2)/(Sqrt[a*(1 +
 Sec[e + f*x])]*(c - c*Sec[e + f*x])) + (Cos[e + f*x]*(Log[1 - 2*Sec[e + f*
x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]
*Sqrt[-1 + Sec[e + f*x]^2]] - Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 + 2
*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[-1 + Sec[e + f*x]^2
]])*(1 + Sec[e + f*x])^(3/2)*Sqrt[-1 + Sec[e + f*x]^2]*Sin[e/2 + (f*x)/2]^2
*Sin[e + f*x])/(2*f*(1 + Cos[e + f*x])*Sqrt[2 - 2*Cos[e + f*x]^2]*Sqrt[1 -
Cos[e + f*x]^2]*Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x])) + (Cos[e +
 f*x]*(-8*Log[1 + Sec[e + f*x]] + 8*Log[Sqrt[Sec[e + f*x]] + Sec[e + f*x]^(
3/2) + Sqrt[1 + Sec[e + f*x]]*Sqrt[-1 + Sec[e + f*x]^2]] + Sqrt[2]*(-Log[1
- 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 +
 Sec[e + f*x]]*Sqrt[-1 + Sec[e + f*x]^2]] + Log[1 - 2*Sec[e + f*x] - 3*Sec[
e + f*x]^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[-1 +
Sec[e + f*x]^2]]))*(1 + Sec[e + f*x])^(3/2)*Sqrt[-1 + Sec[e + f*x]^2]*Sin[e
/2 + (f*x)/2]^2*Sin[e + f*x])/(2*f*(1 - Cos[e + f*x])*(1 - Cos[e + f*x]^2)*
Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x]))
```

Maple [B] time = 0.346, size = 317, normalized size = 2.3

$$\frac{\left(\cos\left(fx+e\right)\right)^{3}}{cfa\left(\sin\left(fx+e\right)\right)^{2}}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(-\cos\left(fx+e\right)-1+\sin\left(fx+e\right)\right)}{4}\sqrt{-2\left(1+\cos\left(fx+e\right)\right)^{-1}}\right)\cos\left(fx+e\right)+\sqrt{2}\,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/c/f/a*(2^(1/2)*arctan(1/4*2^(1/2)*(-2/(1+cos(f*x+e)))^(1/2)*(-cos(f*x+e)-
1+sin(f*x+e)))*cos(f*x+e)+2^(1/2)*arctan(1/4*2^(1/2)*(-2/(1+cos(f*x+e)))^(1
/2)*(cos(f*x+e)+1+sin(f*x+e)))*cos(f*x+e)-2^(1/2)*arctan(1/4*2^(1/2)*(-2/(1
+cos(f*x+e)))^(1/2)*(-cos(f*x+e)-1+sin(f*x+e)))-2^(1/2)*arctan(1/4*2^(1/2)*
(-2/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1+sin(f*x+e)))+sin(f*x+e)*(-2/(1+cos(
f*x+e)))^(1/2)-arctan(1/2*sin(f*x+e)*(-2/(1+cos(f*x+e)))^(1/2))*cos(f*x+e)+
arctan(1/2*sin(f*x+e)*(-2/(1+cos(f*x+e)))^(1/2)))*(1/cos(f*x+e)*a*(1+cos(f*
x+e)))^(1/2)*(1/cos(f*x+e))^(5/2)*cos(f*x+e)^3/sin(f*x+e)^2/(-2/(1+cos(f*x+
e)))^(1/2)
```

Maxima [B] time = 2.07162, size = 1769, normalized size = 12.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqr
t(2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2))*log(2*cos(1/4*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + 2) - (sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + sqrt(2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2
*sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2))*lo
g(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 2) + (sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + sqrt(2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 sqrt(2))*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*
sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (sqrt(2)*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt
(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/
4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (cos(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
```

```
 1)*log(cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 1) + (cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1)*log(cos(1/4*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e)))^2 - 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 1) - 4*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) - 4*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))))/((sqrt(2)*
c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*c*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*c*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*c)*sqrt(a)*f)
```

Fricas [A] time = 0.647244, size = 1229, normalized size = 8.78

$$\left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(fx+e)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\cos(fx+e)}\sin(fx+e)}{\sqrt{a}}-2\cos(fx+e)-3}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e)+2\sqrt{a}\log\left(\frac{a\cos(fx+e)^3-7a\cos(fx+e)^2+}{}\right.}{4acf\sin(fx+e)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(a)*log(-(cos(f*x + e)^2 - 2*sqrt(2)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt(cos(f*x + e))*sin(f*x + e)/sqrt(a) - 2*cos(f*x + e
) - 3)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 2*sqrt(a)*log(
(a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2*cos(f*x + e)
)*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e)/sqrt(cos(f*x
 + e)) + 8*a)/(cos(f*x + e)^3 + cos(f*x + e)^2))*sin(f*x + e) + 4*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x + e)))/(a*c*f*sin(f*x + e)), -1
/2*(sqrt(2)*a*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt(-1/a)*sqrt(cos(f*x + e))/sin(f*x + e))*sin(f*x + e) + 2*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x + e)
)*sin(f*x + e)/(a*cos(f*x + e)^2 - a*cos(f*x + e) - 2*a))*sin(f*x + e) - 2*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x + e)))/(a*c*f*sin(f*x
+ e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sec\left(fx+e\right)^{\frac{5}{2}}}{\sqrt{a\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(-sec(f*x + e)^(5/2)/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c
)), x)
```

3.181 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=116

$$\frac{g\cot(e+fx)\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}{acf}-\frac{g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf}$$

```
[Out] -((g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[2]*Sqrt[g*Sec[e + f
*x]]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[2]*Sqrt[a]*c*f)) + (g*Cot[e + f*x]*S
qrt[g*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])/(a*c*f)
```

Rubi [A] time = 0.302711, antiderivative size = 150, normalized size of antiderivative = 1.29, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3964, 94, 93, 205}

$$\frac{g^{3/2}\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{c}f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{g\tan(e+fx)\sqrt{g\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),
x]
```

```
[Out] -((g*Sqrt[g*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*
Sec[e + f*x]))) + (g^(3/2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[g*Sec[e + f*x]])/(S
qrt[g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c]*f*Sqrt[a +
 a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3964

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a*c*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}dx = -\frac{(acg\tan(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{gx}}{(a+ax)(c-cx)^{3/2}}dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{g\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{\left(ag^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{}{\sqrt{g}\ldots}\right.}{2f\sqrt{a+a\sec(e+fx}\ldots}$$

$$= -\frac{g\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{\left(ag^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{}{ag\ldots}\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{\ldots}}$$

$$= -\frac{g\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)\ldots}{\sqrt{2}\sqrt{c}f\sqrt{a+a\sec(e+fx)}\sqrt{c-\ldots}}$$

Mathematica [B] time = 2.94445, size = 236, normalized size = 2.03

$$-\frac{a\sin^3\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)(g\sec(e+fx))^{5/2}\left(-4\sec(e+fx)+\frac{\sqrt{\tan^2(e+fx)}\left(\log\left(-3\sec^2(e+fx)-2\sec(e+fx)-2\sqrt{2}\sqrt{\tan^2(e+f\ldots}\right.\right.}{\ldots}\right.}{cfg(\sec(e+fx)-1)^2(a(\sec(e+f\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f
*x])),x]
```

```
[Out] -((a*Cos[(e + f*x)/2]*(g*Sec[e + f*x])^(5/2)*Sin[(e + f*x)/2]^3*(-4 - 4*Sec
[e + f*x] + ((Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqrt[Se
c[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x]^2]] - Log[1 - 2*Sec[e
+ f*x] - 3*Sec[e + f*x]^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f
*x]]*Sqrt[Tan[e + f*x]^2]])*Sqrt[Tan[e + f*x]^2])/Sqrt[Sec[(e + f*x)/2]^2])
)/(c*f*g*(-1 + Sec[e + f*x])^2*(a*(1 + Sec[e + f*x]))^(3/2)))
```

Maple [A] time = 0.305, size = 152, normalized size = 1.3

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)\left(\cos\left(fx+e\right)\right)^2}{2fca\left(\sin\left(fx+e\right)\right)^4}\left(-\cos\left(fx+e\right)\text{Arcsinh}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\sqrt{2}+2\sin\left(fx+e\right)\sqrt{\left(1+\cos\left(fx+\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/c/f/a*(-cos(f*x+e)*arcsinh((-1+cos(f*x+e))/sin(f*x+e))*2^(1/2)+2*sin(f
*x+e)*(1/(1+cos(f*x+e)))^(1/2)+arcsinh((-1+cos(f*x+e))/sin(f*x+e))*2^(1/2))
*(g/cos(f*x+e))^(3/2)*(-1+cos(f*x+e))*cos(f*x+e)^2*(1/cos(f*x+e)*a*(1+cos(f
```

```
*x+e)))^(1/2)/(1/(1+cos(f*x+e)))^(3/2)/sin(f*x+e)^4
```

Maxima [B] time = 1.9542, size = 724, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] 1/2*(4*g*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*g*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - (g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + g)*log(cos(1/4*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + (g*cos(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + g)*log(cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/4*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 4*g*sin(1/4*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(g)/((sqrt(2)*c*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*c*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + sqrt(2)*c)*sqrt(a)*f)
```

Fricas [A] time = 0.509675, size = 818, normalized size = 7.05

$$\left[\frac{\sqrt{2}ag\sqrt{\frac{g}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{g}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+g\cos(fx+e)^2-2g\cos(fx+e)-3g}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e)+4g\sqrt{\frac{a\cos(fx+e}{\cos(fx+}}}{4acf\sin(fx+e)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*a*g*sqrt(g/a)*log(-(2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + g*cos(f
*x + e)^2 - 2*g*cos(f*x + e) - 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*
sin(f*x + e) + 4*g*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x +
 e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqrt(2)*a*g*sqrt(-g/a)*arctan
(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x
+ e))*cos(f*x + e)/(g*sin(f*x + e)))*sin(f*x + e) + 2*g*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(-(g*sec(f*x + e))^(3/2)/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e)
 - c)), x)
```

3.182 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=179

$$\frac{g^2\cot(e+fx)\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}{acf}-\frac{2g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}cf}+\frac{g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\ldots}{\sqrt{2}\sqrt{a\sec(e+f\ldots}}\right.}{\sqrt{2}\sqrt{a}cf}$$

```
[Out] (-2*g^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[g*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]])]])/(Sqrt[a]*c*f) + (g^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[g]
*Tan[e + f*x])/(Sqrt[2]*Sqrt[g*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])]])/(S
qrt[2]*Sqrt[a]*c*f) + (g^2*Cot[e + f*x]*Sqrt[g*Sec[e + f*x]]*Sqrt[a + a*Sec
[e + f*x]])/(a*c*f)
```

Rubi [A] time = 0.357504, antiderivative size = 242, normalized size of antiderivative = 1.35, number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3964, 98, 157, 63, 217, 203, 93, 205}

$$\frac{2g^{5/2}\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{g^{5/2}\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{c}f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{g^2\tan(e+fx)\sqrt{g\,\mathrm{se}\ldots}}{f\sqrt{a\sec(e+fx)+a}(c-\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),
x]
```

```
[Out] -((g^2*Sqrt[g*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c -
c*Sec[e + f*x]))) + (2*g^(5/2)*ArcTan[(Sqrt[c]*Sqrt[g*Sec[e + f*x]])/(Sqrt[
g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c]*f*Sqrt[a + a*Sec[e + f
*x]]*Sqrt[c - c*Sec[e + f*x]]) - (g^(5/2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[g*Se
c[e + f*x]])/(Sqrt[g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[2]*Sqr
t[c]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 3964

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a*c*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{(acg\tan(e+fx))\,\text{Subst}\left(\int \frac{(gx)^{3/2}}{(a+ax)(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{(g\tan(e+fx))\,\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{cf\sqrt{a+a\sec(e+}}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{\left(g^3\tan(e+fx)\right)\text{Subst}\left(\int\right.}{f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{\left(2g^2\tan(e+fx)\right)\text{Subst}\left(\int\right.}{f\sqrt{a+a\sec(e+f}}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} - \frac{g^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right.}{\sqrt{2}\sqrt{c}f\sqrt{a+a\sec(e+fx)}\sqrt{}}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{2g^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right.}{\sqrt{c}f\sqrt{a+a\sec(e+fx)}\sqrt{c-}}$$

Mathematica [A] time = 2.30084, size = 328, normalized size = 1.83

$$-\frac{\sin^3(e+fx)\sqrt{\sec(e+fx)+1}(g\sec(e+fx))^{5/2}\left(8\sqrt{\sec(e+fx)}\sqrt{\sec(e+fx)+1}+\sqrt{\tan^2(e+fx)}\left(16\log(\sec(e+f\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f
*x])),x]
```

```
[Out] -((g*Sec[e + f*x])^(5/2)*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]^3*(8*Sqrt[Sec[
e + f*x]]*Sqrt[1 + Sec[e + f*x]] + (16*Log[1 + Sec[e + f*x]] - 16*Log[Sqrt[
Sec[e + f*x]] + Sec[e + f*x]^(3/2) + Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*
x]^2]] + Sqrt[2]*(Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqr
t[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x]^2]] - Log[1 - 2*Se
c[e + f*x] - 3*Sec[e + f*x]^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e
 + f*x]]*Sqrt[Tan[e + f*x]^2]]))*Sqrt[Tan[e + f*x]^2]))/(8*c*f*(-1 + Cos[e
+ f*x])*(1 + Cos[e + f*x])^2*(-1 + Sec[e + f*x])*Sec[e + f*x]^(5/2)*Sqrt[a*
(1 + Sec[e + f*x])])
```

Maple [A] time = 0.334, size = 294, normalized size = 1.6

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^2\left(\cos\left(fx+e\right)\right)^3}{2\,fca\left(\sin\left(fx+e\right)\right)^6}\left(-\cos\left(fx+e\right)\text{Arcsinh}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\sqrt{2}+2\,\cos\left(fx+e\right)\text{Artanh}\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/c/f/a*(-cos(f*x+e)*arcsinh((-1+cos(f*x+e))/sin(f*x+e))*2^(1/2)+2*cos(f
*x+e)*arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1-sin(f*x+e)))-2*cos
(f*x+e)*arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1+sin(f*x+e)))-2*s
in(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)+arcsinh((-1+cos(f*x+e))/sin(f*x+e))*2^(1
/2)-2*arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1-sin(f*x+e)))+2*arc
tanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1+sin(f*x+e))))*(g/cos(f*x+e)
)^(5/2)*(-1+cos(f*x+e))^2*cos(f*x+e)^3*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2
)/(1/(1+cos(f*x+e)))^(5/2)/sin(f*x+e)^6
```

Maxima [B] time = 2.1389, size = 1890, normalized size = 10.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] 1/2*(4*g^2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*g^2*cos(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 4*g^2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (sqrt(
2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*g^2
*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*g^2*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*g^2)*log(2*cos(
1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 2) + (sqrt(2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + sqrt(2)*g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 - 2*sqrt(2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + sqrt(2)*g^2)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*
cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (sqrt(2)*g^2*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*g^2*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*g^2*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*g^2)*log(2*cos(1/4*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2)
 + (sqrt(2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sq
rt(2)*g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2
)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*g^2)*l
og(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 2) + (g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e)))^2 + g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 - 2*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + g^2)*log(c
os(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 1) - (g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*
g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + g^2)*log(cos(1/4
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f
```

```
*x + 2*e))) + 1))*sqrt(g)/((sqrt(2)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + sqrt(2)*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 - 2*sqrt(2)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + sqrt(2)*c)*sqrt(a)*f)
```

Fricas [A] time = 0.913317, size = 1401, normalized size = 7.83

$$\left[\frac{\sqrt{2} a g^{2} \sqrt{\frac{g}{a}} \log\left(\frac{2 \sqrt{2} \sqrt{\frac{g}{a}} \sqrt{\frac{a \cos\left(f x+e\right)+a}{\cos\left(f x+e\right)}} \sqrt{\frac{g}{\cos\left(f x+e\right)}} \cos\left(f x+e\right) \sin\left(f x+e\right)-g \cos\left(f x+e\right)^{2}+2 g \cos\left(f x+e\right)+3 g}{\cos\left(f x+e\right)^{2}+2 \cos\left(f x+e\right)+1}\right) \sin\left(f x+e\right)+2 a g^{2} \sqrt{\frac{g}{a}} \log\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*a*g^2*sqrt(g/a)*log((2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - g*cos(
f*x + e)^2 + 2*g*cos(f*x + e) + 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))
*sin(f*x + e) + 2*a*g^2*sqrt(g/a)*log((g*cos(f*x + e)^3 + 4*(cos(f*x + e)^2
 - 2*cos(f*x + e))*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g
/cos(f*x + e))*sin(f*x + e) - 7*g*cos(f*x + e)^2 + 8*g)/(cos(f*x + e)^3 + c
os(f*x + e)^2))*sin(f*x + e) + 4*g^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), -1/2*(sqrt(2)*a*
g^2*sqrt(-g/a)*arctan(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sqrt(g/cos(f*x + e))*cos(f*x + e)/(g*sin(f*x + e)))*sin(f*x + e) + 2*
a*g^2*sqrt(-g/a)*arctan(2*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(g*cos(f*x + e)^2 - g*cos(
f*x + e) - 2*g))*sin(f*x + e) - 2*g^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(g \sec\left(f x+e\right)\right)^{\frac{5}{2}}}{\sqrt{a \sec\left(f x+e\right)+a}\left(c \sec\left(f x+e\right)-c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(-(g*sec(f*x + e))^(5/2)/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e)
 - c)), x)
```

3.183 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\tan(e+fx)\log(\tan(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] (Log[Tan[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]])
```

Rubi [A] time = 0.255427, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {3963, 2620, 29}

$$\frac{\tan(e+fx)\log(\tan(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (Log[Tan[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]])
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[(
(-(a*c))^(m + 1/2)*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e
 + f*x]]), Int[(g*Csc[e + f*x])^p*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b
, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && Intege
rQ[m + 1/2]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx &= \frac{\tan(e+fx)\int \csc(e+fx)\sec(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tan(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= \frac{\log(\tan(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.788904, size = 94, normalized size = 2.04

$$\frac{4i\left(-1+e^{i(e+fx)}\right)\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\tanh^{-1}\left(e^{2i(e+fx)}\right)}{f\left(1+e^{i(e+fx)}\right)\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
),x]
```

```
[Out] ((4*I)*(-1 + E^(I*(e + f*x)))*ArcTanh[E^((2*I)*(e + f*x))]*Cos[(e + f*x)/2]
^2*Sec[e + f*x])/((1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
 - c*Sec[e + f*x]])
```

Maple [B] time = 0.276, size = 138, normalized size = 3.

$$\frac{\cos\left(fx+e\right)}{af\sin\left(fx+e\right)c}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\left(\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f/a*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(ln((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))+ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e))/s
in(f*x+e)))*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/c
```

Maxima [A] time = 1.88718, size = 76, normalized size = 1.65

$$-\frac{\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)-\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)-1\right)}{\sqrt{a}\sqrt{c}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -(arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) - 1))/(sqrt(a)*sqrt(c)*f)
```

Fricas [B] time = 0.735096, size = 613, normalized size = 13.33

$$\left[-\frac{\sqrt{-ac}\log\left(-\frac{8\left(\left(2\,\cos\left(fx+e\right)^3-\cos\left(fx+e\right)\right)\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(2\,ac\cos\left(fx+e\right)^4-2\,ac\cos\left(fx+e\right)^2+ac\right)\sin\left(fx+e\right)\right)}{\left(\cos\left(fx+e\right)^4-\cos\left(fx+e\right)^2\right)\sin\left(fx+e\right)}\right)}{2\,acf},\ \frac{\sqrt{ac}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(-8*((2*cos(f*x + e)^3 - cos(f*x + e))*sqrt(-a*c)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))
+ (2*a*c*cos(f*x + e)^4 - 2*a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f
*x + e)^4 - cos(f*x + e)^2)*sin(f*x + e)))/(a*c*f), sqrt(a*c)*arctan(sqrt(a
*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)/((2*a*c*cos(f*x + e)^2 - a*c)*sin(f*x + e)))/(a*c*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x)
- 1))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.184 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx$

Optimal. Leaf size=65

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c-d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{d}f\sqrt{c-d}}$$

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c - d]*Sqrt[a + a*S
ec[e + f*x]])])/(Sqrt[c - d]*Sqrt[d]*f)
```

Rubi [A] time = 0.159167, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3967, 208}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c-d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{d}f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c - d]*Sqrt[a + a*S
ec[e + f*x]])])/(Sqrt[c - d]*Sqrt[d]*f)
```

Rule 3967

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b)/f, Subst[Int
[1/(b*c + a*d + d*x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx &= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{ac-ad-dx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\ &= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c-d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{c-d}\sqrt{d}f}\end{aligned}$$

Mathematica [A] time = 0.260163, size = 98, normalized size = 1.51

$$\frac{\sqrt{2}\sqrt{\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c-d}\sqrt{\cos(e+fx)}}\right)}{\sqrt{d}f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - d*Sec[e + f*x]),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c - d]*Sqrt[Cos[e
 + f*x]])]*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/
(Sqrt[c - d]*Sqrt[d]*f)
```

Maple [B] time = 0.272, size = 411, normalized size = 6.3

$$-\frac{1}{f}\left(\ln\left(2\frac{1}{\sqrt{(c+d)(c-d)}\sin\left(fx+e\right)-c\cos\left(fx+e\right)-d\cos\left(fx+e\right)+c+d}\left(\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{-2\frac{d}{c+d}}c\sin\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x)
```

```
[Out] -1/f/(-2*d/(c+d))^(1/2)/((c+d)*(c-d))^(1/2)*(ln(2*((-2*cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*(-2*d/(c+d))^(1/2)*c*sin(f*x+e)+(-2*d/(c+d))^(1/2)*(-2*cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin
(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c
*cos(f*x+e)-d*cos(f*x+e)+c+d))-ln(-2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
(-2*d/(c+d))^(1/2)*c*sin(f*x+e)+(-2*d/(c+d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)-d*sin
(f*x+e)+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)+d
*cos(f*x+e)-c-d)))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/cos(f*x+e)*a*(1+
cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{d\sec\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) - c), x)
```

Fricas [B] time = 1.04891, size = 802, normalized size = 12.34

$$\left[\frac{\sqrt{\frac{a}{cd-d^2}}\log\left(-\frac{\left(ac^2-8\,acd+8\,ad^2\right)\cos\left(fx+e\right)^3+ad^2+\left(ac^2-2\,acd\right)\cos\left(fx+e\right)^2+4\left(\left(c^2d-3\,cd^2+2\,d^3\right)\cos\left(fx+e\right)^2+\left(cd^2-d^3\right)\cos\left(fx+e\right)\right)\sqrt{\frac{a}{cd-d^2}}\sqrt{\frac{a\cos\left(fx+\right.}{\cos\left(fx\right.}}}{c^2\cos\left(fx+e\right)^3+\left(c^2-2\,cd\right)\cos\left(fx+e\right)^2+d^2-\left(2\,cd-d^2\right)\cos\left(fx+e\right)}\right.}{2\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [1/2*sqrt(a/(c*d - d^2))*log(-((a*c^2 - 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 +
 a*d^2 + (a*c^2 - 2*a*c*d)*cos(f*x + e)^2 + 4*((c^2*d - 3*c*d^2 + 2*d^3)*co
s(f*x + e)^2 + (c*d^2 - d^3)*cos(f*x + e))*sqrt(a/(c*d - d^2))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + (6*a*c*d - 7*a*d^2)*cos(f*x + e)
)/(c^2*cos(f*x + e)^3 + (c^2 - 2*c*d)*cos(f*x + e)^2 + d^2 - (2*c*d - d^2)*
cos(f*x + e)))/f, -sqrt(-a/(c*d - d^2))*arctan(2*(c*d - d^2)*sqrt(-a/(c*d -
 d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((
a*c - 2*a*d)*cos(f*x + e)^2 + a*d + (a*c - a*d)*cos(f*x + e)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{c-d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(c - d*sec(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.185 $\int \sec(e + fx)(a + a\sec(e + fx))(c + d\sec(e + fx))^4 \, dx$

Optimal. Leaf size=236

$$\frac{a\left(112c^2d^2 + 95c^3d + 12c^4 + 80cd^3 + 16d^4\right)\tan(e+fx)}{30f} + \frac{a\left(24c^2d^2 + 16c^3d + 8c^4 + 12cd^3 + 3d^4\right)\tanh^{-1}(\sin(e+fx$$

```
[Out] (a*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4)*ArcTanh[Sin[e + f*x]]
)/(8*f) + (a*(12*c^4 + 95*c^3*d + 112*c^2*d^2 + 80*c*d^3 + 16*d^4)*Tan[e +
f*x])/(30*f) + (a*d*(24*c^3 + 130*c^2*d + 116*c*d^2 + 45*d^3)*Sec[e + f*x]*
Tan[e + f*x])/(120*f) + (a*(12*c^2 + 35*c*d + 16*d^2)*(c + d*Sec[e + f*x])^
2*Tan[e + f*x])/(60*f) + (a*(4*c + 5*d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x]
)/(20*f) + (a*(c + d*Sec[e + f*x])^4*Tan[e + f*x])/(5*f)
```

Rubi [A] time = 0.437478, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{a\left(112c^2d^2 + 95c^3d + 12c^4 + 80cd^3 + 16d^4\right)\tan(e+fx)}{30f} + \frac{a\left(24c^2d^2 + 16c^3d + 8c^4 + 12cd^3 + 3d^4\right)\tanh^{-1}(\sin(e+fx$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```


```
[Out] (a*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4)*ArcTanh[Sin[e + f*x]]
)/(8*f) + (a*(12*c^4 + 95*c^3*d + 112*c^2*d^2 + 80*c*d^3 + 16*d^4)*Tan[e +
f*x])/(30*f) + (a*d*(24*c^3 + 130*c^2*d + 116*c*d^2 + 45*d^3)*Sec[e + f*x]*
Tan[e + f*x])/(120*f) + (a*(12*c^2 + 35*c*d + 16*d^2)*(c + d*Sec[e + f*x])^
2*Tan[e + f*x])/(60*f) + (a*(4*c + 5*d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x]
)/(20*f) + (a*(c + d*Sec[e + f*x])^4*Tan[e + f*x])/(5*f)
```


Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx = \frac{a(c+d\sec(e+fx))^4\tan(e+fx)}{5f}+\frac{1}{5}\int \sec(e+fx)(c+d\sec$$

$$=\frac{a(4c+5d)(c+d\sec(e+fx))^3\tan(e+fx)}{20f}+\frac{a(c+d\sec(e+f}{5}$$

$$=\frac{a\left(12c^2+35cd+16d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{60f}+\frac{a(4}{}$$

$$=\frac{ad\left(24c^3+130c^2d+116cd^2+45d^3\right)\sec(e+fx)\tan(e+fx)}{120f}+$$

$$=\frac{ad\left(24c^3+130c^2d+116cd^2+45d^3\right)\sec(e+fx)\tan(e+fx)}{120f}+$$

$$=\frac{a\left(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\right)\tanh^{-1}(\sin(e+fx))}{8f}+$$

$$=\frac{a\left(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\right)\tanh^{-1}(\sin(e+fx))}{8f}+$$

Mathematica [A] time = 1.79959, size = 153, normalized size = 0.65

$$\frac{a\left(15\left(24c^2d^2+16c^3d+8c^4+12cd^3+3d^4\right)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(80d^2\left(3c^2+2cd+d^2\right)\tan^2(e+fx)+15d\right.}{120f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a*(15*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4)*ArcTanh[Sin[e + f
*x]] + Tan[e + f*x]*(120*(c + d)^4 + 15*d*(16*c^3 + 24*c^2*d + 12*c*d^2 + 3
*d^3)*Sec[e + f*x] + 30*d^3*(4*c + d)*Sec[e + f*x]^3 + 80*d^2*(3*c^2 + 2*c*
d + d^2)*Tan[e + f*x]^2 + 24*d^4*Tan[e + f*x]^4)))/(120*f)
```

Maple [A] time = 0.057, size = 431, normalized size = 1.8

$$\frac{ac^4\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+4\frac{ac^3d\tan\left(fx+e\right)}{f}+3\frac{ac^2d^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{f}+3\frac{ac^2d^2\ln\left(\sec\left(fx+e\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x)
```

```
[Out] 1/f*a*c^4*ln(sec(f*x+e)+tan(f*x+e))+4/f*a*c^3*d*tan(f*x+e)+3/f*a*c^2*d^2*se
c(f*x+e)*tan(f*x+e)+3/f*a*c^2*d^2*ln(sec(f*x+e)+tan(f*x+e))+8/3/f*a*c*d^3*t
an(f*x+e)+4/3/f*a*c*d^3*tan(f*x+e)*sec(f*x+e)^2+1/4/f*a*d^4*tan(f*x+e)*sec(
f*x+e)^3+3/8/f*a*d^4*sec(f*x+e)*tan(f*x+e)+3/8/f*a*d^4*ln(sec(f*x+e)+tan(f*
x+e))+1/f*a*c^4*tan(f*x+e)+2/f*a*c^3*d*sec(f*x+e)*tan(f*x+e)+2/f*a*c^3*d*ln
(sec(f*x+e)+tan(f*x+e))+4/f*a*c^2*d^2*tan(f*x+e)+2/f*a*c^2*d^2*tan(f*x+e)*s
ec(f*x+e)^2+1/f*a*c*d^3*tan(f*x+e)*sec(f*x+e)^3+3/2/f*a*c*d^3*sec(f*x+e)*ta
n(f*x+e)+3/2/f*a*c*d^3*ln(sec(f*x+e)+tan(f*x+e))+8/15/f*a*d^4*tan(f*x+e)+1/
5/f*a*d^4*tan(f*x+e)*sec(f*x+e)^4+4/15/f*a*d^4*tan(f*x+e)*sec(f*x+e)^2
```

Maxima [A] time = 0.991393, size = 512, normalized size = 2.17

$$480\left(\tan\left(fx+e\right)^{3}+3\tan\left(fx+e\right)\right)ac^{2}d^{2}+320\left(\tan\left(fx+e\right)^{3}+3\tan\left(fx+e\right)\right)acd^{3}+16\left(3\tan\left(fx+e\right)^{5}+10\,\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/240*(480*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^2*d^2 + 320*(tan(f*x + e)^
3 + 3*tan(f*x + e))*a*c*d^3 + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15
*tan(f*x + e))*a*d^4 - 60*a*c*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(s
in(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin
(f*x + e) - 1)) - 15*a*d^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x
+ e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x +
e) - 1)) - 240*a*c^3*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
 e) + 1) + log(sin(f*x + e) - 1)) - 360*a*c^2*d^2*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a*c^4*lo
g(sec(f*x + e) + tan(f*x + e)) + 240*a*c^4*tan(f*x + e) + 960*a*c^3*d*tan(f
*x + e))/f
```

Fricas [A] time = 0.540925, size = 687, normalized size = 2.91

$$15\left(8\,ac^{4}+16\,ac^{3}d+24\,ac^{2}d^{2}+12\,acd^{3}+3\,ad^{4}\right)\cos\left(fx+e\right)^{5}\log\left(\sin\left(fx+e\right)+1\right)-15\left(8\,ac^{4}+16\,ac^{3}d+24\,ac^{2}d^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/240*(15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4)*cos(
f*x + e)^5*log(sin(f*x + e) + 1) - 15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2
+ 12*a*c*d^3 + 3*a*d^4)*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(24*a*d^4
 + 8*(15*a*c^4 + 60*a*c^3*d + 60*a*c^2*d^2 + 40*a*c*d^3 + 8*a*d^4)*cos(f*x
+ e)^4 + 15*(16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4)*cos(f*x + e)
```

```
^3 + 16*(15*a*c^2*d^2 + 10*a*c*d^3 + 2*a*d^4)*cos(f*x + e)^2 + 30*(4*a*c*d^
3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int c^4\sec\left(e+fx\right)dx+\int c^4\sec^2\left(e+fx\right)dx+\int d^4\sec^5\left(e+fx\right)dx+\int d^4\sec^6\left(e+fx\right)dx+\int 4cd^3\sec^4\left(e+fx\right)d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))**4,x)
```

```
[Out] a*(Integral(c**4*sec(e + f*x), x) + Integral(c**4*sec(e + f*x)**2, x) + Int
egral(d**4*sec(e + f*x)**5, x) + Integral(d**4*sec(e + f*x)**6, x) + Integr
al(4*c*d**3*sec(e + f*x)**4, x) + Integral(4*c*d**3*sec(e + f*x)**5, x) + I
ntegral(6*c**2*d**2*sec(e + f*x)**3, x) + Integral(6*c**2*d**2*sec(e + f*x)
**4, x) + Integral(4*c**3*d*sec(e + f*x)**2, x) + Integral(4*c**3*d*sec(e +
 f*x)**3, x))
```

Giac [B] time = 1.51785, size = 802, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] 1/120*(15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4)*log(
abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2 +
12*a*c*d^3 + 3*a*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(120*a*c^4*tan
(1/2*f*x + 1/2*e)^9 + 240*a*c^3*d*tan(1/2*f*x + 1/2*e)^9 + 360*a*c^2*d^2*ta
n(1/2*f*x + 1/2*e)^9 + 180*a*c*d^3*tan(1/2*f*x + 1/2*e)^9 + 45*a*d^4*tan(1/
2*f*x + 1/2*e)^9 - 480*a*c^4*tan(1/2*f*x + 1/2*e)^7 - 1440*a*c^3*d*tan(1/2*
f*x + 1/2*e)^7 - 1200*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 1160*a*c*d^3*tan(1
/2*f*x + 1/2*e)^7 - 130*a*d^4*tan(1/2*f*x + 1/2*e)^7 + 720*a*c^4*tan(1/2*f*
x + 1/2*e)^5 + 2880*a*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 2400*a*c^2*d^2*tan(1/2
*f*x + 1/2*e)^5 + 1600*a*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 464*a*d^4*tan(1/2*f
*x + 1/2*e)^5 - 480*a*c^4*tan(1/2*f*x + 1/2*e)^3 - 2400*a*c^3*d*tan(1/2*f*x
 + 1/2*e)^3 - 2640*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 1400*a*c*d^3*tan(1/2*
f*x + 1/2*e)^3 - 190*a*d^4*tan(1/2*f*x + 1/2*e)^3 + 120*a*c^4*tan(1/2*f*x +
 1/2*e) + 720*a*c^3*d*tan(1/2*f*x + 1/2*e) + 1080*a*c^2*d^2*tan(1/2*f*x + 1
/2*e) + 780*a*c*d^3*tan(1/2*f*x + 1/2*e) + 195*a*d^4*tan(1/2*f*x + 1/2*e))/
(tan(1/2*f*x + 1/2*e)^2 - 1)^5)/f
```

3.186 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=171

$$\frac{a\left(16c^2d+3c^3+12cd^2+4d^3\right)\tan(e+fx)}{6f}+\frac{a\left(12c^2d+8c^3+12cd^2+3d^3\right)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{ad\left(6c^2+20cd+\right.}{}$$

```
[Out] (a*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*ArcTanh[Sin[e + f*x]])/(8*f) + (a*
(3*c^3 + 16*c^2*d + 12*c*d^2 + 4*d^3)*Tan[e + f*x])/(6*f) + (a*d*(6*c^2 + 2
0*c*d + 9*d^2)*Sec[e + f*x]*Tan[e + f*x])/(24*f) + (a*(3*c + 4*d)*(c + d*Se
c[e + f*x])^2*Tan[e + f*x])/(12*f) + (a*(c + d*Sec[e + f*x])^3*Tan[e + f*x]
)/(4*f)
```

Rubi [A] time = 0.295138, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{a\left(16c^2d+3c^3+12cd^2+4d^3\right)\tan(e+fx)}{6f}+\frac{a\left(12c^2d+8c^3+12cd^2+3d^3\right)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{ad\left(6c^2+20cd+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*ArcTanh[Sin[e + f*x]])/(8*f) + (a*
(3*c^3 + 16*c^2*d + 12*c*d^2 + 4*d^3)*Tan[e + f*x])/(6*f) + (a*d*(6*c^2 + 2
0*c*d + 9*d^2)*Sec[e + f*x]*Tan[e + f*x])/(24*f) + (a*(3*c + 4*d)*(c + d*Se
c[e + f*x])^2*Tan[e + f*x])/(12*f) + (a*(c + d*Sec[e + f*x])^3*Tan[e + f*x]
)/(4*f)
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx = \frac{a(c+d\sec(e+fx))^3\tan(e+fx)}{4f}+\frac{1}{4}\int \sec(e+fx)(c+d\sec$$
$$= \frac{a(3c+4d)(c+d\sec(e+fx))^2\tan(e+fx)}{12f}+\frac{a(c+d\sec(e+f}{4}$$
$$= \frac{ad\left(6c^2+20cd+9d^2\right)\sec(e+fx)\tan(e+fx)}{24f}+\frac{a(3c+4d)(c}{}$$
$$= \frac{ad\left(6c^2+20cd+9d^2\right)\sec(e+fx)\tan(e+fx)}{24f}+\frac{a(3c+4d)(c}{}$$
$$= \frac{a\left(8c^3+12c^2d+12cd^2+3d^3\right)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{ad\left(6c^2\right.}{}$$
$$= \frac{a\left(8c^3+12c^2d+12cd^2+3d^3\right)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a\left(3c^3+\right.}{}$$

Mathematica [A] time = 0.677162, size = 103, normalized size = 0.6

$$\frac{a\left(3\left(12c^2d+8c^3+12cd^2+3d^3\right)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(8d^2(3c+d)\tan^2(e+fx)+9d(2c+d)^2\sec(e+fx)\right.\right.}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(3*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*ArcTanh[Sin[e + f*x]] + Tan[e +
 f*x]*(24*(c + d)^3 + 9*d*(2*c + d)^2*Sec[e + f*x] + 6*d^3*Sec[e + f*x]^3 +
 8*d^2*(3*c + d)*Tan[e + f*x]^2)))/(24*f)
```

Maple [A] time = 0.049, size = 290, normalized size = 1.7

$$\frac{ac^3\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+3\frac{ac^2d\tan\left(fx+e\right)}{f}+\frac{3\,ad^2c\sec\left(fx+e\right)\tan\left(fx+e\right)}{2\,f}+\frac{3\,ad^2c\ln\left(\sec\left(fx+e\right)+\right.}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x)
```

```
[Out] 1/f*a*c^3*ln(sec(f*x+e)+tan(f*x+e))+3/f*a*c^2*d*tan(f*x+e)+3/2/f*a*d^2*c*se
c(f*x+e)*tan(f*x+e)+3/2/f*a*d^2*c*ln(sec(f*x+e)+tan(f*x+e))+2/3/f*a*d^3*tan
(f*x+e)+1/3/f*a*d^3*tan(f*x+e)*sec(f*x+e)^2+1/f*a*c^3*tan(f*x+e)+3/2/f*a*c^
2*d*sec(f*x+e)*tan(f*x+e)+3/2/f*a*c^2*d*ln(sec(f*x+e)+tan(f*x+e))+2/f*a*d^2
*c*tan(f*x+e)+1/f*a*d^2*c*tan(f*x+e)*sec(f*x+e)^2+1/4/f*a*d^3*tan(f*x+e)*se
c(f*x+e)^3+3/8/f*a*d^3*sec(f*x+e)*tan(f*x+e)+3/8/f*a*d^3*ln(sec(f*x+e)+tan(
f*x+e))
```

Maxima [A] time = 0.98131, size = 359, normalized size = 2.1

$$48\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)acd^2+16\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)ad^3-3ad^3\left(\frac{2\left(3\sin\left(fx+e\right)^3-5\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^4-2\sin\left(fx+e\right)^2+1}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(48*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c*d^2 + 16*(tan(f*x + e)^3 + 3
*tan(f*x + e))*a*d^3 - 3*a*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(
f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*
x + e) - 1)) - 36*a*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*
x + e) + 1) + log(sin(f*x + e) - 1)) - 36*a*c*d^2*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 48*a*c^3*log
(sec(f*x + e) + tan(f*x + e)) + 48*a*c^3*tan(f*x + e) + 144*a*c^2*d*tan(f*x
 + e))/f
```

Fricas [A] time = 0.514157, size = 510, normalized size = 2.98

$$3\left(8ac^3+12ac^2d+12acd^2+3ad^3\right)\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-3\left(8ac^3+12ac^2d+12acd^2+3ad^3\right)\cos\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*cos(f*x + e)^4*log(si
n(f*x + e) + 1) - 3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*cos(f*x +
 e)^4*log(-sin(f*x + e) + 1) + 2*(6*a*d^3 + 8*(3*a*c^3 + 9*a*c^2*d + 6*a*c*
d^2 + 2*a*d^3)*cos(f*x + e)^3 + 9*(4*a*c^2*d + 4*a*c*d^2 + a*d^3)*cos(f*x +
 e)^2 + 8*(3*a*c*d^2 + a*d^3)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int c^3\sec\left(e+fx\right)dx+\int c^3\sec^2\left(e+fx\right)dx+\int d^3\sec^4\left(e+fx\right)dx+\int d^3\sec^5\left(e+fx\right)dx+\int 3cd^2\sec^3\left(e+f\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))**3,x)
```

```
[Out] a*(Integral(c**3*sec(e + f*x), x) + Integral(c**3*sec(e + f*x)**2, x) + Int
egral(d**3*sec(e + f*x)**4, x) + Integral(d**3*sec(e + f*x)**5, x) + Integr
al(3*c*d**2*sec(e + f*x)**3, x) + Integral(3*c*d**2*sec(e + f*x)**4, x) + I
ntegral(3*c**2*d*sec(e + f*x)**2, x) + Integral(3*c**2*d*sec(e + f*x)**3, x
))
```

Giac [B] time = 1.62895, size = 539, normalized size = 3.15

$$3\left(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\left(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/24*(3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*log(abs(tan(1/2*f*x +
 1/2*e) + 1)) - 3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*log(abs(tan
(1/2*f*x + 1/2*e) - 1)) - 2*(24*a*c^3*tan(1/2*f*x + 1/2*e)^7 + 36*a*c^2*d*t
an(1/2*f*x + 1/2*e)^7 + 36*a*c*d^2*tan(1/2*f*x + 1/2*e)^7 + 9*a*d^3*tan(1/2
*f*x + 1/2*e)^7 - 72*a*c^3*tan(1/2*f*x + 1/2*e)^5 - 180*a*c^2*d*tan(1/2*f*x
 + 1/2*e)^5 - 84*a*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 49*a*d^3*tan(1/2*f*x + 1/
2*e)^5 + 72*a*c^3*tan(1/2*f*x + 1/2*e)^3 + 252*a*c^2*d*tan(1/2*f*x + 1/2*e)
^3 + 156*a*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 31*a*d^3*tan(1/2*f*x + 1/2*e)^3 -
 24*a*c^3*tan(1/2*f*x + 1/2*e) - 108*a*c^2*d*tan(1/2*f*x + 1/2*e) - 108*a*c
*d^2*tan(1/2*f*x + 1/2*e) - 39*a*d^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1
/2*e)^2 - 1)^4)/f
```

3.187 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=108

$$\frac{2a\left(c^2+3cd+d^2\right)\tan(e+fx)}{3f}+\frac{a\left(2c^2+2cd+d^2\right)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a\tan(e+fx)(c+d\sec(e+fx))^2}{3f}+\frac{ad(2c}{}$$

```
[Out] (a*(2*c^2 + 2*c*d + d^2)*ArcTanh[Sin[e + f*x]])/(2*f) + (2*a*(c^2 + 3*c*d +
 d^2)*Tan[e + f*x])/(3*f) + (a*d*(2*c + 3*d)*Sec[e + f*x]*Tan[e + f*x])/(6*
f) + (a*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f)
```

Rubi [A] time = 0.165014, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{2a\left(c^2+3cd+d^2\right)\tan(e+fx)}{3f}+\frac{a\left(2c^2+2cd+d^2\right)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a\tan(e+fx)(c+d\sec(e+fx))^2}{3f}+\frac{ad(2c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```


```
[Out] (a*(2*c^2 + 2*c*d + d^2)*ArcTanh[Sin[e + f*x]])/(2*f) + (2*a*(c^2 + 3*c*d +
 d^2)*Tan[e + f*x])/(3*f) + (a*d*(2*c + 3*d)*Sec[e + f*x]*Tan[e + f*x])/(6*
f) + (a*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f)
```


Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx = \frac{a(c+d\sec(e+fx))^2\tan(e+fx)}{3f}+\frac{1}{3}\int \sec(e+fx)(c+d\sec$$
$$= \frac{ad(2c+3d)\sec(e+fx)\tan(e+fx)}{6f}+\frac{a(c+d\sec(e+fx))^2\ \mathrm{ta}}{3f}$$
$$= \frac{ad(2c+3d)\sec(e+fx)\tan(e+fx)}{6f}+\frac{a(c+d\sec(e+fx))^2\ \mathrm{ta}}{3f}$$
$$= \frac{a\left(2c^2+2cd+d^2\right)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{ad(2c+3d)\sec(e+}{6f}$$
$$= \frac{a\left(2c^2+2cd+d^2\right)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{2a\left(c^2+3cd+d^2\right)\mathrm{t}}{3f}$$

Mathematica [A] time = 0.413438, size = 75, normalized size = 0.69

$$\frac{a\left(3\left(2c^2+2cd+d^2\right)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(2\left(3(c+d)^2+d^2\tan^2(e+fx)\right)+3d(2c+d)\sec(e+fx)\right)\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a*(3*(2*c^2 + 2*c*d + d^2)*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(3*d*(2*c
+ d)*Sec[e + f*x] + 2*(3*(c + d)^2 + d^2*Tan[e + f*x]^2))))/(6*f)
```

Maple [A] time = 0.042, size = 174, normalized size = 1.6

$$\frac{c^2a\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+2\frac{acd\tan\left(fx+e\right)}{f}+\frac{ad^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}+\frac{ad^2\ln\left(\sec\left(fx+e\right)+\tan\left(\right.\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x)
```

```
[Out] 1/f*c^2*a*ln(sec(f*x+e)+tan(f*x+e))+2/f*a*c*d*tan(f*x+e)+1/2/f*a*d^2*sec(f*
x+e)*tan(f*x+e)+1/2/f*a*d^2*ln(sec(f*x+e)+tan(f*x+e))+1/f*c^2*a*tan(f*x+e)+
1/f*a*c*d*sec(f*x+e)*tan(f*x+e)+1/f*a*c*d*ln(sec(f*x+e)+tan(f*x+e))+2/3/f*a
*d^2*tan(f*x+e)+1/3/f*a*d^2*tan(f*x+e)*sec(f*x+e)^2
```

Maxima [A] time = 0.964594, size = 223, normalized size = 2.06

$$4\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)ad^{2}-6\,acd\left(\frac{2\,\sin\left(fx+e\right)}{\sin\left(fx+e\right)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-3\,ad^{2}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*d^2 - 6*a*c*d*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 3*a*
d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(
f*x + e) - 1)) + 12*a*c^2*log(sec(f*x + e) + tan(f*x + e)) + 12*a*c^2*tan(f
*x + e) + 24*a*c*d*tan(f*x + e))/f
```

Fricas [A] time = 0.494363, size = 371, normalized size = 3.44

$$\frac{3\left(2\,ac^{2}+2\,acd+ad^{2}\right)\cos\left(fx+e\right)^{3}\log\left(\sin\left(fx+e\right)+1\right)-3\left(2\,ac^{2}+2\,acd+ad^{2}\right)\cos\left(fx+e\right)^{3}\log\left(-\sin\left(fx+e\right)\right.}{12\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(2*a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e)^3*log(sin(f*x + e) + 1) -
3*(2*a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*
a*d^2 + 2*(3*a*c^2 + 6*a*c*d + 2*a*d^2)*cos(f*x + e)^2 + 3*(2*a*c*d + a*d^2
)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int c^{2}\sec\left(e+fx\right)dx+\int c^{2}\sec^{2}\left(e+fx\right)dx+\int d^{2}\sec^{3}\left(e+fx\right)dx+\int d^{2}\sec^{4}\left(e+fx\right)dx+\int 2cd\sec^{2}\left(e+fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))**2,x)
```

```
[Out] a*(Integral(c**2*sec(e + f*x), x) + Integral(c**2*sec(e + f*x)**2, x) + Int
egral(d**2*sec(e + f*x)**3, x) + Integral(d**2*sec(e + f*x)**4, x) + Integr
al(2*c*d*sec(e + f*x)**2, x) + Integral(2*c*d*sec(e + f*x)**3, x))
```

Giac [B] time = 1.62735, size = 329, normalized size = 3.05

$$3\left(2\,ac^{2}+2\,acd+ad^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\left(2\,ac^{2}+2\,acd+ad^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(6\,ac^{2}\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] 1/6*(3*(2*a*c^2 + 2*a*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*(
2*a*c^2 + 2*a*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(6*a*c^2*
tan(1/2*f*x + 1/2*e)^5 + 6*a*c*d*tan(1/2*f*x + 1/2*e)^5 + 3*a*d^2*tan(1/2*f
*x + 1/2*e)^5 - 12*a*c^2*tan(1/2*f*x + 1/2*e)^3 - 24*a*c*d*tan(1/2*f*x + 1/
2*e)^3 - 4*a*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*a*c^2*tan(1/2*f*x + 1/2*e) + 18
*a*c*d*tan(1/2*f*x + 1/2*e) + 9*a*d^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x +
1/2*e)^2 - 1)^3)/f
```

3.188 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=56

$$\frac{a(c+d)\tan(e+fx)}{f}+\frac{a(2c+d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{ad\tan(e+fx)\sec(e+fx)}{2f}$$

```
[Out] (a*(2*c + d)*ArcTanh[Sin[e + f*x]])/(2*f) + (a*(c + d)*Tan[e + f*x])/f + (a
*d*Sec[e + f*x]*Tan[e + f*x])/(2*f)
```

Rubi [A] time = 0.0719762, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3997, 3787, 3770, 3767, 8}

$$\frac{a(c+d)\tan(e+fx)}{f}+\frac{a(2c+d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{ad\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*(2*c + d)*ArcTanh[Sin[e + f*x]])/(2*f) + (a*(c + d)*Tan[e + f*x])/f + (a
*d*Sec[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx = \frac{ad\sec(e+fx)\tan(e+fx)}{2f} + \frac{1}{2}\int \sec(e+fx)(a(2c+d)+2a(c$$
$$= \frac{ad\sec(e+fx)\tan(e+fx)}{2f} + (a(c+d))\int \sec^2(e+fx)\,dx + \frac{1}{2}($$
$$= \frac{a(2c+d)\tanh^{-1}(\sin(e+fx))}{2f} + \frac{ad\sec(e+fx)\tan(e+fx)}{2f} -$$
$$= \frac{a(2c+d)\tanh^{-1}(\sin(e+fx))}{2f} + \frac{a(c+d)\tan(e+fx)}{f} + \frac{ad\sec$$

Mathematica [A] time = 0.029623, size = 75, normalized size = 1.34

$$\frac{ac\tan(e+fx)}{f} + \frac{ac\tanh^{-1}(\sin(e+fx))}{f} + \frac{ad\tan(e+fx)}{f} + \frac{ad\tanh^{-1}(\sin(e+fx))}{2f} + \frac{ad\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*c*ArcTanh[Sin[e + f*x]])/f + (a*d*ArcTanh[Sin[e + f*x]])/(2*f) + (a*c*Ta
n[e + f*x])/f + (a*d*Tan[e + f*x])/f + (a*d*Sec[e + f*x]*Tan[e + f*x])/(2*f
)
```

Maple [A] time = 0.035, size = 86, normalized size = 1.5

$$\frac{ac\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f} + \frac{ad\tan\left(fx+e\right)}{f} + \frac{ac\tan\left(fx+e\right)}{f} + \frac{ad\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f} + \frac{ad\ln\left(\sec\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

```
[Out] 1/f*a*c*ln(sec(f*x+e)+tan(f*x+e))+1/f*a*d*tan(f*x+e)+1/f*a*c*tan(f*x+e)+1/2
*a*d*sec(f*x+e)*tan(f*x+e)/f+1/2/f*a*d*ln(sec(f*x+e)+tan(f*x+e))
```

Maxima [A] time = 0.977225, size = 119, normalized size = 2.12

$$-\frac{ad\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1} - \log\left(\sin\left(fx+e\right)+1\right) + \log\left(\sin\left(fx+e\right)-1\right)\right) - 4\,ac\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right) - 4\,ac\tan\left(fx\right.}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] -1/4*(a*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) - 4*a*c*log(sec(f*x + e) + tan(f*x + e)) - 4*a*c*tan(f
```

```
*x + e) - 4*a*d*tan(f*x + e))/f
```

Fricas [A] time = 0.48114, size = 247, normalized size = 4.41

$$\frac{(2\,ac + ad)\cos\left(fx + e\right)^{2}\log\left(\sin\left(fx + e\right) + 1\right) - (2\,ac + ad)\cos\left(fx + e\right)^{2}\log\left(-\sin\left(fx + e\right) + 1\right) + 2\left(ad + 2\,(ac + a\right.}{4\,f\cos\left(fx + e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/4*((2*a*c + a*d)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a*c + a*d)*cos
(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(a*d + 2*(a*c + a*d)*cos(f*x + e))*s
in(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int c\sec\left(e + fx\right)dx + \int c\sec^{2}\left(e + fx\right)dx + \int d\sec^{2}\left(e + fx\right)dx + \int d\sec^{3}\left(e + fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

```
[Out] a*(Integral(c*sec(e + f*x), x) + Integral(c*sec(e + f*x)**2, x) + Integral(
d*sec(e + f*x)**2, x) + Integral(d*sec(e + f*x)**3, x))
```

Giac [B] time = 1.51048, size = 177, normalized size = 3.16

$$\frac{(2\,ac + ad)\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + 1\right|\right) - (2\,ac + ad)\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - 1\right|\right) - \frac{2\left(2\,ac\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{3} + ad\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{3} - \right.}{\left(\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\right.\right.}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((2*a*c + a*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - (2*a*c + a*d)*log(a
bs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(2*a*c*tan(1/2*f*x + 1/2*e)^3 + a*d*tan(1
/2*f*x + 1/2*e)^3 - 2*a*c*tan(1/2*f*x + 1/2*e) - 3*a*d*tan(1/2*f*x + 1/2*e)
)/(tan(1/2*f*x + 1/2*e)^2 - 1)^2)/f
```

3.189 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=69

$$\frac{a\tanh^{-1}(\sin(e+fx))}{df}-\frac{2a\sqrt{c-d}\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{df\sqrt{c+d}}$$

```
[Out] (a*ArcTanh[Sin[e + f*x]])/(d*f) - (2*a*Sqrt[c - d]*ArcTanh[(Sqrt[c - d]*Tan
[(e + f*x)/2])/Sqrt[c + d]])/(d*Sqrt[c + d]*f)
```

Rubi [A] time = 0.144089, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3998, 3770, 3831, 2659, 208}

$$\frac{a\tanh^{-1}(\sin(e+fx))}{df}-\frac{2a\sqrt{c-d}\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{df\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```


```
[Out] (a*ArcTanh[Sin[e + f*x]])/(d*f) - (2*a*Sqrt[c - d]*ArcTanh[(Sqrt[c - d]*Tan
[(e + f*x)/2])/Sqrt[c + d]])/(d*Sqrt[c + d]*f)
```


Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{a\int \sec(e+fx)\,dx}{d} + \frac{(-ac+ad)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{d}$$
$$= \frac{a\tanh^{-1}(\sin(e+fx))}{df} - \frac{(a(c-d))\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d^2}$$
$$= \frac{a\tanh^{-1}(\sin(e+fx))}{df} - \frac{(2a(c-d))\,\text{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx\right.\right.}{d^2f}$$
$$= \frac{a\tanh^{-1}(\sin(e+fx))}{df} - \frac{2a\sqrt{c-d}\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d\sqrt{c+d}f}$$

Mathematica [A] time = 0.184981, size = 107, normalized size = 1.55

$$\frac{a\left(\frac{2(c-d)\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} - \log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*((2*(c - d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c
^2 - d^2] - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]]))/(d*f)
```

Maple [B] time = 0.078, size = 135, normalized size = 2.

$$\frac{a}{fd}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right) - \frac{a}{fd}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right) - 2\frac{ac}{fd\sqrt{(c+d)(c-d)}}\text{Artanh}\left(\frac{\tan\left(1/2\,fx+e/2\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] 1/f*a/d*ln(tan(1/2*f*x+1/2*e)+1)-1/f*a/d*ln(tan(1/2*f*x+1/2*e)-1)-2/f*a/d/(
(c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*c+
2/f*a/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.64202, size = 622, normalized size = 9.01

$$\left[\frac{a\sqrt{\frac{c-d}{c+d}}\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2-2\left(c^2+cd+\left(cd+d^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{c-d}{c+d}}\sin\left(fx+e\right)+2\,c^2-d^2}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2}\right)+a\log\left(\sin\left(fx+e\right)+1\right)-a\log\left(\right.}{2\,df}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [1/2*(a*sqrt((c - d)/(c + d))*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f
*x + e)^2 - 2*(c^2 + c*d + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*
sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)
) + a*log(sin(f*x + e) + 1) - a*log(-sin(f*x + e) + 1))/(d*f), -1/2*(2*a*sq
rt(-(c - d)/(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d))/((
c - d)*sin(f*x + e))) - a*log(sin(f*x + e) + 1) + a*log(-sin(f*x + e) + 1))
/(d*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx+\int\frac{\sec^2\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c + d*sec(e + f*x)), x) + Integral(sec(e + f*x)**
2/(c + d*sec(e + f*x)), x))
```

Giac [B] time = 2.03586, size = 178, normalized size = 2.58

$$\frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{d}-\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{d}+\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,c-2\,d\right)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)\left(ac-ad\right)}{\sqrt{-c^2+d^2}\,d}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d - a*log(abs(tan(1/2*f*x + 1/2*e) -
1))/d + 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(
```

```
1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(a*c - a*d)/(
sqrt(-c^2 + d^2)*d))/f
```

3.190 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=79

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}(c+d)^{3/2}}+\frac{a\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}$$

```
[Out] (2*a*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*(c +
 d)^(3/2)*f) + (a*Tan[e + f*x])/((c + d)*f*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.139065, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}(c+d)^{3/2}}+\frac{a\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*(c +
 d)^(3/2)*f) + (a*Tan[e + f*x])/((c + d)*f*(c + d*Sec[e + f*x]))
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx &= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} - \frac{\int \frac{a(c-d)\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{-c^2+d^2}\\
&= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} + \frac{a\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c+d}\\
&= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} + \frac{a\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d(c+d)}\\
&= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{d(c+d)f}\\
&= \frac{2a\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}(c+d)^{3/2}f} + \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))}
\end{aligned}$$

Mathematica [A] time = 0.225903, size = 75, normalized size = 0.95

$$\frac{a\left(\frac{\sin(e+fx)}{c\cos(e+fx)+d} - \frac{2\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}\right)}{f(c+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a*((-2*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^
2] + Sin[e + f*x]/(d + c*Cos[e + f*x])))/((c + d)*f)
```

Maple [A] time = 0.089, size = 105, normalized size = 1.3

$$4\,\frac{a}{f}\left(-1/2\,\frac{\tan\left(1/2\,fx+e/2\right)}{(c+d)\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2c-\left(\tan\left(1/2\,fx+e/2\right)\right)^2d-c-d\right)}+1/2\,\frac{1}{(c+d)\sqrt{(c+d)(c-d)}}\mathrm{Artanh}\left(\frac{\tan
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 4/f*a*(-1/2*tan(1/2*f*x+1/2*e)/(c+d)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/
2*e)^2*d-c-d)+1/2/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d
)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.509431, size = 780, normalized size = 9.87

$$\left[\frac{\left(ac\cos\left(fx+e\right)+ad\right)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2+2\,\sqrt{c^2-d^2}\left(d\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)+2\,c^2-d^2}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2}\right)+2\left(ac^2-ad^2\right)\mathrm{si}}{2\left(\left(c^4+c^3d-c^2d^2-cd^3\right)f\cos\left(fx+e\right)+\left(c^3d+c^2d^2-cd^3-d^4\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((a*c*cos(f*x + e) + a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c
^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x
 + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(
a*c^2 - a*d^2)*sin(f*x + e))/((c^4 + c^3*d - c^2*d^2 - c*d^3)*f*cos(f*x + e
) + (c^3*d + c^2*d^2 - c*d^3 - d^4)*f), ((a*c*cos(f*x + e) + a*d)*sqrt(-c^2
 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x
+ e))) + (a*c^2 - a*d^2)*sin(f*x + e))/((c^4 + c^3*d - c^2*d^2 - c*d^3)*f*c
os(f*x + e) + (c^3*d + c^2*d^2 - c*d^3 - d^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{\sec\left(e+fx\right)}{c^2+2cd\sec\left(e+fx\right)+d^2\sec^2\left(e+fx\right)}\,dx+\int\frac{\sec^2\left(e+fx\right)}{c^2+2cd\sec\left(e+fx\right)+d^2\sec^2\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)**2)
, x) + Integral(sec(e + f*x)**2/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f
*x)**2), x))
```

Giac [B] time = 1.70837, size = 193, normalized size = 2.44

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)a}{\sqrt{-c^2+d^2}(c+d)}+\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)(c+d)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*
x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*a/(sqrt(-c^2 + d^2)
*(c + d)) + a*tan(1/2*f*x + 1/2*e)/((c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f
*x + 1/2*e)^2 - c - d)*(c + d)))/f
```

3.191 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=131

$$\frac{a(2c-d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{5/2}}+\frac{a(c-2d)\tan(e+fx)}{2f(c-d)(c+d)^2(c+d\sec(e+fx))}+\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

```
[Out] (a*(2*c - d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c - d)^
(3/2)*(c + d)^(5/2)*f) + (a*Tan[e + f*x])/(2*(c + d)*f*(c + d*Sec[e + f*x])
^2) + (a*(c - 2*d)*Tan[e + f*x])/(2*(c - d)*(c + d)^2*f*(c + d*Sec[e + f*x]
))
```

Rubi [A] time = 0.270406, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{a(2c-d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{5/2}}+\frac{a(c-2d)\tan(e+fx)}{2f(c-d)(c+d)^2(c+d\sec(e+fx))}+\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(2*c - d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c - d)^
(3/2)*(c + d)^(5/2)*f) + (a*Tan[e + f*x])/(2*(c + d)*f*(c + d*Sec[e + f*x])
^2) + (a*(c - 2*d)*Tan[e + f*x])/(2*(c - d)*(c + d)^2*f*(c + d*Sec[e + f*x]
))
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} - \frac{\int \frac{\sec(e+fx)(-2a(c-d)-a(c-d)\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx}{2\left(c^2-d^2\right)}$$

$$= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2 f(c+d\sec(e+fx))} + \frac{\int \frac{a(c-d}{c\cdots}}{2\cdots}$$

$$= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2 f(c+d\sec(e+fx))} + \frac{(a(2c-\cdots}{2\cdots}$$

$$= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2 f(c+d\sec(e+fx))} + \frac{(a(2c-\cdots}{2(\cdots}$$

$$= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2 f(c+d\sec(e+fx))} + \frac{(a(2c-\cdots}{\cdots}$$

$$= \frac{a(2c-d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{5/2}f} + \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{\cdots}{2(c-d\cdots}$$

Mathematica [A] time = 1.17543, size = 167, normalized size = 1.27

$$\frac{a(\cos(e+fx)+1)\sec^2\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{c^2-d^2}\sin(e+fx)\left(\left(2c^2-2cd-d^2\right)\cos(e+fx)+d(c-2d)\right)-2(2c-d)(c\cos\cdots\right.}{4f(c-d)(c+d)^2\sqrt{c^2-d^2}(c\cos(e+fx)+d)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(1 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*(-2*(2*c - d)*ArcTanh[((-c + d)*Ta
n[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^2 + Sqrt[c^2 - d^2]*(
(c - 2*d)*d + (2*c^2 - 2*c*d - d^2)*Cos[e + f*x])*Sin[e + f*x]))/(4*(c - d)
*(c + d)^2*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x])^2)
```

Maple [A] time = 0.102, size = 178, normalized size = 1.4

$$4\,\frac{a}{f}\left(\frac{1}{\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2c-\left(\tan\left(1/2\,fx+e/2\right)\right)^2d-c-d\right)^2}\left(-1/4\,\frac{(2\,c-d)\left(\tan\left(1/2\,fx+e/2\right)\right)^3}{c^2+2\,cd+d^2}+1/4\,\frac{(2\,c-3\,d)\cdots}{(c-\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x)
```

```
[Out] 4/f*a*((-1/4*(2*c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/4*(2*c-3*d)/(c+
d)/(c-d)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d
-c-d)^2+1/4*(2*c-d)/(c^3+c^2*d-c*d^2-d^3)/((c+d)*(c-d))^(1/2)*arctanh(tan(1
/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.579293, size = 1604, normalized size = 12.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] [1/4*((2*a*c*d^2 - a*d^3 + (2*a*c^3 - a*c^2*d)*cos(f*x + e)^2 + 2*(2*a*c^2*
d - a*c*d^2)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 -
 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e
) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(a*c^
3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a*d^4 + (2*a*c^4 - 2*a*c^3*d - 3*a*c^2*d^2
+ 2*a*c*d^3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/((c^7 + c^6*d - 2*c^5*d^2
- 2*c^4*d^3 + c^3*d^4 + c^2*d^5)*f*cos(f*x + e)^2 + 2*(c^6*d + c^5*d^2 - 2*
c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c^5*d^2 + c^4*d^3
- 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f), 1/2*((2*a*c*d^2 - a*d^3 + (2*a*c
^3 - a*c^2*d)*cos(f*x + e)^2 + 2*(2*a*c^2*d - a*c*d^2)*cos(f*x + e))*sqrt(-
c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f
*x + e))) + (a*c^3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a*d^4 + (2*a*c^4 - 2*a*c^3
*d - 3*a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/((c^7 + c
^6*d - 2*c^5*d^2 - 2*c^4*d^3 + c^3*d^4 + c^2*d^5)*f*cos(f*x + e)^2 + 2*(c^6
*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c
^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{\sec\left(e+fx\right)}{c^3+3c^2d\sec\left(e+fx\right)+3cd^2\sec^2\left(e+fx\right)+d^3\sec^3\left(e+fx\right)}\,dx+\int \frac{\sec^2\left(e+fx\right)}{c^3+3c^2d\sec\left(e+fx\right)+3cd^2\sec^2\left(e+fx\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f
*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(sec(e + f*x)**2/(c**3 + 3*c**
2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3), x))
```

Giac [B] time = 1.39031, size = 370, normalized size = 2.82

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)(2ac-ad)}{\left(c^3+c^2d-cd^2-d^3\right)\sqrt{-c^2+d^2}}-\frac{2ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3acd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+ad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2a}{\left(c^3+c^2d-cd^2-d^3\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] ((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x
 + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(2*a*c - a*d)/((c^3
+ c^2*d - c*d^2 - d^3)*sqrt(-c^2 + d^2)) - (2*a*c^2*tan(1/2*f*x + 1/2*e)^3
- 3*a*c*d*tan(1/2*f*x + 1/2*e)^3 + a*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a*c^2*t
an(1/2*f*x + 1/2*e) + a*c*d*tan(1/2*f*x + 1/2*e) + 3*a*d^2*tan(1/2*f*x + 1/
2*e))/((c^3 + c^2*d - c*d^2 - d^3)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*
x + 1/2*e)^2 - c - d)^2))/f
```

3.192 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=189

$$\frac{a\left(2c^2-2cd+d^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{5/2}(c+d)^{7/2}}+\frac{a(c-4d)(2c-d)\tan(e+fx)}{6f(c-d)^2(c+d)^3(c+d\sec(e+fx))}+\frac{a(2c-3d)\tan(e+fx)}{6f(c-d)(c+d)^2(c+d\sec(e+f x}$$

```
[Out] (a*(2*c^2 - 2*c*d + d^2)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]
])/((c - d)^(5/2)*(c + d)^(7/2)*f) + (a*Tan[e + f*x])/(3*(c + d)*f*(c + d*S
ec[e + f*x])^3) + (a*(2*c - 3*d)*Tan[e + f*x])/(6*(c - d)*(c + d)^2*f*(c +
d*Sec[e + f*x])^2) + (a*(c - 4*d)*(2*c - d)*Tan[e + f*x])/(6*(c - d)^2*(c +
 d)^3*f*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.454203, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{a\left(2c^2-2cd+d^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{5/2}(c+d)^{7/2}}+\frac{a(c-4d)(2c-d)\tan(e+fx)}{6f(c-d)^2(c+d)^3(c+d\sec(e+fx))}+\frac{a(2c-3d)\tan(e+fx)}{6f(c-d)(c+d)^2(c+d\sec(e+f x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a*(2*c^2 - 2*c*d + d^2)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]
])/((c - d)^(5/2)*(c + d)^(7/2)*f) + (a*Tan[e + f*x])/(3*(c + d)*f*(c + d*S
ec[e + f*x])^3) + (a*(2*c - 3*d)*Tan[e + f*x])/(6*(c - d)*(c + d)^2*f*(c +
d*Sec[e + f*x])^2) + (a*(c - 4*d)*(2*c - d)*Tan[e + f*x])/(6*(c - d)^2*(c +
 d)^3*f*(c + d*Sec[e + f*x]))
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} - \frac{\int \frac{\sec(e+fx)(-3a(c-d)-2a(c-d)\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx}{3\left(c^2-d^2\right)}$$

$$= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} + \int \frac{\sec(\ldots}{\ldots}$$

$$= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} + \frac{a(\ldots}{6(c-\ldots}$$

$$= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} + \frac{a(\ldots}{6(c-\ldots}$$

$$= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} + \frac{a(\ldots}{6(c-\ldots}$$

$$= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} + \frac{a(\ldots}{6(c-\ldots}$$

$$= \frac{a\left(2c^2-2cd+d^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{5/2}(c+d)^{7/2}f} + \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} \ldots$$

Mathematica [A] time = 3.18907, size = 247, normalized size = 1.31

$$-\frac{a(\cos(e+fx)+1)\sec^2\left(\frac{1}{2}(e+fx)\right)\left(6\left(2c^2-2cd+d^2\right)(c\cos(e+fx)+d)^3\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)-\frac{1}{2}\sqrt{c^2-d^2}\,\mathrm{si}\ldots}{12f(c-\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] -(a*(1 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*(6*(2*c^2 - 2*c*d + d^2)*ArcTanh[
((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^3 - (Sqrt
[c^2 - d^2]*(6*c^4 - 12*c^3*d + 2*c^2*d^2 - 15*c*d^3 + 10*d^4 + 6*d*(2*c^3
- 7*c^2*d + 2*c*d^2 + d^3)*Cos[e + f*x] + (6*c^4 - 12*c^3*d - 2*c^2*d^2 + 3
*c*d^3 + 2*d^4)*Cos[2*(e + f*x)])*Sin[e + f*x])/2))/(12*(c - d)^2*(c + d)^3
*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x])^3)
```

Maple [A] time = 0.117, size = 271, normalized size = 1.4

$$4\frac{a}{f}\left(\frac{1}{\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^{2}c-\left(\tan\left(1/2\,fx+e/2\right)\right)^{2}d-c-d\right)^{3}}\left(-1/4\,\frac{\left(2\,c^{2}-2\,cd+d^{2}\right)\left(\tan\left(1/2\,fx+e/2\right)\right)^{5}}{c^{3}+3\,c^{2}d+3\,d^{2}c+d^{3}}+1/3\,\frac{\left(3\,c^{2}\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x)
```

```
[Out] 4/f*a*((-1/4*(2*c^2-2*c*d+d^2)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)
^5+1/3*(3*c^2-6*c*d+d^2)/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1/4*(2*
c^2-6*c*d+3*d^2)/(c+d)/(c^2-2*c*d+d^2)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2
*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3+1/4*(2*c^2-2*c*d+d^2)/(c^5+c^4*d-2*c^
3*d^2-2*c^2*d^3+c*d^4+d^5)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(
c-d)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.691661, size = 2731, normalized size = 14.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] [1/12*(3*(2*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 + (2*a*c^5 - 2*a*c^4*d + a*c^3*d^
2)*cos(f*x + e)^3 + 3*(2*a*c^4*d - 2*a*c^3*d^2 + a*c^2*d^3)*cos(f*x + e)^2
+ 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x + e))*sqrt(c^2 - d^2)*log
((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*
cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*c
os(f*x + e) + d^2)) + 2*(2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*a*c*d^
5 - 4*a*d^6 + (6*a*c^6 - 12*a*c^5*d - 8*a*c^4*d^2 + 15*a*c^3*d^3 + 4*a*c^2*
d^4 - 3*a*c*d^5 - 2*a*d^6)*cos(f*x + e)^2 + 3*(2*a*c^5*d - 7*a*c^4*d^2 + 8*
a*c^2*d^4 - 2*a*c*d^5 - a*d^6)*cos(f*x + e))*sin(f*x + e))/((c^10 + c^9*d -
 3*c^8*d^2 - 3*c^7*d^3 + 3*c^6*d^4 + 3*c^5*d^5 - c^4*d^6 - c^3*d^7)*f*cos(f
*x + e)^3 + 3*(c^9*d + c^8*d^2 - 3*c^7*d^3 - 3*c^6*d^4 + 3*c^5*d^5 + 3*c^4*
d^6 - c^3*d^7 - c^2*d^8)*f*cos(f*x + e)^2 + 3*(c^8*d^2 + c^7*d^3 - 3*c^6*d^
4 - 3*c^5*d^5 + 3*c^4*d^6 + 3*c^3*d^7 - c^2*d^8 - c*d^9)*f*cos(f*x + e) + (
c^7*d^3 + c^6*d^4 - 3*c^5*d^5 - 3*c^4*d^6 + 3*c^3*d^7 + 3*c^2*d^8 - c*d^9 -
 d^10)*f), 1/6*(3*(2*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 + (2*a*c^5 - 2*a*c^4*d +
 a*c^3*d^2)*cos(f*x + e)^3 + 3*(2*a*c^4*d - 2*a*c^3*d^2 + a*c^2*d^3)*cos(f*
```

```
x + e)^2 + 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x + e))*sqrt(-c^2
+ d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x +
 e))) + (2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*a*c*d^5 - 4*a*d^6 + (6
*a*c^6 - 12*a*c^5*d - 8*a*c^4*d^2 + 15*a*c^3*d^3 + 4*a*c^2*d^4 - 3*a*c*d^5
- 2*a*d^6)*cos(f*x + e)^2 + 3*(2*a*c^5*d - 7*a*c^4*d^2 + 8*a*c^2*d^4 - 2*a*
c*d^5 - a*d^6)*cos(f*x + e))*sin(f*x + e))/((c^10 + c^9*d - 3*c^8*d^2 - 3*c
^7*d^3 + 3*c^6*d^4 + 3*c^5*d^5 - c^4*d^6 - c^3*d^7)*f*cos(f*x + e)^3 + 3*(c
^9*d + c^8*d^2 - 3*c^7*d^3 - 3*c^6*d^4 + 3*c^5*d^5 + 3*c^4*d^6 - c^3*d^7 -
c^2*d^8)*f*cos(f*x + e)^2 + 3*(c^8*d^2 + c^7*d^3 - 3*c^6*d^4 - 3*c^5*d^5 +
3*c^4*d^6 + 3*c^3*d^7 - c^2*d^8 - c*d^9)*f*cos(f*x + e) + (c^7*d^3 + c^6*d^
4 - 3*c^5*d^5 - 3*c^4*d^6 + 3*c^3*d^7 + 3*c^2*d^8 - c*d^9 - d^10)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{\sec\left(e+fx\right)}{c^4+4c^3d\sec\left(e+fx\right)+6c^2d^2\sec^2\left(e+fx\right)+4cd^3\sec^3\left(e+fx\right)+d^4\sec^4\left(e+fx\right)}\,dx+\int \frac{}{c^4+4c^3d\sec\left(e+f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**4,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e
+ f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral
(sec(e + f*x)**2/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**
2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x))
```

Giac [B] time = 1.50392, size = 632, normalized size = 3.34

$$-\frac{\frac{3\left(2ac^2-2acd+ad^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(c^5+c^4d-2c^3d^2-2c^2d^3+cd^4+d^5\right)\sqrt{-c^2+d^2}}+\frac{6ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-18ac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+21ac^2d^2\tan}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] -1/3*(3*(2*a*c^2 - 2*a*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2
*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-
c^2 + d^2)))/((c^5 + c^4*d - 2*c^3*d^2 - 2*c^2*d^3 + c*d^4 + d^5)*sqrt(-c^2
 + d^2)) + (6*a*c^4*tan(1/2*f*x + 1/2*e)^5 - 18*a*c^3*d*tan(1/2*f*x + 1/2*e
)^5 + 21*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^5 - 12*a*c*d^3*tan(1/2*f*x + 1/2*e)
^5 + 3*a*d^4*tan(1/2*f*x + 1/2*e)^5 - 12*a*c^4*tan(1/2*f*x + 1/2*e)^3 + 24*
a*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 8*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 24*a*
c*d^3*tan(1/2*f*x + 1/2*e)^3 + 4*a*d^4*tan(1/2*f*x + 1/2*e)^3 + 6*a*c^4*tan
(1/2*f*x + 1/2*e) - 6*a*c^3*d*tan(1/2*f*x + 1/2*e) - 21*a*c^2*d^2*tan(1/2*f
*x + 1/2*e) + 9*a*d^4*tan(1/2*f*x + 1/2*e))/((c^5 + c^4*d - 2*c^3*d^2 - 2*c
^2*d^3 + c*d^4 + d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2
- c - d)^3))/f
```

3.193 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx$

Optimal. Leaf size=327

$$-\frac{a^2\left(-311c^3d^2-448c^2d^3-48c^4d+4c^5-288cd^4-64d^5\right)\tan(e+fx)}{60df}+\frac{a^2\left(84c^2d^2+64c^3d+24c^4+48cd^3+11d^4\right)\tanh}{16f}$$

```
[Out] (a^2*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*ArcTanh[Sin[e + f
*x]])/(16*f) - (a^2*(4*c^5 - 48*c^4*d - 311*c^3*d^2 - 448*c^2*d^3 - 288*c*d
^4 - 64*d^5)*Tan[e + f*x])/(60*d*f) - (a^2*(8*c^4 - 96*c^3*d - 438*c^2*d^2
- 464*c*d^3 - 165*d^4)*Sec[e + f*x]*Tan[e + f*x])/(240*f) - (a^2*(4*c^3 - 4
8*c^2*d - 123*c*d^2 - 64*d^3)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(120*d*f
) - (a^2*(4*c^2 - 48*c*d - 55*d^2)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(12
0*d*f) - (a^2*(c - 12*d)*(c + d*Sec[e + f*x])^4*Tan[e + f*x])/(30*d*f) + (a
^2*(c + d*Sec[e + f*x])^5*Tan[e + f*x])/(6*d*f)
```

Rubi [A] time = 0.42421, antiderivative size = 371, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3987, 100, 153, 147, 50, 63, 217, 203}

$$\frac{a^2\left(84c^2d^2+64c^3d+24c^4+48cd^3+11d^4\right)\tan(e+fx)}{16f}+\frac{a^3\left(84c^2d^2+64c^3d+24c^4+48cd^3+11d^4\right)\tan(e+fx)\tan^{-1}}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*Tan[e + f*x])/(16
*f) + (a^3*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*ArcTan[Sqrt
[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(8*f*Sqrt[a
- a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((24*c^4 + 64*c^3*d + 84*c^2*
d^2 + 48*c*d^3 + 11*d^4)*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(48*f) + (d
*(9*c + 2*d)*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3
0*f) + (d*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(6*f)
 + (d*(a + a*Sec[e + f*x])^2*(2*(52*c^3 + 56*c^2*d + 48*c*d^2 + 9*d^3) + d*
(48*c^2 + 32*c*d + 19*d^2)*Sec[e + f*x])*Tan[e + f*x])/(120*f)
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
```

```
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}(c+dx)^4}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\tan(e+fx)}{6f} + \frac{\tan(\cdots}{}$$

$$= \frac{d(9c+2d)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx}{30f}$$

$$= \frac{d(9c+2d)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx}{30f}$$

$$= \frac{\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\left(a^2+a^2\sec(e+fx)\right.}{48f}$$

$$= \frac{a^2\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\tan(e+fx)}{16f} + \frac{(2\cdots}{}$$

$$= \frac{a^2\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\tan(e+fx)}{16f} + \frac{(2\cdots}{}$$

$$= \frac{a^2\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\tan(e+fx)}{16f} + \frac{(2\cdots}{}$$

$$= \frac{a^2\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\tan(e+fx)}{16f} + \frac{a^3\cdots}{}$$

Mathematica [A] time = 2.0183, size = 460, normalized size = 1.41

$$-\frac{a^2(\cos(e+fx)+1)^2\sec^4\left(\frac{1}{2}(e+fx)\right)\sec^6(e+fx)\left(240\left(84c^2d^2+64c^3d+24c^4+48cd^3+11d^4\right)\cos^6(e+fx)\left(\log\left(\cos\cdots\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] -(a^2*(1 + Cos[e + f*x])^2*Sec[(e + f*x)/2]^4*Sec[e + f*x]^6*(240*(24*c^4 +
 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*Cos[e + f*x]^6*(Log[Cos[(e + f*
x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 2*(
360*c^4 + 2880*c^3*d + 5220*c^2*d^2 + 4080*c*d^3 + 1255*d^4 + 32*(75*c^4 +
310*c^3*d + 480*c^2*d^2 + 336*c*d^3 + 88*d^4)*Cos[e + f*x] + 20*(24*c^4 + 1
92*c^3*d + 324*c^2*d^2 + 240*c*d^3 + 55*d^4)*Cos[2*(e + f*x)] + 1200*c^4*Co
s[3*(e + f*x)] + 4640*c^3*d*Cos[3*(e + f*x)] + 6720*c^2*d^2*Cos[3*(e + f*x)
] + 4032*c*d^3*Cos[3*(e + f*x)] + 896*d^4*Cos[3*(e + f*x)] + 120*c^4*Cos[4*
(e + f*x)] + 960*c^3*d*Cos[4*(e + f*x)] + 1260*c^2*d^2*Cos[4*(e + f*x)] + 7
20*c*d^3*Cos[4*(e + f*x)] + 165*d^4*Cos[4*(e + f*x)] + 240*c^4*Cos[5*(e + f
*x)] + 800*c^3*d*Cos[5*(e + f*x)] + 960*c^2*d^2*Cos[5*(e + f*x)] + 576*c*d^
3*Cos[5*(e + f*x)] + 128*d^4*Cos[5*(e + f*x)])*Sin[e + f*x]))/(15360*f)
```

Maple [A] time = 0.075, size = 602, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x)
```

```
[Out] 3/2/f*a^2*c^4*ln(sec(f*x+e)+tan(f*x+e))+11/16/f*a^2*d^4*ln(sec(f*x+e)+tan(f
*x+e))+2/f*a^2*c^4*tan(f*x+e)+16/15/f*a^2*d^4*tan(f*x+e)+4/f*a^2*c^3*d*ln(s
ec(f*x+e)+tan(f*x+e))+3/f*a^2*c*d^3*ln(sec(f*x+e)+tan(f*x+e))+24/5/f*a^2*c*
d^3*tan(f*x+e)+8/f*a^2*c^2*d^2*tan(f*x+e)+1/6/f*a^2*d^4*tan(f*x+e)*sec(f*x+
e)^5+20/3/f*a^2*c^3*d*tan(f*x+e)+2/5/f*a^2*d^4*tan(f*x+e)*sec(f*x+e)^4+8/15
/f*a^2*d^4*tan(f*x+e)*sec(f*x+e)^2+21/4/f*a^2*c^2*d^2*ln(sec(f*x+e)+tan(f*x
+e))+11/24/f*a^2*d^4*tan(f*x+e)*sec(f*x+e)^3+11/16/f*a^2*d^4*sec(f*x+e)*tan
(f*x+e)+4/3/f*a^2*c^3*d*tan(f*x+e)*sec(f*x+e)^2+3/2/f*a^2*c^2*d^2*tan(f*x+e
)*sec(f*x+e)^3+12/5/f*a^2*c*d^3*tan(f*x+e)*sec(f*x+e)^2+4/f*a^2*c^2*d^2*tan
(f*x+e)*sec(f*x+e)^2+4/5/f*a^2*c*d^3*tan(f*x+e)*sec(f*x+e)^4+2/f*a^2*c*d^3*
tan(f*x+e)*sec(f*x+e)^3+3/f*a^2*c*d^3*sec(f*x+e)*tan(f*x+e)+21/4/f*a^2*c^2*
d^2*sec(f*x+e)*tan(f*x+e)+4/f*a^2*c^3*d*sec(f*x+e)*tan(f*x+e)+1/2*a^2*c^4*s
ec(f*x+e)*tan(f*x+e)/f
```

Maxima [B] time = 1.02794, size = 922, normalized size = 2.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/480*(640*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3*d + 1920*(tan(f*x + e)
^3 + 3*tan(f*x + e))*a^2*c^2*d^2 + 128*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^
3 + 15*tan(f*x + e))*a^2*c*d^3 + 640*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*
c*d^3 + 64*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*d^4
 - 5*a^2*d^4*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(
sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x
+ e) + 1) + 15*log(sin(f*x + e) - 1)) - 180*a^2*c^2*d^2*(2*(3*sin(f*x + e)^
3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x
 + e) + 1) + 3*log(sin(f*x + e) - 1)) - 240*a^2*c*d^3*(2*(3*sin(f*x + e)^3
- 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x +
 e) + 1) + 3*log(sin(f*x + e) - 1)) - 30*a^2*d^4*(2*(3*sin(f*x + e)^3 - 5*s
in(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) +
 1) + 3*log(sin(f*x + e) - 1)) - 120*a^2*c^4*(2*sin(f*x + e)/(sin(f*x + e)^
2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 960*a^2*c^3*d*(2*
sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e
) - 1)) - 720*a^2*c^2*d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*
x + e) + 1) + log(sin(f*x + e) - 1)) + 480*a^2*c^4*log(sec(f*x + e) + tan(f
*x + e)) + 960*a^2*c^4*tan(f*x + e) + 1920*a^2*c^3*d*tan(f*x + e))/f
```

Fricas [A] time = 0.56677, size = 884, normalized size = 2.7

$$\frac{15\left(24\,a^2c^4+64\,a^2c^3d+84\,a^2c^2d^2+48\,a^2cd^3+11\,a^2d^4\right)\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-15\left(24\,a^2c^4+64\,a^2c^3d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(24*a^2*c^4 + 64*a^2*c^3*d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a
^2*d^4)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 15*(24*a^2*c^4 + 64*a^2*c^3*
d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*cos(f*x + e)^6*log(-sin(f*x
 + e) + 1) + 2*(40*a^2*d^4 + 32*(15*a^2*c^4 + 50*a^2*c^3*d + 60*a^2*c^2*d^2
 + 36*a^2*c*d^3 + 8*a^2*d^4)*cos(f*x + e)^5 + 15*(8*a^2*c^4 + 64*a^2*c^3*d
+ 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*cos(f*x + e)^4 + 64*(5*a^2*c^
3*d + 15*a^2*c^2*d^2 + 9*a^2*c*d^3 + 2*a^2*d^4)*cos(f*x + e)^3 + 10*(36*a^2
*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*cos(f*x + e)^2 + 96*(2*a^2*c*d^3 + a^
2*d^4)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int c^4\sec\left(e+fx\right)dx+\int 2c^4\sec^2\left(e+fx\right)dx+\int c^4\sec^3\left(e+fx\right)dx+\int d^4\sec^5\left(e+fx\right)dx+\int 2d^4\sec^6\left(e+fx\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e))**4,x)
```

```
[Out] a**2*(Integral(c**4*sec(e + f*x), x) + Integral(2*c**4*sec(e + f*x)**2, x)
+ Integral(c**4*sec(e + f*x)**3, x) + Integral(d**4*sec(e + f*x)**5, x) + I
ntegral(2*d**4*sec(e + f*x)**6, x) + Integral(d**4*sec(e + f*x)**7, x) + In
tegral(4*c*d**3*sec(e + f*x)**4, x) + Integral(8*c*d**3*sec(e + f*x)**5, x)
 + Integral(4*c*d**3*sec(e + f*x)**6, x) + Integral(6*c**2*d**2*sec(e + f*x
)**3, x) + Integral(12*c**2*d**2*sec(e + f*x)**4, x) + Integral(6*c**2*d**2
*sec(e + f*x)**5, x) + Integral(4*c**3*d*sec(e + f*x)**2, x) + Integral(8*c
**3*d*sec(e + f*x)**3, x) + Integral(4*c**3*d*sec(e + f*x)**4, x))
```

Giac [B] time = 1.4544, size = 1038, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(24*a^2*c^4 + 64*a^2*c^3*d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a
^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(24*a^2*c^4 + 64*a^2*c^3*d
+ 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e)
- 1)) - 2*(360*a^2*c^4*tan(1/2*f*x + 1/2*e)^11 + 960*a^2*c^3*d*tan(1/2*f*x
+ 1/2*e)^11 + 1260*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^11 + 720*a^2*c*d^3*tan(
1/2*f*x + 1/2*e)^11 + 165*a^2*d^4*tan(1/2*f*x + 1/2*e)^11 - 2040*a^2*c^4*ta
n(1/2*f*x + 1/2*e)^9 - 5440*a^2*c^3*d*tan(1/2*f*x + 1/2*e)^9 - 7140*a^2*c^2
*d^2*tan(1/2*f*x + 1/2*e)^9 - 4080*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^9 - 935*a
^2*d^4*tan(1/2*f*x + 1/2*e)^9 + 4560*a^2*c^4*tan(1/2*f*x + 1/2*e)^7 + 13440
*a^2*c^3*d*tan(1/2*f*x + 1/2*e)^7 + 15480*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^
7 + 10272*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^7 + 1986*a^2*d^4*tan(1/2*f*x + 1/2
*e)^7 - 5040*a^2*c^4*tan(1/2*f*x + 1/2*e)^5 - 17280*a^2*c^3*d*tan(1/2*f*x +
 1/2*e)^5 - 19080*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^5 - 11232*a^2*c*d^3*tan(
1/2*f*x + 1/2*e)^5 - 3006*a^2*d^4*tan(1/2*f*x + 1/2*e)^5 + 2760*a^2*c^4*tan
```

```
(1/2*f*x + 1/2*e)^3 + 11200*a^2*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 13980*a^2*c^
2*d^2*tan(1/2*f*x + 1/2*e)^3 + 7440*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 1305
*a^2*d^4*tan(1/2*f*x + 1/2*e)^3 - 600*a^2*c^4*tan(1/2*f*x + 1/2*e) - 2880*a
^2*c^3*d*tan(1/2*f*x + 1/2*e) - 4500*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e) - 312
0*a^2*c*d^3*tan(1/2*f*x + 1/2*e) - 795*a^2*d^4*tan(1/2*f*x + 1/2*e))/(tan(1
/2*f*x + 1/2*e)^2 - 1)^6)/f
```

3.194 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=242

$$-\frac{a^2\left(-44c^2d^2-10c^3d+c^4-40cd^3-12d^4\right)\tan(e+fx)}{10df}+\frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^2\left(c^2-\right.}{}$$

```
[Out] (3*a^2*(2*c + d)*(2*c^2 + 3*c*d + 2*d^2)*ArcTanh[Sin[e + f*x]])/(8*f) - (a^
2*(c^4 - 10*c^3*d - 44*c^2*d^2 - 40*c*d^3 - 12*d^4)*Tan[e + f*x])/(10*d*f)
- (a^2*(2*c^3 - 20*c^2*d - 57*c*d^2 - 30*d^3)*Sec[e + f*x]*Tan[e + f*x])/(4
0*f) - (a^2*(c^2 - 10*c*d - 12*d^2)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(2
0*d*f) - (a^2*(c - 10*d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(20*d*f) + (a
^2*(c + d*Sec[e + f*x])^4*Tan[e + f*x])/(5*d*f)
```

Rubi [A] time = 0.342701, antiderivative size = 277, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 100, 147, 50, 63, 217, 203}

$$\frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f}+\frac{3a^3(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(2c+d)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (3*a^2*(2*c + d)*(2*c^2 + 3*c*d + 2*d^2)*Tan[e + f*x])/(8*f) + (3*a^3*(2*c
+ d)*(2*c^2 + 3*c*d + 2*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Se
c[e + f*x])]]*Tan[e + f*x])/(4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e
+ f*x]]) + ((2*c + d)*(2*c^2 + 3*c*d + 2*d^2)*(a^2 + a^2*Sec[e + f*x])*Tan[
e + f*x])/(8*f) + (d*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2*Tan[e +
f*x])/(5*f) + (d*(a + a*Sec[e + f*x])^2*(2*(8*c^2 + 5*c*d + 2*d^2) + d*(7*c
 + 2*d)*Sec[e + f*x])*Tan[e + f*x])/(20*f)
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}(c+dx)^3}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f} + \frac{\tan(}{}$$

$$= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f} + \frac{d(a+}{}$$

$$= \frac{(2c+d)\left(2c^2+3cd+2d^2\right)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{8f}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{(2c+d)\left(2c^2+\right.}{}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{(2c+d)\left(2c^2+\right.}{}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{(2c+d)\left(2c^2+\right.}{}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{3a^3(2c+d)\left(2c^2\right.}{4f\sqrt{}}$$

Mathematica [A] time = 1.45491, size = 326, normalized size = 1.35

$$-\frac{a^2(\cos(e+fx)+1)^2\sec^4\left(\frac{1}{2}(e+fx)\right)\sec^5(e+fx)\left(120\left(8c^2d+4c^3+7cd^2+2d^3\right)\cos^5(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] -(a^2*(1 + Cos[e + f*x])^2*Sec[(e + f*x)/2]^4*Sec[e + f*x]^5*(120*(4*c^3 +
8*c^2*d + 7*c*d^2 + 2*d^3)*Cos[e + f*x]^5*(Log[Cos[(e + f*x)/2] - Sin[(e +
f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 2*(120*c^3 + 380*c^2
*d + 400*c*d^2 + 152*d^3 + 5*(12*c^3 + 72*c^2*d + 87*c*d^2 + 34*d^3)*Cos[e
+ f*x] + 16*(10*c^3 + 30*c^2*d + 30*c*d^2 + 9*d^3)*Cos[2*(e + f*x)] + 20*c^
3*Cos[3*(e + f*x)] + 120*c^2*d*Cos[3*(e + f*x)] + 105*c*d^2*Cos[3*(e + f*x)
] + 30*d^3*Cos[3*(e + f*x)] + 40*c^3*Cos[4*(e + f*x)] + 100*c^2*d*Cos[4*(e
+ f*x)] + 80*c*d^2*Cos[4*(e + f*x)] + 24*d^3*Cos[4*(e + f*x)])*Sin[e + f*x]
))/(1280*f)
```

Maple [A] time = 0.063, size = 420, normalized size = 1.7

$$\frac{3a^2c^3\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}+5\frac{a^2c^2d\tan\left(fx+e\right)}{f}+\frac{21a^2d^2c\sec\left(fx+e\right)\tan\left(fx+e\right)}{8f}+\frac{21a^2d^2c\ln\left(\sec\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x)
```

```
[Out] 3/2/f*a^2*c^3*ln(sec(f*x+e)+tan(f*x+e))+5/f*a^2*c^2*d*tan(f*x+e)+21/8/f*a^2
*d^2*c*sec(f*x+e)*tan(f*x+e)+21/8/f*a^2*d^2*c*ln(sec(f*x+e)+tan(f*x+e))+6/5
/f*a^2*d^3*tan(f*x+e)+3/5/f*a^2*d^3*tan(f*x+e)*sec(f*x+e)^2+2/f*a^2*c^3*tan
(f*x+e)+3/f*a^2*c^2*d*sec(f*x+e)*tan(f*x+e)+3/f*a^2*c^2*d*ln(sec(f*x+e)+tan
(f*x+e))+4/f*a^2*d^2*c*tan(f*x+e)+2/f*a^2*d^2*c*tan(f*x+e)*sec(f*x+e)^2+1/2
/f*a^2*d^3*tan(f*x+e)*sec(f*x+e)^3+3/4/f*a^2*d^3*sec(f*x+e)*tan(f*x+e)+3/4/
f*a^2*d^3*ln(sec(f*x+e)+tan(f*x+e))+1/2*a^2*c^3*sec(f*x+e)*tan(f*x+e)/f+1/f
*a^2*c^2*d*tan(f*x+e)*sec(f*x+e)^2+3/4/f*a^2*d^2*c*tan(f*x+e)*sec(f*x+e)^3+
1/5/f*a^2*d^3*tan(f*x+e)*sec(f*x+e)^4
```

Maxima [B] time = 0.988417, size = 633, normalized size = 2.62

$$\frac{240\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)a^{2}c^{2}d+480\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)a^{2}cd^{2}+16\left(3\,\tan\left(fx+e\right)^{5}+10\,\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/240*(240*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^2*d + 480*(tan(f*x + e)^
3 + 3*tan(f*x + e))*a^2*c*d^2 + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 +
15*tan(f*x + e))*a^2*d^3 + 80*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*d^3 - 4
5*a^2*c*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(
f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 30*a
^2*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x +
 e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 60*a^2*c^
3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*
x + e) - 1)) - 360*a^2*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin
(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 180*a^2*c*d^2*(2*sin(f*x + e)/(si
n(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a^
2*c^3*log(sec(f*x + e) + tan(f*x + e)) + 480*a^2*c^3*tan(f*x + e) + 720*a^2
*c^2*d*tan(f*x + e))/f
```

Fricas [A] time = 0.530885, size = 667, normalized size = 2.76

$$\frac{15\left(4\,a^{2}c^{3}+8\,a^{2}c^{2}d+7\,a^{2}cd^{2}+2\,a^{2}d^{3}\right)\cos\left(fx+e\right)^{5}\log\left(\sin\left(fx+e\right)+1\right)-15\left(4\,a^{2}c^{3}+8\,a^{2}c^{2}d+7\,a^{2}cd^{2}+2\,a^{2}d^{3}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/80*(15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*d^3)*cos(f*x + e)^5
*log(sin(f*x + e) + 1) - 15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*
d^3)*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(8*a^2*d^3 + 8*(10*a^2*c^3 +
 25*a^2*c^2*d + 20*a^2*c*d^2 + 6*a^2*d^3)*cos(f*x + e)^4 + 5*(4*a^2*c^3 + 2
4*a^2*c^2*d + 21*a^2*c*d^2 + 6*a^2*d^3)*cos(f*x + e)^3 + 8*(5*a^2*c^2*d + 1
```

```
0*a^2*c*d^2 + 3*a^2*d^3)*cos(f*x + e)^2 + 10*(3*a^2*c*d^2 + 2*a^2*d^3)*cos(
f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int c^3\sec\left(e+fx\right)dx+\int 2c^3\sec^2\left(e+fx\right)dx+\int c^3\sec^3\left(e+fx\right)dx+\int d^3\sec^4\left(e+fx\right)dx+\int 2d^3\sec^5\left(e+fx\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**2*(Integral(c**3*sec(e + f*x), x) + Integral(2*c**3*sec(e + f*x)**2, x)
+ Integral(c**3*sec(e + f*x)**3, x) + Integral(d**3*sec(e + f*x)**4, x) + I
ntegral(2*d**3*sec(e + f*x)**5, x) + Integral(d**3*sec(e + f*x)**6, x) + In
tegral(3*c*d**2*sec(e + f*x)**3, x) + Integral(6*c*d**2*sec(e + f*x)**4, x)
 + Integral(3*c*d**2*sec(e + f*x)**5, x) + Integral(3*c**2*d*sec(e + f*x)**
2, x) + Integral(6*c**2*d*sec(e + f*x)**3, x) + Integral(3*c**2*d*sec(e + f
*x)**4, x))
```

Giac [B] time = 1.47218, size = 714, normalized size = 2.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/40*(15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*d^3)*log(abs(tan(1/
2*f*x + 1/2*e) + 1)) - 15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*d^
3)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(60*a^2*c^3*tan(1/2*f*x + 1/2*e)^
9 + 120*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^9 + 105*a^2*c*d^2*tan(1/2*f*x + 1/2*
e)^9 + 30*a^2*d^3*tan(1/2*f*x + 1/2*e)^9 - 280*a^2*c^3*tan(1/2*f*x + 1/2*e)
^7 - 560*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^7 - 490*a^2*c*d^2*tan(1/2*f*x + 1/2
*e)^7 - 140*a^2*d^3*tan(1/2*f*x + 1/2*e)^7 + 480*a^2*c^3*tan(1/2*f*x + 1/2*
e)^5 + 1120*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^5 + 800*a^2*c*d^2*tan(1/2*f*x +
1/2*e)^5 + 288*a^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 360*a^2*c^3*tan(1/2*f*x + 1
/2*e)^3 - 1040*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 790*a^2*c*d^2*tan(1/2*f*x
 + 1/2*e)^3 - 180*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 100*a^2*c^3*tan(1/2*f*x
+ 1/2*e) + 360*a^2*c^2*d*tan(1/2*f*x + 1/2*e) + 375*a^2*c*d^2*tan(1/2*f*x +
 1/2*e) + 130*a^2*d^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^5)
/f
```

3.195 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=176

$$-\frac{a^2\left(-8c^2d+c^3-20cd^2-8d^3\right)\tan(e+fx)}{6df}+\frac{a^2\left(12c^2+16cd+7d^2\right)\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^2\left(2c(c-8d)-21d^2\right)\text{ta}}{24}$$

```
[Out] (a^2*(12*c^2 + 16*c*d + 7*d^2)*ArcTanh[Sin[e + f*x]])/(8*f) - (a^2*(c^3 - 8
*c^2*d - 20*c*d^2 - 8*d^3)*Tan[e + f*x])/(6*d*f) - (a^2*(2*c*(c - 8*d) - 21
*d^2)*Sec[e + f*x]*Tan[e + f*x])/(24*f) - (a^2*(c - 8*d)*(c + d*Sec[e + f*x
])^2*Tan[e + f*x])/(12*d*f) + (a^2*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(4*
d*f)
```

Rubi [A] time = 0.262041, antiderivative size = 234, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 90, 80, 50, 63, 217, 203}

$$\frac{a^2\left(12c^2+16cd+7d^2\right)\tan(e+fx)}{8f}+\frac{a^3\left(12c^2+16cd+7d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{\left(12c^2+16cd+7d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2 + 16*c*d + 7*d^2)*Tan[e + f*x])/(8*f) + (a^3*(12*c^2 + 16*c*d
+ 7*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e
+ f*x])/(4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d*(5*c +
 2*d)*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(12*f) + ((12*c^2 + 16*c*d + 7*d
^2)*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(24*f) + (d*(a + a*Sec[e + f*x])
^2*(c + d*Sec[e + f*x])*Tan[e + f*x])/(4*f)
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}(c+dx)^2}{\sqrt{a-ax}}\,dx, x, \sec(e+f\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))\tan(e+fx)}{4f} + \text{tan}$$

$$= \frac{d(5c+2d)(a+a\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{d(a+a\sec(}{}$$

$$= \frac{d(5c+2d)(a+a\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{(12c^2+16c}{}$$

$$= \frac{a^2\left(12c^2+16cd+7d^2\right)\tan(e+fx)}{8f} + \frac{d(5c+2d)(a+a\sec}{}$$

$$= \frac{a^2\left(12c^2+16cd+7d^2\right)\tan(e+fx)}{8f} + \frac{d(5c+2d)(a+a\sec}{}$$

$$= \frac{a^2\left(12c^2+16cd+7d^2\right)\tan(e+fx)}{8f} + \frac{d(5c+2d)(a+a\sec}{}$$

$$= \frac{a^2\left(12c^2+16cd+7d^2\right)\tan(e+fx)}{8f} + \frac{a^3\left(12c^2+16cd+7\right.}{4f\sqrt{a-a\sec}}$$

Mathematica [B] time = 1.00136, size = 479, normalized size = 2.72

$$-\frac{a^2\sec^4(e+fx)\left(12\left(12c^2+16cd+7d^2\right)\cos(2(e+fx))\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] -(a^2*Sec[e + f*x]^4*(108*c^2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 14
4*c*d*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 63*d^2*Log[Cos[(e + f*x)/2
] - Sin[(e + f*x)/2]] + 12*(12*c^2 + 16*c*d + 7*d^2)*Cos[2*(e + f*x)]*(Log[
Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)
/2]]) + 3*(12*c^2 + 16*c*d + 7*d^2)*Cos[4*(e + f*x)]*(Log[Cos[(e + f*x)/2]
- Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 108*c^2*L
og[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 144*c*d*Log[Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]] - 63*d^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 24*c^2*
Sin[e + f*x] - 96*c*d*Sin[e + f*x] - 90*d^2*Sin[e + f*x] - 96*c^2*Sin[2*(e
+ f*x)] - 224*c*d*Sin[2*(e + f*x)] - 128*d^2*Sin[2*(e + f*x)] - 24*c^2*Sin[
3*(e + f*x)] - 96*c*d*Sin[3*(e + f*x)] - 42*d^2*Sin[3*(e + f*x)] - 48*c^2*S
in[4*(e + f*x)] - 80*c*d*Sin[4*(e + f*x)] - 32*d^2*Sin[4*(e + f*x)]))/(192*
f)
```

Maple [A] time = 0.052, size = 268, normalized size = 1.5

$$\frac{3a^2c^2\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f} + \frac{10a^2cd\tan\left(fx+e\right)}{3f} + \frac{7a^2d^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{8f} + \frac{7a^2d^2\ln\left(\sec\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x)
```

```
[Out] 3/2/f*a^2*c^2*ln(sec(f*x+e)+tan(f*x+e))+10/3/f*a^2*c*d*tan(f*x+e)+7/8/f*a^2
*d^2*sec(f*x+e)*tan(f*x+e)+7/8/f*a^2*d^2*ln(sec(f*x+e)+tan(f*x+e))+2/f*a^2*
c^2*tan(f*x+e)+2/f*a^2*c*d*sec(f*x+e)*tan(f*x+e)+2/f*a^2*c*d*ln(sec(f*x+e)+
tan(f*x+e))+4/3/f*a^2*d^2*tan(f*x+e)+2/3/f*a^2*d^2*tan(f*x+e)*sec(f*x+e)^2+
1/2*a^2*c^2*sec(f*x+e)*tan(f*x+e)/f+2/3/f*a^2*c*d*tan(f*x+e)*sec(f*x+e)^2+1
/4/f*a^2*d^2*tan(f*x+e)*sec(f*x+e)^3
```

Maxima [A] time = 1.01973, size = 437, normalized size = 2.48

$$32\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^2cd+32\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^2d^2-3a^2d^2\left(\frac{2\left(3\sin\left(fx+e\right)^3-5\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^4-2\sin\left(fx+e\right)^2+1}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/48*(32*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c*d + 32*(tan(f*x + e)^3 + 3
*tan(f*x + e))*a^2*d^2 - 3*a^2*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(
sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(si
n(f*x + e) - 1)) - 12*a^2*c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(si
n(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 48*a^2*c*d*(2*sin(f*x + e)/(sin(
f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 12*a^2*d
^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f
*x + e) - 1)) + 48*a^2*c^2*log(sec(f*x + e) + tan(f*x + e)) + 96*a^2*c^2*ta
n(f*x + e) + 96*a^2*c*d*tan(f*x + e))/f
```

Fricas [A] time = 0.510715, size = 490, normalized size = 2.78

$$3\left(12a^2c^2+16a^2cd+7a^2d^2\right)\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-3\left(12a^2c^2+16a^2cd+7a^2d^2\right)\cos\left(fx+e\right)^4\log\left(-\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*cos(f*x + e)^4*log(sin(f*x +
e) + 1) - 3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*cos(f*x + e)^4*log(-sin(f
*x + e) + 1) + 2*(6*a^2*d^2 + 16*(3*a^2*c^2 + 5*a^2*c*d + 2*a^2*d^2)*cos(f*
x + e)^3 + 3*(4*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*cos(f*x + e)^2 + 16*(a^2*
c*d + a^2*d^2)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int c^2\sec\left(e+fx\right)dx+\int 2c^2\sec^2\left(e+fx\right)dx+\int c^2\sec^3\left(e+fx\right)dx+\int d^2\sec^3\left(e+fx\right)dx+\int 2d^2\sec^4\left(e+fx\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(c**2*sec(e + f*x), x) + Integral(2*c**2*sec(e + f*x)**2, x)
+ Integral(c**2*sec(e + f*x)**3, x) + Integral(d**2*sec(e + f*x)**3, x) + I
ntegral(2*d**2*sec(e + f*x)**4, x) + Integral(d**2*sec(e + f*x)**5, x) + In
tegral(2*c*d*sec(e + f*x)**2, x) + Integral(4*c*d*sec(e + f*x)**3, x) + Int
egral(2*c*d*sec(e + f*x)**4, x))
```

Giac [B] time = 1.40344, size = 452, normalized size = 2.57

$$\frac{3\left(12\,a^2c^2+16\,a^2cd+7\,a^2d^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\left(12\,a^2c^2+16\,a^2cd+7\,a^2d^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*log(abs(tan(1/2*f*x + 1/2*e)
+ 1)) - 3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*log(abs(tan(1/2*f*x + 1/2*e
) - 1)) - 2*(36*a^2*c^2*tan(1/2*f*x + 1/2*e)^7 + 48*a^2*c*d*tan(1/2*f*x + 1
/2*e)^7 + 21*a^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 132*a^2*c^2*tan(1/2*f*x + 1/2
*e)^5 - 176*a^2*c*d*tan(1/2*f*x + 1/2*e)^5 - 77*a^2*d^2*tan(1/2*f*x + 1/2*e
)^5 + 156*a^2*c^2*tan(1/2*f*x + 1/2*e)^3 + 272*a^2*c*d*tan(1/2*f*x + 1/2*e)
^3 + 83*a^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 60*a^2*c^2*tan(1/2*f*x + 1/2*e) -
144*a^2*c*d*tan(1/2*f*x + 1/2*e) - 75*a^2*d^2*tan(1/2*f*x + 1/2*e))/(tan(1/
2*f*x + 1/2*e)^2 - 1)^4)/f
```

3.196 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=103

$$\frac{2a^2(3c+2d)\tan(e+fx)}{3f}+\frac{a^2(3c+2d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a^2(3c+2d)\tan(e+fx)\sec(e+fx)}{6f}+\frac{d\tan(e+fx)(a\,\mathrm{se}}{3f}$$

```
[Out] (a^2*(3*c + 2*d)*ArcTanh[Sin[e + f*x]])/(2*f) + (2*a^2*(3*c + 2*d)*Tan[e +
f*x])/(3*f) + (a^2*(3*c + 2*d)*Sec[e + f*x]*Tan[e + f*x])/(6*f) + (d*(a + a
*Sec[e + f*x])^2*Tan[e + f*x])/(3*f)
```

Rubi [A] time = 0.117064, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4001, 3788, 3767, 8, 4046, 3770}

$$\frac{2a^2(3c+2d)\tan(e+fx)}{3f}+\frac{a^2(3c+2d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a^2(3c+2d)\tan(e+fx)\sec(e+fx)}{6f}+\frac{d\tan(e+fx)(a\,\mathrm{se}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*(3*c + 2*d)*ArcTanh[Sin[e + f*x]])/(2*f) + (2*a^2*(3*c + 2*d)*Tan[e +
f*x])/(3*f) + (a^2*(3*c + 2*d)*Sec[e + f*x]*Tan[e + f*x])/(6*f) + (d*(a + a
*Sec[e + f*x])^2*Tan[e + f*x])/(3*f)
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx = \frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3f}+\frac{1}{3}(3c+2d)\int \sec(e+$$

$$= \frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3f}+\frac{1}{3}(3c+2d)\int \sec(e+$$

$$= \frac{a^2(3c+2d)\sec(e+fx)\tan(e+fx)}{6f}+\frac{d(a+a\sec(e+fx))}{3f}$$

$$= \frac{a^2(3c+2d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{2a^2(3c+2d)\tan(e+fx}{3f}$$

Mathematica [B] time = 6.24232, size = 481, normalized size = 4.67

$$a^2\cos^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^2(c+d\sec(e+fx))\left(\frac{4(6c+5d)\sin\left(\frac{fx}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{}{\left(\sin\left(\frac{e}{2}\right)\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*Cos[e + f*x]^3*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x])^2*(c + d*Sec[e +
f*x])*(-6*(3*c + 2*d)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 6*(3*c + 2
*d)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (2*d*Sin[(f*x)/2])/((Cos[e/2
] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) + ((3*c + 7*d)*Cos[e
/2] - (3*c + 5*d)*Sin[e/2])/((Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^2) + (4*(6*c + 5*d)*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])) + (2*d*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/
2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) - ((3*c + 7*d)*Cos[e/2] + (3*c
 + 5*d)*Sin[e/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^2) + (4*(6*c + 5*d)*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]))))/(48*f*(d + c*Cos[e + f*x]))
```

Maple [A] time = 0.041, size = 141, normalized size = 1.4

$$\frac{3a^2c\ln(\sec(fx+e)+\tan(fx+e))}{2f}+\frac{5a^2d\tan(fx+e)}{3f}+2\frac{a^2c\tan(fx+e)}{f}+\frac{a^2d\sec(fx+e)\tan(fx+e)}{f}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x)
```

```
[Out] 3/2/f*a^2*c*ln(sec(f*x+e)+tan(f*x+e))+5/3/f*a^2*d*tan(f*x+e)+2/f*a^2*c*tan(
f*x+e)+1/f*a^2*d*sec(f*x+e)*tan(f*x+e)+1/f*a^2*d*ln(sec(f*x+e)+tan(f*x+e))+
1/2*a^2*c*sec(f*x+e)*tan(f*x+e)/f+1/3/f*a^2*d*tan(f*x+e)*sec(f*x+e)^2
```

Maxima [A] time = 0.98352, size = 225, normalized size = 2.18

$$\frac{4\left(\tan\left(fx+e\right)^{3}+3\tan\left(fx+e\right)\right)a^{2}d-3a^{2}c\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-6a^{2}d\left(\frac{2\sin\left(f\right.}{\sin\left(f\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*d - 3*a^2*c*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 6*a^
2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(
f*x + e) - 1)) + 12*a^2*c*log(sec(f*x + e) + tan(f*x + e)) + 24*a^2*c*tan(f
*x + e) + 12*a^2*d*tan(f*x + e))/f
```

Fricas [A] time = 0.490226, size = 336, normalized size = 3.26

$$\frac{3\left(3a^{2}c+2a^{2}d\right)\cos\left(fx+e\right)^{3}\log\left(\sin\left(fx+e\right)+1\right)-3\left(3a^{2}c+2a^{2}d\right)\cos\left(fx+e\right)^{3}\log\left(-\sin\left(fx+e\right)+1\right)+2\left(2a^{2}d\right.}{12f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(3*a^2*c + 2*a^2*d)*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*(3*a^2
*c + 2*a^2*d)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*a^2*d + 2*(6*a^2
*c + 5*a^2*d)*cos(f*x + e)^2 + 3*(a^2*c + 2*a^2*d)*cos(f*x + e))*sin(f*x +
e))/(f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int c\sec\left(e+fx\right)dx+\int 2c\sec^{2}\left(e+fx\right)dx+\int c\sec^{3}\left(e+fx\right)dx+\int d\sec^{2}\left(e+fx\right)dx+\int 2d\sec^{3}\left(e+fx\right)dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e)),x)
```

```
[Out] a**2*(Integral(c*sec(e + f*x), x) + Integral(2*c*sec(e + f*x)**2, x) + Inte
gral(c*sec(e + f*x)**3, x) + Integral(d*sec(e + f*x)**2, x) + Integral(2*d*
sec(e + f*x)**3, x) + Integral(d*sec(e + f*x)**4, x))
```

Giac [A] time = 1.43254, size = 252, normalized size = 2.45

$$\frac{3\left(3a^2c+2a^2d\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-3\left(3a^2c+2a^2d\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{2\left(9a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+6a^2d\right.}{}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(3*a^2*c + 2*a^2*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*(3*a^2*c
+ 2*a^2*d)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(9*a^2*c*tan(1/2*f*x + 1/
2*e)^5 + 6*a^2*d*tan(1/2*f*x + 1/2*e)^5 - 24*a^2*c*tan(1/2*f*x + 1/2*e)^3 -
 16*a^2*d*tan(1/2*f*x + 1/2*e)^3 + 15*a^2*c*tan(1/2*f*x + 1/2*e) + 18*a^2*d
*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)/f
```

3.197 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=95

$$-\frac{a^2(c-2d)\tanh^{-1}(\sin(e+fx))}{d^2f}+\frac{2a^2(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^2f\sqrt{c+d}}+\frac{a^2\tan(e+fx)}{df}$$

```
[Out] -((a^2*(c - 2*d)*ArcTanh[Sin[e + f*x]])/(d^2*f)) + (2*a^2*(c - d)^(3/2)*Arc
Tanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(d^2*Sqrt[c + d]*f) + (a^
2*Tan[e + f*x])/(d*f)
```

Rubi [B] time = 0.245479, antiderivative size = 208, normalized size of antiderivative = 2.19, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3987, 102, 157, 63, 217, 203, 93, 205}

$$-\frac{2a^3(c-2d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{2a^3(c-d)^{3/2}\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{d^2f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^2\tan(e+fx)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*Tan[e + f*x])/(d*f) - (2*a^3*(c - 2*d)*ArcTan[Sqrt[a - a*Sec[e + f*x]]
/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(d^2*f*Sqrt[a - a*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]]) - (2*a^3*(c - d)^(3/2)*ArcTan[(Sqrt[c + d]*Sqrt[a
 + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(
d^2*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2\tan(e+fx)}{df} + \frac{(a\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{-a^3d+a^3(c-2d)x}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{df\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2\tan(e+fx)}{df} + \frac{\left(a^4(c-2d)\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx, x, \sec(e+\right.}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2\tan(e+fx)}{df} - \frac{\left(2a^3(c-2d)\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}}\,dx, x, \sqrt{a-a\sec}\right.}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2\tan(e+fx)}{df} - \frac{2a^3(c-d)^{3/2}\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^2\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\left(2a^3(\right.}{}$$

$$= \frac{a^2\tan(e+fx)}{df} - \frac{2a^3(c-2d)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2a^3(c-d)^3}{d^2\sqrt{c+d}}$$

Mathematica [C] time = 1.98948, size = 329, normalized size = 3.46

$$a^2\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^2(c\cos(e+fx)+d)\left(-\frac{2i(c-d)^2(\cos(e)-i\sin(e))\tan^{-1}\left(\frac{(\sin(e)+i\cos(e))\left(\tan\left(\frac{fx}{2}\right)(c\cos(e)-d)\right.}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right.}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*Cos[e + f*x]*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x]
)^2*((c - 2*d)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - (c - 2*d)*Log[Cos
[(e + f*x)/2] + Sin[(e + f*x)/2]] - ((2*I)*(c - d)^2*ArcTan[((I*Cos[e] + Si
n[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos
[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I
*Sin[e])^2]) + (d*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])) + (d*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]))))/(4*d^2*f*(c + d*Sec[e + f*x]))
```

Maple [B] time = 0.081, size = 291, normalized size = 3.1

$$-\frac{a^2}{fd}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1}-\frac{a^2c}{fd^2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+2\,\frac{a^2\ln\left(\tan\left(1/2\,fx+e/2\right)+1\right)}{fd}-\frac{a^2}{fd}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x)
```

```
[Out] -1/f*a^2/d/(tan(1/2*f*x+1/2*e)+1)-1/f*a^2/d^2*ln(tan(1/2*f*x+1/2*e)+1)*c+2/
f*a^2/d*ln(tan(1/2*f*x+1/2*e)+1)-1/f*a^2/d/(tan(1/2*f*x+1/2*e)-1)+1/f*a^2/d
^2*ln(tan(1/2*f*x+1/2*e)-1)*c-2/f*a^2/d*ln(tan(1/2*f*x+1/2*e)-1)+2/f*a^2/d^
2/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))
*c^2-4/f*a^2/d/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*
(c-d))^(1/2))*c+2/f*a^2/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d
)/((c+d)*(c-d))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.9532, size = 953, normalized size = 10.03

$$\left[\frac{2\,a^2d\sin\left(fx+e\right)-\left(a^2c-a^2d\right)\sqrt{\frac{c-d}{c+d}}\cos\left(fx+e\right)\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2-2\left(c^2+cd+\left(cd+d^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{c-d}{c+d}}\sin\left(f\right.}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2}\right.}{2\,d^2f\cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/2*(2*a^2*d*sin(f*x + e) - (a^2*c - a^2*d)*sqrt((c - d)/(c + d))*cos(f*x
+ e)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d
+ (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 - d^
2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - (a^2*c - 2*a^2*d)*cos
(f*x + e)*log(sin(f*x + e) + 1) + (a^2*c - 2*a^2*d)*cos(f*x + e)*log(-sin(f
*x + e) + 1))/(d^2*f*cos(f*x + e)), 1/2*(2*a^2*d*sin(f*x + e) + 2*(a^2*c -
a^2*d)*sqrt(-(c - d)/(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c
 + d))/((c - d)*sin(f*x + e)))*cos(f*x + e) - (a^2*c - 2*a^2*d)*cos(f*x + e
)*log(sin(f*x + e) + 1) + (a^2*c - 2*a^2*d)*cos(f*x + e)*log(-sin(f*x + e)
+ 1))/(d^2*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx+\int\frac{2\sec^2\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx+\int\frac{\sec^3\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e)),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c + d*sec(e + f*x)), x) + Integral(2*sec(e + f
*x)**2/(c + d*sec(e + f*x)), x) + Integral(sec(e + f*x)**3/(c + d*sec(e + f
*x)), x))
```

Giac [B] time = 1.28523, size = 274, normalized size = 2.88

$$-\frac{\frac{2\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)d}+\frac{\left(a^2c-2\,a^2d\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{d^2}-\frac{\left(a^2c-2\,a^2d\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{d^2}+\frac{2\left(a^2c^2-2\,a^2cd+a^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,c-2\right.}{\sqrt{-c^2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -(2*a^2*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*d) + (a^2*c - 2*
a^2*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^2 - (a^2*c - 2*a^2*d)*log(abs(t
an(1/2*f*x + 1/2*e) - 1))/d^2 + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) -
d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/(sqrt(-c^2 + d^2)*d^2))/f
```

3.198 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=117

$$-\frac{2a^2\sqrt{c-d}(c+2d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^2f(c+d)^{3/2}}-\frac{a^2(c-d)\tan(e+fx)}{df(c+d)(c+d\sec(e+fx))}+\frac{a^2\tanh^{-1}(\sin(e+fx))}{d^2f}$$

```
[Out] (a^2*ArcTanh[Sin[e + f*x]])/(d^2*f) - (2*a^2*Sqrt[c - d]*(c + 2*d)*ArcTanh[
(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(d^2*(c + d)^(3/2)*f) - (a^2*(
c - d)*Tan[e + f*x])/(d*(c + d)*f*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.25679, antiderivative size = 231, normalized size of antiderivative = 1.97, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3987, 98, 157, 63, 217, 203, 93, 205}

$$\frac{2a^3\sqrt{c-d}(c+2d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{d^2f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{a^2(c-d)\tan(e+fx)}{df(c+d)(c+d\sec(e+fx))}+\frac{2a^3\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a}}{\sqrt{a(se}}\right.}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^3*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e +
f*x])/(d^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^3*Sq
rt[c - d]*(c + 2*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c -
 d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(d^2*(c + d)^(3/2)*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^2*(c - d)*Tan[e + f*x])/(d*(
c + d)*f*(c + d*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
```

```
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{(a\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{-2a^3d-a^3(c+d)x}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx\right.}{d(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\,\mathrm{se}}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} - \frac{\left(a^4\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx, x,\right.}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{\left(2a^3\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}}\,dx, x, \sqrt{a}\right.}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(}}$$

$$= \frac{2a^3\sqrt{c-d}(c+2d)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^2(c-d)\tan(}{d(c+d)f(c+d\,\mathrm{se}}$$

$$= \frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^3\sqrt{c-d}(c+2d)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{}}{\sqrt{c-d}\sqrt{}}\right.}{d^2(c+d)^{3/2}f\sqrt{a-a\sec(e+f}}$$

Mathematica [C] time = 1.47496, size = 312, normalized size = 2.67

$$a^2\sec^4\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^2(c\cos(e+fx)+d)\left(\frac{2\left(c^2+cd-2d^2\right)(\sin(e)+i\cos(e))(c\cos(e+fx)+d)\tan^{-1}\left(\frac{(\sin(e)+i\cos(e))\left(\tan\left(\frac{fx}{2}\right)(c\cos(e\right.}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(}\right.}{(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x])^2*(-((d + c
*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) + (d + c*Cos[e + f
*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (2*(c^2 + c*d - 2*d^2)*ArcT
an[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^
2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f*x])*(I*Cos[e] + Sin
[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((c - d)*d*(d
*Sin[e] - c*Sin[f*x]))/(c*(c + d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2
]))))/(4*d^2*f*(c + d*Sec[e + f*x])^2)
```

Maple [B] time = 0.11, size = 330, normalized size = 2.8

$$\frac{a^2}{fd^2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-\frac{a^2}{fd^2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+2\,\frac{a^2c\tan\left(1/2\,fx+e/2\right)}{fd\,(c+d)\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2c-\left(\tan\left(1/2\,fx+e/2\right)\right)^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 1/f*a^2/d^2*ln(tan(1/2*f*x+1/2*e)+1)-1/f*a^2/d^2*ln(tan(1/2*f*x+1/2*e)-1)+2
/f*a^2/d*c/(c+d)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2
*e)^2*d-c-d)-2/f*a^2/d^2/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*
e)*(c-d)/((c+d)*(c-d))^(1/2))*c^2-2/f*a^2/d*c/(c+d)/((c+d)*(c-d))^(1/2)*arc
tanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))-2/f*a^2/(c+d)*tan(1/2*f*
x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)+4/f*a^2/(c+d)/
((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.987852, size = 1289, normalized size = 11.02

$$\left[\frac{\left(a^2cd + 2\,a^2d^2 + \left(a^2c^2 + 2\,a^2cd\right)\cos\left(fx+e\right)\right)\sqrt{\frac{c-d}{c+d}}\log\left(\frac{2\,cd\cos\left(fx+e\right) - \left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2 - 2\left(c^2+cd+\left(cd+d^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{c-d}{c+d}}\sin}{c^2\cos\left(fx+e\right)^2 + 2\,cd\cos\left(fx+e\right) + d^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*((a^2*c*d + 2*a^2*d^2 + (a^2*c^2 + 2*a^2*c*d)*cos(f*x + e))*sqrt((c -
d)/(c + d))*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2
 + c*d + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c
^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + (a^2*c*d + a^2
*d^2 + (a^2*c^2 + a^2*c*d)*cos(f*x + e))*log(sin(f*x + e) + 1) - (a^2*c*d +
 a^2*d^2 + (a^2*c^2 + a^2*c*d)*cos(f*x + e))*log(-sin(f*x + e) + 1) - 2*(a^
2*c*d - a^2*d^2)*sin(f*x + e))/((c^2*d^2 + c*d^3)*f*cos(f*x + e) + (c*d^3 +
 d^4)*f), -1/2*(2*(a^2*c*d + 2*a^2*d^2 + (a^2*c^2 + 2*a^2*c*d)*cos(f*x + e)
)*sqrt(-(c - d)/(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d)
)/((c - d)*sin(f*x + e))) - (a^2*c*d + a^2*d^2 + (a^2*c^2 + a^2*c*d)*cos(f*
x + e))*log(sin(f*x + e) + 1) + (a^2*c*d + a^2*d^2 + (a^2*c^2 + a^2*c*d)*co
s(f*x + e))*log(-sin(f*x + e) + 1) + 2*(a^2*c*d - a^2*d^2)*sin(f*x + e))/((
c^2*d^2 + c*d^3)*f*cos(f*x + e) + (c*d^3 + d^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{\sec\left(e+fx\right)}{c^2+2cd\sec\left(e+fx\right)+d^2\sec^2\left(e+fx\right)}\,dx+\int\frac{2\sec^2\left(e+fx\right)}{c^2+2cd\sec\left(e+fx\right)+d^2\sec^2\left(e+fx\right)}\,dx+\int\frac{}{c^2+2cd\sec\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)*
*2), x) + Integral(2*sec(e + f*x)**2/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(
e + f*x)**2), x) + Integral(sec(e + f*x)**3/(c**2 + 2*c*d*sec(e + f*x) + d*
*2*sec(e + f*x)**2), x))
```

Giac [B] time = 1.38316, size = 323, normalized size = 2.76

$$\frac{\frac{a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{d^2}-\frac{a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{d^2}+\frac{2\left(a^2c^2+a^2cd-2\,a^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(cd^2+d^3\right)\sqrt{-c^2+d^2}}-}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] (a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^2 - a^2*log(abs(tan(1/2*f*x + 1/2
*e) - 1))/d^2 + 2*(a^2*c^2 + a^2*c*d - 2*a^2*d^2)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x +
1/2*e))/sqrt(-c^2 + d^2)))/((c*d^2 + d^3)*sqrt(-c^2 + d^2)) + 2*(a^2*c*tan(
1/2*f*x + 1/2*e) - a^2*d*tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^2 -
 d*tan(1/2*f*x + 1/2*e)^2 - c - d)*(c*d + d^2)))/f
```

3.199 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=130

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}(c+d)^{5/2}}+\frac{3a^2\tan(e+fx)}{2f(c+d)^2(c+d\sec(e+fx))}+\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{2f(c+d)(c+d\sec(e+fx))^2}$$

```
[Out] (3*a^2*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*(c
 + d)^(5/2)*f) + ((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(2*(c + d)*f*(c +
d*Sec[e + f*x])^2) + (3*a^2*Tan[e + f*x])/(2*(c + d)^2*f*(c + d*Sec[e + f*x
]))
```

Rubi [A] time = 0.196767, antiderivative size = 184, normalized size of antiderivative = 1.42, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3987, 94, 93, 205}

$$-\frac{3a^3\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{f\sqrt{c-d}(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{3a^2\tan(e+fx)}{2f(c+d)^2(c+d\sec(e+fx))}+\frac{\tan(e+fx)\left(a^2\sec(e+f\right.}{2f(c+d)(c+d\sec(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (-3*a^3*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a -
 a*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c - d]*(c + d)^(5/2)*f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((a^2 + a^2*Sec[e + f*x])*Tan[e + f
*x])/(2*(c + d)*f*(c + d*Sec[e + f*x])^2) + (3*a^2*Tan[e + f*x])/(2*(c + d)
^2*f*(c + d*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} - \frac{\left(3a^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+ax}}{\sqrt{a-ax}(c+dx)^2}\,dx,\right.}{2(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\,\text{se}}}$$

$$= \frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{3a^2\tan(e+fx)}{2(c+d)^2f(c+d\sec(e+fx))} - \frac{\left(3a^4\tan(e\right.}{2(c}$$

$$= \frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{3a^2\tan(e+fx)}{2(c+d)^2f(c+d\sec(e+fx))} - \frac{\left(3a^4\tan(e\right.}{(c+}$$

$$= -\frac{3a^3\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{c-d}(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left(a^2+a^2\sec(e+fx)\right.}{2(c+d)f(c+d\,\text{se}}$$

Mathematica [C] time = 1.16088, size = 249, normalized size = 1.92

$$\frac{a^2\sec^4\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)(\sec(e+fx)+1)^2(c\cos(e+fx)+d)\left(-\frac{6i(\cos(e)-i\sin(e))(c\cos(e+fx)+d)^2\tan^{-1}\left(\frac{(\sin(e)+i\cos(e))\left(\tan\left(\frac{fx}{2}\right)\right.}{\sqrt{c^2-d^2}\sqrt{(\cos(e}}\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right.}{8f(c+d)^2(c+d\sec(e+fx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^4*Sec[e + f*x]*(1 + Sec[e + f*x]
)^2*(((-6*I)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f
*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f*x]
)^2*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((
c - d)*(c + d)*Sec[e]*(-(d*Sin[e]) + c*Sin[f*x]))/c^2 + ((d + c*Cos[e + f*x
])*Sec[e]*((c^2 - 4*c*d - 2*d^2)*Sin[e] + c*(4*c + d)*Sin[f*x]))/c^2))/(8*(
c + d)^2*f*(c + d*Sec[e + f*x])^3)
```

Maple [A] time = 0.107, size = 167, normalized size = 1.3

$$8\frac{a^2}{f}\left(1/4\frac{\tan\left(1/2fx+e/2\right)}{(c+d)\left(\left(\tan\left(1/2fx+e/2\right)\right)^2c-\left(\tan\left(1/2fx+e/2\right)\right)^2d-c-d\right)^2}+3/4\frac{1}{c+d}\left(-1/2\frac{\tan}{(c+d)\left(\left(\tan\left(1/2fx+e/2\right)\right.\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x)
```

```
[Out] 8/f*a^2*(1/4*tan(1/2*f*x+1/2*e)/(c+d)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1
/2*e)^2*d-c-d)^2+3/4/(c+d)*(-1/2*tan(1/2*f*x+1/2*e)/(c+d)/(tan(1/2*f*x+1/2*
e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)+1/2/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(ta
n(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.561703, size = 1339, normalized size = 10.3

$$\left[\frac{3\left(a^2c^2\cos\left(fx+e\right)^2+2\,a^2cd\cos\left(fx+e\right)+a^2d^2\right)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2+2\,\sqrt{c^2-d^2}\left(d\cos\left(fx+e\right)+c\right)\sin}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2}\right.}{4\left(\left(c^6+2\,c^5d-2\,c^3d^3-c^2d^4\right)f\cos\left(fx+e\right)^2+2\left(c^5d+2\,c^4d^2-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(3*(a^2*c^2*cos(f*x + e)^2 + 2*a^2*c*d*cos(f*x + e) + a^2*d^2)*sqrt(c^
2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^
2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)
^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2 - 4*
a^2*d^3 + (4*a^2*c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e))*sin
(f*x + e))/((c^6 + 2*c^5*d - 2*c^3*d^3 - c^2*d^4)*f*cos(f*x + e)^2 + 2*(c^5
*d + 2*c^4*d^2 - 2*c^2*d^4 - c*d^5)*f*cos(f*x + e) + (c^4*d^2 + 2*c^3*d^3 -
 2*c*d^5 - d^6)*f), 1/2*(3*(a^2*c^2*cos(f*x + e)^2 + 2*a^2*c*d*cos(f*x + e)
 + a^2*d^2)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/
((c^2 - d^2)*sin(f*x + e))) + (a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2 - 4*a^2*d^
3 + (4*a^2*c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e))*sin(f*x +
 e))/((c^6 + 2*c^5*d - 2*c^3*d^3 - c^2*d^4)*f*cos(f*x + e)^2 + 2*(c^5*d + 2
*c^4*d^2 - 2*c^2*d^4 - c*d^5)*f*cos(f*x + e) + (c^4*d^2 + 2*c^3*d^3 - 2*c*d
^5 - d^6)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{\sec\left(e+fx\right)}{c^3+3c^2d\sec\left(e+fx\right)+3cd^2\sec^2\left(e+fx\right)+d^3\sec^3\left(e+fx\right)}\,dx+\int\frac{2\sec^2\left(e+fx\right)}{c^3+3c^2d\sec\left(e+fx\right)+3cd^2\sec^2\left(e+f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e
+ f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(2*sec(e + f*x)**2/(c**3 +
3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3), x
) + Integral(sec(e + f*x)**3/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e
 + f*x)**2 + d**3*sec(e + f*x)**3), x))
```

Giac [A] time = 1.36723, size = 297, normalized size = 2.28

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)a^2}{\left(c^2+2cd+d^2\right)\sqrt{-c^2+d^2}}+\frac{3a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3a^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-5a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-5a^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)^2\left(c^2+2cd+d^2\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -(3*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*
x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*a^2/((c^2 + 2*c*d +
 d^2)*sqrt(-c^2 + d^2)) + (3*a^2*c*tan(1/2*f*x + 1/2*e)^3 - 3*a^2*d*tan(1/2
*f*x + 1/2*e)^3 - 5*a^2*c*tan(1/2*f*x + 1/2*e) - 5*a^2*d*tan(1/2*f*x + 1/2*
e))/((c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2*(c^2 +
 2*c*d + d^2)))/f
```

3.200 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=213

$$\frac{a^2(3c-2d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{7/2}}+\frac{a^2(3c-2d)\tan(e+fx)}{2f(c-d)(c+d)^3(c+d\sec(e+fx))}+\frac{(3c-2d)\tan(e+fx)\left(a^2\sec(e+fx)\right.}{6f(c-d)(c+d)^2(c+d\sec(e+fx)}$$

```
[Out] (a^2*(3*c - 2*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c -
 d)^(3/2)*(c + d)^(7/2)*f) - (d*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(3*(c^
2 - d^2)*f*(c + d*Sec[e + f*x])^3) + ((3*c - 2*d)*(a^2 + a^2*Sec[e + f*x])*
Tan[e + f*x])/(6*(c - d)*(c + d)^2*f*(c + d*Sec[e + f*x])^2) + (a^2*(3*c -
2*d)*Tan[e + f*x])/(2*(c - d)*(c + d)^3*f*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.28264, antiderivative size = 268, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3987, 96, 94, 93, 205}

$$-\frac{a^3(3c-2d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{3/2}(c+d)^{7/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^2(3c-2d)\tan(e+fx)}{2f(c-d)(c+d)^3(c+d\sec(e+fx))}+\frac{(3c-2d)\tan(e+}{6f(c-d)(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] -((a^3*(3*c - 2*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c -
d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/((c - d)^(3/2)*(c + d)^(7/2)*f*
Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])) - (d*(a + a*Sec[e + f*x
])^2*Tan[e + f*x])/(3*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^3) + ((3*c - 2*d)*
(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(6*(c - d)*(c + d)^2*f*(c + d*Sec[e
+ f*x])^2) + (a^2*(3*c - 2*d)*Tan[e + f*x])/(2*(c - d)*(c + d)^3*f*(c + d*S
ec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}(c+dx)^4}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} - \frac{\left(a^2(3c-2d)\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax}{\sqrt{a-ax}(}\right.}{3\left(c^2-d^2\right)f\sqrt{a-a\sec(e+fx)}\sqrt{a}}$$

$$= -\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{(3c-2d)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2}$$

$$= -\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{(3c-2d)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2}$$

$$= -\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{(3c-2d)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2}$$

$$= -\frac{a^3(3c-2d)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{(c-d)^{3/2}(c+d)^{7/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{d(a+a\sec(e+fx}{3\left(c^2-d^2\right)f(c+a}$$

Mathematica [A] time = 4.73488, size = 211, normalized size = 0.99

$$\frac{a^2(c-d)^2\left(24(3c-2d)(c\cos(e+fx)+d)^3\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)-2\sqrt{c^2-d^2}\sin(e+fx)\left(6\left(6c^2d+c^3-7cd^2-2d^3\right)\right.\right.}{24f(d-c)^3(c+d)^3\sqrt{c^2-d^2}(c\cos(e+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*(c - d)^2*(24*(3*c - 2*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2
 - d^2]]*(d + c*Cos[e + f*x])^3 - 2*Sqrt[c^2 - d^2]*(12*c^3 - 5*c^2*d + 6*c
```

```
*d^2 - 22*d^3 + 6*(c^3 + 6*c^2*d - 7*c*d^2 - 2*d^3)*Cos[e + f*x] + (12*c^3
- 7*c^2*d - 6*c*d^2 - 2*d^3)*Cos[2*(e + f*x)])*Sin[e + f*x]))/(24*(-c + d)^
3*(c + d)^3*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x])^3)
```

Maple [A] time = 0.128, size = 228, normalized size = 1.1

$$8\,\frac{a^2}{f}\left(-\frac{1}{\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2c-\left(\tan\left(1/2\,fx+e/2\right)\right)^2d-c-d\right)^3}\left(1/8\,\frac{(3\,c-2\,d)\,(c-d)\left(\tan\left(1/2\,fx+e/2\right)\right)^5}{c^3+3\,c^2d+3\,d^2c+d^3}-1/3\,\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x)
```

```
[Out] 8/f*a^2*(-(1/8*(3*c-2*d)*(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)
^5-1/3*(3*c-2*d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/8*(5*c-6*d)/(c+d)/(
c-d)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d
)^3+1/8*(3*c-2*d)/(c^4+2*c^3*d-2*c*d^3-d^4)/((c+d)*(c-d))^(1/2)*arctanh(tan
(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.67826, size = 2585, normalized size = 12.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*(3*a^2*c*d^3 - 2*a^2*d^4 + (3*a^2*c^4 - 2*a^2*c^3*d)*cos(f*x + e)^
3 + 3*(3*a^2*c^3*d - 2*a^2*c^2*d^2)*cos(f*x + e)^2 + 3*(3*a^2*c^2*d^2 - 2*a
^2*c*d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*
d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) +
 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(a^2*c^4
*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 + 10*a^2*d^5 + (12*a^2*c^
5 - 7*a^2*c^4*d - 18*a^2*c^3*d^2 + 5*a^2*c^2*d^3 + 6*a^2*c*d^4 + 2*a^2*d^5)
*cos(f*x + e)^2 + 3*(a^2*c^5 + 6*a^2*c^4*d - 8*a^2*c^3*d^2 - 8*a^2*c^2*d^3
+ 7*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e))*sin(f*x + e))/((c^9 + 2*c^8*d - c^
7*d^2 - 4*c^6*d^3 - c^5*d^4 + 2*c^4*d^5 + c^3*d^6)*f*cos(f*x + e)^3 + 3*(c^
8*d + 2*c^7*d^2 - c^6*d^3 - 4*c^5*d^4 - c^4*d^5 + 2*c^3*d^6 + c^2*d^7)*f*co
```

```
s(f*x + e)^2 + 3*(c^7*d^2 + 2*c^6*d^3 - c^5*d^4 - 4*c^4*d^5 - c^3*d^6 + 2*c
^2*d^7 + c*d^8)*f*cos(f*x + e) + (c^6*d^3 + 2*c^5*d^4 - c^4*d^5 - 4*c^3*d^6
 - c^2*d^7 + 2*c*d^8 + d^9)*f), 1/6*(3*(3*a^2*c*d^3 - 2*a^2*d^4 + (3*a^2*c^
4 - 2*a^2*c^3*d)*cos(f*x + e)^3 + 3*(3*a^2*c^3*d - 2*a^2*c^2*d^2)*cos(f*x +
 e)^2 + 3*(3*a^2*c^2*d^2 - 2*a^2*c*d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arct
an(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (a^
2*c^4*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 + 10*a^2*d^5 + (12*a
^2*c^5 - 7*a^2*c^4*d - 18*a^2*c^3*d^2 + 5*a^2*c^2*d^3 + 6*a^2*c*d^4 + 2*a^2
*d^5)*cos(f*x + e)^2 + 3*(a^2*c^5 + 6*a^2*c^4*d - 8*a^2*c^3*d^2 - 8*a^2*c^2
*d^3 + 7*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e))*sin(f*x + e))/((c^9 + 2*c^8*d
 - c^7*d^2 - 4*c^6*d^3 - c^5*d^4 + 2*c^4*d^5 + c^3*d^6)*f*cos(f*x + e)^3 +
3*(c^8*d + 2*c^7*d^2 - c^6*d^3 - 4*c^5*d^4 - c^4*d^5 + 2*c^3*d^6 + c^2*d^7)
*f*cos(f*x + e)^2 + 3*(c^7*d^2 + 2*c^6*d^3 - c^5*d^4 - 4*c^4*d^5 - c^3*d^6
+ 2*c^2*d^7 + c*d^8)*f*cos(f*x + e) + (c^6*d^3 + 2*c^5*d^4 - c^4*d^5 - 4*c^
3*d^6 - c^2*d^7 + 2*c*d^8 + d^9)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{\sec\left(e+fx\right)}{c^4+4c^3d\sec\left(e+fx\right)+6c^2d^2\sec^2\left(e+fx\right)+4cd^3\sec^3\left(e+fx\right)+d^4\sec^4\left(e+fx\right)}\,dx+\int \frac{}{c^4+4c^3d\sec\left(e+fx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**4,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec
(e + f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integ
ral(2*sec(e + f*x)**2/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f
*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral(se
c(e + f*x)**3/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**2 +
 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x))
```

Giac [B] time = 1.44826, size = 567, normalized size = 2.66

$$-\frac{\frac{3\left(3a^2c-2a^2d\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(c^4+2c^3d-2cd^3-d^4\right)\sqrt{-c^2+d^2}}+\frac{9a^2c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-24a^2c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+21a^2cd^2\tan\left(\frac{1}{2}fx-\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*(3*a^2*c - 2*a^2*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d
) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^
2)))/((c^4 + 2*c^3*d - 2*c*d^3 - d^4)*sqrt(-c^2 + d^2)) + (9*a^2*c^3*tan(1/
2*f*x + 1/2*e)^5 - 24*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^5 + 21*a^2*c*d^2*tan(1
/2*f*x + 1/2*e)^5 - 6*a^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 24*a^2*c^3*tan(1/2*f
*x + 1/2*e)^3 + 16*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 24*a^2*c*d^2*tan(1/2*
f*x + 1/2*e)^3 - 16*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^2*c^3*tan(1/2*f*x
 + 1/2*e) + 12*a^2*c^2*d*tan(1/2*f*x + 1/2*e) - 21*a^2*c*d^2*tan(1/2*f*x +
1/2*e) - 18*a^2*d^3*tan(1/2*f*x + 1/2*e))/((c^4 + 2*c^3*d - 2*c*d^3 - d^4)*
```

```
(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))/f
```

3.201 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx$

Optimal. Leaf size=276

$$\frac{a^2\left(12c^2-16cd+7d^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{4f(c-d)^{5/2}(c+d)^{9/2}}+\frac{a^2\left(16c^2d+2c^3-59cd^2+32d^3\right)\tan(e+fx)}{24df(c-d)^2(c+d)^4(c+d\sec(e+fx))}+\frac{a^2\left(2c^2+16cd-21\cdots\right.}{24df(c-d)(c+d)^3(c\cdots}$$

```
[Out] (a^2*(12*c^2 - 16*c*d + 7*d^2)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[
c + d]])/(4*(c - d)^(5/2)*(c + d)^(9/2)*f) - (a^2*(c - d)*Tan[e + f*x])/(4*
d*(c + d)*f*(c + d*Sec[e + f*x])^4) + (a^2*(c + 8*d)*Tan[e + f*x])/(12*d*(c
 + d)^2*f*(c + d*Sec[e + f*x])^3) + (a^2*(2*c^2 + 16*c*d - 21*d^2)*Tan[e +
f*x])/(24*(c - d)*d*(c + d)^3*f*(c + d*Sec[e + f*x])^2) + (a^2*(2*c^3 + 16*
c^2*d - 59*c*d^2 + 32*d^3)*Tan[e + f*x])/(24*(c - d)^2*d*(c + d)^4*f*(c + d
*Sec[e + f*x]))
```

Rubi [A] time = 0.556834, antiderivative size = 330, normalized size of antiderivative = 1.2, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 98, 151, 12, 93, 205}

$$-\frac{a^3\left(12c^2-16cd+7d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{4f(c-d)^{5/2}(c+d)^{9/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^2\left(16c^2d+2c^3-59cd^2+32d^3\right)\tan(e+fx)}{24df(c-d)^2(c+d)^4(c+d\sec(e+fx))}+\frac{a^2\left(2\cdots\right.}{24df(c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] -(a^3*(12*c^2 - 16*c*d + 7*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]
])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(4*(c - d)^(5/2)*(
c + d)^(9/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^2*(c
 - d)*Tan[e + f*x])/(4*d*(c + d)*f*(c + d*Sec[e + f*x])^4) + (a^2*(c + 8*d)
*Tan[e + f*x])/(12*d*(c + d)^2*f*(c + d*Sec[e + f*x])^3) + (a^2*(2*c^2 + 16
*c*d - 21*d^2)*Tan[e + f*x])/(24*(c - d)*d*(c + d)^3*f*(c + d*Sec[e + f*x])
^2) + (a^2*(2*c^3 + 16*c^2*d - 59*c*d^2 + 32*d^3)*Tan[e + f*x])/(24*(c - d)
^2*d*(c + d)^4*f*(c + d*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}(c+dx)^5}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{(a\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{-8a^3d-a^3(c+7d)x}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)^4}\,dx\right.}{4d(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\,\mathrm{se}}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{\tan(e+f}{12ad(c}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{a^2\left(2c^2\right.}{24(c-d)a}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{a^2\left(2c^2\right.}{24(c-d)a}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{a^2\left(2c^2\right.}{24(c-d)a}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{a^2\left(2c^2\right.}{24(c-d)a}$$

$$= -\frac{a^3\left(12c^2-16cd+7d^2\right)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{4(c-d)^{5/2}(c+d)^{9/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^2(c-d)\,\mathrm{t}}{4d(c+d)f(c+}$$

Mathematica [A] time = 9.69683, size = 322, normalized size = 1.17

$$a^2\left(\frac{\sin(e+fx)\left(-16c^3d^2\cos(3(e+fx))+5c^2d^3\cos(3(e+fx))+\left(208c^3d^2-785c^2d^3-172c^4d+144c^5+368cd^4+102d^5\right)\cos(e+fx)+2\left(-227c^3d^2+32c^2d^3+96c^4d+12c^5+4\right.}{(c\cos(e}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*((-24*(12*c^2 - 16*c*d + 7*d^2)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sq
rt[c^2 - d^2]])/Sqrt[c^2 - d^2] + ((24*c^5 + 192*c^4*d - 446*c^3*d^2 + 128*
c^2*d^3 - 148*c*d^4 + 160*d^5 + (144*c^5 - 172*c^4*d + 208*c^3*d^2 - 785*c^
2*d^3 + 368*c*d^4 + 102*d^5)*Cos[e + f*x] + 2*(12*c^5 + 96*c^4*d - 227*c^3*
d^2 + 32*c^2*d^3 + 44*c*d^4 + 16*d^5)*Cos[2*(e + f*x)] + 48*c^5*Cos[3*(e +
f*x)] - 68*c^4*d*Cos[3*(e + f*x)] - 16*c^3*d^2*Cos[3*(e + f*x)] + 5*c^2*d^3
*Cos[3*(e + f*x)] + 16*c*d^4*Cos[3*(e + f*x)] + 6*d^5*Cos[3*(e + f*x)])*Sin
[e + f*x])/(d + c*Cos[e + f*x])^4))/(96*(c - d)^2*(c + d)^4*f)
```

Maple [A] time = 0.146, size = 352, normalized size = 1.3

$$8\,\frac{a^2}{f}\left(-\frac{1}{\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2c-\left(\tan\left(1/2\,fx+e/2\right)\right)^2d-c-d\right)^4}\left(1/32\,\frac{\left(12\,c^2-16\,cd+7\,d^2\right)(c-d)\left(\tan\left(1/2\,fx+e/2\right)\right.}{c^4+4\,c^3d+6\,c^2d^2+4\,cd^3+d^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x)
```

```
[Out] 8/f*a^2*(-(1/32*(12*c^2-16*c*d+7*d^2)*(c-d)/(c^4+4*c^3*d+6*c^2*d^2+4*c*d^3+
d^4)*tan(1/2*f*x+1/2*e)^7-11/96*(12*c^2-16*c*d+7*d^2)/(c^3+3*c^2*d+3*c*d^2+
d^3)*tan(1/2*f*x+1/2*e)^5+1/96*(156*c^2-272*c*d+83*d^2)/(c-d)/(c^2+2*c*d+d^
2)*tan(1/2*f*x+1/2*e)^3-1/32*(20*c^2-48*c*d+25*d^2)/(c+d)/(c^2-2*c*d+d^2)*t
an(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^4+1/
32*(12*c^2-16*c*d+7*d^2)/(c^6+2*c^5*d-c^4*d^2-4*c^3*d^3-c^2*d^4+2*c*d^5+d^6
)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.836016, size = 4093, normalized size = 14.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] [1/48*(3*(12*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6 + (12*a^2*c^6 - 16*a^2*
c^5*d + 7*a^2*c^4*d^2)*cos(f*x + e)^4 + 4*(12*a^2*c^5*d - 16*a^2*c^4*d^2 +
7*a^2*c^3*d^3)*cos(f*x + e)^3 + 6*(12*a^2*c^4*d^2 - 16*a^2*c^3*d^3 + 7*a^2*
c^2*d^4)*cos(f*x + e)^2 + 4*(12*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5)
*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(
f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 -
d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*a^2*c^5*d^2 +
16*a^2*c^4*d^3 - 61*a^2*c^3*d^4 + 16*a^2*c^2*d^5 + 59*a^2*c*d^6 - 32*a^2*d^
7 + (48*a^2*c^7 - 68*a^2*c^6*d - 64*a^2*c^5*d^2 + 73*a^2*c^4*d^3 + 32*a^2*c
^3*d^4 + a^2*c^2*d^5 - 16*a^2*c*d^6 - 6*a^2*d^7)*cos(f*x + e)^3 + (12*a^2*c
^7 + 96*a^2*c^6*d - 239*a^2*c^5*d^2 - 64*a^2*c^4*d^3 + 271*a^2*c^3*d^4 - 16
*a^2*c^2*d^5 - 44*a^2*c*d^6 - 16*a^2*d^7)*cos(f*x + e)^2 + (8*a^2*c^6*d + 6
4*a^2*c^5*d^2 - 208*a^2*c^4*d^3 + 16*a^2*c^3*d^4 + 221*a^2*c^2*d^5 - 80*a^2
*c*d^6 - 21*a^2*d^7)*cos(f*x + e))*sin(f*x + e))/((c^12 + 2*c^11*d - 2*c^10
*d^2 - 6*c^9*d^3 + 6*c^7*d^5 + 2*c^6*d^6 - 2*c^5*d^7 - c^4*d^8)*f*cos(f*x +
 e)^4 + 4*(c^11*d + 2*c^10*d^2 - 2*c^9*d^3 - 6*c^8*d^4 + 6*c^6*d^6 + 2*c^5*
d^7 - 2*c^4*d^8 - c^3*d^9)*f*cos(f*x + e)^3 + 6*(c^10*d^2 + 2*c^9*d^3 - 2*c
^8*d^4 - 6*c^7*d^5 + 6*c^5*d^7 + 2*c^4*d^8 - 2*c^3*d^9 - c^2*d^10)*f*cos(f*
x + e)^2 + 4*(c^9*d^3 + 2*c^8*d^4 - 2*c^7*d^5 - 6*c^6*d^6 + 6*c^4*d^8 + 2*c
^3*d^9 - 2*c^2*d^10 - c*d^11)*f*cos(f*x + e) + (c^8*d^4 + 2*c^7*d^5 - 2*c^6
*d^6 - 6*c^5*d^7 + 6*c^3*d^9 + 2*c^2*d^10 - 2*c*d^11 - d^12)*f), 1/24*(3*(1
```

```
2*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6 + (12*a^2*c^6 - 16*a^2*c^5*d + 7*a
^2*c^4*d^2)*cos(f*x + e)^4 + 4*(12*a^2*c^5*d - 16*a^2*c^4*d^2 + 7*a^2*c^3*d
^3)*cos(f*x + e)^3 + 6*(12*a^2*c^4*d^2 - 16*a^2*c^3*d^3 + 7*a^2*c^2*d^4)*co
s(f*x + e)^2 + 4*(12*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5)*cos(f*x +
e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 -
d^2)*sin(f*x + e))) + (2*a^2*c^5*d^2 + 16*a^2*c^4*d^3 - 61*a^2*c^3*d^4 + 16
*a^2*c^2*d^5 + 59*a^2*c*d^6 - 32*a^2*d^7 + (48*a^2*c^7 - 68*a^2*c^6*d - 64*
a^2*c^5*d^2 + 73*a^2*c^4*d^3 + 32*a^2*c^3*d^4 + a^2*c^2*d^5 - 16*a^2*c*d^6
- 6*a^2*d^7)*cos(f*x + e)^3 + (12*a^2*c^7 + 96*a^2*c^6*d - 239*a^2*c^5*d^2
- 64*a^2*c^4*d^3 + 271*a^2*c^3*d^4 - 16*a^2*c^2*d^5 - 44*a^2*c*d^6 - 16*a^2
*d^7)*cos(f*x + e)^2 + (8*a^2*c^6*d + 64*a^2*c^5*d^2 - 208*a^2*c^4*d^3 + 16
*a^2*c^3*d^4 + 221*a^2*c^2*d^5 - 80*a^2*c*d^6 - 21*a^2*d^7)*cos(f*x + e))*s
in(f*x + e))/((c^12 + 2*c^11*d - 2*c^10*d^2 - 6*c^9*d^3 + 6*c^7*d^5 + 2*c^6
*d^6 - 2*c^5*d^7 - c^4*d^8)*f*cos(f*x + e)^4 + 4*(c^11*d + 2*c^10*d^2 - 2*c
^9*d^3 - 6*c^8*d^4 + 6*c^6*d^6 + 2*c^5*d^7 - 2*c^4*d^8 - c^3*d^9)*f*cos(f*x
 + e)^3 + 6*(c^10*d^2 + 2*c^9*d^3 - 2*c^8*d^4 - 6*c^7*d^5 + 6*c^5*d^7 + 2*c
^4*d^8 - 2*c^3*d^9 - c^2*d^10)*f*cos(f*x + e)^2 + 4*(c^9*d^3 + 2*c^8*d^4 -
2*c^7*d^5 - 6*c^6*d^6 + 6*c^4*d^8 + 2*c^3*d^9 - 2*c^2*d^10 - c*d^11)*f*cos(
f*x + e) + (c^8*d^4 + 2*c^7*d^5 - 2*c^6*d^6 - 6*c^5*d^7 + 6*c^3*d^9 + 2*c^2
*d^10 - 2*c*d^11 - d^12)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{\sec\left(e+fx\right)}{c^5+5c^4d\sec\left(e+fx\right)+10c^3d^2\sec^2\left(e+fx\right)+10c^2d^3\sec^3\left(e+fx\right)+5cd^4\sec^4\left(e+fx\right)+d^5\sec^5\left(e+fx\right)}\,dx+\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**5,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*se
c(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d
**5*sec(e + f*x)**5), x) + Integral(2*sec(e + f*x)**2/(c**5 + 5*c**4*d*sec(
e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*
c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x) + Integral(sec(e + f*x)*
*3/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2*d
**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x))
```

Giac [B] time = 1.60067, size = 998, normalized size = 3.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/12*(3*(12*a^2*c^2 - 16*a^2*c*d + 7*a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/
2*e))/sqrt(-c^2 + d^2)))/((c^6 + 2*c^5*d - c^4*d^2 - 4*c^3*d^3 - c^2*d^4 +
2*c*d^5 + d^6)*sqrt(-c^2 + d^2)) - (36*a^2*c^5*tan(1/2*f*x + 1/2*e)^7 - 156
*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^7 + 273*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^7
- 243*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^7 + 111*a^2*c*d^4*tan(1/2*f*x + 1/2*
e)^7 - 21*a^2*d^5*tan(1/2*f*x + 1/2*e)^7 - 132*a^2*c^5*tan(1/2*f*x + 1/2*e)
```

```
^5 + 308*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^5 - 121*a^2*c^3*d^2*tan(1/2*f*x + 1
/2*e)^5 - 231*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 + 253*a^2*c*d^4*tan(1/2*f*
x + 1/2*e)^5 - 77*a^2*d^5*tan(1/2*f*x + 1/2*e)^5 + 156*a^2*c^5*tan(1/2*f*x
+ 1/2*e)^3 - 116*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 345*a^2*c^3*d^2*tan(1/2
*f*x + 1/2*e)^3 + 199*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 189*a^2*c*d^4*ta
n(1/2*f*x + 1/2*e)^3 - 83*a^2*d^5*tan(1/2*f*x + 1/2*e)^3 - 60*a^2*c^5*tan(1
/2*f*x + 1/2*e) - 36*a^2*c^4*d*tan(1/2*f*x + 1/2*e) + 177*a^2*c^3*d^2*tan(1
/2*f*x + 1/2*e) + 147*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 81*a^2*c*d^4*tan(1
/2*f*x + 1/2*e) - 75*a^2*d^5*tan(1/2*f*x + 1/2*e))/((c^6 + 2*c^5*d - c^4*d^
2 - 4*c^3*d^3 - c^2*d^4 + 2*c*d^5 + d^6)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(
1/2*f*x + 1/2*e)^2 - c - d)^4))/f
```

3.202 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=288

$$\frac{a^3\left(90c^2d+40c^3+78cd^2+23d^3\right)\tan(e+fx)}{16f}+\frac{a^3\left(90c^2d+40c^3+78cd^2+23d^3\right)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{(90c^2d+40c^3}{}$$

```
[Out] (a^3*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*ArcTanh[Sin[e + f*x]])/(16*f)
+ (a^3*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*Tan[e + f*x])/(16*f) + ((40*
c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/
(48*f) + (a*(3*c + 8*d)*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2*Tan[e
 + f*x])/(30*f) + (a*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3*Tan[e +
f*x])/(6*f) + (a*(a + a*Sec[e + f*x])^2*(2*(4*c^3 + 74*c^2*d + 66*c*d^2 + 2
1*d^3) + d*(6*c^2 + 62*c*d + 31*d^2)*Sec[e + f*x])*Tan[e + f*x])/(120*f)
```

Rubi [A] time = 0.427078, antiderivative size = 333, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 100, 147, 50, 63, 217, 203}

$$\frac{a^3\left(90c^2d+40c^3+78cd^2+23d^3\right)\tan(e+fx)}{16f}+\frac{a^4\left(90c^2d+40c^3+78cd^2+23d^3\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*Tan[e + f*x])/(16*f) + (a^4*(4
0*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[
a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(8*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
 a*Sec[e + f*x]]) + (a*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*(a + a*Sec[e
 + f*x])^2*Tan[e + f*x])/(120*f) + ((40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)
*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(48*f) + (d*(a + a*Sec[e + f*x])^3*
(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(6*f) + (d*(a + a*Sec[e + f*x])^3*(70*
c^2 + 54*c*d + 19*d^2 + 4*d*(8*c + 3*d)*Sec[e + f*x])*Tan[e + f*x])/(120*f)
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx = -\frac{\left(a^2\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{(a+ax)^{5/2}(c+dx)^3}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\tan(e+fx)}{6f} + \frac{\tan(}{}$$

$$= \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\tan(e+fx)}{6f} + \frac{d(a+}{}$$

$$= \frac{a\left(40c^3+90c^2d+78cd^2+23d^3\right)(a+a\sec(e+fx))^2\tan(e+}{120f}$$

$$= \frac{a\left(40c^3+90c^2d+78cd^2+23d^3\right)(a+a\sec(e+fx))^2\tan(e+}{120f}$$

$$= \frac{a^3\left(40c^3+90c^2d+78cd^2+23d^3\right)\tan(e+fx)}{16f} + \frac{a\left(40c^3+90\right.}{}$$

$$= \frac{a^3\left(40c^3+90c^2d+78cd^2+23d^3\right)\tan(e+fx)}{16f} + \frac{a\left(40c^3+90\right.}{}$$

$$= \frac{a^3\left(40c^3+90c^2d+78cd^2+23d^3\right)\tan(e+fx)}{16f} + \frac{a\left(40c^3+90\right.}{}$$

$$= \frac{a^3\left(40c^3+90c^2d+78cd^2+23d^3\right)\tan(e+fx)}{16f} + \frac{a^4\left(40c^3+9\right.}{}$$

Mathematica [A] time = 2.77836, size = 380, normalized size = 1.32

$$-\frac{a^3(\cos(e+fx)+1)^3\sec^6\left(\frac{1}{2}(e+fx)\right)\sec^6(e+fx)\left(240\left(90c^2d+40c^3+78cd^2+23d^3\right)\cos^6(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+f\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] -(a^3*(1 + Cos[e + f*x])^3*Sec[(e + f*x)/2]^6*Sec[e + f*x]^6*(240*(40*c^3 +
 90*c^2*d + 78*c*d^2 + 23*d^3)*Cos[e + f*x]^6*(Log[Cos[(e + f*x)/2] - Sin[(
e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 2*(1080*c^3 + 47
70*c^2*d + 5670*c*d^2 + 2275*d^3 + 16*(305*c^3 + 945*c^2*d + 984*c*d^2 + 34
4*d^3)*Cos[e + f*x] + 20*(72*c^3 + 306*c^2*d + 342*c*d^2 + 115*d^3)*Cos[2*(
e + f*x)] + 2360*c^3*Cos[3*(e + f*x)] + 6840*c^2*d*Cos[3*(e + f*x)] + 6384*
c*d^2*Cos[3*(e + f*x)] + 1904*d^3*Cos[3*(e + f*x)] + 360*c^3*Cos[4*(e + f*x
)] + 1350*c^2*d*Cos[4*(e + f*x)] + 1170*c*d^2*Cos[4*(e + f*x)] + 345*d^3*Co
s[4*(e + f*x)] + 440*c^3*Cos[5*(e + f*x)] + 1080*c^2*d*Cos[5*(e + f*x)] + 9
12*c*d^2*Cos[5*(e + f*x)] + 272*d^3*Cos[5*(e + f*x)])*Sin[e + f*x]))/(30720
*f)
```

Maple [A] time = 0.075, size = 523, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x)
```

```
[Out] 34/15/f*a^3*d^3*tan(f*x+e)+17/15/f*a^3*d^3*tan(f*x+e)*sec(f*x+e)^2+3/5/f*a^
3*d^3*tan(f*x+e)*sec(f*x+e)^4+1/3/f*a^3*c^3*tan(f*x+e)*sec(f*x+e)^2+3/2*a^3
*c^3*sec(f*x+e)*tan(f*x+e)/f+1/6/f*a^3*d^3*tan(f*x+e)*sec(f*x+e)^5+39/8/f*a
^3*d^2*c*sec(f*x+e)*tan(f*x+e)+45/8/f*a^3*c^2*d*sec(f*x+e)*tan(f*x+e)+9/4/f
*a^3*d^2*c*tan(f*x+e)*sec(f*x+e)^3+9/f*a^3*c^2*d*tan(f*x+e)+39/8/f*a^3*d^2*
c*ln(sec(f*x+e)+tan(f*x+e))+45/8/f*a^3*c^2*d*ln(sec(f*x+e)+tan(f*x+e))+23/2
4/f*a^3*d^3*tan(f*x+e)*sec(f*x+e)^3+23/16/f*a^3*d^3*sec(f*x+e)*tan(f*x+e)+5
/2/f*a^3*c^3*ln(sec(f*x+e)+tan(f*x+e))+11/3/f*a^3*c^3*tan(f*x+e)+23/16/f*a^
3*d^3*ln(sec(f*x+e)+tan(f*x+e))+3/4/f*a^3*c^2*d*tan(f*x+e)*sec(f*x+e)^3+38/
5/f*a^3*d^2*c*tan(f*x+e)+19/5/f*a^3*d^2*c*tan(f*x+e)*sec(f*x+e)^2+3/f*a^3*c
^2*d*tan(f*x+e)*sec(f*x+e)^2+3/5/f*a^3*d^2*c*tan(f*x+e)*sec(f*x+e)^4
```

Maxima [B] time = 1.01484, size = 946, normalized size = 3.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/480*(160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^3 + 1440*(tan(f*x + e)^3
 + 3*tan(f*x + e))*a^3*c^2*d + 96*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 1
5*tan(f*x + e))*a^3*c*d^2 + 1440*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c*d^
2 + 96*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*d^3 + 1
60*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*d^3 - 5*a^3*d^3*(2*(15*sin(f*x + e
)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)
^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x + e)
 - 1)) - 90*a^3*c^2*d*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^
4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) -
1)) - 270*a^3*c*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4
- 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)
) - 90*a^3*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*s
in(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 3
60*a^3*c^3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + l
og(sin(f*x + e) - 1)) - 1080*a^3*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1)
 - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 360*a^3*c*d^2*(2*sin(f*
x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)
) + 480*a^3*c^3*log(sec(f*x + e) + tan(f*x + e)) + 1440*a^3*c^3*tan(f*x + e
) + 1440*a^3*c^2*d*tan(f*x + e))/f
```

Fricas [A] time = 0.554277, size = 791, normalized size = 2.75

$$15\left(40\,a^3c^3+90\,a^3c^2d+78\,a^3cd^2+23\,a^3d^3\right)\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-15\left(40\,a^3c^3+90\,a^3c^2d+78\,a^3cd^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 + 23*a^3*d^3)*cos(f*x +
 e)^6*log(sin(f*x + e) + 1) - 15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2
+ 23*a^3*d^3)*cos(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(40*a^3*d^3 + 16*(5
5*a^3*c^3 + 135*a^3*c^2*d + 114*a^3*c*d^2 + 34*a^3*d^3)*cos(f*x + e)^5 + 15
*(24*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 + 23*a^3*d^3)*cos(f*x + e)^4 + 1
6*(5*a^3*c^3 + 45*a^3*c^2*d + 57*a^3*c*d^2 + 17*a^3*d^3)*cos(f*x + e)^3 + 1
0*(18*a^3*c^2*d + 54*a^3*c*d^2 + 23*a^3*d^3)*cos(f*x + e)^2 + 144*(a^3*c*d^
2 + a^3*d^3)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int c^3\sec\left(e+fx\right)dx+\int 3c^3\sec^2\left(e+fx\right)dx+\int 3c^3\sec^3\left(e+fx\right)dx+\int c^3\sec^4\left(e+fx\right)dx+\int d^3\sec^4\left(e+fx\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**3*(Integral(c**3*sec(e + f*x), x) + Integral(3*c**3*sec(e + f*x)**2, x)
+ Integral(3*c**3*sec(e + f*x)**3, x) + Integral(c**3*sec(e + f*x)**4, x) +
 Integral(d**3*sec(e + f*x)**4, x) + Integral(3*d**3*sec(e + f*x)**5, x) +
Integral(3*d**3*sec(e + f*x)**6, x) + Integral(d**3*sec(e + f*x)**7, x) + I
ntegral(3*c*d**2*sec(e + f*x)**3, x) + Integral(9*c*d**2*sec(e + f*x)**4, x
) + Integral(9*c*d**2*sec(e + f*x)**5, x) + Integral(3*c*d**2*sec(e + f*x)*
*6, x) + Integral(3*c**2*d*sec(e + f*x)**2, x) + Integral(9*c**2*d*sec(e +
f*x)**3, x) + Integral(9*c**2*d*sec(e + f*x)**4, x) + Integral(3*c**2*d*sec
(e + f*x)**5, x))
```

Giac [B] time = 1.33745, size = 825, normalized size = 2.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 + 23*a^3*d^3)*log(abs(t
an(1/2*f*x + 1/2*e) + 1)) - 15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 +
23*a^3*d^3)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(600*a^3*c^3*tan(1/2*f*x
 + 1/2*e)^11 + 1350*a^3*c^2*d*tan(1/2*f*x + 1/2*e)^11 + 1170*a^3*c*d^2*tan(
1/2*f*x + 1/2*e)^11 + 345*a^3*d^3*tan(1/2*f*x + 1/2*e)^11 - 3400*a^3*c^3*ta
n(1/2*f*x + 1/2*e)^9 - 7650*a^3*c^2*d*tan(1/2*f*x + 1/2*e)^9 - 6630*a^3*c*d
^2*tan(1/2*f*x + 1/2*e)^9 - 1955*a^3*d^3*tan(1/2*f*x + 1/2*e)^9 + 7920*a^3*
c^3*tan(1/2*f*x + 1/2*e)^7 + 17820*a^3*c^2*d*tan(1/2*f*x + 1/2*e)^7 + 15444
*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^7 + 4554*a^3*d^3*tan(1/2*f*x + 1/2*e)^7 - 9
360*a^3*c^3*tan(1/2*f*x + 1/2*e)^5 - 22500*a^3*c^2*d*tan(1/2*f*x + 1/2*e)^5
 - 17964*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 5814*a^3*d^3*tan(1/2*f*x + 1/2*
e)^5 + 5560*a^3*c^3*tan(1/2*f*x + 1/2*e)^3 + 15390*a^3*c^2*d*tan(1/2*f*x +
1/2*e)^3 + 12570*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 3165*a^3*d^3*tan(1/2*f*
x + 1/2*e)^3 - 1320*a^3*c^3*tan(1/2*f*x + 1/2*e) - 4410*a^3*c^2*d*tan(1/2*f
*x + 1/2*e) - 4590*a^3*c*d^2*tan(1/2*f*x + 1/2*e) - 1575*a^3*d^3*tan(1/2*f*
x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^6)/f
```

3.203 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=257

$$\frac{a^3\left(72c^2d^2-15c^3d+2c^4+180cd^3+76d^4\right)\tan(e+fx)}{30d^2f}+\frac{a^3\left(20c^2+30cd+13d^2\right)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^3\left(2c^2-1\right.}{}$$

```
[Out] (a^3*(20*c^2 + 30*c*d + 13*d^2)*ArcTanh[Sin[e + f*x]])/(8*f) + (a^3*(2*c^4
- 15*c^3*d + 72*c^2*d^2 + 180*c*d^3 + 76*d^4)*Tan[e + f*x])/(30*d^2*f) + (a
^3*(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^3)*Sec[e + f*x]*Tan[e + f*x])/(120
*d*f) + (a^3*(2*c^2 - 15*c*d + 76*d^2)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])
/(60*d^2*f) - (a^3*(2*c - 11*d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(20*d^
2*f) + ((a^3 + a^3*Sec[e + f*x])*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(5*d*
f)
```

Rubi [A] time = 0.299609, antiderivative size = 273, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 90, 80, 50, 63, 217, 203}

$$\frac{a^3\left(20c^2+30cd+13d^2\right)\tan(e+fx)}{8f}+\frac{a^4\left(20c^2+30cd+13d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{\left(20c^2+30cd+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*(20*c^2 + 30*c*d + 13*d^2)*Tan[e + f*x])/(8*f) + (a^4*(20*c^2 + 30*c*d
 + 13*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[
e + f*x])/(4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a*(20*
c^2 + 30*c*d + 13*d^2)*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(60*f) + (3*d*(
2*c + d)*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(20*f) + ((20*c^2 + 30*c*d +
13*d^2)*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(24*f) + (d*(a + a*Sec[e + f
*x])^3*(c + d*Sec[e + f*x])*Tan[e + f*x])/(5*f)
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}(c+dx)^2}{\sqrt{a-ax}}\,dx, x, \sec(e+f\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))\tan(e+fx)}{5f} + \frac{\tan}{}$$

$$= \frac{3d(2c+d)(a+a\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{d(a+a\sec(}{}$$

$$= \frac{a\left(20c^2+30cd+13d^2\right)(a+a\sec(e+fx))^2\tan(e+fx)}{60f} +$$

$$= \frac{a\left(20c^2+30cd+13d^2\right)(a+a\sec(e+fx))^2\tan(e+fx)}{60f} +$$

$$= \frac{a^3\left(20c^2+30cd+13d^2\right)\tan(e+fx)}{8f} + \frac{a\left(20c^2+30cd+1\right.}{}$$

$$= \frac{a^3\left(20c^2+30cd+13d^2\right)\tan(e+fx)}{8f} + \frac{a\left(20c^2+30cd+1\right.}{}$$

$$= \frac{a^3\left(20c^2+30cd+13d^2\right)\tan(e+fx)}{8f} + \frac{a\left(20c^2+30cd+1\right.}{}$$

$$= \frac{a^3\left(20c^2+30cd+13d^2\right)\tan(e+fx)}{8f} + \frac{a^4\left(20c^2+30cd+\right.}{4f\sqrt{a-a}}$$

Mathematica [A] time = 2.5544, size = 433, normalized size = 1.68

$$-\frac{a^3(\cos(e+fx)+1)^3\sec^6\left(\frac{1}{2}(e+fx)\right)\sec^5(e+fx)\left(240\left(20c^2+30cd+13d^2\right)\cos^5(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\text{s}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] -(a^3*(1 + Cos[e + f*x])^3*Sec[(e + f*x)/2]^6*Sec[e + f*x]^5*(240*(20*c^2 +
 30*c*d + 13*d^2)*Cos[e + f*x]^5*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
- Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - Sec[e]*(80*(34*c^2 + 60*c*d +
 29*d^2)*Sin[f*x] - 240*(7*c^2 + 10*c*d + 3*d^2)*Sin[2*e + f*x] + 360*c^2*S
in[e + 2*f*x] + 1140*c*d*Sin[e + 2*f*x] + 750*d^2*Sin[e + 2*f*x] + 360*c^2*
Sin[3*e + 2*f*x] + 1140*c*d*Sin[3*e + 2*f*x] + 750*d^2*Sin[3*e + 2*f*x] + 1
840*c^2*Sin[2*e + 3*f*x] + 3360*c*d*Sin[2*e + 3*f*x] + 1520*d^2*Sin[2*e + 3
*f*x] - 360*c^2*Sin[4*e + 3*f*x] - 240*c*d*Sin[4*e + 3*f*x] + 180*c^2*Sin[3
*e + 4*f*x] + 450*c*d*Sin[3*e + 4*f*x] + 195*d^2*Sin[3*e + 4*f*x] + 180*c^2
*Sin[5*e + 4*f*x] + 450*c*d*Sin[5*e + 4*f*x] + 195*d^2*Sin[5*e + 4*f*x] + 4
40*c^2*Sin[4*e + 5*f*x] + 720*c*d*Sin[4*e + 5*f*x] + 304*d^2*Sin[4*e + 5*f*
x])))/(15360*f)
```

Maple [A] time = 0.06, size = 342, normalized size = 1.3

$$\frac{5\,a^3c^2\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}+6\,\frac{a^3cd\tan\left(fx+e\right)}{f}+\frac{13\,a^3d^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{8f}+\frac{13\,a^3d^2\ln\left(\sec\left(fx+\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x)
```

```
[Out] 5/2/f*a^3*c^2*ln(sec(f*x+e)+tan(f*x+e))+6/f*a^3*c*d*tan(f*x+e)+13/8/f*a^3*d
^2*sec(f*x+e)*tan(f*x+e)+13/8/f*a^3*d^2*ln(sec(f*x+e)+tan(f*x+e))+11/3/f*a^
3*c^2*tan(f*x+e)+15/4/f*a^3*c*d*sec(f*x+e)*tan(f*x+e)+15/4/f*a^3*c*d*ln(sec
(f*x+e)+tan(f*x+e))+38/15/f*a^3*d^2*tan(f*x+e)+19/15/f*a^3*d^2*tan(f*x+e)*s
ec(f*x+e)^2+3/2*a^3*c^2*sec(f*x+e)*tan(f*x+e)/f+2/f*a^3*c*d*tan(f*x+e)*sec(
f*x+e)^2+3/4/f*a^3*d^2*tan(f*x+e)*sec(f*x+e)^3+1/3/f*a^3*c^2*tan(f*x+e)*sec
(f*x+e)^2+1/2/f*a^3*c*d*tan(f*x+e)*sec(f*x+e)^3+1/5/f*a^3*d^2*tan(f*x+e)*se
c(f*x+e)^4
```

Maxima [A] time = 1.01397, size = 620, normalized size = 2.41

$$\frac{80\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^3c^2+480\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^3cd+16\left(3\,\tan\left(fx+e\right)^5+10\,\tan\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/240*(80*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^2 + 480*(tan(f*x + e)^3 +
 3*tan(f*x + e))*a^3*c*d + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*ta
n(f*x + e))*a^3*d^2 + 240*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*d^2 - 30*a^
3*c*d*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x +
e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 45*a^3*d^2
*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2
+ 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 180*a^3*c^2*(2*
sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e
) - 1)) - 360*a^3*c*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
e) + 1) + log(sin(f*x + e) - 1)) - 60*a^3*d^2*(2*sin(f*x + e)/(sin(f*x + e)
^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a^3*c^2*log(
sec(f*x + e) + tan(f*x + e)) + 720*a^3*c^2*tan(f*x + e) + 480*a^3*c*d*tan(f
*x + e))/f
```

Fricas [A] time = 0.516725, size = 587, normalized size = 2.28

$$\frac{15\left(20\,a^3c^2+30\,a^3cd+13\,a^3d^2\right)\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\left(20\,a^3c^2+30\,a^3cd+13\,a^3d^2\right)\cos\left(fx+e\right)^5\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*cos(f*x + e)^5*log(sin(f*x
 + e) + 1) - 15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*cos(f*x + e)^5*log(-
sin(f*x + e) + 1) + 2*(24*a^3*d^2 + 8*(55*a^3*c^2 + 90*a^3*c*d + 38*a^3*d^2
)*cos(f*x + e)^4 + 15*(12*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*cos(f*x + e)^3
 + 8*(5*a^3*c^2 + 30*a^3*c*d + 19*a^3*d^2)*cos(f*x + e)^2 + 30*(2*a^3*c*d +
 3*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int c^2\sec\left(e+fx\right)dx+\int 3c^2\sec^2\left(e+fx\right)dx+\int 3c^2\sec^3\left(e+fx\right)dx+\int c^2\sec^4\left(e+fx\right)dx+\int d^2\sec^3\left(e+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c+d*sec(f*x+e))**2,x)
```

```
[Out] a**3*(Integral(c**2*sec(e + f*x), x) + Integral(3*c**2*sec(e + f*x)**2, x)
+ Integral(3*c**2*sec(e + f*x)**3, x) + Integral(c**2*sec(e + f*x)**4, x) +
 Integral(d**2*sec(e + f*x)**3, x) + Integral(3*d**2*sec(e + f*x)**4, x) +
Integral(3*d**2*sec(e + f*x)**5, x) + Integral(d**2*sec(e + f*x)**6, x) + I
ntegral(2*c*d*sec(e + f*x)**2, x) + Integral(6*c*d*sec(e + f*x)**3, x) + In
tegral(6*c*d*sec(e + f*x)**4, x) + Integral(2*c*d*sec(e + f*x)**5, x))
```

Giac [A] time = 1.28398, size = 532, normalized size = 2.07

$$\frac{15\left(20\,a^3c^2+30\,a^3cd+13\,a^3d^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\left(20\,a^3c^2+30\,a^3cd+13\,a^3d^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*log(abs(tan(1/2*f*x + 1/2*
e) + 1)) - 15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*log(abs(tan(1/2*f*x +
1/2*e) - 1)) - 2*(300*a^3*c^2*tan(1/2*f*x + 1/2*e)^9 + 450*a^3*c*d*tan(1/2*
f*x + 1/2*e)^9 + 195*a^3*d^2*tan(1/2*f*x + 1/2*e)^9 - 1400*a^3*c^2*tan(1/2*
f*x + 1/2*e)^7 - 2100*a^3*c*d*tan(1/2*f*x + 1/2*e)^7 - 910*a^3*d^2*tan(1/2*
f*x + 1/2*e)^7 + 2560*a^3*c^2*tan(1/2*f*x + 1/2*e)^5 + 3840*a^3*c*d*tan(1/2
*f*x + 1/2*e)^5 + 1664*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 - 2120*a^3*c^2*tan(1/
2*f*x + 1/2*e)^3 - 3660*a^3*c*d*tan(1/2*f*x + 1/2*e)^3 - 1330*a^3*d^2*tan(1
/2*f*x + 1/2*e)^3 + 660*a^3*c^2*tan(1/2*f*x + 1/2*e) + 1470*a^3*c*d*tan(1/2
*f*x + 1/2*e) + 765*a^3*d^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 -
 1)^5)/f
```

3.204 $\int \sec(e + fx)(a + a\sec(e + fx))^3(c + d\sec(e + fx))\,dx$

Optimal. Leaf size=125

$$\frac{a^3(4c+3d)\tan^3(e+fx)}{12f}+\frac{a^3(4c+3d)\tan(e+fx)}{f}+\frac{5a^3(4c+3d)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{3a^3(4c+3d)\tan(e+fx)\sec(e+fx)}{8f}$$

```
[Out] (5*a^3*(4*c + 3*d)*ArcTanh[Sin[e + f*x]])/(8*f) + (a^3*(4*c + 3*d)*Tan[e +
f*x])/f + (3*a^3*(4*c + 3*d)*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (d*(a + a*S
ec[e + f*x])^3*Tan[e + f*x])/(4*f) + (a^3*(4*c + 3*d)*Tan[e + f*x]^3)/(12*f
)
```

Rubi [A] time = 0.152805, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4001, 3791, 3770, 3767, 8, 3768}

$$\frac{a^3(4c+3d)\tan^3(e+fx)}{12f}+\frac{a^3(4c+3d)\tan(e+fx)}{f}+\frac{5a^3(4c+3d)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{3a^3(4c+3d)\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x]),x]
```

```
[Out] (5*a^3*(4*c + 3*d)*ArcTanh[Sin[e + f*x]])/(8*f) + (a^3*(4*c + 3*d)*Tan[e +
f*x])/f + (3*a^3*(4*c + 3*d)*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (d*(a + a*S
ec[e + f*x])^3*Tan[e + f*x])/(4*f) + (a^3*(4*c + 3*d)*Tan[e + f*x]^3)/(12*f
)
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx = \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f} + \frac{1}{4}(4c+3d)\int \sec(e+$$

$$= \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f} + \frac{1}{4}(4c+3d)\int \left(a^3\sec(\right.$$

$$= \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f} + \frac{1}{4}\left(a^3(4c+3d)\right)\int \sec$$

$$= \frac{a^3(4c+3d)\tanh^{-1}(\sin(e+fx))}{4f} + \frac{3a^3(4c+3d)\sec(e+fx}{8f}$$

$$= \frac{5a^3(4c+3d)\tanh^{-1}(\sin(e+fx))}{8f} + \frac{a^3(4c+3d)\tan(e+fx}{f}$$

Mathematica [B] time = 1.30439, size = 273, normalized size = 2.18

$$-\frac{a^3(\cos(e+fx)+1)^3\sec^6\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\left(120(4c+3d)\cos^4(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x]),x]
```

```
[Out] -(a^3*(1 + Cos[e + f*x])^3*Sec[(e + f*x)/2]^6*Sec[e + f*x]^4*(120*(4*c + 3*
d)*Cos[e + f*x]^4*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]]) - Sec[e]*(-24*(11*c + 9*d)*Sin[e] + (36*c + 69
*d)*Sin[f*x] + 36*c*Sin[2*e + f*x] + 69*d*Sin[2*e + f*x] + 280*c*Sin[e + 2*
f*x] + 264*d*Sin[e + 2*f*x] - 72*c*Sin[3*e + 2*f*x] - 24*d*Sin[3*e + 2*f*x]
 + 36*c*Sin[2*e + 3*f*x] + 45*d*Sin[2*e + 3*f*x] + 36*c*Sin[4*e + 3*f*x] +
45*d*Sin[4*e + 3*f*x] + 88*c*Sin[3*e + 4*f*x] + 72*d*Sin[3*e + 4*f*x])))/(1
536*f)
```

Maple [A] time = 0.047, size = 188, normalized size = 1.5

$$\frac{5a^3c\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{2f}+3\frac{a^3d\tan\left(fx+e\right)}{f}+\frac{11a^3c\tan\left(fx+e\right)}{3f}+\frac{15a^3d\sec\left(fx+e\right)\tan\left(fx+e\right)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x)
```

```
[Out] 5/2/f*a^3*c*ln(sec(f*x+e)+tan(f*x+e))+3/f*a^3*d*tan(f*x+e)+11/3/f*a^3*c*tan
(f*x+e)+15/8/f*a^3*d*sec(f*x+e)*tan(f*x+e)+15/8/f*a^3*d*ln(sec(f*x+e)+tan(f
*x+e))+3/2*a^3*c*sec(f*x+e)*tan(f*x+e)/f+1/f*a^3*d*tan(f*x+e)*sec(f*x+e)^2+
1/3/f*a^3*c*tan(f*x+e)*sec(f*x+e)^2+1/4/f*a^3*d*tan(f*x+e)*sec(f*x+e)^3
```

Maxima [B] time = 0.99547, size = 354, normalized size = 2.83

$$16\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^3c+48\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^3d-3a^3d\left(\frac{2\left(3\sin\left(fx+e\right)^3-5\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^4-2\sin\left(fx+e\right)^2+1}-3\,\mathrm{lc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c + 48*(tan(f*x + e)^3 + 3*t
an(f*x + e))*a^3*d - 3*a^3*d*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*
x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x
+ e) - 1)) - 36*a^3*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
e) + 1) + log(sin(f*x + e) - 1)) - 36*a^3*d*(2*sin(f*x + e)/(sin(f*x + e)^2
 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 48*a^3*c*log(sec(f
*x + e) + tan(f*x + e)) + 144*a^3*c*tan(f*x + e) + 48*a^3*d*tan(f*x + e))/f
```

Fricas [A] time = 0.499817, size = 393, normalized size = 3.14

$$\frac{15\left(4a^3c+3a^3d\right)\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-15\left(4a^3c+3a^3d\right)\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)+2\left(6a^3\right.}{48f\cos\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/48*(15*(4*a^3*c + 3*a^3*d)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 15*(4*a
^3*c + 3*a^3*d)*cos(f*x + e)^4*log(-sin(f*x + e) + 1) + 2*(6*a^3*d + 8*(11*
a^3*c + 9*a^3*d)*cos(f*x + e)^3 + 9*(4*a^3*c + 5*a^3*d)*cos(f*x + e)^2 + 8*
(a^3*c + 3*a^3*d)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int c\sec\left(e+fx\right)dx+\int 3c\sec^2\left(e+fx\right)dx+\int 3c\sec^3\left(e+fx\right)dx+\int c\sec^4\left(e+fx\right)dx+\int d\sec^2\left(e+fx\right)dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c+d*sec(f*x+e)),x)
```

```
[Out] a**3*(Integral(c*sec(e + f*x), x) + Integral(3*c*sec(e + f*x)**2, x) + Inte
gral(3*c*sec(e + f*x)**3, x) + Integral(c*sec(e + f*x)**4, x) + Integral(d*
```

```
sec(e + f*x)**2, x) + Integral(3*d*sec(e + f*x)**3, x) + Integral(3*d*sec(e
 + f*x)**4, x) + Integral(d*sec(e + f*x)**5, x))
```

Giac [A] time = 1.20533, size = 301, normalized size = 2.41

$$15\left(4a^3c + 3a^3d\right)\log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - 15\left(4a^3c + 3a^3d\right)\log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right) - \frac{2\left(60a^3c\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^7 + 4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/24*(15*(4*a^3*c + 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(4*a^3
*c + 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(60*a^3*c*tan(1/2*f*x
+ 1/2*e)^7 + 45*a^3*d*tan(1/2*f*x + 1/2*e)^7 - 220*a^3*c*tan(1/2*f*x + 1/2*
e)^5 - 165*a^3*d*tan(1/2*f*x + 1/2*e)^5 + 292*a^3*c*tan(1/2*f*x + 1/2*e)^3
+ 219*a^3*d*tan(1/2*f*x + 1/2*e)^3 - 132*a^3*c*tan(1/2*f*x + 1/2*e) - 147*a
^3*d*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

3.205 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=153

$$\frac{a^3\left(c^2-3cd+3d^2\right)\tanh^{-1}(\sin(e+fx))}{d^3f}-\frac{a^3(c-3d)\tan(e+fx)}{d^2f}-\frac{2a^3(c-d)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^3f\sqrt{c+d}}+\frac{a^3\tanh^{-1}(\ldots}{2\ldots}$$

```
[Out] (a^3*ArcTanh[Sin[e + f*x]])/(2*d*f) + (a^3*(c^2 - 3*c*d + 3*d^2)*ArcTanh[Si
n[e + f*x]])/(d^3*f) - (2*a^3*(c - d)^(5/2)*ArcTanh[(Sqrt[c - d]*Tan[(e + f
*x)/2])/Sqrt[c + d]])/(d^3*Sqrt[c + d]*f) - (a^3*(c - 3*d)*Tan[e + f*x])/(d
^2*f) + (a^3*Sec[e + f*x]*Tan[e + f*x])/(2*d*f)
```

Rubi [A] time = 0.324612, antiderivative size = 257, normalized size of antiderivative = 1.68, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {3987, 102, 154, 157, 63, 217, 203, 93, 205}

$$\frac{a^4\left(2c^2-6cd+7d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f}+\frac{2a^4(c-d)^{5/2}\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c-\ldots}}{\sqrt{c-\ldots}}\right)}{d^3f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\,\mathrm{se}\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x]),x]
```

```
[Out] -(a^3*(2*c - 5*d)*Tan[e + f*x])/(2*d^2*f) + (a^4*(2*c^2 - 6*c*d + 7*d^2)*Ar
cTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(d^
3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^4*(c - d)^(5/
2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Se
c[e + f*x]])]*Tan[e + f*x])/(d^3*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqr
t[a + a*Sec[e + f*x]]) + ((a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(2*d*f)
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
```

```
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2df} + \frac{(a\tan(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{a+ax}\left(-a^3(c+2d)+a^3(\cdots\right.}{\sqrt{a-ax}(c+dx)}\right.}{2df\sqrt{a-a\sec(e+fx)}\sqrt{a+a\cdots}}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2df} - \frac{\tan(e+fx)\,\text{Su}\cdots}{2d^2\cdots}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2df} + \frac{\left(a^5(c-d)^3\tan\cdots\right.}{d\cdots}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2df} + \frac{\left(2a^5(c-d)^3\tan\cdots\right.}{\cdots}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{2a^4(c-d)^{5/2}\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{a^4\left(2c^2-6cd+7d^2\right)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+\cdots}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] time = 2.28753, size = 419, normalized size = 2.74

$$\frac{a^3\cos^2(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^3(c\cos(e+fx)+d)\left(-2\left(2c^2-6cd+7d^2\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^3*Cos[e + f*x]^2*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*
x])^3*(-2*(2*c^2 - 6*c*d + 7*d^2)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
+ 2*(2*c^2 - 6*c*d + 7*d^2)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (8*(
c - d)^3*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/
2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(I*Cos[e] + Sin[e]))/(S
qrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) + d^2/(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^2 - (4*(c - 3*d)*d*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])) - d^2/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^2 - (4*(c - 3*d)*d*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]))))/(32*d^3*f*(c + d*Sec[e + f*x]))
```

Maple [B] time = 0.097, size = 491, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x)
```

```
[Out] -1/2/f*a^3/d/(tan(1/2*f*x+1/2*e)+1)^2+1/f*a^3/d^2/(tan(1/2*f*x+1/2*e)+1)*c-
5/2/f*a^3/d/(tan(1/2*f*x+1/2*e)+1)+1/f*a^3/d^3*ln(tan(1/2*f*x+1/2*e)+1)*c^2
-3/f*a^3/d^2*ln(tan(1/2*f*x+1/2*e)+1)*c+7/2/f*a^3/d*ln(tan(1/2*f*x+1/2*e)+1
)+1/2/f*a^3/d/(tan(1/2*f*x+1/2*e)-1)^2+1/f*a^3/d^2/(tan(1/2*f*x+1/2*e)-1)*c
-5/2/f*a^3/d/(tan(1/2*f*x+1/2*e)-1)-1/f*a^3/d^3*ln(tan(1/2*f*x+1/2*e)-1)*c^
2+3/f*a^3/d^2*ln(tan(1/2*f*x+1/2*e)-1)*c-7/2/f*a^3/d*ln(tan(1/2*f*x+1/2*e)-
1)-2/f*a^3/d^3/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*
(c-d))^(1/2))*c^3+6/f*a^3/d^2/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e
)*(c-d)/((c+d)*(c-d))^(1/2))*c^2-6/f*a^3/d/((c+d)*(c-d))^(1/2)*arctanh(tan(
1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*c+2/f*a^3/((c+d)*(c-d))^(1/2)*arc
tanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52238, size = 1235, normalized size = 8.07

$$\left[\frac{2\left(a^3c^2 - 2\,a^3cd + a^3d^2\right)\sqrt{\frac{c-d}{c+d}}\cos\left(fx+e\right)^2\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2-2\left(c^2+cd+\left(cd+d^2\right)\cos\left(fx+e\right)\right)\sqrt{\frac{c-d}{c+d}}\sin\left(fx+e\right)+2\,c}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/4*(2*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*sqrt((c - d)/(c + d))*cos(f*x + e)^
2*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d + (
c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 - d^2)/
(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + (2*a^3*c^2 - 6*a^3*c*d +
 7*a^3*d^2)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a^3*c^2 - 6*a^3*c*d +
 7*a^3*d^2)*cos(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(a^3*d^2 - 2*(a^3*c*d
 - 3*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/(d^3*f*cos(f*x + e)^2), -1/4*(4*(
a^3*c^2 - 2*a^3*c*d + a^3*d^2)*sqrt(-(c - d)/(c + d))*arctan(-(d*cos(f*x +
e) + c)*sqrt(-(c - d)/(c + d))/((c - d)*sin(f*x + e)))*cos(f*x + e)^2 - (2*
a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*cos(f*x + e)^2*log(sin(f*x + e) + 1) + (2*
a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*cos(f*x + e)^2*log(-sin(f*x + e) + 1) - 2*
(a^3*d^2 - 2*(a^3*c*d - 3*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/(d^3*f*cos(f
*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{3}\left(\int \frac{\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx+\int \frac{3\sec^{2}\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx+\int \frac{3\sec^{3}\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx+\int \frac{\sec^{4}\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e)),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c + d*sec(e + f*x)), x) + Integral(3*sec(e + f
*x)**2/(c + d*sec(e + f*x)), x) + Integral(3*sec(e + f*x)**3/(c + d*sec(e +
 f*x)), x) + Integral(sec(e + f*x)**4/(c + d*sec(e + f*x)), x))
```

Giac [B] time = 1.30457, size = 398, normalized size = 2.6

$$\frac{\frac{\left(2a^{3}c^{2}-6a^{3}cd+7a^{3}d^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{d^{3}}-\frac{\left(2a^{3}c^{2}-6a^{3}cd+7a^{3}d^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{d^{3}}+\frac{4\left(a^{3}c^{3}-3a^{3}c^{2}d+3a^{3}cd^{2}-a^{3}d^{3}\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2}{\sqrt{-c^{2}+d^{2}}d}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/2*((2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1))
/d^3 - (2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*log(abs(tan(1/2*f*x + 1/2*e) - 1
))/d^3 + 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(pi*floor(1/2*(f
*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1
/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/(sqrt(-c^2 + d^2)*d^3) + 2*(2*a^3*c*tan
(1/2*f*x + 1/2*e)^3 - 5*a^3*d*tan(1/2*f*x + 1/2*e)^3 - 2*a^3*c*tan(1/2*f*x
+ 1/2*e) + 7*a^3*d*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*d^
2))/f
```

3.206 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=161

$$\frac{2a^3c\tan(e+fx)}{d^2f(c+d)}-\frac{a^3(2c-3d)\tanh^{-1}(\sin(e+fx))}{d^3f}+\frac{2a^3(c-d)^{3/2}(2c+3d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^3f(c+d)^{3/2}}-\frac{(c-d)\tan(}{df(c}$$

```
[Out] -((a^3*(2*c - 3*d)*ArcTanh[Sin[e + f*x]])/(d^3*f)) + (2*a^3*(c - d)^(3/2)*(
2*c + 3*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(d^3*(c + d
)^(3/2)*f) + (2*a^3*c*Tan[e + f*x])/(d^2*(c + d)*f) - ((c - d)*(a^3 + a^3*S
ec[e + f*x])*Tan[e + f*x])/(d*(c + d)*f*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.354823, antiderivative size = 274, normalized size of antiderivative = 1.7, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {3987, 98, 154, 157, 63, 217, 203, 93, 205}

$$\frac{2a^3c\tan(e+fx)}{d^2f(c+d)}-\frac{2a^4(2c-3d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{2a^4(c-d)^{3/2}(2c+3d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c-d}}\right)}{d^3f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^3*c*Tan[e + f*x])/(d^2*(c + d)*f) - (2*a^4*(2*c - 3*d)*ArcTan[Sqrt[a -
 a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(d^3*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*a^4*(c - d)^(3/2)*(2*c + 3*d
)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec
[e + f*x]])]*Tan[e + f*x])/(d^3*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) - ((c - d)*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(
d*(c + d)*f*(c + d*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{\sqrt{a+ax}(\cdots}{\sqrt{\cdots}}\right.}{d(c+d)f\sqrt{a-a\sec(e+\cdots}}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} - \frac{\tan(e+fx)\,\text{S}\cdots}{d^2(c+\cdots}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{\left(a^5(2c-3d)\,\text{t}\cdots\right.}{d\cdots}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} - \frac{\left(2a^4(2c-3d)\cdots\right.}{\cdots}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{2a^4(c-d)^{3/2}(2c+3d)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+\cdots}{d^3(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{2a^4(2c-3d)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2a^4(c-\cdots}{d^3(\cdots}$$

Mathematica [C] time = 4.05094, size = 455, normalized size = 2.83

$$\frac{a^3\cos(e+fx)\sec^6\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^3(c\cos(e+fx)+d)\left(-\frac{2i(2c+3d)(c-d)^2(\cos(e)-i\sin(e))(c\cos(e+fx)+d)\tan^{-1}\left(\frac{(\sin(\cdots}{\cdots}\right)}{(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin\cdots}}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*Cos[e + f*x]*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*x]
)^3*((2*c - 3*d)*(d + c*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]] + (-2*c + 3*d)*(d + c*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x
)/2]] - ((2*I)*(c - d)^2*(2*c + 3*d)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e]
+ (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^
2])]*(d + c*Cos[e + f*x])*(Cos[e] - I*Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqr
t[(Cos[e] - I*Sin[e])^2]) + ((c - d)^2*d*(-(d*Sin[e]) + c*Sin[f*x]))/(c*(c
+ d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])) + (d*(d + c*Cos[e + f*x])
*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
) + (d*(d + c*Cos[e + f*x])*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]))))/(8*d^3*f*(c + d*Sec[e + f*x])^2)
```

Maple [B] time = 0.107, size = 548, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x)
```

```
[Out] -1/f*a^3/d^2/(tan(1/2*f*x+1/2*e)+1)-2/f*a^3/d^3*ln(tan(1/2*f*x+1/2*e)+1)*c+
3/f*a^3/d^2*ln(tan(1/2*f*x+1/2*e)+1)-1/f*a^3/d^2/(tan(1/2*f*x+1/2*e)-1)+2/f
*a^3/d^3*ln(tan(1/2*f*x+1/2*e)-1)*c-3/f*a^3/d^2*ln(tan(1/2*f*x+1/2*e)-1)-2/
f*a^3/d^2/(c+d)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*
e)^2*d-c-d)*c^2+4/f*a^3/d/(c+d)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-
tan(1/2*f*x+1/2*e)^2*d-c-d)*c-2/f*a^3/(c+d)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x
+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)+4/f*a^3/d^3/(c+d)/((c+d)*(c-d))^(1/
2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*c^3-2/f*a^3/d^2/(c
+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2
))*c^2-8/f*a^3/d/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)
/((c+d)*(c-d))^(1/2))*c+6/f*a^3/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f
*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.57318, size = 1891, normalized size = 11.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(((2*a^3*c^3 + a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*a^3*c^2*d
 + a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*sqrt((c - d)/(c + d))*log((2*c*d*co
s(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d + (c*d + d^2)*cos(
f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x +
e)^2 + 2*c*d*cos(f*x + e) + d^2)) + ((2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*
cos(f*x + e)^2 + (2*a^3*c^2*d - a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*log(si
n(f*x + e) + 1) - ((2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (
2*a^3*c^2*d - a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) -
 2*(a^3*c*d^2 + a^3*d^3 + (2*a^3*c^2*d - a^3*c*d^2 + a^3*d^3)*cos(f*x + e))
*sin(f*x + e))/((c^2*d^3 + c*d^4)*f*cos(f*x + e)^2 + (c*d^4 + d^5)*f*cos(f*
x + e)), 1/2*(2*((2*a^3*c^3 + a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*
a^3*c^2*d + a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*sqrt(-(c - d)/(c + d))*arc
tan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d))/((c - d)*sin(f*x + e))) -
((2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*a^3*c^2*d - a^3*
c*d^2 - 3*a^3*d^3)*cos(f*x + e))*log(sin(f*x + e) + 1) + ((2*a^3*c^3 - a^3*
c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*a^3*c^2*d - a^3*c*d^2 - 3*a^3*d^3)
*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(a^3*c*d^2 + a^3*d^3 + (2*a^3*c^2
*d - a^3*c*d^2 + a^3*d^3)*cos(f*x + e))*sin(f*x + e))/((c^2*d^3 + c*d^4)*f*
cos(f*x + e)^2 + (c*d^4 + d^5)*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int \frac{\sec\left(e+fx\right)}{c^2+2cd\sec\left(e+fx\right)+d^2\sec^2\left(e+fx\right)}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{c^2+2cd\sec\left(e+fx\right)+d^2\sec^2\left(e+fx\right)}\,dx+\int \frac{3}{c^2+2cd\sec\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**2,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)*
*2), x) + Integral(3*sec(e + f*x)**2/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(
e + f*x)**2), x) + Integral(3*sec(e + f*x)**3/(c**2 + 2*c*d*sec(e + f*x) +
d**2*sec(e + f*x)**2), x) + Integral(sec(e + f*x)**4/(c**2 + 2*c*d*sec(e +
f*x) + d**2*sec(e + f*x)**2), x))
```

Giac [B] time = 1.30229, size = 444, normalized size = 2.76

$$-\frac{\frac{2\left(2a^3c^3-a^3c^2d-4a^3cd^2+3a^3d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(cd^3+d^4\right)\sqrt{-c^2+d^2}}+\frac{4\left(a^3c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-a^3cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-\cdots\right.}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-\cdots\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -(2*(2*a^3*c^3 - a^3*c^2*d - 4*a^3*c*d^2 + 3*a^3*d^3)*(pi*floor(1/2*(f*x +
e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*
x + 1/2*e))/sqrt(-c^2 + d^2)))/((c*d^3 + d^4)*sqrt(-c^2 + d^2)) + 4*(a^3*c^
2*tan(1/2*f*x + 1/2*e)^3 - a^3*c*d*tan(1/2*f*x + 1/2*e)^3 - a^3*c^2*tan(1/2
*f*x + 1/2*e) - a^3*d^2*tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^4 -
d*tan(1/2*f*x + 1/2*e)^4 - 2*c*tan(1/2*f*x + 1/2*e)^2 + c + d)*(c*d^2 + d^3
)) + (2*a^3*c - 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^3 - (2*a^3*c
- 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/d^3)/f
```

3.207 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=188

$$-\frac{a^3\sqrt{c-d}\left(2c^2+6cd+7d^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^3f(c+d)^{5/2}}-\frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2f(c+d)^2(c+d\sec(e+fx))}-\frac{(c-d)\tan(e+fx)\left(a^3\sec(e\right.}{2df(c+d)(c+d\sec(e}$$

```
[Out] (a^3*ArcTanh[Sin[e + f*x]])/(d^3*f) - (a^3*Sqrt[c - d]*(2*c^2 + 6*c*d + 7*d
^2)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(d^3*(c + d)^(5/2)
*f) - ((c - d)*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(2*d*(c + d)*f*(c + d
*Sec[e + f*x])^2) - (a^3*(c - d)*(2*c + 5*d)*Tan[e + f*x])/(2*d^2*(c + d)^2
*f*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.394444, antiderivative size = 301, normalized size of antiderivative = 1.6, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {3987, 98, 149, 157, 63, 217, 203, 93, 205}

$$\frac{a^4\sqrt{c-d}\left(2c^2+6cd+7d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{d^3f(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2f(c+d)^2(c+d\sec(e+fx))}-\frac{(c-d)\tan(e+fx}{2df(c+d)(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^4*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e +
f*x])/(d^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^4*Sqrt
[c - d]*(2*c^2 + 6*c*d + 7*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]
])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(d^3*(c + d)^(5/2)
*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)*(a^3 + a^3
*Sec[e + f*x])*Tan[e + f*x])/(2*d*(c + d)*f*(c + d*Sec[e + f*x])^2) - (a^3*
(c - d)*(2*c + 5*d)*Tan[e + f*x])/(2*d^2*(c + d)^2*f*(c + d*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} + \frac{(a\tan(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{a+ax}\left(a^3(c\right.}{\sqrt{a-}}\right.}{2d(c+d)f\sqrt{a-a\sec(e+}}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2(c+d)^2f(c+d\sec(e+fx))} + $$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2(c+d)^2f(c+d\sec(e+fx))} - $$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2(c+d)^2f(c+d\sec(e+fx))} + $$

$$= \frac{a^4\sqrt{c-d}\left(2c^2+6cd+7d^2\right)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)\left(a^3\right.}{2d(c+}$$

$$= \frac{2a^4\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a^4\sqrt{c-d}\left(2c^2+6cd+7d^2\right)\tan^{-1}\left(\right.}{d^3(c+d)^{5/2}f\sqrt{a-a\sec(e+}}$$

Mathematica [C] time = 3.37798, size = 393, normalized size = 2.09

$$a^3\sec^6\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^3(c\cos(e+fx)+d)\left(\frac{d(c-d)\sec(e)\left(\left(5c^2d^2+6c^3d+2c^4+12cd^3+2d^4\right)\sin(e)-c\left(-d\left(c^2+6cd+2d^2\right)\sin(2e+fx)\right.\right.}{c^2(c+d)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*x])^3*(-4*(d +
c*Cos[e + f*x])^2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*(d + c*Cos[e
 + f*x])^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (4*(2*c^3 + 4*c^2*d +
 c*d^2 - 7*d^3)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan
[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f
*x])^2*(I*Cos[e] + Sin[e]))/((c + d)^2*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin
[e])^2]) + ((c - d)*d*Sec[e]*((2*c^4 + 6*c^3*d + 5*c^2*d^2 + 12*c*d^3 + 2*d
^4)*Sin[e] - c*(d*(7*c^2 + 18*c*d + 2*d^2)*Sin[f*x] - d*(c^2 + 6*c*d + 2*d^
2)*Sin[2*e + f*x] + c*(2*c^2 + 6*c*d + d^2)*Sin[e + 2*f*x]))))/(c^2*(c + d)^
2)))/(32*d^3*f*(c + d*Sec[e + f*x])^3)
```

Maple [B] time = 0.127, size = 768, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x)
```

```
[Out] 1/f*a^3/d^3*ln(tan(1/2*f*x+1/2*e)+1)-1/f*a^3/d^3*ln(tan(1/2*f*x+1/2*e)-1)+2
/f*a^3/d^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c^2+2*c*d
+d^2)*tan(1/2*f*x+1/2*e)^3*c^3+1/f*a^3/d/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*
x+1/2*e)^2*d-c-d)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*c^2-8/f*a^3*c/(tan
(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c^2+2*c*d+d^2)*tan(1/2*f
*x+1/2*e)^3-2/f*a^3/d^2/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)
^2/(c+d)*tan(1/2*f*x+1/2*e)*c^2-5/f*a^3/d*c/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2
*f*x+1/2*e)^2*d-c-d)^2/(c+d)*tan(1/2*f*x+1/2*e)-2/f*a^3/d^3/(c^2+2*c*d+d^2)
/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*
c^3-4/f*a^3/d^2/(c^2+2*c*d+d^2)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2
*e)*(c-d)/((c+d)*(c-d))^(1/2))*c^2-1/f*a^3/d*c/(c^2+2*c*d+d^2)/((c+d)*(c-d)
)^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))+5/f*a^3*d/(ta
n(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/(c^2+2*c*d+d^2)*tan(1/2*
f*x+1/2*e)^3+7/f*a^3/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2/
(c+d)*tan(1/2*f*x+1/2*e)+7/f*a^3/(c^2+2*c*d+d^2)/((c+d)*(c-d))^(1/2)*arctan
h(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.68489, size = 2534, normalized size = 13.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*((2*a^3*c^2*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4 + (2*a^3*c^4 + 6*a^3*c^3*d +
 7*a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(2*a^3*c^3*d + 6*a^3*c^2*d^2 + 7*a^3*c*d
^3)*cos(f*x + e))*sqrt((c - d)/(c + d))*log((2*c*d*cos(f*x + e) - (c^2 - 2*
d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)
/(c + d))*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x +
 e) + d^2)) + 2*(a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4 + (a^3*c^4 + 2*a^3*c^3
*d + a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(a^3*c^3*d + 2*a^3*c^2*d^2 + a^3*c*d^3
)*cos(f*x + e))*log(sin(f*x + e) + 1) - 2*(a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*
d^4 + (a^3*c^4 + 2*a^3*c^3*d + a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(a^3*c^3*d +
 2*a^3*c^2*d^2 + a^3*c*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) - 2*(3*a^3
*c^2*d^2 + 3*a^3*c*d^3 - 6*a^3*d^4 + (2*a^3*c^3*d + 4*a^3*c^2*d^2 - 5*a^3*c
*d^3 - a^3*d^4)*cos(f*x + e))*sin(f*x + e))/((c^4*d^3 + 2*c^3*d^4 + c^2*d^5
)*f*cos(f*x + e)^2 + 2*(c^3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c^2*
d^5 + 2*c*d^6 + d^7)*f), -1/2*((2*a^3*c^2*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4 + (
2*a^3*c^4 + 6*a^3*c^3*d + 7*a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(2*a^3*c^3*d +
```

```
6*a^3*c^2*d^2 + 7*a^3*c*d^3)*cos(f*x + e))*sqrt(-(c - d)/(c + d))*arctan(-(
d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d))/((c - d)*sin(f*x + e))) - (a^3*c
^2*d^2 + 2*a^3*c*d^3 + a^3*d^4 + (a^3*c^4 + 2*a^3*c^3*d + a^3*c^2*d^2)*cos(
f*x + e)^2 + 2*(a^3*c^3*d + 2*a^3*c^2*d^2 + a^3*c*d^3)*cos(f*x + e))*log(si
n(f*x + e) + 1) + (a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4 + (a^3*c^4 + 2*a^3*c
^3*d + a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(a^3*c^3*d + 2*a^3*c^2*d^2 + a^3*c*d
^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + (3*a^3*c^2*d^2 + 3*a^3*c*d^3 - 6
*a^3*d^4 + (2*a^3*c^3*d + 4*a^3*c^2*d^2 - 5*a^3*c*d^3 - a^3*d^4)*cos(f*x +
e))*sin(f*x + e))/((c^4*d^3 + 2*c^3*d^4 + c^2*d^5)*f*cos(f*x + e)^2 + 2*(c^
3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c^2*d^5 + 2*c*d^6 + d^7)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int \frac{\sec\left(e+fx\right)}{c^3+3c^2d\sec\left(e+fx\right)+3cd^2\sec^2\left(e+fx\right)+d^3\sec^3\left(e+fx\right)}\,dx+\int \frac{3\sec^2\left(e+fx\right)}{c^3+3c^2d\sec\left(e+fx\right)+3cd^2\sec^2\left(e+fx\right)+\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e
+ f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(3*sec(e + f*x)**2/(c**3 +
3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3), x
) + Integral(3*sec(e + f*x)**3/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec
(e + f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(sec(e + f*x)**4/(c**3 +
 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3),
x))
```

Giac [B] time = 1.36028, size = 528, normalized size = 2.81

$$\frac{a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{d^3}-\frac{a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{d^3}+\frac{\left(2a^3c^3+4a^3c^2d+a^3cd^2-7a^3d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}\ldots\right.}{\sqrt{-c^2+d^2}}\right.\right.}{\left(c^2d^3+2cd^4+d^5\right)\sqrt{-c^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] (a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^3 - a^3*log(abs(tan(1/2*f*x + 1/2
*e) - 1))/d^3 + (2*a^3*c^3 + 4*a^3*c^2*d + a^3*c*d^2 - 7*a^3*d^3)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) -
d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^2*d^3 + 2*c*d^4 + d^5)*sqrt(
-c^2 + d^2)) + (2*a^3*c^3*tan(1/2*f*x + 1/2*e)^3 + a^3*c^2*d*tan(1/2*f*x +
1/2*e)^3 - 8*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 5*a^3*d^3*tan(1/2*f*x + 1/2
*e)^3 - 2*a^3*c^3*tan(1/2*f*x + 1/2*e) - 7*a^3*c^2*d*tan(1/2*f*x + 1/2*e) +
 2*a^3*c*d^2*tan(1/2*f*x + 1/2*e) + 7*a^3*d^3*tan(1/2*f*x + 1/2*e))/((c^2*d
^2 + 2*c*d^3 + d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 -
c - d)^2))/f
```

3.208 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=178

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}(c+d)^{7/2}}+\frac{5a^3(c+4d)\tan(e+fx)}{6df(c+d)^3(c+d\sec(e+fx))}-\frac{5a^3(c-d)\tan(e+fx)}{6df(c+d)^2(c+d\sec(e+fx))^2}+\frac{a\tan(e+fx)(a\,s}{3f(c+d)(c+c}$$

```
[Out] (5*a^3*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*(c
 + d)^(7/2)*f) + (a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(3*(c + d)*f*(c +
d*Sec[e + f*x])^3) - (5*a^3*(c - d)*Tan[e + f*x])/(6*d*(c + d)^2*f*(c + d*S
ec[e + f*x])^2) + (5*a^3*(c + 4*d)*Tan[e + f*x])/(6*d*(c + d)^3*f*(c + d*Se
c[e + f*x]))
```

Rubi [A] time = 0.223182, antiderivative size = 227, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3987, 94, 93, 205}

$$-\frac{5a^4\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{f\sqrt{c-d}(c+d)^{7/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{5a^3\tan(e+fx)}{2f(c+d)^3(c+d\sec(e+fx))}+\frac{5\tan(e+fx)\left(a^3\sec(e+\right.}{6f(c+d)^2(c+d\sec(e}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (-5*a^4*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a -
 a*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c - d]*(c + d)^(7/2)*f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a*(a + a*Sec[e + f*x])^2*Tan[e + f
*x])/(3*(c + d)*f*(c + d*Sec[e + f*x])^3) + (5*(a^3 + a^3*Sec[e + f*x])*Tan
[e + f*x])/(6*(c + d)^2*f*(c + d*Sec[e + f*x])^2) + (5*a^3*Tan[e + f*x])/(2
*(c + d)^3*f*(c + d*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}(c+dx)^4}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} - \frac{\left(5a^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}(c+dx)^3}\,dx,\right.}{3(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec}}$$

$$= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{5\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{6(c+d)^2f(c+d\sec(e+fx))^2} - \frac{\left(5a^4\right.}{}$$

$$= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{5\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{6(c+d)^2f(c+d\sec(e+fx))^2} + \frac{}{2(c+}$$

$$= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{5\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{6(c+d)^2f(c+d\sec(e+fx))^2} + \frac{}{2(c+}$$

$$= -\frac{5a^4\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{c-d}(c+d)^{7/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a(a+a\sec(e+fx))^2}{3(c+d)f(c+d\sec}$$

Mathematica [C] time = 3.51964, size = 398, normalized size = 2.24

$$\frac{a^3\sec^6\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)(\sec(e+fx)+1)^3(c\cos(e+fx)+d)\left(\frac{c\sec(e)\left(-3\left(30c^2d^2-3c^3d+6c^4+18cd^3+4d^4\right)\sin(2e+fx)+c\left(3\left(38c^2d+\right.\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*Sec[e + f*x]*(1 + Sec[e + f*x]
)^3*(((-120*I)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[
(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f*
x])^3*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) +
(c*Sec[e]*(6*(8*c^4 + 6*c^3*d + 30*c^2*d^2 + 9*c*d^3 + 2*d^4)*Sin[f*x] - 3*
(6*c^4 - 3*c^3*d + 30*c^2*d^2 + 18*c*d^3 + 4*d^4)*Sin[2*e + f*x] + c*(3*(3*
c^3 + 38*c^2*d + 12*c*d^2 + 2*d^3)*Sin[e + 2*f*x] + 3*(3*c^3 - 6*c^2*d - 6*
c*d^2 - 2*d^3)*Sin[3*e + 2*f*x] + c*(22*c^2 + 9*c*d + 2*d^2)*Sin[2*e + 3*f*
x])) - 2*d*(66*c^4 + 27*c^3*d + 50*c^2*d^2 + 18*c*d^3 + 4*d^4)*Tan[e])/c^3)
)/(192*(c + d)^3*f*(c + d*Sec[e + f*x])^4)
```

Maple [A] time = 0.128, size = 227, normalized size = 1.3

$$16\,{\frac {{a}^{3}}{f} \left(-1/6\,{\frac {\tan \left(1/2\,fx+e/2 \right) }{ \left(c+d \right) \left(\left(\tan \left(1/2\,fx+e/2 \right) \right) ^{2}c- \left(\tan \left(1/2\,fx+e/2 \right) \right) ^{2}d-c-d \right) ^{3}}}-5/6\,{\frac {1}{c+d} \left(-1/4\,{\frac {\tan \left(1/2\,fx+e/2 \right) }{ \left(c+d \right) \left(\left(\tan \left(1/2\,fx \right.\right.\right.}}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x)
```

```
[Out] 16/f*a^3*(-1/6*tan(1/2*f*x+1/2*e)/(c+d)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x
+1/2*e)^2*d-c-d)^3-5/6/(c+d)*(-1/4*tan(1/2*f*x+1/2*e)/(c+d)/(tan(1/2*f*x+1/
2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2-3/4/(c+d)*(-1/2*tan(1/2*f*x+1/2*e)/(
c+d)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)+1/2/(c+d)/((c+d)*(
c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2)))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.642665, size = 2178, normalized size = 12.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(15*(a^3*c^3*cos(f*x + e)^3 + 3*a^3*c^2*d*cos(f*x + e)^2 + 3*a^3*c*d^
2*cos(f*x + e) + a^3*d^3)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 -
2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e)
 + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*a^3
*c^4 + 9*a^3*c^3*d + 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a^3*d^4 + (22*a^3*c^
4 + 9*a^3*c^3*d - 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 2*a^3*d^4)*cos(f*x + e)^2
+ 3*(3*a^3*c^4 + 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d^4)*cos(f*x + e))*sin
(f*x + e))/((c^8 + 3*c^7*d + 2*c^6*d^2 - 2*c^5*d^3 - 3*c^4*d^4 - c^3*d^5)*f
*cos(f*x + e)^3 + 3*(c^7*d + 3*c^6*d^2 + 2*c^5*d^3 - 2*c^4*d^4 - 3*c^3*d^5
- c^2*d^6)*f*cos(f*x + e)^2 + 3*(c^6*d^2 + 3*c^5*d^3 + 2*c^4*d^4 - 2*c^3*d^
5 - 3*c^2*d^6 - c*d^7)*f*cos(f*x + e) + (c^5*d^3 + 3*c^4*d^4 + 2*c^3*d^5 -
2*c^2*d^6 - 3*c*d^7 - d^8)*f), 1/6*(15*(a^3*c^3*cos(f*x + e)^3 + 3*a^3*c^2*
d*cos(f*x + e)^2 + 3*a^3*c*d^2*cos(f*x + e) + a^3*d^3)*sqrt(-c^2 + d^2)*arc
tan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (2
*a^3*c^4 + 9*a^3*c^3*d + 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a^3*d^4 + (22*a^
3*c^4 + 9*a^3*c^3*d - 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 2*a^3*d^4)*cos(f*x + e
)^2 + 3*(3*a^3*c^4 + 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d^4)*cos(f*x + e))
```

```
*sin(f*x + e))/((c^8 + 3*c^7*d + 2*c^6*d^2 - 2*c^5*d^3 - 3*c^4*d^4 - c^3*d^
5)*f*cos(f*x + e)^3 + 3*(c^7*d + 3*c^6*d^2 + 2*c^5*d^3 - 2*c^4*d^4 - 3*c^3*
d^5 - c^2*d^6)*f*cos(f*x + e)^2 + 3*(c^6*d^2 + 3*c^5*d^3 + 2*c^4*d^4 - 2*c^
3*d^5 - 3*c^2*d^6 - c*d^7)*f*cos(f*x + e) + (c^5*d^3 + 3*c^4*d^4 + 2*c^3*d^
5 - 2*c^2*d^6 - 3*c*d^7 - d^8)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int \frac{\sec\left(e+fx\right)}{c^4+4c^3d\sec\left(e+fx\right)+6c^2d^2\sec^2\left(e+fx\right)+4cd^3\sec^3\left(e+fx\right)+d^4\sec^4\left(e+fx\right)}\,dx+\int \frac{}{c^4+4c^3d\sec\left(e+fx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**4,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec
(e + f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integ
ral(3*sec(e + f*x)**2/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f
*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral(3*
sec(e + f*x)**3/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**2
 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral(sec(e +
f*x)**4/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**2 + 4*c*d
**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x))
```

Giac [A] time = 1.45993, size = 432, normalized size = 2.43

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)a^3}{\left(c^3+3c^2d+3cd^2+d^3\right)\sqrt{-c^2+d^2}}+\frac{15a^3c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-30a^3cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+15a^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-40}{\left(c^3+3c^2d+3\right.}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(15*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1
/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*a^3/((c^3 + 3*
c^2*d + 3*c*d^2 + d^3)*sqrt(-c^2 + d^2)) + (15*a^3*c^2*tan(1/2*f*x + 1/2*e)
^5 - 30*a^3*c*d*tan(1/2*f*x + 1/2*e)^5 + 15*a^3*d^2*tan(1/2*f*x + 1/2*e)^5
- 40*a^3*c^2*tan(1/2*f*x + 1/2*e)^3 + 40*a^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 3
3*a^3*c^2*tan(1/2*f*x + 1/2*e) + 66*a^3*c*d*tan(1/2*f*x + 1/2*e) + 33*a^3*d
^2*tan(1/2*f*x + 1/2*e))/((c^3 + 3*c^2*d + 3*c*d^2 + d^3)*(c*tan(1/2*f*x +
1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))/f
```

3.209 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

Optimal. Leaf size=266

$$\frac{5a^3(4c-3d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{4f(c-d)^{3/2}(c+d)^{9/2}}+\frac{5a^3(4c-3d)(c+4d)\tan(e+fx)}{24df(c-d)(c+d)^4(c+d\sec(e+fx))}-\frac{5a^3(4c-3d)\tan(e+fx)}{24df(c+d)^3(c+d\sec(e+fx))^2}-$$

```
[Out] (5*a^3*(4*c - 3*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(4*
(c - d)^(3/2)*(c + d)^(9/2)*f) - (d*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(4
*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^4) + (a*(4*c - 3*d)*(a + a*Sec[e + f*x]
)^2*Tan[e + f*x])/(12*(c - d)*(c + d)^2*f*(c + d*Sec[e + f*x])^3) - (5*a^3*
(4*c - 3*d)*Tan[e + f*x])/(24*d*(c + d)^3*f*(c + d*Sec[e + f*x])^2) + (5*a^
3*(4*c - 3*d)*(c + 4*d)*Tan[e + f*x])/(24*(c - d)*d*(c + d)^4*f*(c + d*Sec[
e + f*x]))
```

Rubi [A] time = 0.33683, antiderivative size = 327, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3987, 96, 94, 93, 205}

$$-\frac{5a^4(4c-3d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{4f(c-d)^{3/2}(c+d)^{9/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{5a^3(4c-3d)\tan(e+fx)}{8f(c-d)(c+d)^4(c+d\sec(e+fx))}+\frac{5(4c-3d)\tan(}{24f(c-d)(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (-5*a^4*(4*c - 3*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c -
 d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(4*(c - d)^(3/2)*(c + d)^(9/2)
*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d*(a + a*Sec[e + f
*x])^3*Tan[e + f*x])/(4*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^4) + (a*(4*c - 3
*d)*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(12*(c - d)*(c + d)^2*f*(c + d*Sec
[e + f*x])^3) + (5*(4*c - 3*d)*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(24*(
c - d)*(c + d)^3*f*(c + d*Sec[e + f*x])^2) + (5*a^3*(4*c - 3*d)*Tan[e + f*x
])/(8*(c - d)*(c + d)^4*f*(c + d*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}(c+dx)^5}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4\left(c^2-d^2\right)f(c+d\sec(e+fx))^4} - \frac{\left(a^2(4c-3d)\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax}{\sqrt{a-ax}(}\right.}{4\left(c^2-d^2\right)f\sqrt{a-a\sec(e+fx)}\sqrt{a}}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4\left(c^2-d^2\right)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+fx))^2\tan(e+fx)}{12(c-d)(c+d)^2f(c+d\sec(e+fx))^3}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4\left(c^2-d^2\right)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+fx))^2\tan(e+fx)}{12(c-d)(c+d)^2f(c+d\sec(e+fx))^3}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4\left(c^2-d^2\right)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+fx))^2\tan(e+fx)}{12(c-d)(c+d)^2f(c+d\sec(e+fx))^3}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4\left(c^2-d^2\right)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+fx))^2\tan(e+fx)}{12(c-d)(c+d)^2f(c+d\sec(e+fx))^3}$$

$$= -\frac{5a^4(4c-3d)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{4(c-d)^{3/2}(c+d)^{9/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{d(a+a\sec(e+f}{4\left(c^2-d^2\right)f(c+}$$

Mathematica [A] time = 9.66236, size = 274, normalized size = 1.03

$$\frac{a^3\left(-\frac{\sin(e+fx)\left(37c^2d^2\cos(3(e+fx))+\left(-577c^2d^2-84c^3d-296c^4+984cd^3+198d^4\right)\cos(e+fx)+\left(384c^2d^2-470c^3d-72c^4+200cd^3+48d^4\right)\cos(2(e+fx))+336c^2d^2+36}{(c\cos(e+fx)+d)^4}\right.}{96f(c-d)(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*((-120*(4*c - 3*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]
])/Sqrt[c^2 - d^2] - ((-72*c^4 - 478*c^3*d + 336*c^2*d^2 + 28*c*d^3 + 336*d
^4 + (-296*c^4 - 84*c^3*d - 577*c^2*d^2 + 984*c*d^3 + 198*d^4)*Cos[e + f*x]
 + (-72*c^4 - 470*c^3*d + 384*c^2*d^2 + 200*c*d^3 + 48*d^4)*Cos[2*(e + f*x)
] - 88*c^4*Cos[3*(e + f*x)] + 36*c^3*d*Cos[3*(e + f*x)] + 37*c^2*d^2*Cos[3*
(e + f*x)] + 24*c*d^3*Cos[3*(e + f*x)] + 6*d^4*Cos[3*(e + f*x)])*Sin[e + f*
x])/(d + c*Cos[e + f*x])^4))/(96*(c - d)*(c + d)^4*f)
```

Maple [A] time = 0.156, size = 303, normalized size = 1.1

$$16\,{\frac {{a}^{3}}{f} \left({\frac {1}{ \left(\left(\tan \left(1/2\,fx+e/2 \right) \right) ^{2}c- \left(\tan \left(1/2\,fx+e/2 \right) \right) ^{2}d-c-d \right) ^{4}} \left(-{\frac { \left(20\,c-15\,d \right) \left({c}^{2}-2\,cd+{d}^{2} \right) \left(\tan \left(1/2\,fx+e/2 \right) \right) }{64\,{c}^{4}+256\,{c}^{3}d+384\,{c}^{2}{d}^{2}+256\,c{d}^{3}+64\,{d}^{4}}}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x)
```

```
[Out] 16/f*a^3*((-5/64*(4*c-3*d)*(c^2-2*c*d+d^2)/(c^4+4*c^3*d+6*c^2*d^2+4*c*d^3+d
^4)*tan(1/2*f*x+1/2*e)^7+55/192*(c-d)*(4*c-3*d)/(c^3+3*c^2*d+3*c*d^2+d^3)*t
an(1/2*f*x+1/2*e)^5-73/192*(4*c-3*d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1
/64*(44*c-49*d)/(c+d)/(c-d)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan
(1/2*f*x+1/2*e)^2*d-c-d)^4+5/64*(4*c-3*d)/(c^5+3*c^4*d+2*c^3*d^2-2*c^2*d^3-
3*c*d^4-d^5)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c
-d))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.804176, size = 3699, normalized size = 13.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] [1/48*(15*(4*a^3*c*d^4 - 3*a^3*d^5 + (4*a^3*c^5 - 3*a^3*c^4*d)*cos(f*x + e)
^4 + 4*(4*a^3*c^4*d - 3*a^3*c^3*d^2)*cos(f*x + e)^3 + 6*(4*a^3*c^3*d^2 - 3*
a^3*c^2*d^3)*cos(f*x + e)^2 + 4*(4*a^3*c^2*d^3 - 3*a^3*c*d^4)*cos(f*x + e))
*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2
*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(
f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*a^3*c^5*d + 12*a^3*c^4*d^2 +
 41*a^3*c^3*d^3 - 84*a^3*c^2*d^4 - 43*a^3*c*d^5 + 72*a^3*d^6 + (88*a^3*c^6
- 36*a^3*c^5*d - 125*a^3*c^4*d^2 + 12*a^3*c^3*d^3 + 31*a^3*c^2*d^4 + 24*a^3
*c*d^5 + 6*a^3*d^6)*cos(f*x + e)^3 + (36*a^3*c^6 + 235*a^3*c^5*d - 228*a^3*
c^4*d^2 - 335*a^3*c^3*d^3 + 168*a^3*c^2*d^4 + 100*a^3*c*d^5 + 24*a^3*d^6)*c
os(f*x + e)^2 + (8*a^3*c^6 + 48*a^3*c^5*d + 164*a^3*c^4*d^2 - 276*a^3*c^3*d
^3 - 217*a^3*c^2*d^4 + 228*a^3*c*d^5 + 45*a^3*d^6)*cos(f*x + e))*sin(f*x +
e))/((c^11 + 3*c^10*d + c^9*d^2 - 5*c^8*d^3 - 5*c^7*d^4 + c^6*d^5 + 3*c^5*d
^6 + c^4*d^7)*f*cos(f*x + e)^4 + 4*(c^10*d + 3*c^9*d^2 + c^8*d^3 - 5*c^7*d^
4 - 5*c^6*d^5 + c^5*d^6 + 3*c^4*d^7 + c^3*d^8)*f*cos(f*x + e)^3 + 6*(c^9*d^
2 + 3*c^8*d^3 + c^7*d^4 - 5*c^6*d^5 - 5*c^5*d^6 + c^4*d^7 + 3*c^3*d^8 + c^2
*d^9)*f*cos(f*x + e)^2 + 4*(c^8*d^3 + 3*c^7*d^4 + c^6*d^5 - 5*c^5*d^6 - 5*c
^4*d^7 + c^3*d^8 + 3*c^2*d^9 + c*d^10)*f*cos(f*x + e) + (c^7*d^4 + 3*c^6*d^
5 + c^5*d^6 - 5*c^4*d^7 - 5*c^3*d^8 + c^2*d^9 + 3*c*d^10 + d^11)*f), 1/24*(
15*(4*a^3*c*d^4 - 3*a^3*d^5 + (4*a^3*c^5 - 3*a^3*c^4*d)*cos(f*x + e)^4 + 4*
(4*a^3*c^4*d - 3*a^3*c^3*d^2)*cos(f*x + e)^3 + 6*(4*a^3*c^3*d^2 - 3*a^3*c^2
*d^3)*cos(f*x + e)^2 + 4*(4*a^3*c^2*d^3 - 3*a^3*c*d^4)*cos(f*x + e))*sqrt(-
c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f
*x + e))) + (2*a^3*c^5*d + 12*a^3*c^4*d^2 + 41*a^3*c^3*d^3 - 84*a^3*c^2*d^4
 - 43*a^3*c*d^5 + 72*a^3*d^6 + (88*a^3*c^6 - 36*a^3*c^5*d - 125*a^3*c^4*d^2
 + 12*a^3*c^3*d^3 + 31*a^3*c^2*d^4 + 24*a^3*c*d^5 + 6*a^3*d^6)*cos(f*x + e)
^3 + (36*a^3*c^6 + 235*a^3*c^5*d - 228*a^3*c^4*d^2 - 335*a^3*c^3*d^3 + 168*
a^3*c^2*d^4 + 100*a^3*c*d^5 + 24*a^3*d^6)*cos(f*x + e)^2 + (8*a^3*c^6 + 48*
a^3*c^5*d + 164*a^3*c^4*d^2 - 276*a^3*c^3*d^3 - 217*a^3*c^2*d^4 + 228*a^3*c
*d^5 + 45*a^3*d^6)*cos(f*x + e))*sin(f*x + e))/((c^11 + 3*c^10*d + c^9*d^2
- 5*c^8*d^3 - 5*c^7*d^4 + c^6*d^5 + 3*c^5*d^6 + c^4*d^7)*f*cos(f*x + e)^4 +
 4*(c^10*d + 3*c^9*d^2 + c^8*d^3 - 5*c^7*d^4 - 5*c^6*d^5 + c^5*d^6 + 3*c^4*
d^7 + c^3*d^8)*f*cos(f*x + e)^3 + 6*(c^9*d^2 + 3*c^8*d^3 + c^7*d^4 - 5*c^6*
d^5 - 5*c^5*d^6 + c^4*d^7 + 3*c^3*d^8 + c^2*d^9)*f*cos(f*x + e)^2 + 4*(c^8*
d^3 + 3*c^7*d^4 + c^6*d^5 - 5*c^5*d^6 - 5*c^4*d^7 + c^3*d^8 + 3*c^2*d^9 + c
*d^10)*f*cos(f*x + e) + (c^7*d^4 + 3*c^6*d^5 + c^5*d^6 - 5*c^4*d^7 - 5*c^3*
d^8 + c^2*d^9 + 3*c*d^10 + d^11)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int \frac{\sec\left(e+fx\right)}{c^5+5c^4d\sec\left(e+fx\right)+10c^3d^2\sec^2\left(e+fx\right)+10c^2d^3\sec^3\left(e+fx\right)+5cd^4\sec^4\left(e+fx\right)+d^5\sec^5\left(e+fx\right)}\,dx+\dots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**5,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*se
c(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d
**5*sec(e + f*x)**5), x) + Integral(3*sec(e + f*x)**2/(c**5 + 5*c**4*d*sec(
e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*
c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x) + Integral(3*sec(e + f*x
)**3/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2
*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x
) + Integral(sec(e + f*x)**4/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*s
ec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 +
```

```
d**5*sec(e + f*x)**5), x))
```

Giac [B] time = 1.53497, size = 845, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/12*(15*(4*a^3*c - 3*a^3*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2
*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 +
 d^2)))/((c^5 + 3*c^4*d + 2*c^3*d^2 - 2*c^2*d^3 - 3*c*d^4 - d^5)*sqrt(-c^2
+ d^2)) - (60*a^3*c^4*tan(1/2*f*x + 1/2*e)^7 - 225*a^3*c^3*d*tan(1/2*f*x +
1/2*e)^7 + 315*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 195*a^3*c*d^3*tan(1/2*f
*x + 1/2*e)^7 + 45*a^3*d^4*tan(1/2*f*x + 1/2*e)^7 - 220*a^3*c^4*tan(1/2*f*x
 + 1/2*e)^5 + 385*a^3*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 55*a^3*c^2*d^2*tan(1/2
*f*x + 1/2*e)^5 - 385*a^3*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 165*a^3*d^4*tan(1/
2*f*x + 1/2*e)^5 + 292*a^3*c^4*tan(1/2*f*x + 1/2*e)^3 + 73*a^3*c^3*d*tan(1/
2*f*x + 1/2*e)^3 - 511*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 73*a^3*c*d^3*ta
n(1/2*f*x + 1/2*e)^3 + 219*a^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 132*a^3*c^4*tan
(1/2*f*x + 1/2*e) - 249*a^3*c^3*d*tan(1/2*f*x + 1/2*e) + 45*a^3*c^2*d^2*tan
(1/2*f*x + 1/2*e) + 309*a^3*c*d^3*tan(1/2*f*x + 1/2*e) + 147*a^3*d^4*tan(1/
2*f*x + 1/2*e))/((c^5 + 3*c^4*d + 2*c^3*d^2 - 2*c^2*d^3 - 3*c*d^4 - d^5)*(c
*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^4))/f
```

3.210 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=183

$$\frac{d\left(-12c^2d+8c^3+12cd^2-3d^3\right)\tanh^{-1}(\sin(e+fx))}{2af}-\frac{d\tan(e+fx)\left(d\left(6c^2-20cd+9d^2\right)\sec(e+fx)+4\left(-16c^2d+3c^3\right.\right.}{6af}$$

```
[Out] (d*(8*c^3 - 12*c^2*d + 12*c*d^2 - 3*d^3)*ArcTanh[Sin[e + f*x]])/(2*a*f) - (
(3*c - 4*d)*d*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*a*f) + ((c - d)*(c +
d*Sec[e + f*x])^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])) - (d*(4*(3*c^3 - 1
6*c^2*d + 12*c*d^2 - 4*d^3) + d*(6*c^2 - 20*c*d + 9*d^2)*Sec[e + f*x])*Tan[
e + f*x])/(6*a*f)
```

Rubi [A] time = 0.335273, antiderivative size = 236, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 98, 153, 147, 63, 217, 203}

$$\frac{d\left(-12c^2d+8c^3+12cd^2-3d^3\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{d\tan(e+fx)\left(d\left(6c^2-20cd+9d^2\right)\sec(e+fx)+4\right.}{6af}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (d*(8*c^3 - 12*c^2*d + 12*c*d^2 - 3*d^3)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sq
rt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a
+ a*Sec[e + f*x]]) - ((3*c - 4*d)*d*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3
*a*f) + ((c - d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x
])) - (d*(4*(3*c^3 - 16*c^2*d + 12*c*d^2 - 4*d^3) + d*(6*c^2 - 20*c*d + 9*d
^2)*Sec[e + f*x])*Tan[e + f*x])/(6*a*f)
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^4}{\sqrt{a-ax}(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^2\left(-a^2(4c-3d)d\cdots\right.}{\sqrt{a-ax}\sqrt{a\cdots}}\right.}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\cdots}}$$

$$= -\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af} + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e\cdots}{f(a+a\sec(e+fx))}$$

$$= -\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af} + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e\cdots}{f(a+a\sec(e+fx))}$$

$$= -\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af} + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e\cdots}{f(a+a\sec(e+fx))}$$

$$= -\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af} + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e\cdots}{f(a+a\sec(e+fx))}$$

$$= \frac{d\left(8c^3-12c^2d+12cd^2-3d^3\right)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(3c-4d)d(c\cdots}{}$$

Mathematica [B] time = 6.45251, size = 1243, normalized size = 6.79

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

```
[Out] ((-8*c^3*d + 12*c^2*d^2 - 12*c*d^3 + 3*d^4)*Cos[e/2 + (f*x)/2]^2*Cos[e + f*
x]^3*Log[Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x])^4)/(
f*(d + c*Cos[e + f*x])^4*(a + a*Sec[e + f*x])) + ((8*c^3*d - 12*c^2*d^2 + 1
2*c*d^3 - 3*d^4)*Cos[e/2 + (f*x)/2]^2*Cos[e + f*x]^3*Log[Cos[e/2 + (f*x)/2]
 + Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x])^4)/(f*(d + c*Cos[e + f*x])^4*(a
 + a*Sec[e + f*x])) + (Cos[e/2 + (f*x)/2]*Sec[e/2]*Sec[e]*(c + d*Sec[e + f*
x])^4*(-18*c^4*Sin[(f*x)/2] + 72*c^3*d*Sin[(f*x)/2] - 36*c^2*d^2*Sin[(f*x)/
2] + 24*c*d^3*Sin[(f*x)/2] + 6*d^4*Sin[(f*x)/2] + 18*c^4*Sin[(3*f*x)/2] - 7
2*c^3*d*Sin[(3*f*x)/2] + 180*c^2*d^2*Sin[(3*f*x)/2] - 108*c*d^3*Sin[(3*f*x)
/2] + 39*d^4*Sin[(3*f*x)/2] - 72*c^2*d^2*Sin[e - (f*x)/2] + 48*c*d^3*Sin[e
- (f*x)/2] - 24*d^4*Sin[e - (f*x)/2] - 36*c^2*d^2*Sin[e + (f*x)/2] + 24*c*d
^3*Sin[e + (f*x)/2] - 6*d^4*Sin[e + (f*x)/2] - 18*c^4*Sin[2*e + (f*x)/2] +
72*c^3*d*Sin[2*e + (f*x)/2] - 144*c^2*d^2*Sin[2*e + (f*x)/2] + 96*c*d^3*Sin
[2*e + (f*x)/2] - 24*d^4*Sin[2*e + (f*x)/2] + 72*c^2*d^2*Sin[e + (3*f*x)/2]
 - 36*c*d^3*Sin[e + (3*f*x)/2] + 21*d^4*Sin[e + (3*f*x)/2] + 18*c^4*Sin[2*e
 + (3*f*x)/2] - 72*c^3*d*Sin[2*e + (3*f*x)/2] + 72*c^2*d^2*Sin[2*e + (3*f*x
)/2] - 36*c*d^3*Sin[2*e + (3*f*x)/2] + 9*d^4*Sin[2*e + (3*f*x)/2] - 36*c^2*
d^2*Sin[3*e + (3*f*x)/2] + 36*c*d^3*Sin[3*e + (3*f*x)/2] - 9*d^4*Sin[3*e +
(3*f*x)/2] + 36*c^2*d^2*Sin[e + (5*f*x)/2] - 12*c*d^3*Sin[e + (5*f*x)/2] +
7*d^4*Sin[e + (5*f*x)/2] - 6*c^4*Sin[2*e + (5*f*x)/2] + 24*c^3*d*Sin[2*e +
(5*f*x)/2] + 12*c*d^3*Sin[2*e + (5*f*x)/2] + d^4*Sin[2*e + (5*f*x)/2] + 12*
c*d^3*Sin[3*e + (5*f*x)/2] - 3*d^4*Sin[3*e + (5*f*x)/2] - 6*c^4*Sin[4*e + (
5*f*x)/2] + 24*c^3*d*Sin[4*e + (5*f*x)/2] - 36*c^2*d^2*Sin[4*e + (5*f*x)/2]
 + 36*c*d^3*Sin[4*e + (5*f*x)/2] - 9*d^4*Sin[4*e + (5*f*x)/2] + 6*c^4*Sin[2
*e + (7*f*x)/2] - 24*c^3*d*Sin[2*e + (7*f*x)/2] + 72*c^2*d^2*Sin[2*e + (7*f
```

```
*x)/2] - 48*c*d^3*Sin[2*e + (7*f*x)/2] + 16*d^4*Sin[2*e + (7*f*x)/2] + 36*c
^2*d^2*Sin[3*e + (7*f*x)/2] - 24*c*d^3*Sin[3*e + (7*f*x)/2] + 10*d^4*Sin[3*
e + (7*f*x)/2] + 6*c^4*Sin[4*e + (7*f*x)/2] - 24*c^3*d*Sin[4*e + (7*f*x)/2]
 + 36*c^2*d^2*Sin[4*e + (7*f*x)/2] - 24*c*d^3*Sin[4*e + (7*f*x)/2] + 6*d^4*
Sin[4*e + (7*f*x)/2]))/(48*f*(d + c*Cos[e + f*x])^4*(a + a*Sec[e + f*x]))
```

Maple [B] time = 0.079, size = 596, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x)
```

```
[Out] 6/a/f*d^3/(tan(1/2*f*x+1/2*e)+1)*c-2/a/f*d^3/(tan(1/2*f*x+1/2*e)+1)^2*c-4/a
/f*tan(1/2*f*x+1/2*e)*c*d^3+4/a/f*ln(tan(1/2*f*x+1/2*e)+1)*c^3*d-6/a/f*ln(t
an(1/2*f*x+1/2*e)+1)*c^2*d^2+6/a/f*ln(tan(1/2*f*x+1/2*e)+1)*c*d^3+6/a/f*tan
(1/2*f*x+1/2*e)*c^2*d^2-4/a/f*ln(tan(1/2*f*x+1/2*e)-1)*c^3*d+6/a/f*ln(tan(1
/2*f*x+1/2*e)-1)*c^2*d^2-6/a/f*ln(tan(1/2*f*x+1/2*e)-1)*c*d^3-6/a/f*d^2/(ta
n(1/2*f*x+1/2*e)-1)*c^2+6/a/f*d^3/(tan(1/2*f*x+1/2*e)-1)*c+2/a/f*d^3/(tan(1
/2*f*x+1/2*e)-1)^2*c-5/2/a/f*d^4/(tan(1/2*f*x+1/2*e)-1)-1/a/f*d^4/(tan(1/2*
f*x+1/2*e)-1)^2+1/a/f*tan(1/2*f*x+1/2*e)*d^4-1/3/a/f*d^4/(tan(1/2*f*x+1/2*e
)+1)^3-3/2/a/f*ln(tan(1/2*f*x+1/2*e)+1)*d^4-5/2/a/f*d^4/(tan(1/2*f*x+1/2*e)
+1)+1/a/f*d^4/(tan(1/2*f*x+1/2*e)+1)^2-1/3/a/f*d^4/(tan(1/2*f*x+1/2*e)-1)^3
+1/f*c^4/a*tan(1/2*f*x+1/2*e)+3/2/a/f*ln(tan(1/2*f*x+1/2*e)-1)*d^4-4/a/f*ta
n(1/2*f*x+1/2*e)*c^3*d-6/a/f*d^2/(tan(1/2*f*x+1/2*e)+1)*c^2
```

Maxima [B] time = 1.02911, size = 805, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/6*(d^4*(2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 16*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a - 3*a*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - a*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6) - 9*log(sin(f*x + e)/(cos(f*x + e) + 1) +
1)/a + 9*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 6*sin(f*x + e)/(a*(co
s(f*x + e) + 1))) - 12*c*d^3*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e
) + 1) + 1)/a + 3*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x +
e)/(a*(cos(f*x + e) + 1))) - 36*c^2*d^2*(log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a
- a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)
/(a*(cos(f*x + e) + 1))) + 24*c^3*d*(log(sin(f*x + e)/(cos(f*x + e) + 1) +
1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*
x + e) + 1))) + 6*c^4*sin(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 0.51661, size = 693, normalized size = 3.79

$$\frac{3\left(\left(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^4+\left(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^3\right)\log\left(\sin\left(fx+e\right)+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*((8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^4 + (8*c^3*
d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^3)*log(sin(f*x + e) + 1) -
3*((8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^4 + (8*c^3*d - 12
*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) + 2*(2*
d^4 + 2*(3*c^4 - 12*c^3*d + 36*c^2*d^2 - 24*c*d^3 + 8*d^4)*cos(f*x + e)^3 +
 (36*c^2*d^2 - 12*c*d^3 + 7*d^4)*cos(f*x + e)^2 + (12*c*d^3 - d^4)*cos(f*x
+ e))*sin(f*x + e))/(a*f*cos(f*x + e)^4 + a*f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^4\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{d^4\sec^5\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{4cd^3\sec^4\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{6c^2d^2\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{4c^3d\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c**4*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d**4*sec(e +
f*x)**5/(sec(e + f*x) + 1), x) + Integral(4*c*d**3*sec(e + f*x)**4/(sec(e +
 f*x) + 1), x) + Integral(6*c**2*d**2*sec(e + f*x)**3/(sec(e + f*x) + 1), x
) + Integral(4*c**3*d*sec(e + f*x)**2/(sec(e + f*x) + 1), x))/a
```

Giac [B] time = 1.35509, size = 487, normalized size = 2.66

$$\frac{\frac{3\left(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{3\left(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}+\frac{6\left(c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-4\,c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*log(abs(tan(1/2*f*x + 1/2*
e) + 1))/a - 3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*log(abs(tan(1/2*f*
x + 1/2*e) - 1))/a + 6*(c^4*tan(1/2*f*x + 1/2*e) - 4*c^3*d*tan(1/2*f*x + 1/
2*e) + 6*c^2*d^2*tan(1/2*f*x + 1/2*e) - 4*c*d^3*tan(1/2*f*x + 1/2*e) + d^4*
tan(1/2*f*x + 1/2*e))/a - 2*(36*c^2*d^2*tan(1/2*f*x + 1/2*e)^5 - 36*c*d^3*t
an(1/2*f*x + 1/2*e)^5 + 15*d^4*tan(1/2*f*x + 1/2*e)^5 - 72*c^2*d^2*tan(1/2*
f*x + 1/2*e)^3 + 48*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 16*d^4*tan(1/2*f*x + 1/2
*e)^3 + 36*c^2*d^2*tan(1/2*f*x + 1/2*e) - 12*c*d^3*tan(1/2*f*x + 1/2*e) + 9
*d^4*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a))/f
```

3.211 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{3d\left(2c^2-2cd+d^2\right)\tanh^{-1}(\sin(e+fx))}{2af}-\frac{d\tan(e+fx)\left(4\left(c^2-3cd+d^2\right)+d(2c-3d)\sec(e+fx)\right)}{2af}+\frac{(c-d)\tan(e}{f(a}$$

```
[Out] (3*d*(2*c^2 - 2*c*d + d^2)*ArcTanh[Sin[e + f*x]])/(2*a*f) + ((c - d)*(c + d
*Sec[e + f*x])^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])) - (d*(4*(c^2 - 3*c*
d + d^2) + (2*c - 3*d)*d*Sec[e + f*x])*Tan[e + f*x])/(2*a*f)
```

Rubi [A] time = 0.253739, antiderivative size = 171, normalized size of antiderivative = 1.46, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 98, 147, 63, 217, 203}

$$\frac{3d\left(2c^2-2cd+d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{d\tan(e+fx)\left(4\left(c^2-3cd+d^2\right)+d(2c-3d)\sec(e+fx)\right)}{2af}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (3*d*(2*c^2 - 2*c*d + d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[
e + f*x])]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*
x]]) + ((c - d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]
)) - (d*(4*(c^2 - 3*c*d + d^2) + (2*c - 3*d)*d*Sec[e + f*x])*Tan[e + f*x])/
(2*a*f)
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
```

```
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^3}{\sqrt{a-ax}(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)\left(-a^2(3c-2d)d+\cdots}{\sqrt{a-ax}\sqrt{a-\cdots}}\right.}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\cdots}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{d\left(4\left(c^2-3cd+d^2\right)+(2c-3d)d\sec(e\cdots\right.}{2af}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{d\left(4\left(c^2-3cd+d^2\right)+(2c-3d)d\sec(e\cdots\right.}{2af}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{d\left(4\left(c^2-3cd+d^2\right)+(2c-3d)d\sec(e\cdots\right.}{2af}$$

$$= \frac{3d\left(2c^2-2cd+d^2\right)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(c+d\sec(e+fx\cdots}{f(a+a\sec(e+\cdots}$$

Mathematica [B] time = 2.5219, size = 275, normalized size = 2.35

$$\frac{\cos^6\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left(\left(\tan^2\left(\frac{1}{2}(e+fx)\right)-1\right)\left(-2\left(-3c^2d+c^3+9cd^2-3d^3\right)\tan\left(\frac{1}{2}(e+fx)\right)+3d\left(2c^2-2cd+d\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (Cos[(e + f*x)/2]^6*Sec[e + f*x]^2*(16*d^3*Csc[e + f*x]^3*Sin[(e + f*x)/2]^
4 + (-1 + Tan[(e + f*x)/2]^2)*(3*d*(2*c^2 - 2*c*d + d^2)*(Log[Cos[(e + f*x)
/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 2*(c^
3 - 3*c^2*d + 9*c*d^2 - 3*d^3)*Tan[(e + f*x)/2] - 3*d*(2*c^2 - 2*c*d + d^2)
*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2]])*Tan[(e + f*x)/2]^2 + 2*(c - d)^3*Tan[(e + f*x)/2]^3)))/(a*f*(1
+ Cos[e + f*x]))
```

Maple [B] time = 0.069, size = 371, normalized size = 3.2

$$\frac{c^3}{fa}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-3\,\frac{\tan\left(1/2\,fx+e/2\right)c^2d}{fa}+3\,\frac{\tan\left(1/2\,fx+e/2\right)cd^2}{fa}-\frac{d^3}{fa}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{d^3}{2\,fa}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x)
```

```
[Out] 1/f*c^3/a*tan(1/2*f*x+1/2*e)-3/a/f*tan(1/2*f*x+1/2*e)*c^2*d+3/a/f*tan(1/2*f
*x+1/2*e)*c*d^2-1/a/f*tan(1/2*f*x+1/2*e)*d^3-1/2/a/f*d^3/(tan(1/2*f*x+1/2*e
)+1)^2+3/a/f*ln(tan(1/2*f*x+1/2*e)+1)*c^2*d-3/a/f*ln(tan(1/2*f*x+1/2*e)+1)*
c*d^2+3/2/a/f*ln(tan(1/2*f*x+1/2*e)+1)*d^3-3/a/f*d^2/(tan(1/2*f*x+1/2*e)+1)
*c+3/2/a/f*d^3/(tan(1/2*f*x+1/2*e)+1)+1/2/a/f*d^3/(tan(1/2*f*x+1/2*e)-1)^2-
3/a/f*ln(tan(1/2*f*x+1/2*e)-1)*c^2*d+3/a/f*ln(tan(1/2*f*x+1/2*e)-1)*c*d^2-3
/2/a/f*ln(tan(1/2*f*x+1/2*e)-1)*d^3-3/a/f*d^2/(tan(1/2*f*x+1/2*e)-1)*c+3/2/
a/f*d^3/(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.02283, size = 524, normalized size = 4.48

$$-\frac{d^3\left(\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a-\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}+\frac{2\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+6\,cd^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(d^3*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a + 3*l
og(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x + e)/(a*(cos(f*x + e)
 + 1))) + 6*c*d^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x
 + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1)
)) - 6*c^2*d*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)
/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - 2*c^3*s
in(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 0.500271, size = 512, normalized size = 4.38

$$\frac{3\left(\left(2c^2d-2cd^2+d^3\right)\cos\left(fx+e\right)^3+\left(2c^2d-2cd^2+d^3\right)\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(\left(2c^2d-2cd^2+d^3\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/4*(3*((2*c^2*d - 2*c*d^2 + d^3)*cos(f*x + e)^3 + (2*c^2*d - 2*c*d^2 + d^3
)*cos(f*x + e)^2)*log(sin(f*x + e) + 1) - 3*((2*c^2*d - 2*c*d^2 + d^3)*cos(
f*x + e)^3 + (2*c^2*d - 2*c*d^2 + d^3)*cos(f*x + e)^2)*log(-sin(f*x + e) +
1) + 2*(d^3 + 2*(c^3 - 3*c^2*d + 6*c*d^2 - 2*d^3)*cos(f*x + e)^2 + (6*c*d^2
 - d^3)*cos(f*x + e))*sin(f*x + e))/(a*f*cos(f*x + e)^3 + a*f*cos(f*x + e)^
2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{c^3\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\frac{d^3\sec^4\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\frac{3cd^2\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\frac{3c^2d\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c**3*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d**3*sec(e +
f*x)**4/(sec(e + f*x) + 1), x) + Integral(3*c*d**2*sec(e + f*x)**3/(sec(e +
 f*x) + 1), x) + Integral(3*c**2*d*sec(e + f*x)**2/(sec(e + f*x) + 1), x))/
a
```

Giac [B] time = 1.24518, size = 311, normalized size = 2.66

$$\frac{\frac{3\left(2c^2d-2cd^2+d^3\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a}-\frac{3\left(2c^2d-2cd^2+d^3\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a}+\frac{2\left(c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-3c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+3cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.\right.}{a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/2*(3*(2*c^2*d - 2*c*d^2 + d^3)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 3*(
2*c^2*d - 2*c*d^2 + d^3)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a + 2*(c^3*tan(
1/2*f*x + 1/2*e) - 3*c^2*d*tan(1/2*f*x + 1/2*e) + 3*c*d^2*tan(1/2*f*x + 1/2
*e) - d^3*tan(1/2*f*x + 1/2*e))/a - 2*(6*c*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*d
^3*tan(1/2*f*x + 1/2*e)^3 - 6*c*d^2*tan(1/2*f*x + 1/2*e) + d^3*tan(1/2*f*x
+ 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a))/f
```

3.212 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)} dx$

Optimal. Leaf size=68

$$\frac{d(2c-d)\tanh^{-1}(\sin(e+fx))}{af}+\frac{(c-d)^2\tan(e+fx)}{f(a\sec(e+fx)+a)}+\frac{d^2\tan(e+fx)}{af}$$

```
[Out] ((2*c - d)*d*ArcTanh[Sin[e + f*x]])/(a*f) + (d^2*Tan[e + f*x])/(a*f) + ((c
- d)^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rubi [A] time = 0.14553, antiderivative size = 125, normalized size of antiderivative = 1.84, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 89, 80, 63, 217, 203}

$$\frac{2d(2c-d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)^2\tan(e+fx)}{f(a\sec(e+fx)+a)}+\frac{d^2\tan(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (d^2*Tan[e + f*x])/(a*f) + ((c - d)^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])
) + (2*(2*c - d)*d*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x]
)]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{(c+dx)^2}{\sqrt{a-ax}(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{\tan(e+fx)\operatorname{Subst}\left(\int \frac{a^3(2c-d)d+a^3d^2x}{\sqrt{a-ax}\sqrt{a+ax}}\,dx, x, \sec(e+fx)\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(a(2c-d)d\tan(e+fx))\operatorname{Subst}\left(\int \frac{}{\sqrt{a-a}\ldots}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+\ldots}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{(2(2c-d)d\tan(e+fx))\operatorname{Subst}\left(\int \frac{1}{\sqrt{2a-}\ldots}\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+\ldots}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{(2(2c-d)d\tan(e+fx))\operatorname{Subst}\left(\int \frac{1}{1+x^2}\ldots\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\,\mathrm{s}\ldots}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{2(2c-d)d\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e\ldots}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+\ldots}}$$

Mathematica [B] time = 1.56551, size = 237, normalized size = 3.49

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\cos(e+fx)(c+d\sec(e+fx))^2\left((c-d)^2\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+d\cos\left(\frac{1}{2}(e+fx)\right)\left(\frac{}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right.\right.}\ldots\right.\right.}{af(\sec\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*Cos[e + f*x]*(c + d*Sec[e + f*x])^2*((c - d)^2*Sec[e/2]
*Sin[(f*x)/2] + d*Cos[(e + f*x)/2]*(-((2*c - d)*(Log[Cos[(e + f*x)/2] - Sin
[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) + (d*Sin[f*x])/
((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))))/(a*f*(d + c*Cos[e + f*x])
```

```
^2*(1 + Sec[e + f*x]))
```

Maple [B] time = 0.051, size = 196, normalized size = 2.9

$$\frac{c^2}{fa}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2\,\frac{cd\tan\left(1/2\,fx+e/2\right)}{fa}+\frac{d^2}{fa}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{d^2}{fa}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{-1}+2\,\frac{d\ln\left(\tan\left(1/2\,fx+e\right.\right.}{fa}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x)
```

```
[Out] 1/a/f*tan(1/2*f*x+1/2*e)*c^2-2/a/f*c*d*tan(1/2*f*x+1/2*e)+1/a/f*tan(1/2*f*x
+1/2*e)*d^2-1/a/f*d^2/(tan(1/2*f*x+1/2*e)+1)+2/a/f*d*ln(tan(1/2*f*x+1/2*e)+
1)*c-1/a/f*d^2*ln(tan(1/2*f*x+1/2*e)+1)-1/a/f*d^2/(tan(1/2*f*x+1/2*e)-1)-2/
a/f*d*ln(tan(1/2*f*x+1/2*e)-1)*c+1/a/f*d^2*ln(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 0.968144, size = 301, normalized size = 4.43

$$-\frac{d^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-2\,cd\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin\right.}{\cos}\right.}{}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -(d^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f
*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - 2*c*d*(
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e)
 + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - c^2*sin(f*x + e)/(a*(
cos(f*x + e) + 1)))/f
```

Fricas [B] time = 0.480677, size = 370, normalized size = 5.44

$$\frac{\left(\left(2\,cd-d^2\right)\cos\left(fx+e\right)^2+\left(2\,cd-d^2\right)\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-\left(\left(2\,cd-d^2\right)\cos\left(fx+e\right)^2+\left(2\,cd-d^2\right)\right.}{2\left(af\cos\left(fx+e\right)^2+af\cos\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/2*(((2*c*d - d^2)*cos(f*x + e)^2 + (2*c*d - d^2)*cos(f*x + e))*log(sin(f*
x + e) + 1) - ((2*c*d - d^2)*cos(f*x + e)^2 + (2*c*d - d^2)*cos(f*x + e))*l
og(-sin(f*x + e) + 1) + 2*(d^2 + (c^2 - 2*c*d + 2*d^2)*cos(f*x + e))*sin(f*
x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^2 \sec(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{d^2 \sec^3(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{2cd \sec^2(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c**2*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d**2*sec(e +
f*x)**3/(sec(e + f*x) + 1), x) + Integral(2*c*d*sec(e + f*x)**2/(sec(e + f*
x) + 1), x))/a
```

Giac [B] time = 1.29584, size = 193, normalized size = 2.84

$$\frac{\frac{(2cd-d^2)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a} - \frac{(2cd-d^2)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a} - \frac{2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a} + \frac{c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] ((2*c*d - d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - (2*c*d - d^2)*log(abs
(tan(1/2*f*x + 1/2*e) - 1))/a - 2*d^2*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x +
1/2*e)^2 - 1)*a) + (c^2*tan(1/2*f*x + 1/2*e) - 2*c*d*tan(1/2*f*x + 1/2*e) +
 d^2*tan(1/2*f*x + 1/2*e))/a)/f
```

3.213 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=43

$$\frac{(c-d)\tan(e+fx)}{f(a\sec(e+fx)+a)}+\frac{d\tanh^{-1}(\sin(e+fx))}{af}$$

```
[Out] (d*ArcTanh[Sin[e + f*x]])/(a*f) + ((c - d)*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rubi [A] time = 0.084563, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3998, 3770, 3794}

$$\frac{(c-d)\tan(e+fx)}{f(a\sec(e+fx)+a)}+\frac{d\tanh^{-1}(\sin(e+fx))}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (d*ArcTanh[Sin[e + f*x]])/(a*f) + ((c - d)*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx &= (c-d)\int \frac{\sec(e+fx)}{a+a\sec(e+fx)}\,dx+\frac{d\int \sec(e+fx)\,dx}{a}\\ &= \frac{d\tanh^{-1}(\sin(e+fx))}{af}+\frac{(c-d)\tan(e+fx)}{f(a+a\sec(e+fx))}\end{aligned}$$

Mathematica [B] time = 0.262645, size = 109, normalized size = 2.53

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left((c-d)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+d\cos\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(\right.\right.\right.\right.}{af(\cos(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(d*Cos[(e + f*x)/2]*(-Log[Cos[(e + f*x)/2] - Sin[(e + f
*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (c - d)*Sec[e/2]*Sin[
(f*x)/2]))/(a*f*(1 + Cos[e + f*x]))
```

Maple [A] time = 0.043, size = 78, normalized size = 1.8

$$\frac{c}{fa}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{d}{fa}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{d}{fa}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\frac{d}{fa}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] 1/a/f*c*tan(1/2*f*x+1/2*e)-1/a/f*tan(1/2*f*x+1/2*e)*d-1/a/f*d*ln(tan(1/2*f*
x+1/2*e)-1)+1/a/f*d*ln(tan(1/2*f*x+1/2*e)+1)
```

Maxima [B] time = 0.986664, size = 134, normalized size = 3.12

$$\frac{d\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+\frac{c\sin(fx+e)}{a(\cos(fx+e)+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] (d*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x
+ e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + c*sin(f*x + e)/(a
*(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 0.476746, size = 197, normalized size = 4.58

$$\frac{(d\cos(fx+e)+d)\log(\sin(fx+e)+1)-(d\cos(fx+e)+d)\log(-\sin(fx+e)+1)+2(c-d)\sin(fx+e)}{2(af\cos(fx+e)+af)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/2*((d*cos(f*x + e) + d)*log(sin(f*x + e) + 1) - (d*cos(f*x + e) + d)*log(
-sin(f*x + e) + 1) + 2*(c - d)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx + \int \frac{d\sec^{2}\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d*sec(e + f*x)**
2/(sec(e + f*x) + 1), x))/a
```

Giac [A] time = 1.2826, size = 100, normalized size = 2.33

$$\frac{\frac{d\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a} - \frac{d\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a} + \frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (d*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - d*log(abs(tan(1/2*f*x + 1/2*e) -
1))/a + (c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/a)/f
```

3.214 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=83

$$\frac{\tan(e+fx)}{f(c-d)(a\sec(e+fx)+a)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{af(c-d)^{3/2}\sqrt{c+d}}$$

```
[Out] (-2*d*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(a*(c - d)^(3/2)
*Sqrt[c + d]*f) + Tan[e + f*x]/((c - d)*f*(a + a*Sec[e + f*x]))
```

Rubi [A] time = 0.155081, antiderivative size = 134, normalized size of antiderivative = 1.61, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3987, 96, 93, 205}

$$\frac{2d\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{3/2}\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{\tan(e+fx)}{f(c-d)(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/((c - d)*f*(a + a*Sec[e + f*x])) + (2*d*ArcTan[(Sqrt[c + d]*Sq
rt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x
])/((c - d)^(3/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
 f*x]])
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{3/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))} + \frac{(ad\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+d}\right.}{(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+}}$$

$$= \frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))} + \frac{(2ad\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{ac-ad-(-ac-ad)}\right.}{(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+}}$$

$$= \frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))} + \frac{2d\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(}{(c-d)^{3/2}\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+}}$$

Mathematica [C] time = 0.663608, size = 160, normalized size = 1.93

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+\frac{2d(\sin(e)+i\cos(e))\cos\left(\frac{1}{2}(e+fx)\right)\tan^{-1}\left(\frac{(\sin(e)+i\cos(e))\left(\tan\left(\frac{fx}{2}\right)(c\cos(e)-d)+c\sin(e)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)}{af(c-d)(\cos(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*((2*d*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*C
os[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[(e
 + f*x)/2]*(I*Cos[e] + Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2
]) + Sec[e/2]*Sin[(f*x)/2]))/(a*(c - d)*f*(1 + Cos[e + f*x]))
```

Maple [A] time = 0.065, size = 74, normalized size = 0.9

$$\frac{1}{fa}\left(\frac{1}{c-d}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2\frac{d}{(c-d)\sqrt{(c+d)(c-d)}}\text{Artanh}\left(\frac{\tan\left(1/2\,fx+e/2\right)(c-d)}{\sqrt{(c+d)(c-d)}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] 1/f/a*(1/(c-d)*tan(1/2*f*x+1/2*e)-2*d/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(tan
(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.505809, size = 783, normalized size = 9.43

$$\left[-\frac{\sqrt{c^2-d^2}\left(d\cos\left(fx+e\right)+d\right)\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2+2\,\sqrt{c^2-d^2}\left(d\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)+2\,c^2-d^2}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2}\right)-2\left(c^2-d^2\right)\sin(}{2\left(\left(ac^3-ac^2d-acd^2+ad^3\right)f\cos\left(fx+e\right)+\left(ac^3-ac^2d-acd^2+ad^3\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [-1/2*(sqrt(c^2 - d^2)*(d*cos(f*x + e) + d)*log((2*c*d*cos(f*x + e) - (c^2
- 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x +
e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(c^2
 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x + e) +
 (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f), -(sqrt(-c^2 + d^2)*(d*cos(f*x + e)
 + d)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x +
e))) - (c^2 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos
(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec\left(e+fx\right)}{c\sec\left(e+fx\right)+c+d\sec^2\left(e+fx\right)+d\sec\left(e+fx\right)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/(c*sec(e + f*x) + c + d*sec(e + f*x)**2 + d*sec(e + f
*x)), x)/a
```

Giac [A] time = 1.2391, size = 154, normalized size = 1.86

$$\frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d}{(ac-ad)\sqrt{-c^2+d^2}}+\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac-ad}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x
 + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d/((a*c - a*d)*sqrt(
-c^2 + d^2)) + tan(1/2*f*x + 1/2*e)/(a*c - a*d))/f
```

3.215 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=145

$$-\frac{d\tan(e+fx)}{f\left(c^2-d^2\right)(a\sec(e+fx)+a)(c+d\sec(e+fx))}-\frac{2d(2c+d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{af(c-d)^{5/2}(c+d)^{3/2}}+\frac{(c+2d)\tan(e+fx)}{f(c-d)^2(c+d)(a\sec(e+f$$

```
[Out] (-2*d*(2*c + d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(a*(c
- d)^(5/2)*(c + d)^(3/2)*f) + ((c + 2*d)*Tan[e + f*x])/((c - d)^2*(c + d)*f
*(a + a*Sec[e + f*x])) - (d*Tan[e + f*x])/((c^2 - d^2)*f*(a + a*Sec[e + f*x
])*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.252992, antiderivative size = 196, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 103, 152, 12, 93, 205}

$$-\frac{d\tan(e+fx)}{f\left(c^2-d^2\right)(a\sec(e+fx)+a)(c+d\sec(e+fx))}+\frac{2d(2c+d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{5/2}(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{}{f(c-d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((c + 2*d)*Tan[e + f*x])/((c - d)^2*(c + d)*f*(a + a*Sec[e + f*x])) + (2*d*
(2*c + d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a
 - a*Sec[e + f*x]])]*Tan[e + f*x])/((c - d)^(5/2)*(c + d)^(3/2)*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d*Tan[e + f*x])/((c^2 - d^2)*f
*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{3/2}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{\left(c^2-d^2\right)f(a+a\sec(e+fx))(c+d\sec(e+fx))} - \frac{\tan(e+fx)\operatorname{Subs}\ldots}{\left(c^2-d^2\right)f\sqrt{\ldots}}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} - \frac{d\tan(e+fx)}{\left(c^2-d^2\right)f(a+a\sec(e+fx))(c\ldots}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} - \frac{d\tan(e+fx)}{\left(c^2-d^2\right)f(a+a\sec(e+fx))(c\ldots}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} - \frac{d\tan(e+fx)}{\left(c^2-d^2\right)f(a+a\sec(e+fx))(c\ldots}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} + \frac{2d(2c+d)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+a\ldots}}{\sqrt{c-d}\sqrt{a-a\ldots}}\right.}{(c-d)^{5/2}(c+d)^{3/2}f\sqrt{a-a\sec(e\ldots}}$$

Mathematica [C] time = 3.3504, size = 286, normalized size = 1.97

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)(c\cos(e+fx)+d)\left(\frac{2d(2c+d)(\sin(e)+i\cos(e))\cos\left(\frac{1}{2}(e+fx)\right)(c\cos(e+fx)+d)\tan^{-1}\left(\frac{(\sin(e)+i\cos(e))\left(\tan\left(\frac{fx}{2}\right)(c\ldots\right.}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\ldots}}\right.}{(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\ldots\right.}{af(c-d)^2(\sec(e+fx)+1)(c+d\sec(e\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])*Sec[e + f*x]^3*((2*d*(2*c + d)*Arc
Tan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c
^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x
])*(I*Cos[e] + Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2
]) + (d + c*Cos[e + f*x])*Sec[e/2]*Sin[(f*x)/2] + (d^2*Cos[(e + f*x)/2]*(-(
d*Sin[e]) + c*Sin[f*x]))/(c*(c + d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e
/2]))))/(a*(c - d)^2*f*(1 + Sec[e + f*x])*(c + d*Sec[e + f*x])^2)
```

Maple [A] time = 0.082, size = 146, normalized size = 1.

$$\frac{1}{fa}\left(\frac{1}{c^2-2\,cd+d^2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4\,\frac{d}{(c-d)^2}\left(-1/2\,\frac{d\tan\left(1/2\,fx+e/2\right)}{(c+d)\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2c-\left(\tan\left(1/2\,fx+e/2\right)\right)^2d-c-d\right)}-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 1/f/a*(1/(c^2-2*c*d+d^2)*tan(1/2*f*x+1/2*e)+4*d/(c-d)^2*(-1/2*d/(c+d)*tan(1
/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)-1/2*(2*c+
d)/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))
^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.563112, size = 1497, normalized size = 10.32

$$\left[\frac{\left(2\,cd^2+d^3+\left(2\,c^2d+cd^2\right)\cos\left(fx+e\right)^2+\left(2\,c^2d+3\,cd^2+d^3\right)\cos\left(fx+e\right)\right)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2}{c^2\cos\left(fx+e\right.}\right.}{2\left(\left(ac^6-ac^5d-2\,ac^4d^2+2\,ac^3d^3+ac^2d^4-acd^5\right)f\cos\left(fx+e\right)^2+\left(ac^6-3\,ac^4d^2+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((2*c*d^2 + d^3 + (2*c^2*d + c*d^2)*cos(f*x + e)^2 + (2*c^2*d + 3*c*d^
2 + d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d
```

```
^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) +
2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(c^3*d +
2*c^2*d^2 - c*d^3 - 2*d^4 + (c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e))*sin(f
*x + e))/((a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c*d^
5)*f*cos(f*x + e)^2 + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f*cos(f*x
 + e) + (a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)
*f), -((2*c*d^2 + d^3 + (2*c^2*d + c*d^2)*cos(f*x + e)^2 + (2*c^2*d + 3*c*d
^2 + d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*
x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^
4 + (c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e))*sin(f*x + e))/((a*c^6 - a*c^5
*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c*d^5)*f*cos(f*x + e)^2 + (a
*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f*cos(f*x + e) + (a*c^5*d - a*c^4
*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(e+fx)}{c^2\sec(e+fx)+c^2+2cd\sec^2(e+fx)+2cd\sec(e+fx)+d^2\sec^3(e+fx)+d^2\sec^2(e+fx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sec(e + f*x) + c**2 + 2*c*d*sec(e + f*x)**2 + 2
*c*d*sec(e + f*x) + d**2*sec(e + f*x)**3 + d**2*sec(e + f*x)**2), x)/a
```

Giac [A] time = 1.30121, size = 308, normalized size = 2.12

$$-\frac{\frac{2\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{(ac^3-ac^2d-acd^2+ad^3)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)}-\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)(2\,cd+d^2)}{(ac^3-ac^2d-acd^2+ad^3)\sqrt{-c^2+d^2}}-}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -(2*d^2*tan(1/2*f*x + 1/2*e)/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*(c*tan(1/
2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)) - 2*(pi*floor(1/2*(f*
x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/
2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(2*c*d + d^2)/((a*c^3 - a*c^2*d - a*c*d^
2 + a*d^3)*sqrt(-c^2 + d^2)) - tan(1/2*f*x + 1/2*e)/(a*c^2 - 2*a*c*d + a*d^
2))/f
```

3.216 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=207

$$-\frac{3d\left(2c^2+2cd+d^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{af(c-d)^{7/2}(c+d)^{5/2}}+\frac{d(2c+d)(c+4d)\tan(e+fx)}{2af(c-d)^3(c+d)^2(c+d\sec(e+fx))}+\frac{d(2c+3d)\tan(e+fx}{2af(c-d)^2(c+d)(c+d\sec(e}$$

```
[Out] (-3*d*(2*c^2 + 2*c*d + d^2)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c +
 d]])/(a*(c - d)^(7/2)*(c + d)^(5/2)*f) + (d*(2*c + 3*d)*Tan[e + f*x])/(2*a
*(c - d)^2*(c + d)*f*(c + d*Sec[e + f*x])^2) + Tan[e + f*x]/((c - d)*f*(a +
 a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2) + (d*(2*c + d)*(c + 4*d)*Tan[e + f
*x])/(2*a*(c - d)^3*(c + d)^2*f*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.4, antiderivative size = 268, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 103, 151, 152, 12, 93, 205}

$$\frac{3d\left(2c^2+2cd+d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{7/2}(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{d(4c+d)\tan(e+fx)}{2f\left(c^2-d^2\right)^2(a\sec(e+fx)+a)(c+d\sec(e+fx))}-\frac{}{2f\left(c^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c + d)*(c + 4*d)*Tan[e + f*x])/(2*(c - d)^3*(c + d)^2*f*(a + a*Sec[e +
f*x])) + (3*d*(2*c^2 + 2*c*d + d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e +
f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/((c - d)^(7/2)
*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d*Ta
n[e + f*x])/(2*(c^2 - d^2)*f*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2) -
 (d*(4*c + d)*Tan[e + f*x])/(2*(c^2 - d^2)^2*f*(a + a*Sec[e + f*x])*(c + d*
Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{3/2}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))(c+d\sec(e+fx))^2} - \frac{\tan(e+fx)\,\text{Subs}}{2\left(c^2-d^2\right)f}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))(c+d\sec(e+fx))^2} - \frac{d(4}{2\left(c^2-d^2\right)^2 f(a+}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2f(a+a\sec(e+fx))} - \frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))(c}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2f(a+a\sec(e+fx))} - \frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))(c}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2f(a+a\sec(e+fx))} - \frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))(c}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2f(a+a\sec(e+fx))} + \frac{3d\left(2c^2+2cd+d^2\right)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{}}{\sqrt{c-d}\sqrt{}}\right.}{(c-d)^{7/2}(c+d)^{5/2}f\sqrt{a-a\sec(e}}$$

Mathematica [C] time = 6.78144, size = 1422, normalized size = 6.87

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c^2 + 2*c*d + d^2)*Cos[e/2 + (f*x)/2]^2*(d + c*Cos[e + f*x])^3*Sec[e +
f*x]^4*(((-6*I)*d*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e
] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]])
)*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2 - d^2]*f*
Sqrt[Cos[2*e] - I*Sin[2*e]]) - (6*d*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 -
 d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2
*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Sin[e])/(
Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^3*(c + d)^2*(a +
 a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3) + (Cos[e/2 + (f*x)/2]*(d + c*Cos[e
 + f*x])*Sec[e/2]*Sec[e]*Sec[e + f*x]^4*(8*c^5*d*Sin[(f*x)/2] + 10*c^4*d^2*
Sin[(f*x)/2] - 11*c^3*d^3*Sin[(f*x)/2] - 17*c^2*d^4*Sin[(f*x)/2] - 2*c*d^5*
Sin[(f*x)/2] + 2*d^6*Sin[(f*x)/2] - 8*c^5*d*Sin[(3*f*x)/2] - 22*c^4*d^2*Sin
[(3*f*x)/2] - 27*c^3*d^3*Sin[(3*f*x)/2] - 5*c^2*d^4*Sin[(3*f*x)/2] + 2*c*d^
5*Sin[(3*f*x)/2] + 4*c^6*Sin[e - (f*x)/2] + 8*c^5*d*Sin[e - (f*x)/2] + 18*c
^4*d^2*Sin[e - (f*x)/2] + 35*c^3*d^3*Sin[e - (f*x)/2] + 25*c^2*d^4*Sin[e -
(f*x)/2] + 2*c*d^5*Sin[e - (f*x)/2] - 2*d^6*Sin[e - (f*x)/2] - 4*c^6*Sin[e
+ (f*x)/2] - 8*c^5*d*Sin[e + (f*x)/2] - 6*c^4*d^2*Sin[e + (f*x)/2] - 7*c^3*
d^3*Sin[e + (f*x)/2] + 5*c^2*d^4*Sin[e + (f*x)/2] + 2*c*d^5*Sin[e + (f*x)/2
] - 2*d^6*Sin[e + (f*x)/2] + 8*c^5*d*Sin[2*e + (f*x)/2] + 22*c^4*d^2*Sin[2*
e + (f*x)/2] + 17*c^3*d^3*Sin[2*e + (f*x)/2] + 13*c^2*d^4*Sin[2*e + (f*x)/2
] + 2*c*d^5*Sin[2*e + (f*x)/2] - 2*d^6*Sin[2*e + (f*x)/2] + 2*c^6*Sin[e + (
3*f*x)/2] + 4*c^5*d*Sin[e + (3*f*x)/2] - 4*c^4*d^2*Sin[e + (3*f*x)/2] - 19*
c^3*d^3*Sin[e + (3*f*x)/2] - 5*c^2*d^4*Sin[e + (3*f*x)/2] + 2*c*d^5*Sin[e +
```

```
(3*f*x)/2] - 8*c^5*d*Sin[2*e + (3*f*x)/2] - 16*c^4*d^2*Sin[2*e + (3*f*x)/2
] - c^3*d^3*Sin[2*e + (3*f*x)/2] + 2*c^2*d^4*Sin[2*e + (3*f*x)/2] - 2*c*d^5
*Sin[2*e + (3*f*x)/2] + 2*c^6*Sin[3*e + (3*f*x)/2] + 4*c^5*d*Sin[3*e + (3*f
*x)/2] + 2*c^4*d^2*Sin[3*e + (3*f*x)/2] + 7*c^3*d^3*Sin[3*e + (3*f*x)/2] +
2*c^2*d^4*Sin[3*e + (3*f*x)/2] - 2*c*d^5*Sin[3*e + (3*f*x)/2] - 2*c^6*Sin[e
 + (5*f*x)/2] - 4*c^5*d*Sin[e + (5*f*x)/2] - 8*c^4*d^2*Sin[e + (5*f*x)/2] -
 2*c^3*d^3*Sin[e + (5*f*x)/2] + c^2*d^4*Sin[e + (5*f*x)/2] - 6*c^4*d^2*Sin[
2*e + (5*f*x)/2] - 2*c^3*d^3*Sin[2*e + (5*f*x)/2] + c^2*d^4*Sin[2*e + (5*f*
x)/2] - 2*c^6*Sin[3*e + (5*f*x)/2] - 4*c^5*d*Sin[3*e + (5*f*x)/2] - 2*c^4*d
^2*Sin[3*e + (5*f*x)/2]))/(8*c^2*(-c + d)^3*(c + d)^2*f*(a + a*Sec[e + f*x]
)*(c + d*Sec[e + f*x])^3)
```

Maple [A] time = 0.1, size = 221, normalized size = 1.1

$$\frac{1}{fa}\left(\frac{1}{c^3-3\,c^2d+3\,d^2c-d^3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\,\frac{d}{(c-d)^3}\left(\frac{1}{\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2c-\left(\tan\left(1/2\,fx+e/2\right)\right)^2d-c-d\right)^2}\left(-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x)
```

```
[Out] 1/f/a*(1/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)+2*d/(c-d)^3*((-3/2*d*
(2*c^2-c*d-d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*(6*c+d)/(c+d)*ta
n(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2-3/2
*(2*c^2+2*c*d+d^2)/(c^2+2*c*d+d^2)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+
1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.692185, size = 2865, normalized size = 13.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] [-1/4*(3*(2*c^2*d^3 + 2*c*d^4 + d^5 + (2*c^4*d + 2*c^3*d^2 + c^2*d^3)*cos(f
*x + e)^3 + (2*c^4*d + 6*c^3*d^2 + 5*c^2*d^3 + 2*c*d^4)*cos(f*x + e)^2 + (4
*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*
```

```
c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(
f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f
*x + e) + d^2)) - 2*(2*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d^4 - 9*c*d^5 - 4*d^6 +
(2*c^6 + 4*c^5*d + 6*c^4*d^2 - 2*c^3*d^3 - 9*c^2*d^4 - 2*c*d^5 + d^6)*cos(f
*x + e)^2 + (4*c^5*d + 14*c^4*d^2 + 7*c^3*d^3 - 13*c^2*d^4 - 11*c*d^5 - d^6
)*cos(f*x + e))*sin(f*x + e))/((a*c^9 - a*c^8*d - 3*a*c^7*d^2 + 3*a*c^6*d^3
 + 3*a*c^5*d^4 - 3*a*c^4*d^5 - a*c^3*d^6 + a*c^2*d^7)*f*cos(f*x + e)^3 + (a
*c^9 + a*c^8*d - 5*a*c^7*d^2 - 3*a*c^6*d^3 + 9*a*c^5*d^4 + 3*a*c^4*d^5 - 7*
a*c^3*d^6 - a*c^2*d^7 + 2*a*c*d^8)*f*cos(f*x + e)^2 + (2*a*c^8*d - a*c^7*d^
2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a
*c*d^8 + a*d^9)*f*cos(f*x + e) + (a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a
*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f), -1/2*(3*(2*c^2*
d^3 + 2*c*d^4 + d^5 + (2*c^4*d + 2*c^3*d^2 + c^2*d^3)*cos(f*x + e)^3 + (2*c
^4*d + 6*c^3*d^2 + 5*c^2*d^3 + 2*c*d^4)*cos(f*x + e)^2 + (4*c^3*d^2 + 6*c^2
*d^3 + 4*c*d^4 + d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^
2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (2*c^4*d^2 + 9*c^3*d^
3 + 2*c^2*d^4 - 9*c*d^5 - 4*d^6 + (2*c^6 + 4*c^5*d + 6*c^4*d^2 - 2*c^3*d^3
- 9*c^2*d^4 - 2*c*d^5 + d^6)*cos(f*x + e)^2 + (4*c^5*d + 14*c^4*d^2 + 7*c^3
*d^3 - 13*c^2*d^4 - 11*c*d^5 - d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^9 - a
*c^8*d - 3*a*c^7*d^2 + 3*a*c^6*d^3 + 3*a*c^5*d^4 - 3*a*c^4*d^5 - a*c^3*d^6
+ a*c^2*d^7)*f*cos(f*x + e)^3 + (a*c^9 + a*c^8*d - 5*a*c^7*d^2 - 3*a*c^6*d^
3 + 9*a*c^5*d^4 + 3*a*c^4*d^5 - 7*a*c^3*d^6 - a*c^2*d^7 + 2*a*c*d^8)*f*cos(
f*x + e)^2 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4*d
^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e) + (a*c^7*d
^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*
c*d^8 + a*d^9)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(e+fx)}{c^3\sec(e+fx)+c^3+3c^2d\sec^2(e+fx)+3c^2d\sec(e+fx)+3cd^2\sec^3(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^4(e+fx)+d^3\sec^3(e+fx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral(sec(e + f*x)/(c**3*sec(e + f*x) + c**3 + 3*c**2*d*sec(e + f*x)**2
+ 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**3 + 3*c*d**2*sec(e + f*x)*
*2 + d**3*sec(e + f*x)**4 + d**3*sec(e + f*x)**3), x)/a
```

Giac [A] time = 1.30518, size = 505, normalized size = 2.44

$$-\frac{\frac{3\left(2c^2d+2cd^2+d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(ac^5-ac^4d-2ac^3d^2+2ac^2d^3+acd^4-ad^5\right)\sqrt{-c^2+d^2}}-\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac^3-3ac^2d+3acd^2-ad^3}+\frac{6c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3cd^3\tan\left(\frac{1}{2}\cdots\right.}{\left(ac^5-ac^4d-\cdots\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] -(3*(2*c^2*d + 2*c*d^2 + d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c +
2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2
```

```
+ d^2)))/((a*c^5 - a*c^4*d - 2*a*c^3*d^2 + 2*a*c^2*d^3 + a*c*d^4 - a*d^5)*s
qrt(-c^2 + d^2)) - tan(1/2*f*x + 1/2*e)/(a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*
d^3) + (6*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*c*d^3*tan(1/2*f*x + 1/2*e)^3 -
 3*d^4*tan(1/2*f*x + 1/2*e)^3 - 6*c^2*d^2*tan(1/2*f*x + 1/2*e) - 7*c*d^3*ta
n(1/2*f*x + 1/2*e) - d^4*tan(1/2*f*x + 1/2*e))/((a*c^5 - a*c^4*d - 2*a*c^3*
d^2 + 2*a*c^2*d^3 + a*c*d^4 - a*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*
f*x + 1/2*e)^2 - c - d)^2))/f
```

3.217 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=258

$$\frac{5d^2(2c-d)\left(2c^2-3cd+2d^2\right)\tanh^{-1}(\sin(e+fx))}{2a^2f}-\frac{d\left(c^2+10cd-12d^2\right)\tan(e+fx)(c+d\sec(e+fx))^2}{3a^2f}-\frac{d\tan(e+fx)}{}$$

```
[Out] (5*(2*c - d)*d^2*(2*c^2 - 3*c*d + 2*d^2)*ArcTanh[Sin[e + f*x]])/(2*a^2*f) -
 (d*(c^2 + 10*c*d - 12*d^2)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*a^2*f)
+ ((c - d)*(c + 10*d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(3*f*(a^2 + a^2*
Sec[e + f*x])) + ((c - d)*(c + d*Sec[e + f*x])^4*Tan[e + f*x])/(3*f*(a + a*
Sec[e + f*x])^2) - (d*(4*(c^4 + 10*c^3*d - 44*c^2*d^2 + 40*c*d^3 - 12*d^4)
+ d*(2*c^3 + 20*c^2*d - 57*c*d^2 + 30*d^3)*Sec[e + f*x])*Tan[e + f*x])/(6*a
^2*f)
```

Rubi [A] time = 0.437776, antiderivative size = 315, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3987, 98, 150, 153, 147, 63, 217, 203}

$$-\frac{d\left(c^2+10cd-12d^2\right)\tan(e+fx)(c+d\sec(e+fx))^2}{3a^2f}-\frac{d\tan(e+fx)\left(d\left(20c^2d+2c^3-57cd^2+30d^3\right)\sec(e+fx)+4\left(\right.\right.}{6a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (5*(2*c - d)*d^2*(2*c^2 - 3*c*d + 2*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sq
rt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[
a + a*Sec[e + f*x]]) - (d*(c^2 + 10*c*d - 12*d^2)*(c + d*Sec[e + f*x])^2*Ta
n[e + f*x])/(3*a^2*f) + ((c - d)*(c + 10*d)*(c + d*Sec[e + f*x])^3*Tan[e +
f*x])/(3*f*(a^2 + a^2*Sec[e + f*x])) + ((c - d)*(c + d*Sec[e + f*x])^4*Tan[
e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (d*(4*(c^4 + 10*c^3*d - 44*c^2*d^2
 + 40*c*d^3 - 12*d^4) + d*(2*c^3 + 20*c^2*d - 57*c*d^2 + 30*d^3)*Sec[e + f*
x])*Tan[e + f*x])/(6*a^2*f)
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{(c+dx)^5}{\sqrt{a-ax}(a+ax)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\operatorname{Subst}\left(\int \frac{(c+dx)^3\left(-a^2\left(c^2+6cd\right.\right.}{\sqrt{a-ax}(a}\right.}{3af\sqrt{a-a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+10d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\sec(e+fx))^4\tan}{3f(a+a\sec(e+fx))}$$

$$= -\frac{d\left(c^2+10cd-12d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(c+10d)(c+d\,\mathrm{s}}{3f\left(a^2+a^2\right.}$$

$$= -\frac{d\left(c^2+10cd-12d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(c+10d)(c+d\,\mathrm{s}}{3f\left(a^2+a^2\right.}$$

$$= -\frac{d\left(c^2+10cd-12d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(c+10d)(c+d\,\mathrm{s}}{3f\left(a^2+a^2\right.}$$

$$= -\frac{d\left(c^2+10cd-12d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(c+10d)(c+d\,\mathrm{s}}{3f\left(a^2+a^2\right.}$$

$$= \frac{5(2c-d)d^2\left(2c^2-3cd+2d^2\right)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{d\left(c^2+10cd\right.}{}$$

Mathematica [A] time = 4.15357, size = 446, normalized size = 1.73

$$\frac{240d^2\left(8c^2d-4c^3-7cd^2+2d^3\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (240*d^2*(-4*c^3 + 8*c^2*d - 7*c*d^2 + 2*d^3)*Cos[(e + f*x)/2]^4*(Log[Cos[(
e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])
 + 2*Cos[(e + f*x)/2]*(6*c^5 + 15*c^4*d - 120*c^3*d^2 + 420*c^2*d^3 - 300*c
*d^4 + 104*d^5 + (6*c^5 + 60*c^4*d - 300*c^3*d^2 + 840*c^2*d^3 - 585*c*d^4
+ 190*d^5)*Cos[e + f*x] + 4*(2*c^5 + 5*c^4*d - 40*c^3*d^2 + 130*c^2*d^3 - 9
5*c*d^4 + 30*d^5)*Cos[2*(e + f*x)] + 2*c^5*Cos[3*(e + f*x)] + 20*c^4*d*Cos[
3*(e + f*x)] - 100*c^3*d^2*Cos[3*(e + f*x)] + 280*c^2*d^3*Cos[3*(e + f*x)]
- 215*c*d^4*Cos[3*(e + f*x)] + 66*d^5*Cos[3*(e + f*x)] + 2*c^5*Cos[4*(e + f
*x)] + 5*c^4*d*Cos[4*(e + f*x)] - 40*c^3*d^2*Cos[4*(e + f*x)] + 100*c^2*d^3
*Cos[4*(e + f*x)] - 80*c*d^4*Cos[4*(e + f*x)] + 24*d^5*Cos[4*(e + f*x)])*Se
c[e + f*x]^3*Sin[(e + f*x)/2])/(24*a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [B] time = 0.099, size = 766, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x)
```

```
[Out] 1/2/f/a^2*tan(1/2*f*x+1/2*e)*c^5+9/2/f/a^2*tan(1/2*f*x+1/2*e)*d^5-1/3/f/a^2
*d^5/(tan(1/2*f*x+1/2*e)+1)^3-5/f/a^2*ln(tan(1/2*f*x+1/2*e)+1)*d^5-5/f/a^2*
d^5/(tan(1/2*f*x+1/2*e)+1)+3/2/f/a^2*d^5/(tan(1/2*f*x+1/2*e)+1)^2-1/3/f/a^2
*d^5/(tan(1/2*f*x+1/2*e)-1)^3+5/f/a^2*ln(tan(1/2*f*x+1/2*e)-1)*d^5-1/6/f/a^
2*tan(1/2*f*x+1/2*e)^3*c^5+1/6/f/a^2*tan(1/2*f*x+1/2*e)^3*d^5-5/f/a^2*d^5/(
tan(1/2*f*x+1/2*e)-1)-3/2/f/a^2*d^5/(tan(1/2*f*x+1/2*e)-1)^2-5/2/f/a^2*d^4/
(tan(1/2*f*x+1/2*e)+1)^2*c+5/6/f/a^2*tan(1/2*f*x+1/2*e)^3*c^4*d-5/3/f/a^2*t
an(1/2*f*x+1/2*e)^3*c^3*d^2+5/3/f/a^2*tan(1/2*f*x+1/2*e)^3*c^2*d^3-5/6/f/a^
2*tan(1/2*f*x+1/2*e)^3*c*d^4+5/2/f/a^2*tan(1/2*f*x+1/2*e)*c^4*d-15/f/a^2*ta
n(1/2*f*x+1/2*e)*c^3*d^2+25/f/a^2*tan(1/2*f*x+1/2*e)*c^2*d^3+20/f/a^2*ln(ta
n(1/2*f*x+1/2*e)-1)*c^2*d^3-10/f/a^2*ln(tan(1/2*f*x+1/2*e)-1)*c^3*d^2+10/f/
a^2*ln(tan(1/2*f*x+1/2*e)+1)*c^3*d^2-20/f/a^2*ln(tan(1/2*f*x+1/2*e)+1)*c^2*
d^3+35/2/f/a^2*ln(tan(1/2*f*x+1/2*e)+1)*c*d^4-10/f/a^2*d^3/(tan(1/2*f*x+1/2
*e)+1)*c^2+25/2/f/a^2*d^4/(tan(1/2*f*x+1/2*e)+1)*c-35/2/f/a^2*ln(tan(1/2*f*
x+1/2*e)-1)*c*d^4-10/f/a^2*d^3/(tan(1/2*f*x+1/2*e)-1)*c^2+25/2/f/a^2*d^4/(t
an(1/2*f*x+1/2*e)-1)*c+5/2/f/a^2*d^4/(tan(1/2*f*x+1/2*e)-1)^2*c-35/2/f/a^2*
tan(1/2*f*x+1/2*e)*c*d^4
```

Maxima [B] time = 1.04235, size = 1042, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(d^5*(4*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^2 - 3*a^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (27*sin(f*x + e)/(cos(f*x + e) +
 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 30*log(sin(f*x + e)/(cos(f
*x + e) + 1) + 1)/a^2 + 30*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) -
5*c*d^4*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*
x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) + 1)
 + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 10*c^2*d^3*(
(15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/
a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)
/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) - 10*c^3*d^2*((9*sin(f*x + e)/(co
s(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x
+ e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) -
1)/a^2) + 5*c^4*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(
f*x + e) + 1)^3)/a^2 + c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [A] time = 0.539205, size = 1037, normalized size = 4.02

$$15\left(\left(4\,c^{3}d^{2}-8\,c^{2}d^{3}+7\,cd^{4}-2\,d^{5}\right)\cos\left(fx+e\right)^{5}+2\left(4\,c^{3}d^{2}-8\,c^{2}d^{3}+7\,cd^{4}-2\,d^{5}\right)\cos\left(fx+e\right)^{4}+\left(4\,c^{3}d^{2}-8\,c^{2}d^{3}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(15*((4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^5 + 2*(4*c
^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^4 + (4*c^3*d^2 - 8*c^2*d
^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 15*((4*c^3*d^
2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^5 + 2*(4*c^3*d^2 - 8*c^2*d^3
+ 7*c*d^4 - 2*d^5)*cos(f*x + e)^4 + (4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^
5)*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) + 2*(2*d^5 + 2*(2*c^5 + 5*c^4*d -
 40*c^3*d^2 + 100*c^2*d^3 - 80*c*d^4 + 24*d^5)*cos(f*x + e)^4 + (2*c^5 + 20
*c^4*d - 100*c^3*d^2 + 280*c^2*d^3 - 215*c*d^4 + 66*d^5)*cos(f*x + e)^3 + 6
*(10*c^2*d^3 - 5*c*d^4 + 2*d^5)*cos(f*x + e)^2 + (15*c*d^4 - 2*d^5)*cos(f*x
 + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^5 + 2*a^2*f*cos(f*x + e)^4 + a^2*f
*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^5 \sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{d^5 \sec^6(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{5cd^4 \sec^5(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{10c^2d^3 \sec^4(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + }{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**5*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**5*sec(e + f*x)**6/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(5*c*d**4*sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(10*c**2*d**3*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) +
 1), x) + Integral(10*c**3*d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e
+ f*x) + 1), x) + Integral(5*c**4*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*se
c(e + f*x) + 1), x))/a**2
```

Giac [B] time = 1.29513, size = 716, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(15*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*log(abs(tan(1/2*f*x + 1/2
*e) + 1))/a^2 - 15*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*log(abs(tan(1/
2*f*x + 1/2*e) - 1))/a^2 - 2*(60*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 75*c*d^4*
tan(1/2*f*x + 1/2*e)^5 + 30*d^5*tan(1/2*f*x + 1/2*e)^5 - 120*c^2*d^3*tan(1/
2*f*x + 1/2*e)^3 + 120*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 40*d^5*tan(1/2*f*x +
1/2*e)^3 + 60*c^2*d^3*tan(1/2*f*x + 1/2*e) - 45*c*d^4*tan(1/2*f*x + 1/2*e)
+ 18*d^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a^2) - (a^4*
c^5*tan(1/2*f*x + 1/2*e)^3 - 5*a^4*c^4*d*tan(1/2*f*x + 1/2*e)^3 + 10*a^4*c^
3*d^2*tan(1/2*f*x + 1/2*e)^3 - 10*a^4*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 5*a^
4*c*d^4*tan(1/2*f*x + 1/2*e)^3 - a^4*d^5*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^5
```

```
*tan(1/2*f*x + 1/2*e) - 15*a^4*c^4*d*tan(1/2*f*x + 1/2*e) + 90*a^4*c^3*d^2*
tan(1/2*f*x + 1/2*e) - 150*a^4*c^2*d^3*tan(1/2*f*x + 1/2*e) + 105*a^4*c*d^4
*tan(1/2*f*x + 1/2*e) - 27*a^4*d^5*tan(1/2*f*x + 1/2*e))/a^6)/f
```

3.218 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=193

$$\frac{d^2\left(12c^2-16cd+7d^2\right)\tanh^{-1}(\sin(e+fx))}{2a^2f}-\frac{d\tan(e+fx)\left(d\left(2c^2+16cd-21d^2\right)\sec(e+fx)+4\left(8c^2d+c^3-20cd^2+\right.\right.}{6a^2f}$$

```
[Out] (d^2*(12*c^2 - 16*c*d + 7*d^2)*ArcTanh[Sin[e + f*x]])/(2*a^2*f) + ((c - d)*
(c + 8*d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x]
)) + ((c - d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x]
)^2) - (d*(4*(c^3 + 8*c^2*d - 20*c*d^2 + 8*d^3) + d*(2*c^2 + 16*c*d - 21*d^
2)*Sec[e + f*x])*Tan[e + f*x])/(6*a^2*f)
```

Rubi [A] time = 0.320114, antiderivative size = 249, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 98, 150, 147, 63, 217, 203}

$$-\frac{d\tan(e+fx)\left(d\left(2c^2+16cd-21d^2\right)\sec(e+fx)+4\left(8c^2d+c^3-20cd^2+8d^3\right)\right)}{6a^2f}+\frac{(c-d)(c+8d)\tan(e+fx)(c+d\sec}{3f\left(a^2\sec(e+fx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (d^2*(12*c^2 - 16*c*d + 7*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 +
Sec[e + f*x])]]*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[
e + f*x]]) + ((c - d)*(c + 8*d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(
a^2 + a^2*Sec[e + f*x])) + ((c - d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(3
*f*(a + a*Sec[e + f*x])^2) - (d*(4*(c^3 + 8*c^2*d - 20*c*d^2 + 8*d^3) + d*(
2*c^2 + 16*c*d - 21*d^2)*Sec[e + f*x])*Tan[e + f*x])/(6*a^2*f)
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^4}{\sqrt{a-ax}(a+ax)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^2\left(-a^2\left(c^2+5cd\right.\right.}{\sqrt{a-ax}(a}\right.}{3af\sqrt{a-a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\sec(e+fx))^3\tan}{3f(a+a\sec(e+fx))^2}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\sec(e+fx))^3\tan}{3f(a+a\sec(e+fx))^2}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\sec(e+fx))^3\tan}{3f(a+a\sec(e+fx))^2}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\sec(e+fx))^3\tan}{3f(a+a\sec(e+fx))^2}$$

$$= \frac{d^2\left(12c^2-16cd+7d^2\right)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(c+8d)(c+d}{3f\left(a^2+\right.}$$

Mathematica [A] time = 2.83186, size = 310, normalized size = 1.61

$$\frac{2\sin\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left(-24c^2d^2\cos(3(e+fx))+6\left(-12c^2d^2+2c^3d+c^4+28cd^3-10d^4\right)\cos(e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-24*d^2*(12*c^2 - 16*c*d + 7*d^2)*Cos[(e + f*x)/2]^4*(Log[Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2*Cos[(e
 + f*x)/2]*(2*c^4 + 16*c^3*d - 60*c^2*d^2 + 112*c*d^3 - 37*d^4 + 6*(c^4 + 2
*c^3*d - 12*c^2*d^2 + 28*c*d^3 - 10*d^4)*Cos[e + f*x] + (2*c^4 + 16*c^3*d -
 60*c^2*d^2 + 112*c*d^3 - 43*d^4)*Cos[2*(e + f*x)] + 2*c^4*Cos[3*(e + f*x)]
 + 4*c^3*d*Cos[3*(e + f*x)] - 24*c^2*d^2*Cos[3*(e + f*x)] + 40*c*d^3*Cos[3*
(e + f*x)] - 16*d^4*Cos[3*(e + f*x)])*Sec[e + f*x]^2*Sin[(e + f*x)/2])/(12*
a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [B] time = 0.083, size = 514, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -1/6/f/a^2*tan(1/2*f*x+1/2*e)^3*c^4+2/3/f/a^2*tan(1/2*f*x+1/2*e)^3*c^3*d-1/
f/a^2*tan(1/2*f*x+1/2*e)^3*c^2*d^2+2/3/f/a^2*tan(1/2*f*x+1/2*e)^3*c*d^3-1/6
```

```
/f/a^2*tan(1/2*f*x+1/2*e)^3*d^4+1/2/f/a^2*tan(1/2*f*x+1/2*e)*c^4+2/f/a^2*ta
n(1/2*f*x+1/2*e)*c^3*d-9/f/a^2*tan(1/2*f*x+1/2*e)*c^2*d^2+10/f/a^2*tan(1/2*
f*x+1/2*e)*c*d^3-7/2/f/a^2*tan(1/2*f*x+1/2*e)*d^4+6/f/a^2*ln(tan(1/2*f*x+1/
2*e)+1)*c^2*d^2-8/f/a^2*ln(tan(1/2*f*x+1/2*e)+1)*c*d^3+7/2/f/a^2*ln(tan(1/2
*f*x+1/2*e)+1)*d^4-4/f/a^2*d^3/(tan(1/2*f*x+1/2*e)+1)*c+5/2/f/a^2*d^4/(tan(
1/2*f*x+1/2*e)+1)-1/2/f/a^2*d^4/(tan(1/2*f*x+1/2*e)+1)^2-4/f/a^2*d^3/(tan(1
/2*f*x+1/2*e)-1)*c+5/2/f/a^2*d^4/(tan(1/2*f*x+1/2*e)-1)+1/2/f/a^2*d^4/(tan(
1/2*f*x+1/2*e)-1)^2-6/f/a^2*ln(tan(1/2*f*x+1/2*e)-1)*c^2*d^2+8/f/a^2*ln(tan
(1/2*f*x+1/2*e)-1)*c*d^3-7/2/f/a^2*ln(tan(1/2*f*x+1/2*e)-1)*d^4
```

Maxima [B] time = 1.04037, size = 724, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(d^4*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(
f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) +
1) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) - 4*c*d^3*((
15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a
^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 6*c^2*d^2*((9*sin(f*x + e)/(cos(
f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x +
e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)
/a^2) - 4*c^3*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/a^2 - c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^
3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [A] time = 0.518773, size = 841, normalized size = 4.36

$$\frac{3\left(\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)\cos\left(fx+e\right)^4+2\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)\cos\left(fx+e\right)^3+\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)\cos\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*((12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x + e)^4 + 2*(12*c^2*d^2 - 1
6*c*d^3 + 7*d^4)*cos(f*x + e)^3 + (12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x +
 e)^2)*log(sin(f*x + e) + 1) - 3*((12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x +
 e)^4 + 2*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x + e)^3 + (12*c^2*d^2 - 16
*c*d^3 + 7*d^4)*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) + 2*(3*d^4 + 4*(c^4
+ 2*c^3*d - 12*c^2*d^2 + 20*c*d^3 - 8*d^4)*cos(f*x + e)^3 + (2*c^4 + 16*c^3
*d - 60*c^2*d^2 + 112*c*d^3 - 43*d^4)*cos(f*x + e)^2 + 6*(4*c*d^3 - d^4)*co
s(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^4 + 2*a^2*f*cos(f*x + e)^3 +
a^2*f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^4\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{d^4\sec^5(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{4cd^3\sec^4(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{6c^2d^2\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx +}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**4*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**4*sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(4*c*d**3*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(6*c**2*d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) +
1), x) + Integral(4*c**3*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x
) + 1), x))/a**2
```

Giac [B] time = 1.30644, size = 508, normalized size = 2.63

$$\frac{\frac{3(12c^2d^2-16cd^3+7d^4)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2} - \frac{3(12c^2d^2-16cd^3+7d^4)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2} - \frac{6\left(8cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-5d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-8cd^3}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a
^2 - 3*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a
^2 - 6*(8*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 5*d^4*tan(1/2*f*x + 1/2*e)^3 - 8*c
*d^3*tan(1/2*f*x + 1/2*e) + 3*d^4*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2
*e)^2 - 1)^2*a^2) - (a^4*c^4*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c^3*d*tan(1/2*f
*x + 1/2*e)^3 + 6*a^4*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c*d^3*tan(1/2*
f*x + 1/2*e)^3 + a^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^4*tan(1/2*f*x + 1
/2*e) - 12*a^4*c^3*d*tan(1/2*f*x + 1/2*e) + 54*a^4*c^2*d^2*tan(1/2*f*x + 1/
2*e) - 60*a^4*c*d^3*tan(1/2*f*x + 1/2*e) + 21*a^4*d^4*tan(1/2*f*x + 1/2*e))
/a^6)/f
```

3.219 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{\tan(e+fx)\left(4c^2d+c^3-d^2(c-4d)\sec(e+fx)-12cd^2+10d^3\right)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{d^2(3c-2d)\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{(c-d)\tan(e+\ldots}{3f(a\,\mathrm{se}\ldots}$$

```
[Out] ((3*c - 2*d)*d^2*ArcTanh[Sin[e + f*x]])/(a^2*f) + ((c - d)*(c + d*Sec[e + f
*x])^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) + ((c^3 + 4*c^2*d - 12*c*
d^2 + 10*d^3 - (c - 4*d)*d^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a^2 + a^2*Se
c[e + f*x]))
```

Rubi [A] time = 0.233037, antiderivative size = 193, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 98, 143, 63, 217, 203}

$$\frac{\tan(e+fx)\left(4c^2d+c^3-d^2(c-4d)\sec(e+fx)-12cd^2+10d^3\right)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2d^2(3c-2d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*(3*c - 2*d)*d^2*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x]
)]]*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) +
 ((c - d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
 + ((c^3 + 4*c^2*d - 12*c*d^2 + 10*d^3 - (c - 4*d)*d^2*Sec[e + f*x])*Tan[e
+ f*x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
```

```
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{(c+dx)^3}{\sqrt{a-ax}(a+ax)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\operatorname{Subst}\left(\int \frac{(c+dx)\left(-a^2\left(c^2+4cd-\right.\right.}{\sqrt{a-ax}(a+}\right.}{3af\sqrt{a-a\sec(e+fx)}\sqrt{}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\left(c^3+4c^2d-12cd^2+10d^3-(c-4d)d^2\right.}{3f\left(a^2+a^2\sec(e+\right.}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\left(c^3+4c^2d-12cd^2+10d^3-(c-4d)d^2\right.}{3f\left(a^2+a^2\sec(e+\right.}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\left(c^3+4c^2d-12cd^2+10d^3-(c-4d)d^2\right.}{3f\left(a^2+a^2\sec(e+\right.}$$

$$= \frac{2(3c-2d)d^2\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(c+d\sec(e+fx))^2\tan}{3f(a+a\sec(e+fx))^2}$$

Mathematica [B] time = 1.52317, size = 294, normalized size = 2.21

$$2\cos^6\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(2\left(3c^2d+2c^3-12cd^2+13d^3\right)\tan\left(\frac{1}{2}(e+fx)\right)+6d^2(2d-3c)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*Cos[(e + f*x)/2]^6*Sec[e + f*x]*(6*d^2*(-3*c + 2*d)*(Log[Cos[(e + f*x)/2
] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 8*(c -
d)^3*Csc[e + f*x]^3*Sin[(e + f*x)/2]^4 + 32*(c - d)^3*Csc[e + f*x]^5*Sin[(e
 + f*x)/2]^8 + 2*(2*c^3 + 3*c^2*d - 12*c*d^2 + 13*d^3)*Tan[(e + f*x)/2] + 6
*(3*c - 2*d)*d^2*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]])*Tan[(e + f*x)/2]^2 - 2*(c - d)^2*(2*c + 7*d)*Ta
n[(e + f*x)/2]^3))/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [B] time = 0.064, size = 316, normalized size = 2.4

$$-\frac{c^3}{6fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+\frac{c^2d}{2fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-\frac{d^2c}{2fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+\frac{d^3}{6fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+\frac{c^3}{2fa^2}\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x)
```

```
[Out] -1/6/f/a^2*tan(1/2*f*x+1/2*e)^3*c^3+1/2/f/a^2*tan(1/2*f*x+1/2*e)^3*c^2*d-1/
2/f/a^2*tan(1/2*f*x+1/2*e)^3*c*d^2+1/6/f/a^2*tan(1/2*f*x+1/2*e)^3*d^3+1/2/f
/a^2*tan(1/2*f*x+1/2*e)*c^3+3/2/f/a^2*tan(1/2*f*x+1/2*e)*c^2*d-9/2/f/a^2*ta
n(1/2*f*x+1/2*e)*c*d^2+5/2/f/a^2*tan(1/2*f*x+1/2*e)*d^3-1/f/a^2*d^3/(tan(1/
2*f*x+1/2*e)+1)+3/f/a^2*d^2*ln(tan(1/2*f*x+1/2*e)+1)*c-2/f/a^2*d^3*ln(tan(1
/2*f*x+1/2*e)+1)-1/f/a^2*d^3/(tan(1/2*f*x+1/2*e)-1)-3/f/a^2*d^2*ln(tan(1/2*
f*x+1/2*e)-1)*c+2/f/a^2*d^3*ln(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.00243, size = 462, normalized size = 3.47

$$\frac{d^3\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}+\frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)-3cd^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}}{}\right.}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(d^3*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(si
n(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) - 3*c*d^2*((9*sin(f*x +
 e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(s
in(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e)
+ 1) - 1)/a^2) + 3*c^2*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^
3/(cos(f*x + e) + 1)^3)/a^2 + c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(
f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [B] time = 0.499902, size = 632, normalized size = 4.75

$$3\left(\left(3cd^2-2d^3\right)\cos\left(fx+e\right)^3+2\left(3cd^2-2d^3\right)\cos\left(fx+e\right)^2+\left(3cd^2-2d^3\right)\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*((3*c*d^2 - 2*d^3)*cos(f*x + e)^3 + 2*(3*c*d^2 - 2*d^3)*cos(f*x + e)
^2 + (3*c*d^2 - 2*d^3)*cos(f*x + e))*log(sin(f*x + e) + 1) - 3*((3*c*d^2 -
2*d^3)*cos(f*x + e)^3 + 2*(3*c*d^2 - 2*d^3)*cos(f*x + e)^2 + (3*c*d^2 - 2*d
^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(3*d^3 + (2*c^3 + 3*c^2*d - 12
*c*d^2 + 10*d^3)*cos(f*x + e)^2 + (c^3 + 6*c^2*d - 15*c*d^2 + 14*d^3)*cos(f
*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)^2 + a^2
*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^3 \sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{d^3 \sec^4(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{3cd^2 \sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{3c^2 d \sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**3*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**3*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(3*c*d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(3*c**2*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),
 x))/a**2
```

Giac [B] time = 1.2318, size = 354, normalized size = 2.66

$$-\frac{\frac{12\,d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a^2} - \frac{6\left(3cd^2-2d^3\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2} + \frac{6\left(3cd^2-2d^3\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2} + \frac{a^4c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 3a^4c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.}{}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*d^3*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^2) - 6*(3
*c*d^2 - 2*d^3)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 + 6*(3*c*d^2 - 2*d^3
)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^2 + (a^4*c^3*tan(1/2*f*x + 1/2*e)^3
- 3*a^4*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 3*a^4*c*d^2*tan(1/2*f*x + 1/2*e)^3 -
 a^4*d^3*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^3*tan(1/2*f*x + 1/2*e) - 9*a^4*c^
2*d*tan(1/2*f*x + 1/2*e) + 27*a^4*c*d^2*tan(1/2*f*x + 1/2*e) - 15*a^4*d^3*t
an(1/2*f*x + 1/2*e))/a^6)/f
```

3.220 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=89

$$\frac{(c+5d)(c-d)\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{d^2\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{(c-d)^2\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

```
[Out] (d^2*ArcTanh[Sin[e + f*x]])/(a^2*f) + ((c - d)^2*Tan[e + f*x])/(3*f*(a + a*
Sec[e + f*x])^2) + ((c - d)*(c + 5*d)*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e +
 f*x]))
```

Rubi [A] time = 0.153959, antiderivative size = 149, normalized size of antiderivative = 1.67, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 89, 78, 63, 217, 203}

$$\frac{(c+5d)(c-d)\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{(c-d)^2\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}+\frac{2d^2\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] ((c - d)^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) + (2*d^2*ArcTan[Sqrt[
a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(a*f*Sqrt[a -
 a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((c - d)*(c + 5*d)*Tan[e + f*x
])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^2}{\sqrt{a-ax}(a+ax)^{5/2}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{a^3\left(c^2+4cd-2d^2\right)+3a^3d^2x}{\sqrt{a-ax}(a+ax)^{3/2}}\,dx,x,\sec(e+ \ldots\right.}{3a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c-d)(c+5d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} - \frac{\left(d^2\tan(e+fx)\right)\text{Subst}\left(\ldots\right.}{f\sqrt{a-a\sec(e+\ldots}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c-d)(c+5d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{\left(2d^2\tan(e+fx)\right)\text{Subst}\ldots}{af\sqrt{a-a\sec(\ldots}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c-d)(c+5d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{\left(2d^2\tan(e+fx)\right)\text{Subst}\ldots}{af\sqrt{a-a\sec(e+\ldots}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{2d^2\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(c+\ldots}{3f\left(a^2+a\ldots\right.}$$

Mathematica [B] time = 0.736878, size = 181, normalized size = 2.03

$$-\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(-4\left(c^2+cd-2d^2\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\cos^2\left(\frac{1}{2}(e+fx)\right)+(c-d)^2\tan\left(\frac{e}{2}\right)\cos\left(\frac{1}{2}(e+fx)\right)+(c-d)^2\sec\ldots\right.}{3a^2f(\cos(e\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-2*Cos[(e + f*x)/2]*(6*d^2*Cos[(e + f*x)/2]^3*(Log[Cos[(e + f*x)/2] - Sin[
(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (c - d)^2*Sec[e
```

```
/2]*Sin[(f*x)/2] - 4*(c^2 + c*d - 2*d^2)*Cos[(e + f*x)/2]^2*Sec[e/2]*Sin[(f
*x)/2] + (c - d)^2*Cos[(e + f*x)/2]*Tan[e/2]))/(3*a^2*f*(1 + Cos[e + f*x])^
2)
```

Maple [A] time = 0.074, size = 170, normalized size = 1.9

$$\frac{d^2}{fa^2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{cd}{3\,fa^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+\frac{cd}{fa^2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{c^2}{2\,fa^2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{d^2}{fa^2}\ln\left(\tan\left(\frac{fx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x)
```

```
[Out] 1/f/a^2*d^2*ln(tan(1/2*f*x+1/2*e)+1)+1/3/f/a^2*tan(1/2*f*x+1/2*e)^3*c*d+1/f
/a^2*c*d*tan(1/2*f*x+1/2*e)+1/2/f/a^2*tan(1/2*f*x+1/2*e)*c^2-1/f/a^2*d^2*ln
(tan(1/2*f*x+1/2*e)-1)-1/6/f/a^2*tan(1/2*f*x+1/2*e)^3*c^2-1/6/f/a^2*tan(1/2
*f*x+1/2*e)^3*d^2-3/2/f/a^2*tan(1/2*f*x+1/2*e)*d^2
```

Maxima [B] time = 1.00558, size = 263, normalized size = 2.96

$$-\frac{d^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)-\frac{2\,cd\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}-\frac{c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(\ldots}{(\cos(f\ldots}\right.}{a^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(d^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) - 2*c*d*(3*sin(f*x + e)/(cos(f*x + e)
 + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - c^2*(3*sin(f*x + e)/(cos
(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [A] time = 0.481805, size = 383, normalized size = 4.3

$$\frac{3\left(d^2\cos\left(fx+e\right)^2+2\,d^2\cos\left(fx+e\right)+d^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(d^2\cos\left(fx+e\right)^2+2\,d^2\cos\left(fx+e\right)+d^2\right)\log}{6\left(a^2f\cos\left(fx+e\right)^2+2\,a^2f\cos\left(fx+e\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*log(sin(f*x + e) + 1
) - 3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*log(-sin(f*x + e) + 1
) + 2*(c^2 + 4*c*d - 5*d^2 + 2*(c^2 + c*d - 2*d^2)*cos(f*x + e))*sin(f*x +
```

```
e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^2 \sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{d^2 \sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{2cd \sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**2*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(2*c*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a*
*2
```

Giac [A] time = 1.35398, size = 224, normalized size = 2.52

$$\frac{\frac{6\,d^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}-\frac{6\,d^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}-\frac{a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,a^4cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+a^4d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-}{a^6}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*d^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 6*d^2*log(abs(tan(1/2*f
*x + 1/2*e) - 1))/a^2 - (a^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 2*a^4*c*d*tan(1/2
*f*x + 1/2*e)^3 + a^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^2*tan(1/2*f*x +
1/2*e) - 6*a^4*c*d*tan(1/2*f*x + 1/2*e) + 9*a^4*d^2*tan(1/2*f*x + 1/2*e))/a
^6)/f
```

3.221 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=65

$$\frac{(c+2d)\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{(c-d)\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

```
[Out] ((c - d)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) + ((c + 2*d)*Tan[e + f*
x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rubi [A] time = 0.0865492, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {4000, 3794}

$$\frac{(c+2d)\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{(c-d)\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] ((c - d)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) + ((c + 2*d)*Tan[e + f*
x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx &= \frac{(c-d)\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{(c+2d)\int \frac{\sec(e+fx)}{a+a\sec(e+fx)}\,dx}{3a}\\ &= \frac{(c-d)\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{(c+2d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}\end{aligned}$$

Mathematica [A] time = 0.210091, size = 76, normalized size = 1.17

$$\frac{\sec\left(\frac{e}{2}\right)\cos\left(\frac{1}{2}(e+fx)\right)\left((2c+d)\sin\left(e+\frac{3fx}{2}\right)+3(c+d)\sin\left(\frac{fx}{2}\right)-3c\sin\left(e+\frac{fx}{2}\right)\right)}{3a^2f(\cos(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (Cos[(e + f*x)/2]*Sec[e/2]*(3*(c + d)*Sin[(f*x)/2] - 3*c*Sin[e + (f*x)/2] +
 (2*c + d)*Sin[e + (3*f*x)/2]))/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [A] time = 0.051, size = 60, normalized size = 0.9

$$\frac{1}{2fa^2}\left(-\frac{c}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+\frac{d}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

```
[Out] 1/2/f/a^2*(-1/3*tan(1/2*f*x+1/2*e)^3*c+1/3*tan(1/2*f*x+1/2*e)^3*d+c*tan(1/2
*f*x+1/2*e)+tan(1/2*f*x+1/2*e)*d)
```

Maxima [A] time = 0.97264, size = 126, normalized size = 1.94

$$\frac{\frac{d\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}+\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/6*(d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
1)^3)/a^2 + c*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3)/a^2)/f
```

Fricas [A] time = 0.435586, size = 144, normalized size = 2.22

$$\frac{\left((2c+d)\cos\left(fx+e\right)+c+2d\right)\sin\left(fx+e\right)}{3\left(a^2f\cos\left(fx+e\right)^2+2a^2f\cos\left(fx+e\right)+a^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*((2*c + d)*cos(f*x + e) + c + 2*d)*sin(f*x + e)/(a^2*f*cos(f*x + e)^2 +
 2*a^2*f*cos(f*x + e) + a^2*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{d\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integ
ral(d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A] time = 1.14865, size = 86, normalized size = 1.32

$$-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3\,c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-3\,d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{6\,a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -1/6*(c*tan(1/2*f*x + 1/2*e)^3 - d*tan(1/2*f*x + 1/2*e)^3 - 3*c*tan(1/2*f*x
 + 1/2*e) - 3*d*tan(1/2*f*x + 1/2*e))/(a^2*f)
```

3.222 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))} \, dx$

Optimal. Leaf size=129

$$\frac{2d^2\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^2f(c-d)^{5/2}\sqrt{c+d}}+\frac{(c-4d)\tan(e+fx)}{3f(c-d)^2\left(a^2\sec(e+fx)+a^2\right)}+\frac{\tan(e+fx)}{3f(c-d)(a\sec(e+fx)+a)^2}$$

```
[Out] (2*d^2*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(a^2*(c - d)^(5
/2)*Sqrt[c + d]*f) + Tan[e + f*x]/(3*(c - d)*f*(a + a*Sec[e + f*x])^2) + ((
c - 4*d)*Tan[e + f*x])/(3*(c - d)^2*f*(a^2 + a^2*Sec[e + f*x]))
```

Rubi [A] time = 0.241377, antiderivative size = 183, normalized size of antiderivative = 1.42, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 104, 152, 12, 93, 205}

$$\frac{(c-4d)\tan(e+fx)}{3f(c-d)^2\left(a^2\sec(e+fx)+a^2\right)}-\frac{2d^2\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{af(c-d)^{5/2}\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{\tan(e+fx)}{3f(c-d)(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/(3*(c - d)*f*(a + a*Sec[e + f*x])^2) - (2*d^2*ArcTan[(Sqrt[c +
 d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e
 + f*x])/(a*(c - d)^(5/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a
*Sec[e + f*x]]) + ((c - 4*d)*Tan[e + f*x])/(3*(c - d)^2*f*(a^2 + a^2*Sec[e
+ f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{5/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{-a^2(c-3d)-a^2dx}{\sqrt{a-ax}(a+ax)^{3/2}(c+\cdots}\right.}{3a(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a\cdots}}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\left(a^2+a^2\sec(e+fx)\right)} - \frac{\tan\cdots}{\cdots}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\left(a^2+a^2\sec(e+fx)\right)} - \frac{\left(d^2\cdots\right.}{\cdots}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\left(a^2+a^2\sec(e+fx)\right)} - \frac{\left(2d\cdots\right.}{\cdots}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} - \frac{2d^2\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\cdots}{a(c-d)^{5/2}\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\cdots}$$

Mathematica [C] time = 1.62861, size = 209, normalized size = 1.62

$$\frac{\cos\left(\frac{1}{2}(e+fx)\right)\left(\sec\left(\frac{e}{2}\right)\left(-3(c-2d)\sin\left(e+\frac{fx}{2}\right)+(2c-5d)\sin\left(e+\frac{3fx}{2}\right)+3(c-3d)\sin\left(\frac{fx}{2}\right)\right)-\frac{24id^2(\cos(e)-i\sin(e))\cos\cdots}{\cdots}\right.}{3a^2f(c-d)^2(\cos(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

```
[Out] (Cos[(e + f*x)/2]*(((-24*I)*d^2*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d
 + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*
Cos[(e + f*x)/2]^3*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*S
in[e])^2]) + Sec[e/2]*(3*(c - 3*d)*Sin[(f*x)/2] - 3*(c - 2*d)*Sin[e + (f*x)
/2] + (2*c - 5*d)*Sin[e + (3*f*x)/2])))/(3*a^2*(c - d)^2*f*(1 + Cos[e + f*x
])^2)
```

Maple [A] time = 0.07, size = 122, normalized size = 1.

$$\frac{1}{2\,fa^2}\left(-\frac{1}{(c-d)^2}\left(\frac{c}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-\frac{d}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3\,\tan\left(1/2\,fx+e/2\right)d\right)+4\,\frac{d^{2}}{(c-d)^2\,\sqrt{(c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x)
```

```
[Out] 1/2/f/a^2*(-1/(c-d)^2*(1/3*tan(1/2*f*x+1/2*e)^3*c-1/3*tan(1/2*f*x+1/2*e)^3*
d-c*tan(1/2*f*x+1/2*e)+3*tan(1/2*f*x+1/2*e)*d)+4*d^2/(c-d)^2/((c+d)*(c-d))^
(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.530989, size = 1307, normalized size = 10.13

$$\left[\frac{3\left(d^2\cos\left(fx+e\right)^2+2\,d^2\cos\left(fx+e\right)+d^2\right)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2+2\,\sqrt{c^2-d^2}\left(d\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)+2\,c^{2}}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2}\right.}{6\left(\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4\right)f\cos\left(fx+e\right)^2+2\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/6*(3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*sqrt(c^2 - d^2)*log
((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*
cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*c
os(f*x + e) + d^2)) + 2*(c^3 - 4*c^2*d - c*d^2 + 4*d^3 + (2*c^3 - 5*c^2*d -
 2*c*d^2 + 5*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a
```

```
^2*c*d^3 - a^2*d^4)*f*cos(f*x + e)^2 + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d
^3 - a^2*d^4)*f*cos(f*x + e) + (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d
^4)*f), 1/3*(3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*sqrt(-c^2 +
d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e
))) + (c^3 - 4*c^2*d - c*d^2 + 4*d^3 + (2*c^3 - 5*c^2*d - 2*c*d^2 + 5*d^3)*
cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4
)*f*cos(f*x + e)^2 + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*co
s(f*x + e) + (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(e+fx)}{c\sec^2(e+fx)+2c\sec(e+fx)+c+d\sec^3(e+fx)+2d\sec^2(e+fx)+d\sec(e+fx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/(c*sec(e + f*x)**2 + 2*c*sec(e + f*x) + c + d*sec(e +
 f*x)**3 + 2*d*sec(e + f*x)**2 + d*sec(e + f*x)), x)/a**2
```

Giac [B] time = 1.15912, size = 348, normalized size = 2.7

$$-\frac{\frac{12\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d^2}{\left(a^2c^2-2a^2cd+a^2d^2\right)\sqrt{-c^2+d^2}}+\frac{a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2a^4cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+a^4d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3a^4c^2}{a^6c^3-3a^6c^2d+3a^6c}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/6*(12*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1
/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d^2/((a^2*c^2
- 2*a^2*c*d + a^2*d^2)*sqrt(-c^2 + d^2)) + (a^4*c^2*tan(1/2*f*x + 1/2*e)^3
- 2*a^4*c*d*tan(1/2*f*x + 1/2*e)^3 + a^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*a^4
*c^2*tan(1/2*f*x + 1/2*e) + 12*a^4*c*d*tan(1/2*f*x + 1/2*e) - 9*a^4*d^2*tan
(1/2*f*x + 1/2*e))/(a^6*c^3 - 3*a^6*c^2*d + 3*a^6*c*d^2 - a^6*d^3))/f
```

3.223 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=211

$$\frac{d\left(c^2-6cd-10d^2\right)\tan(e+fx)}{3a^2f(c-d)^3(c+d)(c+d\sec(e+fx))}+\frac{2d^2(3c+2d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^2f(c-d)^{7/2}(c+d)^{3/2}}+\frac{(c-6d)\tan(e+fx)}{3a^2f(c-d)^2(\sec(e+fx)+1)(c+d\sec(e+fx))}$$

```
[Out] (2*d^2*(3*c + 2*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(a^
2*(c - d)^(7/2)*(c + d)^(3/2)*f) + (d*(c^2 - 6*c*d - 10*d^2)*Tan[e + f*x])/
(3*a^2*(c - d)^3*(c + d)*f*(c + d*Sec[e + f*x])) + ((c - 6*d)*Tan[e + f*x])
/(3*a^2*(c - d)^2*f*(1 + Sec[e + f*x])*(c + d*Sec[e + f*x])) + Tan[e + f*x]
/(3*(c - d)*f*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.373458, antiderivative size = 260, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 103, 152, 12, 93, 205}

$$\frac{\left(c^2-6cd-10d^2\right)\tan(e+fx)}{3f(c-d)^3(c+d)\left(a^2\sec(e+fx)+a^2\right)}-\frac{d\tan(e+fx)}{f\left(c^2-d^2\right)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))}-\frac{2d^2(3c+2d)\tan(e+f}{af(c-d)^{7/2}(c+d)^{3/2}\sqrt{a-}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((c + 4*d)*Tan[e + f*x])/(3*(c - d)^2*(c + d)*f*(a + a*Sec[e + f*x])^2) - (
2*d^2*(3*c + 2*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d
]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(a*(c - d)^(7/2)*(c + d)^(3/2)*f
*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((c^2 - 6*c*d - 10*d^
2)*Tan[e + f*x])/(3*(c - d)^3*(c + d)*f*(a^2 + a^2*Sec[e + f*x])) - (d*Tan[
e + f*x])/((c^2 - d^2)*f*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{5/2}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{\left(c^2-d^2\right)f(a+a\sec(e+fx))^2(c+d\sec(e+fx))} - \frac{\tan(e+fx)\,\text{Su}}{\left(c^2-d^2\right)}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} - \frac{d\tan(e+f}{\left(c^2-d^2\right)f(a+a\sec(e+fx)}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} + \frac{\left(c^2-6cd-10d^2\right)\tan(e}{3(c-d)^3(c+d)f\left(a^2+a^2\sec\right.}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} + \frac{\left(c^2-6cd-10d^2\right)\tan(e}{3(c-d)^3(c+d)f\left(a^2+a^2\sec\right.}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} + \frac{\left(c^2-6cd-10d^2\right)\tan(e}{3(c-d)^3(c+d)f\left(a^2+a^2\sec\right.}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} - \frac{2d^2(3c+2d)\tan^{-1}\left(\frac{\sqrt{c+d}}{\sqrt{c-d}}\right.}{a(c-d)^{7/2}(c+d)^{3/2}f\sqrt{a-a\,\text{s}}}$$

Mathematica [C] time = 3.91091, size = 376, normalized size = 1.78

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)(c\cos(e+fx)+d)\left(\frac{12d^2(3c+2d)(\sin(e)+i\cos(e))\cos^3\left(\frac{1}{2}(e+fx)\right)(c\cos(e+fx)+d)\tan^{-1}\left(\frac{(\sin(e)+i\cos(e))\left(\tan\left(\frac{fx}{2}\right)(c\right.}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-}}\right.}{(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])*Sec[e + f*x]^4*((12*d^2*(3*c + 2*d
)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(S
qrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[(e + f*x)/2]^3*(d + c*Cos[
e + f*x])*(I*Cos[e] + Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Si
n[e])^2]) + (c - d)*(d + c*Cos[e + f*x])*Sec[e/2]*Sin[(f*x)/2] - 4*(c - 4*d
)*Cos[(e + f*x)/2]^2*(d + c*Cos[e + f*x])*Sec[e/2]*Sin[(f*x)/2] + (6*d^3*Co
s[(e + f*x)/2]^3*(-(d*Sin[e]) + c*Sin[f*x]))/(c*(c + d)*(Cos[e/2] - Sin[e/2
])*(Cos[e/2] + Sin[e/2])) + (c - d)*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])*T
an[e/2]))/(3*a^2*(-c + d)^3*f*(1 + Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2)
```

Maple [A] time = 0.09, size = 203, normalized size = 1.

$$\frac{1}{2fa^2}\left(-\frac{1}{(c^2-2cd+d^2)(c-d)}\left(\frac{c}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-\frac{d}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+5\tan\left(1/2\,fx+e/2\right)d\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 1/2/f/a^2*(-1/(c^2-2*c*d+d^2)/(c-d)*(1/3*tan(1/2*f*x+1/2*e)^3*c-1/3*tan(1/2
*f*x+1/2*e)^3*d-c*tan(1/2*f*x+1/2*e)+5*tan(1/2*f*x+1/2*e)*d)-4*d^2/(c-d)^3*
(-d/(c+d)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d
-c-d)-(3*c+2*d)/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/
((c+d)*(c-d))^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.645012, size = 2631, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/6*(3*(3*c*d^3 + 2*d^4 + (3*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^3 + (6*c^2*d
^2 + 7*c*d^3 + 2*d^4)*cos(f*x + e)^2 + (3*c^2*d^2 + 8*c*d^3 + 4*d^4)*cos(f*
x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e
)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c
^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(c^4*d - 6*c^3*d^2 - 11*
c^2*d^3 + 6*c*d^4 + 10*d^5 + (2*c^5 - 6*c^4*d - 10*c^3*d^2 + 3*c^2*d^3 + 8*
c*d^4 + 3*d^5)*cos(f*x + e)^2 + (c^5 - 4*c^4*d - 14*c^3*d^2 - 10*c^2*d^3 +
13*c*d^4 + 14*d^5)*cos(f*x + e))*sin(f*x + e))/((a^2*c^7 - 2*a^2*c^6*d - a^
2*c^5*d^2 + 4*a^2*c^4*d^3 - a^2*c^3*d^4 - 2*a^2*c^2*d^5 + a^2*c*d^6)*f*cos(
f*x + e)^3 + (2*a^2*c^7 - 3*a^2*c^6*d - 4*a^2*c^5*d^2 + 7*a^2*c^4*d^3 + 2*a
^2*c^3*d^4 - 5*a^2*c^2*d^5 + a^2*d^7)*f*cos(f*x + e)^2 + (a^2*c^7 - 5*a^2*c
^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*d^6 + 2*a^
2*d^7)*f*cos(f*x + e) + (a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^
3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f), 1/3*(3*(3*c*d^3 + 2*d^4 +
(3*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^3 + (6*c^2*d^2 + 7*c*d^3 + 2*d^4)*cos(f*
x + e)^2 + (3*c^2*d^2 + 8*c*d^3 + 4*d^4)*cos(f*x + e))*sqrt(-c^2 + d^2)*arc
tan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (c
^4*d - 6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*d^5 + (2*c^5 - 6*c^4*d - 10*c^
3*d^2 + 3*c^2*d^3 + 8*c*d^4 + 3*d^5)*cos(f*x + e)^2 + (c^5 - 4*c^4*d - 14*c
^3*d^2 - 10*c^2*d^3 + 13*c*d^4 + 14*d^5)*cos(f*x + e))*sin(f*x + e))/((a^2*
c^7 - 2*a^2*c^6*d - a^2*c^5*d^2 + 4*a^2*c^4*d^3 - a^2*c^3*d^4 - 2*a^2*c^2*d
^5 + a^2*c*d^6)*f*cos(f*x + e)^3 + (2*a^2*c^7 - 3*a^2*c^6*d - 4*a^2*c^5*d^2
 + 7*a^2*c^4*d^3 + 2*a^2*c^3*d^4 - 5*a^2*c^2*d^5 + a^2*d^7)*f*cos(f*x + e)^
2 + (a^2*c^7 - 5*a^2*c^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^
5 - 3*a^2*c*d^6 + 2*a^2*d^7)*f*cos(f*x + e) + (a^2*c^6*d - 2*a^2*c^5*d^2 -
a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(e+fx)}{c^2\sec^2(e+fx)+2c^2\sec(e+fx)+c^2+2cd\sec^3(e+fx)+4cd\sec^2(e+fx)+2cd\sec(e+fx)+d^2\sec^4(e+fx)+2d^2\sec^3(e+fx)+d^2\sec^2(e+fx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sec(e + f*x)**2 + 2*c**2*sec(e + f*x) + c**2 +
2*c*d*sec(e + f*x)**3 + 4*c*d*sec(e + f*x)**2 + 2*c*d*sec(e + f*x) + d**2*s
ec(e + f*x)**4 + 2*d**2*sec(e + f*x)**3 + d**2*sec(e + f*x)**2), x)/a**2
```

Giac [B] time = 1.25995, size = 662, normalized size = 3.14

$$\frac{12\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)}+\frac{12\left(3\,cd^2+2\,d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\right.}{\sqrt{-c^2+d^2}}\right.\right.}{\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4\right)\sqrt{-c^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(12*d^3*tan(1/2*f*x + 1/2*e)/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^
2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)) + 12*
(3*c*d^2 + 2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arcta
n(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((a
^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*sqrt(-c^2 + d^2)) - (a^4*c^4*
tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c^2*d^2
*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c*d^3*tan(1/2*f*x + 1/2*e)^3 + a^4*d^4*tan(
1/2*f*x + 1/2*e)^3 - 3*a^4*c^4*tan(1/2*f*x + 1/2*e) + 24*a^4*c^3*d*tan(1/2*
f*x + 1/2*e) - 54*a^4*c^2*d^2*tan(1/2*f*x + 1/2*e) + 48*a^4*c*d^3*tan(1/2*f
*x + 1/2*e) - 15*a^4*d^4*tan(1/2*f*x + 1/2*e))/(a^6*c^6 - 6*a^6*c^5*d + 15*
a^6*c^4*d^2 - 20*a^6*c^3*d^3 + 15*a^6*c^2*d^4 - 6*a^6*c*d^5 + a^6*d^6))/f
```

3.224 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=284

$$\frac{d^2\left(12c^2+16cd+7d^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^2f(c-d)^{9/2}(c+d)^{5/2}}+\frac{d\left(-16c^2d+2c^3-59cd^2-32d^3\right)\tan(e+fx)}{6a^2f(c-d)^4(c+d)^2(c+d\sec(e+fx))}+\frac{d\left(2c^2-16cd-\right.}{6a^2f(c-d)^3(c+}$$

```
[Out] (d^2*(12*c^2 + 16*c*d + 7*d^2)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[
c + d]])/(a^2*(c - d)^(9/2)*(c + d)^(5/2)*f) + (d*(2*c^2 - 16*c*d - 21*d^2)
*Tan[e + f*x])/(6*a^2*(c - d)^3*(c + d)*f*(c + d*Sec[e + f*x])^2) + ((c - 8
*d)*Tan[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sec[e + f*x])*(c + d*Sec[e + f*x]
)^2) + Tan[e + f*x]/(3*(c - d)*f*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]
)^2) + (d*(2*c^3 - 16*c^2*d - 59*c*d^2 - 32*d^3)*Tan[e + f*x])/(6*a^2*(c -
d)^4*(c + d)^2*f*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.55765, antiderivative size = 346, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 103, 151, 152, 12, 93, 205}

$$\frac{\left(-16c^2d+2c^3-59cd^2-32d^3\right)\tan(e+fx)}{6f(c-d)^4(c+d)^2\left(a^2\sec(e+fx)+a^2\right)}-\frac{d^2\left(12c^2+16cd+7d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{af(c-d)^{9/2}(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{}{2f\left(c^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c^2 + 22*c*d + 11*d^2)*Tan[e + f*x])/(6*(c - d)^3*(c + d)^2*f*(a + a*Se
c[e + f*x])^2) - (d^2*(12*c^2 + 16*c*d + 7*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a
+ a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(a
*(c - d)^(9/2)*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
f*x]]) + ((2*c^3 - 16*c^2*d - 59*c*d^2 - 32*d^3)*Tan[e + f*x])/(6*(c - d)^4
*(c + d)^2*f*(a^2 + a^2*Sec[e + f*x])) - (d*Tan[e + f*x])/(2*(c^2 - d^2)*f*
(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2) - (d*(5*c + 2*d)*Tan[e + f*x
])/(2*(c^2 - d^2)^2*f*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
```

```
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{5/2}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} - \frac{\tan(e+fx)}{2\left(c^2-\right.}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} - \frac{}{2\left(c^2-d^2\right)^2}$$

$$= \frac{\left(2c^2+22cd+11d^2\right)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} - \frac{d\tan(e+}{2\left(c^2-d^2\right)f(a+a\sec(e+f}$$

$$= \frac{\left(2c^2+22cd+11d^2\right)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} + \frac{\left(2c^3-16c^2d-59cd^2-32d^3\right.}{6(c-d)^4(c+d)^2f\left(a^2+a^2\right.}$$

$$= \frac{\left(2c^2+22cd+11d^2\right)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} + \frac{\left(2c^3-16c^2d-59cd^2-32d^3\right.}{6(c-d)^4(c+d)^2f\left(a^2+a^2\right.}$$

$$= \frac{\left(2c^2+22cd+11d^2\right)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} + \frac{\left(2c^3-16c^2d-59cd^2-32d^3\right.}{6(c-d)^4(c+d)^2f\left(a^2+a^2\right.}$$

$$= \frac{\left(2c^2+22cd+11d^2\right)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} - \frac{d^2\left(12c^2+16cd+7d^2\right)\tan^{-}}{a(c-d)^{9/2}(c+d)^{5/2}f\sqrt{a-}}$$

Mathematica [C] time = 7.29495, size = 2220, normalized size = 7.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((12*c^2 + 16*c*d + 7*d^2)*Cos[e/2 + (f*x)/2]^4*(d + c*Cos[e + f*x])^3*Sec[
e + f*x]^5*(((-4*I)*d^2*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[C
os[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[
2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2 - d
^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (4*d^2*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sq
rt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sq
rt[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*S
in[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^4*(c + d
)^2*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3) + (Cos[e/2 + (f*x)/2]*(d
 + c*Cos[e + f*x])*Sec[e/2]*Sec[e]*Sec[e + f*x]^5*(-16*c^7*Sin[(f*x)/2] + 1
4*c^6*d*Sin[(f*x)/2] + 220*c^5*d^2*Sin[(f*x)/2] + 334*c^4*d^3*Sin[(f*x)/2]
+ 54*c^3*d^4*Sin[(f*x)/2] - 156*c^2*d^5*Sin[(f*x)/2] - 48*c*d^6*Sin[(f*x)/2
] + 18*d^7*Sin[(f*x)/2] + 14*c^7*Sin[(3*f*x)/2] - 16*c^6*d*Sin[(3*f*x)/2] -
 226*c^5*d^2*Sin[(3*f*x)/2] - 532*c^4*d^3*Sin[(3*f*x)/2] - 583*c^3*d^4*Sin[
(3*f*x)/2] - 232*c^2*d^5*Sin[(3*f*x)/2] - 6*c*d^6*Sin[(3*f*x)/2] + 6*d^7*Si
n[(3*f*x)/2] - 12*c^7*Sin[e - (f*x)/2] + 20*c^6*d*Sin[e - (f*x)/2] + 236*c^
5*d^2*Sin[e - (f*x)/2] + 628*c^4*d^3*Sin[e - (f*x)/2] + 778*c^3*d^4*Sin[e -
 (f*x)/2] + 420*c^2*d^5*Sin[e - (f*x)/2] + 48*c*d^6*Sin[e - (f*x)/2] - 18*d
^7*Sin[e - (f*x)/2] + 12*c^7*Sin[e + (f*x)/2] - 20*c^6*d*Sin[e + (f*x)/2] -
```

```
 236*c^5*d^2*Sin[e + (f*x)/2] - 460*c^4*d^3*Sin[e + (f*x)/2] - 310*c^3*d^4*
Sin[e + (f*x)/2] + 39*c^2*d^5*Sin[e + (f*x)/2] + 48*c*d^6*Sin[e + (f*x)/2]
- 18*d^7*Sin[e + (f*x)/2] - 16*c^7*Sin[2*e + (f*x)/2] + 14*c^6*d*Sin[2*e +
(f*x)/2] + 220*c^5*d^2*Sin[2*e + (f*x)/2] + 502*c^4*d^3*Sin[2*e + (f*x)/2]
+ 522*c^3*d^4*Sin[2*e + (f*x)/2] + 303*c^2*d^5*Sin[2*e + (f*x)/2] + 48*c*d^
6*Sin[2*e + (f*x)/2] - 18*d^7*Sin[2*e + (f*x)/2] - 6*c^7*Sin[e + (3*f*x)/2]
 + 6*c^6*d*Sin[e + (3*f*x)/2] + 126*c^5*d^2*Sin[e + (3*f*x)/2] + 114*c^4*d^
3*Sin[e + (3*f*x)/2] - 159*c^3*d^4*Sin[e + (3*f*x)/2] - 144*c^2*d^5*Sin[e +
 (3*f*x)/2] - 6*c*d^6*Sin[e + (3*f*x)/2] + 6*d^7*Sin[e + (3*f*x)/2] + 14*c^
7*Sin[2*e + (3*f*x)/2] - 16*c^6*d*Sin[2*e + (3*f*x)/2] - 226*c^5*d^2*Sin[2*
e + (3*f*x)/2] - 412*c^4*d^3*Sin[2*e + (3*f*x)/2] - 235*c^3*d^4*Sin[2*e + (
3*f*x)/2] - 7*c^2*d^5*Sin[2*e + (3*f*x)/2] + 6*c*d^6*Sin[2*e + (3*f*x)/2] -
 6*d^7*Sin[2*e + (3*f*x)/2] - 6*c^7*Sin[3*e + (3*f*x)/2] + 6*c^6*d*Sin[3*e
+ (3*f*x)/2] + 126*c^5*d^2*Sin[3*e + (3*f*x)/2] + 234*c^4*d^3*Sin[3*e + (3*
f*x)/2] + 189*c^3*d^4*Sin[3*e + (3*f*x)/2] + 81*c^2*d^5*Sin[3*e + (3*f*x)/2
] + 6*c*d^6*Sin[3*e + (3*f*x)/2] - 6*d^7*Sin[3*e + (3*f*x)/2] + 6*c^7*Sin[e
 + (5*f*x)/2] - 14*c^6*d*Sin[e + (5*f*x)/2] - 134*c^5*d^2*Sin[e + (5*f*x)/2
] - 274*c^4*d^3*Sin[e + (5*f*x)/2] - 193*c^3*d^4*Sin[e + (5*f*x)/2] - 27*c^
2*d^5*Sin[e + (5*f*x)/2] + 6*c*d^6*Sin[e + (5*f*x)/2] - 6*c^7*Sin[2*e + (5*
f*x)/2] + 12*c^6*d*Sin[2*e + (5*f*x)/2] + 42*c^5*d^2*Sin[2*e + (5*f*x)/2] -
 48*c^4*d^3*Sin[2*e + (5*f*x)/2] - 105*c^3*d^4*Sin[2*e + (5*f*x)/2] - 27*c^
2*d^5*Sin[2*e + (5*f*x)/2] + 6*c*d^6*Sin[2*e + (5*f*x)/2] + 6*c^7*Sin[3*e +
 (5*f*x)/2] - 14*c^6*d*Sin[3*e + (5*f*x)/2] - 134*c^5*d^2*Sin[3*e + (5*f*x)
/2] - 202*c^4*d^3*Sin[3*e + (5*f*x)/2] - 61*c^3*d^4*Sin[3*e + (5*f*x)/2] +
12*c^2*d^5*Sin[3*e + (5*f*x)/2] - 6*c*d^6*Sin[3*e + (5*f*x)/2] - 6*c^7*Sin[
4*e + (5*f*x)/2] + 12*c^6*d*Sin[4*e + (5*f*x)/2] + 42*c^5*d^2*Sin[4*e + (5*
f*x)/2] + 24*c^4*d^3*Sin[4*e + (5*f*x)/2] + 27*c^3*d^4*Sin[4*e + (5*f*x)/2]
 + 12*c^2*d^5*Sin[4*e + (5*f*x)/2] - 6*c*d^6*Sin[4*e + (5*f*x)/2] + 4*c^7*S
in[2*e + (7*f*x)/2] - 14*c^6*d*Sin[2*e + (7*f*x)/2] - 40*c^5*d^2*Sin[2*e +
(7*f*x)/2] - 46*c^4*d^3*Sin[2*e + (7*f*x)/2] - 12*c^3*d^4*Sin[2*e + (7*f*x)
/2] + 3*c^2*d^5*Sin[2*e + (7*f*x)/2] - 24*c^4*d^3*Sin[3*e + (7*f*x)/2] - 12
*c^3*d^4*Sin[3*e + (7*f*x)/2] + 3*c^2*d^5*Sin[3*e + (7*f*x)/2] + 4*c^7*Sin[
4*e + (7*f*x)/2] - 14*c^6*d*Sin[4*e + (7*f*x)/2] - 40*c^5*d^2*Sin[4*e + (7*
f*x)/2] - 22*c^4*d^3*Sin[4*e + (7*f*x)/2]))/(48*c^2*(-c + d)^4*(c + d)^2*f*
(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3)
```

Maple [A] time = 0.105, size = 280, normalized size = 1.

$$\frac{1}{2fa^2}\left(-\frac{1}{\left(c^3-3c^2d+3d^2c-d^3\right)(c-d)}\left(\frac{c}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-\frac{d}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+7\tan\left(1/2fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x)
```

```
[Out] 1/2/f/a^2*(-1/(c^3-3*c^2*d+3*c*d^2-d^3)/(c-d)*(1/3*tan(1/2*f*x+1/2*e)^3*c-1
/3*tan(1/2*f*x+1/2*e)^3*d-c*tan(1/2*f*x+1/2*e)+7*tan(1/2*f*x+1/2*e)*d)-8*d^
2/(c-d)^4*((-1/4*d*(8*c^2-3*c*d-5*d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3
+1/4*d*(8*c+3*d)/(c+d)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*
f*x+1/2*e)^2*d-c-d)^2-1/4*(12*c^2+16*c*d+7*d^2)/(c^2+2*c*d+d^2)/((c+d)*(c-d
))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.78321, size = 4329, normalized size = 15.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*(12*c^2*d^4 + 16*c*d^5 + 7*d^6 + (12*c^4*d^2 + 16*c^3*d^3 + 7*c^2*
d^4)*cos(f*x + e)^4 + 2*(12*c^4*d^2 + 28*c^3*d^3 + 23*c^2*d^4 + 7*c*d^5)*co
s(f*x + e)^3 + (12*c^4*d^2 + 64*c^3*d^3 + 83*c^2*d^4 + 44*c*d^5 + 7*d^6)*co
s(f*x + e)^2 + 2*(12*c^3*d^3 + 28*c^2*d^4 + 23*c*d^5 + 7*d^6)*cos(f*x + e))
*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2
*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(
f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*c^5*d^2 - 16*c^4*d^3 - 61*c^
3*d^4 - 16*c^2*d^5 + 59*c*d^6 + 32*d^7 + (4*c^7 - 14*c^6*d - 44*c^5*d^2 - 3
2*c^4*d^3 + 28*c^3*d^4 + 49*c^2*d^5 + 12*c*d^6 - 3*d^7)*cos(f*x + e)^3 + (2
*c^7 - 8*c^6*d - 68*c^5*d^2 - 140*c^4*d^3 - 23*c^3*d^4 + 142*c^2*d^5 + 89*c
*d^6 + 6*d^7)*cos(f*x + e)^2 + (4*c^6*d - 28*c^5*d^2 - 118*c^4*d^3 - 106*c^
3*d^4 + 71*c^2*d^5 + 134*c*d^6 + 43*d^7)*cos(f*x + e))*sin(f*x + e))/((a^2*
c^10 - 2*a^2*c^9*d - 2*a^2*c^8*d^2 + 6*a^2*c^7*d^3 - 6*a^2*c^5*d^5 + 2*a^2*
c^4*d^6 + 2*a^2*c^3*d^7 - a^2*c^2*d^8)*f*cos(f*x + e)^4 + 2*(a^2*c^10 - a^2
*c^9*d - 4*a^2*c^8*d^2 + 4*a^2*c^7*d^3 + 6*a^2*c^6*d^4 - 6*a^2*c^5*d^5 - 4*
a^2*c^4*d^6 + 4*a^2*c^3*d^7 + a^2*c^2*d^8 - a^2*c*d^9)*f*cos(f*x + e)^3 + (
a^2*c^10 + 2*a^2*c^9*d - 9*a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 22*a^2*c^6*d^4 - 2
2*a^2*c^4*d^6 + 4*a^2*c^3*d^7 + 9*a^2*c^2*d^8 - 2*a^2*c*d^9 - a^2*d^10)*f*c
os(f*x + e)^2 + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4
+ 6*a^2*c^5*d^5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9
 - a^2*d^10)*f*cos(f*x + e) + (a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4
+ 6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f
), 1/6*(3*(12*c^2*d^4 + 16*c*d^5 + 7*d^6 + (12*c^4*d^2 + 16*c^3*d^3 + 7*c^2
*d^4)*cos(f*x + e)^4 + 2*(12*c^4*d^2 + 28*c^3*d^3 + 23*c^2*d^4 + 7*c*d^5)*c
os(f*x + e)^3 + (12*c^4*d^2 + 64*c^3*d^3 + 83*c^2*d^4 + 44*c*d^5 + 7*d^6)*c
os(f*x + e)^2 + 2*(12*c^3*d^3 + 28*c^2*d^4 + 23*c*d^5 + 7*d^6)*cos(f*x + e)
)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^
2)*sin(f*x + e))) + (2*c^5*d^2 - 16*c^4*d^3 - 61*c^3*d^4 - 16*c^2*d^5 + 59*
c*d^6 + 32*d^7 + (4*c^7 - 14*c^6*d - 44*c^5*d^2 - 32*c^4*d^3 + 28*c^3*d^4 +
 49*c^2*d^5 + 12*c*d^6 - 3*d^7)*cos(f*x + e)^3 + (2*c^7 - 8*c^6*d - 68*c^5*
d^2 - 140*c^4*d^3 - 23*c^3*d^4 + 142*c^2*d^5 + 89*c*d^6 + 6*d^7)*cos(f*x +
e)^2 + (4*c^6*d - 28*c^5*d^2 - 118*c^4*d^3 - 106*c^3*d^4 + 71*c^2*d^5 + 134
*c*d^6 + 43*d^7)*cos(f*x + e))*sin(f*x + e))/((a^2*c^10 - 2*a^2*c^9*d - 2*a
^2*c^8*d^2 + 6*a^2*c^7*d^3 - 6*a^2*c^5*d^5 + 2*a^2*c^4*d^6 + 2*a^2*c^3*d^7
- a^2*c^2*d^8)*f*cos(f*x + e)^4 + 2*(a^2*c^10 - a^2*c^9*d - 4*a^2*c^8*d^2 +
 4*a^2*c^7*d^3 + 6*a^2*c^6*d^4 - 6*a^2*c^5*d^5 - 4*a^2*c^4*d^6 + 4*a^2*c^3*
d^7 + a^2*c^2*d^8 - a^2*c*d^9)*f*cos(f*x + e)^3 + (a^2*c^10 + 2*a^2*c^9*d -
```

```
 9*a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 22*a^2*c^6*d^4 - 22*a^2*c^4*d^6 + 4*a^2*c^
3*d^7 + 9*a^2*c^2*d^8 - 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^2 + 2*(a^2*c
^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*
c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a^2*d^10)*f*cos(f*x +
 e) + (a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*
c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31008, size = 1048, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(12*c^2*d^2 + 16*c*d^3 + 7*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sg
n(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/s
qrt(-c^2 + d^2)))/((a^2*c^6 - 2*a^2*c^5*d - a^2*c^4*d^2 + 4*a^2*c^3*d^3 - a
^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*sqrt(-c^2 + d^2)) - (a^4*c^6*tan(1/2*f*
x + 1/2*e)^3 - 6*a^4*c^5*d*tan(1/2*f*x + 1/2*e)^3 + 15*a^4*c^4*d^2*tan(1/2*
f*x + 1/2*e)^3 - 20*a^4*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^4*c^2*d^4*tan
(1/2*f*x + 1/2*e)^3 - 6*a^4*c*d^5*tan(1/2*f*x + 1/2*e)^3 + a^4*d^6*tan(1/2*
f*x + 1/2*e)^3 - 3*a^4*c^6*tan(1/2*f*x + 1/2*e) + 36*a^4*c^5*d*tan(1/2*f*x
+ 1/2*e) - 135*a^4*c^4*d^2*tan(1/2*f*x + 1/2*e) + 240*a^4*c^3*d^3*tan(1/2*f
*x + 1/2*e) - 225*a^4*c^2*d^4*tan(1/2*f*x + 1/2*e) + 108*a^4*c*d^5*tan(1/2*
f*x + 1/2*e) - 21*a^4*d^6*tan(1/2*f*x + 1/2*e))/(a^6*c^9 - 9*a^6*c^8*d + 36
*a^6*c^7*d^2 - 84*a^6*c^6*d^3 + 126*a^6*c^5*d^4 - 126*a^6*c^4*d^5 + 84*a^6*
c^3*d^6 - 36*a^6*c^2*d^7 + 9*a^6*c*d^8 - a^6*d^9) + 6*(8*c^2*d^3*tan(1/2*f*
x + 1/2*e)^3 - 3*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 5*d^5*tan(1/2*f*x + 1/2*e)^
3 - 8*c^2*d^3*tan(1/2*f*x + 1/2*e) - 11*c*d^4*tan(1/2*f*x + 1/2*e) - 3*d^5*
tan(1/2*f*x + 1/2*e))/((a^2*c^6 - 2*a^2*c^5*d - a^2*c^4*d^2 + 4*a^2*c^3*d^3
 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1
/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

3.225 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=363

$$-\frac{2d\left(107c^3d^2-472c^2d^3+18c^4d+2c^5+456cd^4-136d^5\right)\tan(e+fx)}{15a^3f}+\frac{d^3\left(-90c^2d+40c^3+78cd^2-23d^3\right)\tanh^{-1}(s}{2a^3f}$$

```
[Out] (d^3*(40*c^3 - 90*c^2*d + 78*c*d^2 - 23*d^3)*ArcTanh[Sin[e + f*x]])/(2*a^3*
f) - (2*d*(2*c^5 + 18*c^4*d + 107*c^3*d^2 - 472*c^2*d^3 + 456*c*d^4 - 136*d
^5)*Tan[e + f*x])/(15*a^3*f) - (d^2*(4*c^4 + 36*c^3*d + 216*c^2*d^2 - 626*c
*d^3 + 345*d^4)*Sec[e + f*x]*Tan[e + f*x])/(30*a^3*f) - (d*(2*c^3 + 18*c^2*
d + 111*c*d^2 - 136*d^3)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(15*a^3*f) +
((c - d)*(2*c^2 + 18*c*d + 115*d^2)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(1
5*f*(a^3 + a^3*Sec[e + f*x])) + ((c - d)*(2*c + 13*d)*(c + d*Sec[e + f*x])^
4*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((c - d)*(c + d*Sec[e + f
*x])^5*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3)
```

Rubi [A] time = 0.538618, antiderivative size = 405, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3987, 98, 150, 153, 147, 63, 217, 203}

$$\frac{d^3\left(-90c^2d+40c^3+78cd^2-23d^3\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)\left(2c^2+18cd+115d^2\right)\tan(e+fx)(c+}{15f\left(a^3\sec(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (d^3*(40*c^3 - 90*c^2*d + 78*c*d^2 - 23*d^3)*ArcTan[Sqrt[a - a*Sec[e + f*x]
]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]]) - (d*(2*c^3 + 18*c^2*d + 111*c*d^2 - 136*d^3)*(c
 + d*Sec[e + f*x])^2*Tan[e + f*x])/(15*a^3*f) + ((c - d)*(2*c^2 + 18*c*d +
115*d^2)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x]
)) + ((c - d)*(2*c + 13*d)*(c + d*Sec[e + f*x])^4*Tan[e + f*x])/(15*a*f*(a
+ a*Sec[e + f*x])^2) + ((c - d)*(c + d*Sec[e + f*x])^5*Tan[e + f*x])/(5*f*(
a + a*Sec[e + f*x])^3) - (d*(4*(2*c^5 + 18*c^4*d + 107*c^3*d^2 - 472*c^2*d^
3 + 456*c*d^4 - 136*d^5) + d*(4*c^4 + 36*c^3*d + 216*c^2*d^2 - 626*c*d^3 +
345*d^4)*Sec[e + f*x])*Tan[e + f*x])/(30*a^3*f)
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^6}{\sqrt{a-ax}(a+ax)^{7/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^4\left(-a^2(2c^2\right.}{\sqrt{a-}}\right.}{5af\sqrt{a-a\sec(e+}}$$

$$= \frac{(c-d)(2c+13d)(c+d\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))}{5f(a+a\sec(e+}$$

$$= \frac{(c-d)\left(2c^2+18cd+115d^2\right)(c+d\sec(e+fx))^3\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c-d)(2c+1}{1}$$

$$= -\frac{d\left(2c^3+18c^2d+111cd^2-136d^3\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15a^3f} + \frac{(c-d)}{}$$

$$= -\frac{d\left(2c^3+18c^2d+111cd^2-136d^3\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15a^3f} + \frac{(c-d)}{}$$

$$= -\frac{d\left(2c^3+18c^2d+111cd^2-136d^3\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15a^3f} + \frac{(c-d)}{}$$

$$= -\frac{d\left(2c^3+18c^2d+111cd^2-136d^3\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15a^3f} + \frac{(c-d)}{}$$

$$= \frac{d^3\left(40c^3-90c^2d+78cd^2-23d^3\right)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{d\left(2c^3\right.}{}$$

Mathematica [B] time = 6.70421, size = 1338, normalized size = 3.69

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (4*(-40*c^3*d^3 + 90*c^2*d^4 - 78*c*d^5 + 23*d^6)*Cos[e/2 + (f*x)/2]^6*Cos[
e + f*x]^3*Log[Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x]
)^6)/(f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) - (4*(-40*c^3*d^3 +
90*c^2*d^4 - 78*c*d^5 + 23*d^6)*Cos[e/2 + (f*x)/2]^6*Cos[e + f*x]^3*Log[Cos
[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x])^6)/(f*(d + c*Cos
[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (2*Cos[e/2 + (f*x)/2]^2*Cos[e + f*x]
^3*Sec[e/2]*(c + d*Sec[e + f*x])^6*(c^6*Sin[e/2] - 6*c^5*d*Sin[e/2] + 15*c^
4*d^2*Sin[e/2] - 20*c^3*d^3*Sin[e/2] + 15*c^2*d^4*Sin[e/2] - 6*c*d^5*Sin[e/
2] + d^6*Sin[e/2]))/(5*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (
8*Cos[e/2 + (f*x)/2]^4*Cos[e + f*x]^3*Sec[e/2]*(c + d*Sec[e + f*x])^6*(-4*c
^6*Sin[e/2] + 9*c^5*d*Sin[e/2] + 15*c^4*d^2*Sin[e/2] - 70*c^3*d^3*Sin[e/2]
+ 90*c^2*d^4*Sin[e/2] - 51*c*d^5*Sin[e/2] + 11*d^6*Sin[e/2]))/(15*f*(d + c*
Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (2*Cos[e/2 + (f*x)/2]*Cos[e + f*x
]^3*Sec[e/2]*(c + d*Sec[e + f*x])^6*(c^6*Sin[(f*x)/2] - 6*c^5*d*Sin[(f*x)/2
] + 15*c^4*d^2*Sin[(f*x)/2] - 20*c^3*d^3*Sin[(f*x)/2] + 15*c^2*d^4*Sin[(f*x
```

```
)/2] - 6*c*d^5*Sin[(f*x)/2] + d^6*Sin[(f*x)/2]))/(5*f*(d + c*Cos[e + f*x])^
6*(a + a*Sec[e + f*x])^3) + (8*Cos[e/2 + (f*x)/2]^3*Cos[e + f*x]^3*Sec[e/2]
*(c + d*Sec[e + f*x])^6*(-4*c^6*Sin[(f*x)/2] + 9*c^5*d*Sin[(f*x)/2] + 15*c^
4*d^2*Sin[(f*x)/2] - 70*c^3*d^3*Sin[(f*x)/2] + 90*c^2*d^4*Sin[(f*x)/2] - 51
*c*d^5*Sin[(f*x)/2] + 11*d^6*Sin[(f*x)/2]))/(15*f*(d + c*Cos[e + f*x])^6*(a
 + a*Sec[e + f*x])^3) + (8*Cos[e/2 + (f*x)/2]^5*Cos[e + f*x]^3*Sec[e/2]*(c
+ d*Sec[e + f*x])^6*(7*c^6*Sin[(f*x)/2] + 18*c^5*d*Sin[(f*x)/2] + 30*c^4*d^
2*Sin[(f*x)/2] - 440*c^3*d^3*Sin[(f*x)/2] + 855*c^2*d^4*Sin[(f*x)/2] - 642*
c*d^5*Sin[(f*x)/2] + 172*d^6*Sin[(f*x)/2]))/(15*f*(d + c*Cos[e + f*x])^6*(a
 + a*Sec[e + f*x])^3) + (8*d^6*Cos[e/2 + (f*x)/2]^6*Sec[e]*(c + d*Sec[e + f
*x])^6*Sin[f*x])/(3*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) - (4*C
os[e/2 + (f*x)/2]^6*Cos[e + f*x]^2*Sec[e]*(c + d*Sec[e + f*x])^6*(-18*c*d^5
*Sin[e] + 9*d^6*Sin[e] - 90*c^2*d^4*Sin[f*x] + 108*c*d^5*Sin[f*x] - 40*d^6*
Sin[f*x]))/(3*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (4*Cos[e/2
 + (f*x)/2]^6*Cos[e + f*x]*Sec[e]*(c + d*Sec[e + f*x])^6*(2*d^6*Sin[e] + 18
*c*d^5*Sin[f*x] - 9*d^6*Sin[f*x]))/(3*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e
 + f*x])^3)
```

Maple [B] time = 0.119, size = 956, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 1/20/f*c^6/a^3*tan(1/2*f*x+1/2*e)^5-1/6/f*c^6/a^3*tan(1/2*f*x+1/2*e)^3+1/4/
f*c^6/a^3*tan(1/2*f*x+1/2*e)-3/10/f/a^3*tan(1/2*f*x+1/2*e)^5*c*d^5+5/2/f/a^
3*tan(1/2*f*x+1/2*e)^3*c^4*d^2+15/2/f/a^3*tan(1/2*f*x+1/2*e)^3*c^2*d^4+3/2/
f/a^3*tan(1/2*f*x+1/2*e)*c^5*d+15/4/f/a^3*tan(1/2*f*x+1/2*e)*c^4*d^2+255/4/
f/a^3*tan(1/2*f*x+1/2*e)*c^2*d^4-93/2/f/a^3*tan(1/2*f*x+1/2*e)*c*d^5+3/f/a^
3*d^5/(tan(1/2*f*x+1/2*e)-1)^2*c-23/2/f/a^3*ln(tan(1/2*f*x+1/2*e)+1)*d^6+1/
20/f/a^3*tan(1/2*f*x+1/2*e)^5*d^6+5/6/f/a^3*tan(1/2*f*x+1/2*e)^3*d^6+49/4/f
/a^3*tan(1/2*f*x+1/2*e)*d^6-1/3/f/a^3*d^6/(tan(1/2*f*x+1/2*e)+1)^3-1/3/f/a^
3*d^6/(tan(1/2*f*x+1/2*e)-1)^3-17/2/f/a^3*d^6/(tan(1/2*f*x+1/2*e)+1)+2/f/a^
3*d^6/(tan(1/2*f*x+1/2*e)+1)^2-17/2/f/a^3*d^6/(tan(1/2*f*x+1/2*e)-1)-2/f/a^
3*d^6/(tan(1/2*f*x+1/2*e)-1)^2+23/2/f/a^3*ln(tan(1/2*f*x+1/2*e)-1)*d^6-15/f
/a^3*d^4/(tan(1/2*f*x+1/2*e)+1)*c^2+21/f/a^3*d^5/(tan(1/2*f*x+1/2*e)+1)*c-3
/f/a^3*d^5/(tan(1/2*f*x+1/2*e)+1)^2*c-1/f/a^3*tan(1/2*f*x+1/2*e)^5*c^3*d^3-
20/3/f/a^3*tan(1/2*f*x+1/2*e)^3*c^3*d^3-4/f/a^3*tan(1/2*f*x+1/2*e)^3*c*d^5-
35/f/a^3*c^3*d^3*tan(1/2*f*x+1/2*e)-15/f/a^3*d^4/(tan(1/2*f*x+1/2*e)-1)*c^2
+21/f/a^3*d^5/(tan(1/2*f*x+1/2*e)-1)*c+20/f/a^3*ln(tan(1/2*f*x+1/2*e)+1)*c^
3*d^3-45/f/a^3*ln(tan(1/2*f*x+1/2*e)+1)*c^2*d^4+39/f/a^3*ln(tan(1/2*f*x+1/2
*e)+1)*c*d^5-20/f/a^3*ln(tan(1/2*f*x+1/2*e)-1)*c^3*d^3+45/f/a^3*ln(tan(1/2*
f*x+1/2*e)-1)*c^2*d^4-39/f/a^3*ln(tan(1/2*f*x+1/2*e)-1)*c*d^5-3/10/f/a^3*ta
n(1/2*f*x+1/2*e)^5*c^5*d+3/4/f/a^3*tan(1/2*f*x+1/2*e)^5*c^4*d^2+3/4/f/a^3*t
an(1/2*f*x+1/2*e)^5*c^2*d^4
```

Maxima [B] time = 1.12129, size = 1277, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(d^6*(20*(33*sin(f*x + e)/(cos(f*x + e) + 1) - 76*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 51*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3 - 3*a^3*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (735*sin(f*x + e)/(cos(f*x +
e) + 1) + 50*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5)/a^3 - 690*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 690
*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) - 6*c*d^5*(60*(5*sin(f*x + e
)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^3 - 2*a^3*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 390*log(sin(f*x + e
)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1
)/a^3) + 45*c^2*d^4*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a
^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)/a^3) - 20*c^3*d^3*((105*sin(f*x + e)/(cos(f*x + e)
+ 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 15*c^4*d^2*(15*sin(f*x + e)/(co
s(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5)/a^3 + c^6*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
/a^3 + 18*c^5*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*
x + e) + 1)^5)/a^3)/f
```

Fricas [A] time = 0.629447, size = 1470, normalized size = 4.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/60*(15*((40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^6 + 3*
(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^5 + 3*(40*c^3*d^
3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^4 + (40*c^3*d^3 - 90*c^2*d
^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 15*((40*c^3
*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^6 + 3*(40*c^3*d^3 - 90*
c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^5 + 3*(40*c^3*d^3 - 90*c^2*d^4 +
78*c*d^5 - 23*d^6)*cos(f*x + e)^4 + (40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 2
3*d^6)*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) + 2*(10*d^6 + 2*(7*c^6 + 18*c
^5*d + 30*c^4*d^2 - 440*c^3*d^3 + 1080*c^2*d^4 - 912*c*d^5 + 272*d^6)*cos(f
*x + e)^5 + 3*(4*c^6 + 36*c^5*d + 60*c^4*d^2 - 680*c^3*d^3 + 1710*c^2*d^4 -
 1434*c*d^5 + 429*d^6)*cos(f*x + e)^4 + (4*c^6 + 36*c^5*d + 210*c^4*d^2 - 1
280*c^3*d^3 + 3510*c^2*d^4 - 2874*c*d^5 + 869*d^6)*cos(f*x + e)^3 + 5*(90*c
^2*d^4 - 54*c*d^5 + 19*d^6)*cos(f*x + e)^2 + 15*(6*c*d^5 - d^6)*cos(f*x + e
))*sin(f*x + e))/(a^3*f*cos(f*x + e)^6 + 3*a^3*f*cos(f*x + e)^5 + 3*a^3*f*c
os(f*x + e)^4 + a^3*f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c^6 \sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{d^6 \sec^7(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{6cd^5 \sec^6(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**6/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**6*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**6*sec(e + f*x)**7/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(6*c*d**5*sec(e + f*x)**6/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(15*c
**2*d**4*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f
*x) + 1), x) + Integral(20*c**3*d**3*sec(e + f*x)**4/(sec(e + f*x)**3 + 3*s
ec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(15*c**4*d**2*sec(e + f*
x)**3/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Inte
gral(6*c**5*d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x))/a**3
```

Giac [B] time = 1.47798, size = 950, normalized size = 2.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(30*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*log(abs(tan(1/2*f*x
+ 1/2*e) + 1))/a^3 - 30*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*log(a
bs(tan(1/2*f*x + 1/2*e) - 1))/a^3 - 20*(90*c^2*d^4*tan(1/2*f*x + 1/2*e)^5 -
 126*c*d^5*tan(1/2*f*x + 1/2*e)^5 + 51*d^6*tan(1/2*f*x + 1/2*e)^5 - 180*c^2
*d^4*tan(1/2*f*x + 1/2*e)^3 + 216*c*d^5*tan(1/2*f*x + 1/2*e)^3 - 76*d^6*tan
(1/2*f*x + 1/2*e)^3 + 90*c^2*d^4*tan(1/2*f*x + 1/2*e) - 90*c*d^5*tan(1/2*f*
x + 1/2*e) + 33*d^6*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a
^3) + (3*a^12*c^6*tan(1/2*f*x + 1/2*e)^5 - 18*a^12*c^5*d*tan(1/2*f*x + 1/2*
e)^5 + 45*a^12*c^4*d^2*tan(1/2*f*x + 1/2*e)^5 - 60*a^12*c^3*d^3*tan(1/2*f*x
 + 1/2*e)^5 + 45*a^12*c^2*d^4*tan(1/2*f*x + 1/2*e)^5 - 18*a^12*c*d^5*tan(1/
2*f*x + 1/2*e)^5 + 3*a^12*d^6*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^6*tan(1/2*
f*x + 1/2*e)^3 + 150*a^12*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 400*a^12*c^3*d^3
*tan(1/2*f*x + 1/2*e)^3 + 450*a^12*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 - 240*a^1
2*c*d^5*tan(1/2*f*x + 1/2*e)^3 + 50*a^12*d^6*tan(1/2*f*x + 1/2*e)^3 + 15*a^
12*c^6*tan(1/2*f*x + 1/2*e) + 90*a^12*c^5*d*tan(1/2*f*x + 1/2*e) + 225*a^12
*c^4*d^2*tan(1/2*f*x + 1/2*e) - 2100*a^12*c^3*d^3*tan(1/2*f*x + 1/2*e) + 38
25*a^12*c^2*d^4*tan(1/2*f*x + 1/2*e) - 2790*a^12*c*d^5*tan(1/2*f*x + 1/2*e)
 + 735*a^12*d^6*tan(1/2*f*x + 1/2*e))/a^15)/f
```

3.226 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=287

$$-\frac{2d\left(72c^2d^2+15c^3d+2c^4-180cd^3+76d^4\right)\tan(e+fx)}{15a^3f}+\frac{d^3\left(20c^2-30cd+13d^2\right)\tanh^{-1}(\sin(e+fx))}{2a^3f}+\frac{(c-d)\left(2\right.}{}$$

```
[Out] (d^3*(20*c^2 - 30*c*d + 13*d^2)*ArcTanh[Sin[e + f*x]])/(2*a^3*f) - (2*d*(2*
c^4 + 15*c^3*d + 72*c^2*d^2 - 180*c*d^3 + 76*d^4)*Tan[e + f*x])/(15*a^3*f)
- (d^2*(4*c^3 + 30*c^2*d + 146*c*d^2 - 195*d^3)*Sec[e + f*x]*Tan[e + f*x])/
(30*a^3*f) + ((c - d)*(2*c^2 + 15*c*d + 76*d^2)*(c + d*Sec[e + f*x])^2*Tan[
e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x])) + ((c - d)*(2*c + 11*d)*(c + d*Se
c[e + f*x])^3*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((c - d)*(c +
 d*Sec[e + f*x])^4*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3)
```

Rubi [A] time = 0.416137, antiderivative size = 329, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 98, 150, 147, 63, 217, 203}

$$\frac{d^3\left(20c^2-30cd+13d^2\right)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)\left(2c^2+15cd+76d^2\right)\tan(e+fx)(c+d\sec(e+f}{15f\left(a^3\sec(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (d^3*(20*c^2 - 30*c*d + 13*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 +
 Sec[e + f*x])]]*Tan[e + f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]]) + ((c - d)*(2*c^2 + 15*c*d + 76*d^2)*(c + d*Sec[e + f*x])^2*T
an[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x])) + ((c - d)*(2*c + 11*d)*(c + d
*Sec[e + f*x])^3*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((c - d)*(
c + d*Sec[e + f*x])^4*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) - (d*(4*(2
*c^4 + 15*c^3*d + 72*c^2*d^2 - 180*c*d^3 + 76*d^4) + d*(4*c^3 + 30*c^2*d +
146*c*d^2 - 195*d^3)*Sec[e + f*x])*Tan[e + f*x])/(30*a^3*f)
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^5}{\sqrt{a-ax}(a+ax)^{7/2}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^3\left(-a^2(2c-\right.}{\sqrt{a-}}\right.}{5af\sqrt{a-a\sec(e+f}}$$

$$= \frac{(c-d)(2c+11d)(c+d\sec(e+fx))^3\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))}{5f(a+a\sec(e+}$$

$$= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c-d)(2c+11}{15}$$

$$= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c-d)(2c+11}{15}$$

$$= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c-d)(2c+11}{15}$$

$$= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c-d)(2c+11}{15}$$

$$= \frac{d^3\left(20c^2-30cd+13d^2\right)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)\left(2c^2+15\right.}{}$$

Mathematica [A] time = 2.14813, size = 439, normalized size = 1.53

$$\frac{2\sin\left(\frac{1}{2}(e+fx)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left(120c^3d^2\cos(3(e+fx))+20c^3d^2\cos(4(e+fx))-1020c^2d^3\cos(3(e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-480*d^3*(20*c^2 - 30*c*d + 13*d^2)*Cos[(e + f*x)/2]^6*(Log[Cos[(e + f*x)/
2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2*Cos[
(e + f*x)/2]*(29*c^5 + 105*c^4*d + 340*c^3*d^2 - 1940*c^2*d^3 + 3420*c*d^4
- 1354*d^5 + 3*(12*c^5 + 90*c^4*d + 120*c^3*d^2 - 1020*c^2*d^3 + 1910*c*d^4
 - 777*d^5)*Cos[e + f*x] + 6*(6*c^5 + 20*c^4*d + 60*c^3*d^2 - 360*c^2*d^3 +
 630*c*d^4 - 261*d^5)*Cos[2*(e + f*x)] + 12*c^5*Cos[3*(e + f*x)] + 90*c^4*d
*Cos[3*(e + f*x)] + 120*c^3*d^2*Cos[3*(e + f*x)] - 1020*c^2*d^3*Cos[3*(e +
f*x)] + 1710*c*d^4*Cos[3*(e + f*x)] - 717*d^5*Cos[3*(e + f*x)] + 7*c^5*Cos[
4*(e + f*x)] + 15*c^4*d*Cos[4*(e + f*x)] + 20*c^3*d^2*Cos[4*(e + f*x)] - 22
0*c^2*d^3*Cos[4*(e + f*x)] + 360*c*d^4*Cos[4*(e + f*x)] - 152*d^5*Cos[4*(e
+ f*x)])*Sec[e + f*x]^2*Sin[(e + f*x)/2])/(120*a^3*f*(1 + Cos[e + f*x])^3)
```

Maple [B] time = 0.104, size = 679, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 1/20/f*c^5/a^3*tan(1/2*f*x+1/2*e)^5-1/6/f*c^5/a^3*tan(1/2*f*x+1/2*e)^3+1/4/
f*c^5/a^3*tan(1/2*f*x+1/2*e)+7/2/f/a^3*d^5/(tan(1/2*f*x+1/2*e)+1)+1/2/f/a^3
*d^5/(tan(1/2*f*x+1/2*e)-1)^2-13/2/f/a^3*ln(tan(1/2*f*x+1/2*e)-1)*d^5+7/2/f
/a^3*d^5/(tan(1/2*f*x+1/2*e)-1)-1/20/f/a^3*tan(1/2*f*x+1/2*e)^5*d^5-2/3/f/a
^3*tan(1/2*f*x+1/2*e)^3*d^5-31/4/f/a^3*tan(1/2*f*x+1/2*e)*d^5-1/2/f/a^3*d^5
/(tan(1/2*f*x+1/2*e)+1)^2+13/2/f/a^3*ln(tan(1/2*f*x+1/2*e)+1)*d^5-10/f/a^3*
ln(tan(1/2*f*x+1/2*e)-1)*c^2*d^3-1/2/f/a^3*tan(1/2*f*x+1/2*e)^5*c^2*d^3+1/4
/f/a^3*tan(1/2*f*x+1/2*e)^5*c*d^4-35/2/f/a^3*tan(1/2*f*x+1/2*e)*c^2*d^3+85/
4/f/a^3*tan(1/2*f*x+1/2*e)*c*d^4+10/f/a^3*ln(tan(1/2*f*x+1/2*e)+1)*c^2*d^3-
15/f/a^3*ln(tan(1/2*f*x+1/2*e)+1)*c*d^4-5/f/a^3*d^4/(tan(1/2*f*x+1/2*e)+1)*
c-1/4/f/a^3*tan(1/2*f*x+1/2*e)^5*c^4*d+1/2/f/a^3*tan(1/2*f*x+1/2*e)^5*c^3*d
^2+15/f/a^3*ln(tan(1/2*f*x+1/2*e)-1)*c*d^4-5/f/a^3*d^4/(tan(1/2*f*x+1/2*e)-
1)*c+5/3/f/a^3*tan(1/2*f*x+1/2*e)^3*c^3*d^2-10/3/f/a^3*tan(1/2*f*x+1/2*e)^3
*c^2*d^3+5/2/f/a^3*tan(1/2*f*x+1/2*e)^3*c*d^4+5/4/f/a^3*tan(1/2*f*x+1/2*e)*
c^4*d+5/2/f/a^3*tan(1/2*f*x+1/2*e)*c^3*d^2
```

Maxima [B] time = 1.08155, size = 930, normalized size = 3.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(d^5*(60*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)/(a^3 - 2*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 4
0*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5)/a^3 - 390*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*
x + e)/(cos(f*x + e) + 1) - 1)/a^3) - 15*c*d^4*(40*sin(f*x + e)/((a^3 - a^3
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e
)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e
)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)
/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 10*c^2*d^3*((105*
sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x +
 e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) - 10*c
^3*d^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - c^5*(15*sin(f*x + e
)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5)/a^3 - 15*c^4*d*(5*sin(f*x + e)/(cos(f*x + e) +
1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A] time = 0.57135, size = 1200, normalized size = 4.18

$$\frac{15\left(\left(20\,c^2d^3-30\,cd^4+13\,d^5\right)\cos\left(fx+e\right)^5+3\left(20\,c^2d^3-30\,cd^4+13\,d^5\right)\cos\left(fx+e\right)^4+3\left(20\,c^2d^3-30\,cd^4+13\,d^5\right)\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/60*(15*((20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^5 + 3*(20*c^2*d^3 -
 30*c*d^4 + 13*d^5)*cos(f*x + e)^4 + 3*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos
(f*x + e)^3 + (20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^2)*log(sin(f*x
+ e) + 1) - 15*((20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^5 + 3*(20*c^2
*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^4 + 3*(20*c^2*d^3 - 30*c*d^4 + 13*d^
5)*cos(f*x + e)^3 + (20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^2)*log(-s
in(f*x + e) + 1) + 2*(15*d^5 + 2*(7*c^5 + 15*c^4*d + 20*c^3*d^2 - 220*c^2*d
^3 + 360*c*d^4 - 152*d^5)*cos(f*x + e)^4 + 3*(4*c^5 + 30*c^4*d + 40*c^3*d^2
 - 340*c^2*d^3 + 570*c*d^4 - 239*d^5)*cos(f*x + e)^3 + (4*c^5 + 30*c^4*d +
140*c^3*d^2 - 640*c^2*d^3 + 1170*c*d^4 - 479*d^5)*cos(f*x + e)^2 + 15*(10*c
*d^4 - 3*d^5)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^5 + 3*a^3*f*c
os(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + a^3*f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c^5\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{d^5\sec^6(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{5cd^4\sec^5(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**5*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**5*sec(e + f*x)**6/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(5*c*d**4*sec(e + f*x)**5/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(10*c
**2*d**3*sec(e + f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f
*x) + 1), x) + Integral(10*c**3*d**2*sec(e + f*x)**3/(sec(e + f*x)**3 + 3*s
ec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(5*c**4*d*sec(e + f*x)**
2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A] time = 1.30357, size = 713, normalized size = 2.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(30*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*log(abs(tan(1/2*f*x + 1/2*e) + 1)
)/a^3 - 30*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*log(abs(tan(1/2*f*x + 1/2*e) -
1))/a^3 - 60*(10*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 7*d^5*tan(1/2*f*x + 1/2*e)^
3 - 10*c*d^4*tan(1/2*f*x + 1/2*e) + 5*d^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f
*x + 1/2*e)^2 - 1)^2*a^3) + (3*a^12*c^5*tan(1/2*f*x + 1/2*e)^5 - 15*a^12*c^
4*d*tan(1/2*f*x + 1/2*e)^5 + 30*a^12*c^3*d^2*tan(1/2*f*x + 1/2*e)^5 - 30*a^
12*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 + 15*a^12*c*d^4*tan(1/2*f*x + 1/2*e)^5 -
3*a^12*d^5*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^5*tan(1/2*f*x + 1/2*e)^3 + 10
0*a^12*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 200*a^12*c^2*d^3*tan(1/2*f*x + 1/2*
e)^3 + 150*a^12*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 40*a^12*d^5*tan(1/2*f*x + 1/
```

```
2*e)^3 + 15*a^12*c^5*tan(1/2*f*x + 1/2*e) + 75*a^12*c^4*d*tan(1/2*f*x + 1/2
*e) + 150*a^12*c^3*d^2*tan(1/2*f*x + 1/2*e) - 1050*a^12*c^2*d^3*tan(1/2*f*x
 + 1/2*e) + 1275*a^12*c*d^4*tan(1/2*f*x + 1/2*e) - 465*a^12*d^5*tan(1/2*f*x
 + 1/2*e))/a^15)/f
```

3.227 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=205

$$\frac{\tan(e+fx)\left(-d^2\left(2c^2+10cd-27d^2\right)\sec(e+fx)+21c^2d^2+8c^3d+2c^4-88cd^3+72d^4\right)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{d^3(4c-3d)\tanh^{-1}(\sin(e}{a^3f}$$

```
[Out] ((4*c - 3*d)*d^3*ArcTanh[Sin[e + f*x]])/(a^3*f) + ((c - d)*(2*c + 9*d)*(c +
 d*Sec[e + f*x])^2*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((c - d)
*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((2*c^
4 + 8*c^3*d + 21*c^2*d^2 - 88*c*d^3 + 72*d^4 - d^2*(2*c^2 + 10*c*d - 27*d^2
)*Sec[e + f*x])*Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x]))
```

Rubi [A] time = 0.280187, antiderivative size = 265, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 98, 150, 143, 63, 217, 203}

$$\frac{\tan(e+fx)\left(-d^2\left(2c^2+10cd-27d^2\right)\sec(e+fx)+21c^2d^2+8c^3d+2c^4-88cd^3+72d^4\right)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{2d^3(4c-3d)\tan(e+fx)}{a^2f\sqrt{a-a\sec(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*(4*c - 3*d)*d^3*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x]
)]]*Tan[e + f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
 + ((c - d)*(2*c + 9*d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(15*a*f*(a + a
*Sec[e + f*x])^2) + ((c - d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(5*f*(a +
 a*Sec[e + f*x])^3) + ((2*c^4 + 8*c^3*d + 21*c^2*d^2 - 88*c*d^3 + 72*d^4 -
d^2*(2*c^2 + 10*c*d - 27*d^2)*Sec[e + f*x])*Tan[e + f*x])/(15*f*(a^3 + a^3*
Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^4}{\sqrt{a-ax}(a+ax)^{7/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(c+dx)^2\left(-a^2\left(2c^2\right.\right.}{\sqrt{a-}}\right.}{5af\sqrt{a-a\sec(e+}}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^3}{5f(a+a\sec(e+f}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^3}{5f(a+a\sec(e+f}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^3}{5f(a+a\sec(e+f}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^3}{5f(a+a\sec(e+f}$$

$$= \frac{2(4c-3d)d^3\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(2c+9d)(c+d\sec(e}{15af(a+a\sec(}$$

Mathematica [A] time = 2.37366, size = 292, normalized size = 1.42

$$2\cos\left(\frac{1}{2}(e+fx)\right)\left(4(c-d)^2\left(7c^2+26cd+57d^2\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\cos^4\left(\frac{1}{2}(e+fx)\right)-60d^3\cos^5\left(\frac{1}{2}(e+fx)\right)\left((4c-3d)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(3*(c - d)^4*Sec[e/2]*Sin[(f*x)/2] - 8*(c - d)^3*(2*c +
 3*d)*Cos[(e + f*x)/2]^2*Sec[e/2]*Sin[(f*x)/2] + 4*(c - d)^2*(7*c^2 + 26*c*
d + 57*d^2)*Cos[(e + f*x)/2]^4*Sec[e/2]*Sin[(f*x)/2] - 60*d^3*Cos[(e + f*x)
/2]^5*((4*c - 3*d)*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]]) - d*Sec[e]*Sec[e + f*x]*Sin[f*x]) + 3*(c - d)
^4*Cos[(e + f*x)/2]*Tan[e/2] - 8*(c - d)^3*(2*c + 3*d)*Cos[(e + f*x)/2]^3*T
an[e/2]))/(15*a^3*f*(1 + Cos[e + f*x])^3)
```

Maple [B] time = 0.079, size = 454, normalized size = 2.2

$$\frac{c^4}{20fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5-\frac{c^3d}{5fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5+\frac{3c^2d^2}{10fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5-\frac{cd^3}{5fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5+\frac{d^4}{20fa^3}\left(\text{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 1/20/f/a^3*tan(1/2*f*x+1/2*e)^5*c^4-1/5/f/a^3*tan(1/2*f*x+1/2*e)^5*c^3*d+3/
10/f/a^3*tan(1/2*f*x+1/2*e)^5*c^2*d^2-1/5/f/a^3*tan(1/2*f*x+1/2*e)^5*c*d^3+
```

```
1/20/f/a^3*tan(1/2*f*x+1/2*e)^5*d^4-1/6/f/a^3*tan(1/2*f*x+1/2*e)^3*c^4+1/f/
a^3*tan(1/2*f*x+1/2*e)^3*c^2*d^2-4/3/f/a^3*tan(1/2*f*x+1/2*e)^3*c*d^3+1/2/f
/a^3*tan(1/2*f*x+1/2*e)^3*d^4+1/4/f/a^3*tan(1/2*f*x+1/2*e)*c^4+1/f/a^3*tan(
1/2*f*x+1/2*e)*c^3*d+3/2/f/a^3*tan(1/2*f*x+1/2*e)*c^2*d^2-7/f/a^3*tan(1/2*f
*x+1/2*e)*c*d^3+17/4/f/a^3*tan(1/2*f*x+1/2*e)*d^4-1/f/a^3*d^4/(tan(1/2*f*x+
1/2*e)+1)+4/f/a^3*d^3*ln(tan(1/2*f*x+1/2*e)+1)*c-3/f/a^3*d^4*ln(tan(1/2*f*x
+1/2*e)+1)-1/f/a^3*d^4/(tan(1/2*f*x+1/2*e)-1)-4/f/a^3*d^3*ln(tan(1/2*f*x+1/
2*e)-1)*c+3/f/a^3*d^4*ln(tan(1/2*f*x+1/2*e)-1)
```

Maxima [B] time = 1.05262, size = 641, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*d^4*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a^3) - 4*c*d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e
)/(cos(f*x + e) + 1) - 1)/a^3) + 6*c^2*d^2*(15*sin(f*x + e)/(cos(f*x + e) +
 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5)/a^3 + c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 12*c^
3*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^
5)/a^3)/f
```

Fricas [A] time = 0.533384, size = 910, normalized size = 4.44

$$\frac{15\left(\left(4\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^4+3\left(4\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^3+3\left(4\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^2+\left(4\,cd^3-3\,d^4\right)\cos\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*((4*c*d^3 - 3*d^4)*cos(f*x + e)^4 + 3*(4*c*d^3 - 3*d^4)*cos(f*x +
e)^3 + 3*(4*c*d^3 - 3*d^4)*cos(f*x + e)^2 + (4*c*d^3 - 3*d^4)*cos(f*x + e))
*log(sin(f*x + e) + 1) - 15*((4*c*d^3 - 3*d^4)*cos(f*x + e)^4 + 3*(4*c*d^3
- 3*d^4)*cos(f*x + e)^3 + 3*(4*c*d^3 - 3*d^4)*cos(f*x + e)^2 + (4*c*d^3 - 3
*d^4)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(15*d^4 + (7*c^4 + 12*c^3*d
+ 12*c^2*d^2 - 88*c*d^3 + 72*d^4)*cos(f*x + e)^3 + 3*(2*c^4 + 12*c^3*d + 12
*c^2*d^2 - 68*c*d^3 + 57*d^4)*cos(f*x + e)^2 + (2*c^4 + 12*c^3*d + 42*c^2*d
^2 - 128*c*d^3 + 117*d^4)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^4
 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c^4 \sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{d^4 \sec^5(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{4cd^3 \sec^4(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**4*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**4*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(4*c*d**3*sec(e + f*x)**4/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(6*c*
*2*d**2*sec(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*
x) + 1), x) + Integral(4*c**3*d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A] time = 1.22684, size = 529, normalized size = 2.58

$$-\frac{\frac{120\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^3} - \frac{60\left(4\,cd^3-3\,d^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3} + \frac{60\left(4\,cd^3-3\,d^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3} - \frac{3\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-12\,a^{12}c^3d\,\mathrm{ta}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(120*d^4*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^3) - 60
*(4*c*d^3 - 3*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 + 60*(4*c*d^3 - 3
*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^3 - (3*a^12*c^4*tan(1/2*f*x + 1/
2*e)^5 - 12*a^12*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 18*a^12*c^2*d^2*tan(1/2*f*x
 + 1/2*e)^5 - 12*a^12*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 3*a^12*d^4*tan(1/2*f*x
 + 1/2*e)^5 - 10*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 60*a^12*c^2*d^2*tan(1/2*
f*x + 1/2*e)^3 - 80*a^12*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*d^4*tan(1/2
*f*x + 1/2*e)^3 + 15*a^12*c^4*tan(1/2*f*x + 1/2*e) + 60*a^12*c^3*d*tan(1/2*
f*x + 1/2*e) + 90*a^12*c^2*d^2*tan(1/2*f*x + 1/2*e) - 420*a^12*c*d^3*tan(1/
2*f*x + 1/2*e) + 255*a^12*d^4*tan(1/2*f*x + 1/2*e))/a^15)/f
```

3.228 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=133

$$\frac{d^3\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{(c-d)\tan(e+fx)\left(\left(2c^2+11cd+29d^2\right)\sec(e+fx)+2\left(2c^2+8cd+11d^2\right)\right)}{15af(a\sec(e+fx)+a)^2}+\frac{(c-d)\tan(e+}{5f(a\,\mathrm{se}}$$

```
[Out] (d^3*ArcTanh[Sin[e + f*x]])/(a^3*f) + ((c - d)*(c + d*Sec[e + f*x])^2*Tan[e
 + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((c - d)*(2*(2*c^2 + 8*c*d + 11*d^2
) + (2*c^2 + 11*c*d + 29*d^2)*Sec[e + f*x])*Tan[e + f*x])/(15*a*f*(a + a*Se
c[e + f*x])^2)
```

Rubi [A] time = 0.200819, antiderivative size = 193, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 98, 145, 63, 217, 203}

$$\frac{2d^3\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)\tan(e+fx)\left(\left(2c^2+11cd+29d^2\right)\sec(e+fx)+2\left(2c^2+8cd+11d^2\right)\right)}{15af(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*d^3*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e +
f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((c - d)*
(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((c - d
)*(2*(2*c^2 + 8*c*d + 11*d^2) + (2*c^2 + 11*c*d + 29*d^2)*Sec[e + f*x])*Tan
[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2)
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
```

```
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{(c+dx)^3}{\sqrt{a-ax}(a+ax)^{7/2}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\,\mathrm{Subst}\left(\int \frac{(c+dx)\left(-a^2\left(2c^2+\right.\right.}{\sqrt{a-ax}}\right.}{5af\sqrt{a-a\sec(e+fx}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(c-d)\left(2\left(2c^2+8cd+11d^2\right)+\left(2c^2\right.\right.}{15af(a+}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(c-d)\left(2\left(2c^2+8cd+11d^2\right)+\left(2c^2\right.\right.}{15af(a+}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(c-d)\left(2\left(2c^2+8cd+11d^2\right)+\left(2c^2\right.\right.}{15af(a+}$$

$$= \frac{2d^3\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(c+d\sec(e+fx))^2\tan(e}{5f(a+a\sec(e+fx))^3}$$

Mathematica [B] time = 1.48399, size = 295, normalized size = 2.22

$$\frac{(c-d)\sec\left(\frac{e}{2}\right)\cos\left(\frac{1}{2}(e+fx)\right)\left(-15\left(2c^2+5cd+5d^2\right)\sin\left(e+\frac{fx}{2}\right)+5\left(8c^2+17cd+29d^2\right)\sin\left(\frac{fx}{2}\right)+20c^2\sin\left(e+\frac{3f}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-240*d^3*Cos[(e + f*x)/2]^6*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Lo
g[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (c - d)*Cos[(e + f*x)/2]*Sec[e/2]
*(5*(8*c^2 + 17*c*d + 29*d^2)*Sin[(f*x)/2] - 15*(2*c^2 + 5*c*d + 5*d^2)*Sin
[e + (f*x)/2] + 20*c^2*Sin[e + (3*f*x)/2] + 65*c*d*Sin[e + (3*f*x)/2] + 95*
d^2*Sin[e + (3*f*x)/2] - 15*c^2*Sin[2*e + (3*f*x)/2] - 15*c*d*Sin[2*e + (3*
f*x)/2] - 15*d^2*Sin[2*e + (3*f*x)/2] + 7*c^2*Sin[2*e + (5*f*x)/2] + 16*c*d
*Sin[2*e + (5*f*x)/2] + 22*d^2*Sin[2*e + (5*f*x)/2]))/(30*a^3*f*(1 + Cos[e
+ f*x])^3)
```

Maple [B] time = 0.071, size = 286, normalized size = 2.2

$$-\frac{3c^2d}{20fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5+\frac{3d^2c}{20fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5+\frac{d^2c}{2fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+\frac{3c^2d}{4fa^3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{3d^2c}{4fa^3}\tan\left(\frac{fx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x)
```

```
[Out] -3/20/f/a^3*tan(1/2*f*x+1/2*e)^5*c^2*d+3/20/f/a^3*tan(1/2*f*x+1/2*e)^5*c*d^
2+1/2/f/a^3*tan(1/2*f*x+1/2*e)^3*c*d^2+3/4/f/a^3*tan(1/2*f*x+1/2*e)*c^2*d+3
/4/f/a^3*tan(1/2*f*x+1/2*e)*c*d^2-1/6/f/a^3*tan(1/2*f*x+1/2*e)^3*c^3-1/3/f/
a^3*tan(1/2*f*x+1/2*e)^3*d^3+1/4/f/a^3*tan(1/2*f*x+1/2*e)*c^3-7/4/f/a^3*tan
(1/2*f*x+1/2*e)*d^3+1/f/a^3*d^3*ln(tan(1/2*f*x+1/2*e)+1)-1/f/a^3*d^3*ln(tan
(1/2*f*x+1/2*e)-1)+1/20/f/a^3*tan(1/2*f*x+1/2*e)^5*c^3-1/20/f/a^3*tan(1/2*f
*x+1/2*e)^5*d^3
```

Maxima [B] time = 1.02522, size = 414, normalized size = 3.11

$$-\frac{d^3\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)-\frac{3cd^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right.}{a^3}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*
x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1)
 - 1)/a^3) - 3*c*d^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - c^3*(
15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 9*c^2*d*(5*sin(f*x + e)/(co
s(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A] time = 0.507776, size = 601, normalized size = 4.52

$$15\left(d^3\cos\left(fx+e\right)^3+3\,d^3\cos\left(fx+e\right)^2+3\,d^3\cos\left(fx+e\right)+d^3\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(d^3\cos\left(fx+e\right)^3+3\,d^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2 + 3*d^3*cos(f*x + e) +
d^3)*log(sin(f*x + e) + 1) - 15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2
+ 3*d^3*cos(f*x + e) + d^3)*log(-sin(f*x + e) + 1) + 2*(2*c^3 + 9*c^2*d + 2
1*c*d^2 - 32*d^3 + (7*c^3 + 9*c^2*d + 6*c*d^2 - 22*d^3)*cos(f*x + e)^2 + 3*
(2*c^3 + 9*c^2*d + 6*c*d^2 - 17*d^3)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos
(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{c^3\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\frac{d^3\sec^4\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\frac{3cd^2\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**3*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**3*sec(e + f*x)**4/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(3*c*d**2*sec(e + f*x)**3/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(3*c*
*2*d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x)
+ 1), x))/a**3
```

Giac [B] time = 1.34942, size = 367, normalized size = 2.76

$$\frac{\frac{60\,d^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{60\,d^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}+\frac{3\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-9\,a^{12}c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+9\,a^{12}cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-3\,a^{12}d^3\,t}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(60*d^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 60*d^3*log(abs(tan(1/
2*f*x + 1/2*e) - 1))/a^3 + (3*a^12*c^3*tan(1/2*f*x + 1/2*e)^5 - 9*a^12*c^2*
d*tan(1/2*f*x + 1/2*e)^5 + 9*a^12*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 3*a^12*d^3
*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^3*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c*d^
2*tan(1/2*f*x + 1/2*e)^3 - 20*a^12*d^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^3
*tan(1/2*f*x + 1/2*e) + 45*a^12*c^2*d*tan(1/2*f*x + 1/2*e) + 45*a^12*c*d^2*
tan(1/2*f*x + 1/2*e) - 105*a^12*d^3*tan(1/2*f*x + 1/2*e))/a^15)/f
```

3.229 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=115

$$\frac{\left(2c^2+6cd+7d^2\right)\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{(c-d)^2\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}+\frac{2(c+4d)(c-d)\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}$$

```
[Out] ((c - d)^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (2*(c - d)*(c + 4*d
)*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((2*c^2 + 6*c*d + 7*d^2)*
Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x]))
```

Rubi [A] time = 0.162603, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3987, 89, 78, 37}

$$\frac{\left(2c^2+6cd+7d^2\right)\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{(c-d)^2\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}+\frac{2(c+4d)(c-d)\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((c - d)^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (2*(c - d)*(c + 4*d
)*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((2*c^2 + 6*c*d + 7*d^2)*
Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^2}{\sqrt{a-ax}(a+ax)^{7/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{a^3\left(2c^2+6cd-3d^2\right)+5a^3d^2x}{\sqrt{a-ax}(a+ax)^{5/2}}\,dx, x, \text{sec}\right.}{5a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{2(c-d)(c+4d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} - \frac{\left(\left(2c^2+6cd+7d^2\right)\text{t}\right.}{15f}$$

$$= \frac{(c-d)^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{2(c-d)(c+4d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{\left(2c^2+6cd+7d^2\right)\text{ta}}{15f\left(a^3+a^3\sec(e\right.}$$

Mathematica [A] time = 0.470205, size = 180, normalized size = 1.57

$$\frac{\sec\left(\frac{e}{2}\right)\cos\left(\frac{1}{2}(e+fx)\right)\left(10\left(4c^2+3cd+2d^2\right)\sin\left(\frac{fx}{2}\right)+20c^2\sin\left(e+\frac{3fx}{2}\right)-15c^2\sin\left(2e+\frac{3fx}{2}\right)+7c^2\sin\left(2e+\frac{5fx}{2}\right)\right.}{30a^3f(\cos(e+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (Cos[(e + f*x)/2]*Sec[e/2]*(10*(4*c^2 + 3*c*d + 2*d^2)*Sin[(f*x)/2] - 30*c*
(c + d)*Sin[e + (f*x)/2] + 20*c^2*Sin[e + (3*f*x)/2] + 30*c*d*Sin[e + (3*f*
x)/2] + 10*d^2*Sin[e + (3*f*x)/2] - 15*c^2*Sin[2*e + (3*f*x)/2] + 7*c^2*Sin
[2*e + (5*f*x)/2] + 6*c*d*Sin[2*e + (5*f*x)/2] + 2*d^2*Sin[2*e + (5*f*x)/2]
))/(30*a^3*f*(1 + Cos[e + f*x])^3)
```

Maple [A] time = 0.064, size = 128, normalized size = 1.1

$$\frac{1}{4fa^3}\left(\frac{c^2}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5 - \frac{2cd}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5 + \frac{d^2}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5 - \frac{2c^2}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3 + \frac{2d^2}{3}\left(\tan\left(\frac{fx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 1/4/f/a^3*(1/5*tan(1/2*f*x+1/2*e)^5*c^2-2/5*tan(1/2*f*x+1/2*e)^5*c*d+1/5*ta
n(1/2*f*x+1/2*e)^5*d^2-2/3*tan(1/2*f*x+1/2*e)^3*c^2+2/3*tan(1/2*f*x+1/2*e)^
3*d^2+tan(1/2*f*x+1/2*e)*c^2+2*c*d*tan(1/2*f*x+1/2*e)+tan(1/2*f*x+1/2*e)*d^
2)
```

Maxima [A] time = 1.0211, size = 248, normalized size = 2.16

$$\frac{\frac{d^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{c^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{6\,cd\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(d^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^2*(15*sin(f*x
+ e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 6*c*d*(5*sin(f*x + e)/(cos(f*x + e) +
1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A] time = 0.45978, size = 270, normalized size = 2.35

$$\frac{\left(\left(7\,c^2+6\,cd+2\,d^2\right)\cos\left(fx+e\right)^2+2\,c^2+6\,cd+7\,d^2+6\left(c^2+3\,cd+d^2\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{15\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*((7*c^2 + 6*c*d + 2*d^2)*cos(f*x + e)^2 + 2*c^2 + 6*c*d + 7*d^2 + 6*(c
^2 + 3*c*d + d^2)*cos(f*x + e))*sin(f*x + e)/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{c^2\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\frac{d^2\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\frac{2cd\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,d}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**2*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**2*sec(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(2*c*d*sec(e + f*x)**2/(sec(e
 + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A] time = 1.24547, size = 185, normalized size = 1.61

$$\frac{3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-6\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-10\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+10\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{60\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(3*c^2*tan(1/2*f*x + 1/2*e)^5 - 6*c*d*tan(1/2*f*x + 1/2*e)^5 + 3*d^2*t
an(1/2*f*x + 1/2*e)^5 - 10*c^2*tan(1/2*f*x + 1/2*e)^3 + 10*d^2*tan(1/2*f*x
+ 1/2*e)^3 + 15*c^2*tan(1/2*f*x + 1/2*e) + 30*c*d*tan(1/2*f*x + 1/2*e) + 15
*d^2*tan(1/2*f*x + 1/2*e))/(a^3*f)
```

3.230 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(2c+3d)\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{(2c+3d)\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}+\frac{(c-d)\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

```
[Out] ((c - d)*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((2*c + 3*d)*Tan[e +
f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((2*c + 3*d)*Tan[e + f*x])/(15*f*(a
^3 + a^3*Sec[e + f*x]))
```

Rubi [A] time = 0.115281, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4000, 3796, 3794}

$$\frac{(2c+3d)\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{(2c+3d)\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}+\frac{(c-d)\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((c - d)*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((2*c + 3*d)*Tan[e +
f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((2*c + 3*d)*Tan[e + f*x])/(15*f*(a
^3 + a^3*Sec[e + f*x]))
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{(c-d)\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(2c+3d)\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx}{5a}$$
$$= \frac{(c-d)\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(2c+3d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(2c+3d)\int \frac{\sec(e+fx)}{a+a\sec(e+fx)}\,d}{15a^2}$$
$$= \frac{(c-d)\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(2c+3d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(2c+3d)\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right.}$$

Mathematica [A] time = 0.337327, size = 135, normalized size = 1.32

$$\frac{\sec\left(\frac{e}{2}\right)\cos\left(\frac{1}{2}(e+fx)\right)\left(-15(2c+d)\sin\left(e+\frac{fx}{2}\right)+5(8c+3d)\sin\left(\frac{fx}{2}\right)+20c\sin\left(e+\frac{3fx}{2}\right)-15c\sin\left(2e+\frac{3fx}{2}\right)+7c\,\mathrm{s}\right.}{30a^3f(\cos(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (Cos[(e + f*x)/2]*Sec[e/2]*(5*(8*c + 3*d)*Sin[(f*x)/2] - 15*(2*c + d)*Sin[e
 + (f*x)/2] + 20*c*Sin[e + (3*f*x)/2] + 15*d*Sin[e + (3*f*x)/2] - 15*c*Sin[
2*e + (3*f*x)/2] + 7*c*Sin[2*e + (5*f*x)/2] + 3*d*Sin[2*e + (5*f*x)/2]))/(3
0*a^3*f*(1 + Cos[e + f*x])^3)
```

Maple [A] time = 0.053, size = 64, normalized size = 0.6

$$\frac{1}{4fa^3}\left(\frac{c-d}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5-\frac{2c}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x)
```

```
[Out] 1/4/f/a^3*(1/5*(c-d)*tan(1/2*f*x+1/2*e)^5-2/3*tan(1/2*f*x+1/2*e)^3*c+c*tan(
1/2*f*x+1/2*e)+tan(1/2*f*x+1/2*e)*d)
```

Maxima [A] time = 1.01488, size = 155, normalized size = 1.52

$$\frac{\frac{c\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{3d\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/60*(c*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 3*d*(5*sin(f*x + e
)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A] time = 0.451149, size = 227, normalized size = 2.23

$$\frac{\left((7\,c+3\,d)\cos\left(fx+e\right)^{2}+3\,(2\,c+3\,d)\cos\left(fx+e\right)+2\,c+3\,d\right)\sin\left(fx+e\right)}{15\left(a^{3}f\cos\left(fx+e\right)^{3}+3\,a^{3}f\cos\left(fx+e\right)^{2}+3\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*((7*c + 3*d)*cos(f*x + e)^2 + 3*(2*c + 3*d)*cos(f*x + e) + 2*c + 3*d)*
sin(f*x + e)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f
*x + e) + a^3*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{c\sec\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{d\sec^{2}\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f
*x) + 1), x) + Integral(d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)
**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A] time = 1.2609, size = 108, normalized size = 1.06

$$\frac{3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-3\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-10\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+15\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{60\,a^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/60*(3*c*tan(1/2*f*x + 1/2*e)^5 - 3*d*tan(1/2*f*x + 1/2*e)^5 - 10*c*tan(1/
2*f*x + 1/2*e)^3 + 15*c*tan(1/2*f*x + 1/2*e) + 15*d*tan(1/2*f*x + 1/2*e))/(
a^3*f)
```

3.231 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=181

$$\frac{\left(2c^2-9cd+22d^2\right)\tan(e+fx)}{15f(c-d)^3\left(a^3\sec(e+fx)+a^3\right)}-\frac{2d^3\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^3f(c-d)^{7/2}\sqrt{c+d}}+\frac{(2c-7d)\tan(e+fx)}{15af(c-d)^2(a\sec(e+fx)+a)^2}+\frac{\tan(}{5f(c-d)(a\,\mathrm{se}}$$

```
[Out] (-2*d^3*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(a^3*(c - d)^(
7/2)*Sqrt[c + d]*f) + Tan[e + f*x]/(5*(c - d)*f*(a + a*Sec[e + f*x])^3) + (
(2*c - 7*d)*Tan[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sec[e + f*x])^2) + ((2*c
^2 - 9*c*d + 22*d^2)*Tan[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Sec[e + f*x])
)
```

Rubi [A] time = 0.322949, antiderivative size = 235, normalized size of antiderivative = 1.3, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 104, 152, 12, 93, 205}

$$\frac{\left(2c^2-9cd+22d^2\right)\tan(e+fx)}{15f(c-d)^3\left(a^3\sec(e+fx)+a^3\right)}+\frac{2d^3\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2f(c-d)^{7/2}\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(2c-7d)\tan(e}{15af(c-d)^2(a\sec(e}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/(5*(c - d)*f*(a + a*Sec[e + f*x])^3) + ((2*c - 7*d)*Tan[e + f*
x])/(15*a*(c - d)^2*f*(a + a*Sec[e + f*x])^2) + (2*d^3*ArcTan[(Sqrt[c + d]*
Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f
*x])/(a^2*(c - d)^(7/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]]) + ((2*c^2 - 9*c*d + 22*d^2)*Tan[e + f*x])/(15*(c - d)^3*f*(a^
3 + a^3*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{7/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\operatorname{Subst}\left(\int \frac{-a^2(2c-5d)-2a^2dx}{\sqrt{a-ax}(a+ax)^{5/2}(c+dx)}\right.}{5a(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+}}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2} - \frac{\tan(e}{}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2} + \frac{\left(2c^2\right.}{15(c}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2} + \frac{\left(2c^2\right.}{15(c}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2} + \frac{\left(2c^2\right.}{15(c}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2} + \frac{}{a^2(c}$$

Mathematica [C] time = 3.146, size = 345, normalized size = 1.91

$$\frac{\cos\left(\frac{1}{2}(e+fx)\right)\left(\sec\left(\frac{e}{2}\right)\left(-15\left(2c^2-7cd+9d^2\right)\sin\left(e+\frac{fx}{2}\right)+5\left(8c^2-27cd+37d^2\right)\sin\left(\frac{fx}{2}\right)+20c^2\sin\left(e+\frac{3fx}{2}\right)-1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])),x]
```

```
[Out] (Cos[(e + f*x)/2]*((480*d^3*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c
*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[
(e + f*x)/2]^5*(I*Cos[e] + Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e
])^2]) + Sec[e/2]*(5*(8*c^2 - 27*c*d + 37*d^2)*Sin[(f*x)/2] - 15*(2*c^2 - 7
*c*d + 9*d^2)*Sin[e + (f*x)/2] + 20*c^2*Sin[e + (3*f*x)/2] - 75*c*d*Sin[e +
 (3*f*x)/2] + 115*d^2*Sin[e + (3*f*x)/2] - 15*c^2*Sin[2*e + (3*f*x)/2] + 45
*c*d*Sin[2*e + (3*f*x)/2] - 45*d^2*Sin[2*e + (3*f*x)/2] + 7*c^2*Sin[2*e + (
5*f*x)/2] - 24*c*d*Sin[2*e + (5*f*x)/2] + 32*d^2*Sin[2*e + (5*f*x)/2])))/(3
0*a^3*(c - d)^3*f*(1 + Cos[e + f*x])^3)
```

Maple [A] time = 0.084, size = 203, normalized size = 1.1

$$\frac{1}{4fa^3}\left(\frac{1}{(c-d)^3}\left(\frac{c^2}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5-\frac{2cd}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5+\frac{d^2}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5-\frac{2c^2}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3+2\left(\tan\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x)
```

```
[Out] 1/4/f/a^3*(1/(c-d)^3*(1/5*tan(1/2*f*x+1/2*e)^5*c^2-2/5*tan(1/2*f*x+1/2*e)^5
*c*d+1/5*tan(1/2*f*x+1/2*e)^5*d^2-2/3*tan(1/2*f*x+1/2*e)^3*c^2+2*tan(1/2*f*
x+1/2*e)^3*c*d-4/3*tan(1/2*f*x+1/2*e)^3*d^2+tan(1/2*f*x+1/2*e)*c^2-4*c*d*ta
n(1/2*f*x+1/2*e)+7*tan(1/2*f*x+1/2*e)*d^2)-8*d^3/(c-d)^3/((c+d)*(c-d))^(1/2
)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.571845, size = 2187, normalized size = 12.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [-1/30*(15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2 + 3*d^3*cos(f*x + e)
+ d^3)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)
^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^
2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(2*c^4 - 9*c^3*d + 20*c^2
*d^2 + 9*c*d^3 - 22*d^4 + (7*c^4 - 24*c^3*d + 25*c^2*d^2 + 24*c*d^3 - 32*d^
4)*cos(f*x + e)^2 + 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2 + 9*c*d^3 - 17*d^4)*cos
(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^
2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^3*c^4*d
+ 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 +
 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a
^3*d^5)*f*cos(f*x + e) + (a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2
*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f), -1/15*(15*(d^3*cos(f*x + e)^3 + 3*d^3*cos
(f*x + e)^2 + 3*d^3*cos(f*x + e) + d^3)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2
+ d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (2*c^4 - 9*c^3*d
+ 20*c^2*d^2 + 9*c*d^3 - 22*d^4 + (7*c^4 - 24*c^3*d + 25*c^2*d^2 + 24*c*d^3
 - 32*d^4)*cos(f*x + e)^2 + 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2 + 9*c*d^3 - 17*
d^4)*cos(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 +
2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^
3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x
+ e)^2 + 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c
*d^4 + a^3*d^5)*f*cos(f*x + e) + (a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2
*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(e+fx)}{c\sec^3(e+fx)+3c\sec^2(e+fx)+3c\sec(e+fx)+c+d\sec^4(e+fx)+3d\sec^3(e+fx)+3d\sec^2(e+fx)+d\sec(e+fx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/(c*sec(e + f*x)**3 + 3*c*sec(e + f*x)**2 + 3*c*sec(e
+ f*x) + c + d*sec(e + f*x)**4 + 3*d*sec(e + f*x)**3 + 3*d*sec(e + f*x)**2
+ d*sec(e + f*x)), x)/a**3
```

Giac [B] time = 1.1888, size = 660, normalized size = 3.65

$$-\frac{\frac{120\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d^3}{\left(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3\right)\sqrt{-c^2+d^2}} - \frac{3a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-12a^{12}c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+18a^{12}c^2d^2\tan\left(\frac{1}{2}fx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/60*(120*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*t
an(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d^3/((a^3*
c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*sqrt(-c^2 + d^2)) - (3*a^12*c^4*
tan(1/2*f*x + 1/2*e)^5 - 12*a^12*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 18*a^12*c^2
*d^2*tan(1/2*f*x + 1/2*e)^5 - 12*a^12*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 3*a^12
*d^4*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 50*a^12*
c^3*d*tan(1/2*f*x + 1/2*e)^3 - 90*a^12*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 + 70*
a^12*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 20*a^12*d^4*tan(1/2*f*x + 1/2*e)^3 + 15
*a^12*c^4*tan(1/2*f*x + 1/2*e) - 90*a^12*c^3*d*tan(1/2*f*x + 1/2*e) + 240*a
^12*c^2*d^2*tan(1/2*f*x + 1/2*e) - 270*a^12*c*d^3*tan(1/2*f*x + 1/2*e) + 10
5*a^12*d^4*tan(1/2*f*x + 1/2*e))/(a^15*c^5 - 5*a^15*c^4*d + 10*a^15*c^3*d^2
 - 10*a^15*c^2*d^3 + 5*a^15*c*d^4 - a^15*d^5))/f
```

3.232 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=288

$$\frac{d\left(-12c^2d+2c^3+43cd^2+72d^3\right)\tan(e+fx)}{15a^3f(c-d)^4(c+d)(c+d\sec(e+fx))}+\frac{\left(2c^2-12cd+45d^2\right)\tan(e+fx)}{15f(c-d)^3\left(a^3\sec(e+fx)+a^3\right)(c+d\sec(e+fx))}-\frac{2d^3(4c+3d)\tan\ldots}{a^3f(c-\ldots}$$

```
[Out] (-2*d^3*(4*c + 3*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(a
^3*(c - d)^(9/2)*(c + d)^(3/2)*f) + (d*(2*c^3 - 12*c^2*d + 43*c*d^2 + 72*d^
3)*Tan[e + f*x])/(15*a^3*(c - d)^4*(c + d)*f*(c + d*Sec[e + f*x])) + Tan[e
+ f*x]/(5*(c - d)*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])) + ((2*c -
9*d)*Tan[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sec[e + f*x])^2*(c + d*Sec[e +
f*x])) + ((2*c^2 - 12*c*d + 45*d^2)*Tan[e + f*x])/(15*(c - d)^3*f*(a^3 + a^
3*Sec[e + f*x])*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.486091, antiderivative size = 325, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3987, 103, 152, 12, 93, 205}

$$\frac{\left(-12c^2d+2c^3+43cd^2+72d^3\right)\tan(e+fx)}{15f(c-d)^4(c+d)\left(a^3\sec(e+fx)+a^3\right)}+\frac{2d^3(4c+3d)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2f(c-d)^{9/2}(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{\ldots}{f\left(c^2-d^2\right)\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((c + 6*d)*Tan[e + f*x])/(5*(c - d)^2*(c + d)*f*(a + a*Sec[e + f*x])^3) + (
(2*c^2 - 10*c*d - 27*d^2)*Tan[e + f*x])/(15*a*(c - d)^3*(c + d)*f*(a + a*Se
c[e + f*x])^2) + (2*d^3*(4*c + 3*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e +
f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(a^2*(c - d)^(
9/2)*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (
(2*c^3 - 12*c^2*d + 43*c*d^2 + 72*d^3)*Tan[e + f*x])/(15*(c - d)^4*(c + d)*
f*(a^3 + a^3*Sec[e + f*x])) - (d*Tan[e + f*x])/((c^2 - d^2)*f*(a + a*Sec[e
+ f*x])^3*(c + d*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
```

```
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{7/2}(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{\left(c^2-d^2\right)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))} - \frac{\tan(e+fx)\,\text{Subst}}{\left(c^2-d^2\right)f\sqrt{a}}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} - \frac{d\tan(e+fx)}{\left(c^2-d^2\right)f(a+a\sec(e+fx))^3(c}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{\left(2c^2-10cd-27d^2\right)\tan(e+}{15a(c-d)^3(c+d)f(a+a\sec(e-}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{\left(2c^2-10cd-27d^2\right)\tan(e+}{15a(c-d)^3(c+d)f(a+a\sec(e-}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{\left(2c^2-10cd-27d^2\right)\tan(e+}{15a(c-d)^3(c+d)f(a+a\sec(e-}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{\left(2c^2-10cd-27d^2\right)\tan(e+}{15a(c-d)^3(c+d)f(a+a\sec(e-}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{\left(2c^2-10cd-27d^2\right)\tan(e+}{15a(c-d)^3(c+d)f(a+a\sec(e-}$$

Mathematica [C] time = 7.01324, size = 1772, normalized size = 6.15

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((4*c + 3*d)*Cos[e/2 + (f*x)/2]^6*(d + c*Cos[e + f*x])^2*Sec[e + f*x]^5*(((
16*I)*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Si
n[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]))*((-I)*
d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos
[2*e] - I*Sin[2*e]]) + (16*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]
*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] -
 I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Sin[e])/(Sqrt[
c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^4*(c + d)*(a + a*Sec[
e + f*x])^3*(c + d*Sec[e + f*x])^2) + (Cos[e/2 + (f*x)/2]*(d + c*Cos[e + f*
x])*Sec[e/2]*Sec[e]*Sec[e + f*x]^5*(-55*c^5*Sin[(f*x)/2] + 135*c^4*d*Sin[(f
*x)/2] - 20*c^3*d^2*Sin[(f*x)/2] - 810*c^2*d^3*Sin[(f*x)/2] - 450*c*d^4*Sin
[(f*x)/2] + 150*d^5*Sin[(f*x)/2] + 47*c^5*Sin[(3*f*x)/2] - 137*c^4*d*Sin[(3
*f*x)/2] + 88*c^3*d^2*Sin[(3*f*x)/2] + 812*c^2*d^3*Sin[(3*f*x)/2] + 690*c*d
^4*Sin[(3*f*x)/2] + 75*d^5*Sin[(3*f*x)/2] - 50*c^5*Sin[e - (f*x)/2] + 130*c
^4*d*Sin[e - (f*x)/2] - 10*c^3*d^2*Sin[e - (f*x)/2] - 1030*c^2*d^3*Sin[e -
(f*x)/2] - 990*c*d^4*Sin[e - (f*x)/2] - 150*d^5*Sin[e - (f*x)/2] + 50*c^5*S
in[e + (f*x)/2] - 130*c^4*d*Sin[e + (f*x)/2] + 10*c^3*d^2*Sin[e + (f*x)/2]
+ 1030*c^2*d^3*Sin[e + (f*x)/2] + 765*c*d^4*Sin[e + (f*x)/2] - 150*d^5*Sin[
e + (f*x)/2] - 55*c^5*Sin[2*e + (f*x)/2] + 135*c^4*d*Sin[2*e + (f*x)/2] - 2
0*c^3*d^2*Sin[2*e + (f*x)/2] - 810*c^2*d^3*Sin[2*e + (f*x)/2] - 675*c*d^4*S
```

```
in[2*e + (f*x)/2] - 150*d^5*Sin[2*e + (f*x)/2] - 30*c^5*Sin[e + (3*f*x)/2]
+ 90*c^4*d*Sin[e + (3*f*x)/2] - 60*c^3*d^2*Sin[e + (3*f*x)/2] - 360*c^2*d^3
*Sin[e + (3*f*x)/2] - 30*c*d^4*Sin[e + (3*f*x)/2] + 75*d^5*Sin[e + (3*f*x)/
2] + 47*c^5*Sin[2*e + (3*f*x)/2] - 137*c^4*d*Sin[2*e + (3*f*x)/2] + 88*c^3*
d^2*Sin[2*e + (3*f*x)/2] + 812*c^2*d^3*Sin[2*e + (3*f*x)/2] + 525*c*d^4*Sin
[2*e + (3*f*x)/2] - 75*d^5*Sin[2*e + (3*f*x)/2] - 30*c^5*Sin[3*e + (3*f*x)/
2] + 90*c^4*d*Sin[3*e + (3*f*x)/2] - 60*c^3*d^2*Sin[3*e + (3*f*x)/2] - 360*
c^2*d^3*Sin[3*e + (3*f*x)/2] - 195*c*d^4*Sin[3*e + (3*f*x)/2] - 75*d^5*Sin[
3*e + (3*f*x)/2] + 20*c^5*Sin[e + (5*f*x)/2] - 76*c^4*d*Sin[e + (5*f*x)/2]
+ 106*c^3*d^2*Sin[e + (5*f*x)/2] + 346*c^2*d^3*Sin[e + (5*f*x)/2] + 219*c*d
^4*Sin[e + (5*f*x)/2] + 15*d^5*Sin[e + (5*f*x)/2] - 15*c^5*Sin[2*e + (5*f*x
)/2] + 45*c^4*d*Sin[2*e + (5*f*x)/2] - 30*c^3*d^2*Sin[2*e + (5*f*x)/2] - 90
*c^2*d^3*Sin[2*e + (5*f*x)/2] + 75*c*d^4*Sin[2*e + (5*f*x)/2] + 15*d^5*Sin[
2*e + (5*f*x)/2] + 20*c^5*Sin[3*e + (5*f*x)/2] - 76*c^4*d*Sin[3*e + (5*f*x)
/2] + 106*c^3*d^2*Sin[3*e + (5*f*x)/2] + 346*c^2*d^3*Sin[3*e + (5*f*x)/2] +
 144*c*d^4*Sin[3*e + (5*f*x)/2] - 15*d^5*Sin[3*e + (5*f*x)/2] - 15*c^5*Sin[
4*e + (5*f*x)/2] + 45*c^4*d*Sin[4*e + (5*f*x)/2] - 30*c^3*d^2*Sin[4*e + (5*
f*x)/2] - 90*c^2*d^3*Sin[4*e + (5*f*x)/2] - 15*d^5*Sin[4*e + (5*f*x)/2] + 7
*c^5*Sin[2*e + (7*f*x)/2] - 27*c^4*d*Sin[2*e + (7*f*x)/2] + 38*c^3*d^2*Sin[
2*e + (7*f*x)/2] + 72*c^2*d^3*Sin[2*e + (7*f*x)/2] + 15*c*d^4*Sin[2*e + (7*
f*x)/2] + 15*c*d^4*Sin[3*e + (7*f*x)/2] + 7*c^5*Sin[4*e + (7*f*x)/2] - 27*c
^4*d*Sin[4*e + (7*f*x)/2] + 38*c^3*d^2*Sin[4*e + (7*f*x)/2] + 72*c^2*d^3*Si
n[4*e + (7*f*x)/2]))/(120*c*(-c + d)^4*(c + d)*f*(a + a*Sec[e + f*x])^3*(c
+ d*Sec[e + f*x])^2)
```

Maple [A] time = 0.104, size = 284, normalized size = 1.

$$\frac{1}{4fa^3}\left(\frac{1}{\left(c^2-2cd+d^2\right)(c-d)^2}\left(\frac{c^2}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5-\frac{2cd}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5+\frac{d^2}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5-\frac{2c^2}{3}\left(\tan\left(\frac{fx}{2}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 1/4/f/a^3*(1/(c^2-2*c*d+d^2)/(c-d)^2*(1/5*tan(1/2*f*x+1/2*e)^5*c^2-2/5*tan(
1/2*f*x+1/2*e)^5*c*d+1/5*tan(1/2*f*x+1/2*e)^5*d^2-2/3*tan(1/2*f*x+1/2*e)^3*
c^2+8/3*tan(1/2*f*x+1/2*e)^3*c*d-2*tan(1/2*f*x+1/2*e)^3*d^2+tan(1/2*f*x+1/2
*e)*c^2-6*c*d*tan(1/2*f*x+1/2*e)+17*tan(1/2*f*x+1/2*e)*d^2)+16*d^3/(c-d)^4*
(-1/2*d/(c+d)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)
^2*d-c-d)-1/2*(4*c+3*d)/(c+d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e
)*(c-d)/((c+d)*(c-d))^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.706203, size = 3638, normalized size = 12.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [1/30*(15*(4*c*d^4 + 3*d^5 + (4*c^2*d^3 + 3*c*d^4)*cos(f*x + e)^4 + (12*c^2
*d^3 + 13*c*d^4 + 3*d^5)*cos(f*x + e)^3 + 3*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*c
os(f*x + e)^2 + (4*c^2*d^3 + 15*c*d^4 + 9*d^5)*cos(f*x + e))*sqrt(c^2 - d^2
)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2
)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*
c*d*cos(f*x + e) + d^2)) + 2*(2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c^2*d^
4 - 43*c*d^5 - 72*d^6 + (7*c^6 - 27*c^5*d + 31*c^4*d^2 + 99*c^3*d^3 - 23*c^
2*d^4 - 72*c*d^5 - 15*d^6)*cos(f*x + e)^3 + (6*c^6 - 29*c^5*d + 51*c^4*d^2
+ 193*c^3*d^3 + 60*c^2*d^4 - 164*c*d^5 - 117*d^6)*cos(f*x + e)^2 + (2*c^6 -
 6*c^5*d + 5*c^4*d^2 + 147*c^3*d^3 + 164*c^2*d^4 - 141*c*d^5 - 171*d^6)*cos
(f*x + e))*sin(f*x + e))/((a^3*c^8 - 3*a^3*c^7*d + a^3*c^6*d^2 + 5*a^3*c^5*
d^3 - 5*a^3*c^4*d^4 - a^3*c^3*d^5 + 3*a^3*c^2*d^6 - a^3*c*d^7)*f*cos(f*x +
e)^4 + (3*a^3*c^8 - 8*a^3*c^7*d + 16*a^3*c^5*d^3 - 10*a^3*c^4*d^4 - 8*a^3*c
^3*d^5 + 8*a^3*c^2*d^6 - a^3*d^8)*f*cos(f*x + e)^3 + 3*(a^3*c^8 - 2*a^3*c^7
*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*
c*d^7 - a^3*d^8)*f*cos(f*x + e)^2 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^
3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x +
e) + (a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d
^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f), -1/15*(15*(4*c*d^4 + 3*d^5 +
(4*c^2*d^3 + 3*c*d^4)*cos(f*x + e)^4 + (12*c^2*d^3 + 13*c*d^4 + 3*d^5)*cos(
f*x + e)^3 + 3*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e)^2 + (4*c^2*d^3 +
15*c*d^4 + 9*d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(
d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (2*c^5*d - 12*c^4*d^2 + 4
1*c^3*d^3 + 84*c^2*d^4 - 43*c*d^5 - 72*d^6 + (7*c^6 - 27*c^5*d + 31*c^4*d^2
 + 99*c^3*d^3 - 23*c^2*d^4 - 72*c*d^5 - 15*d^6)*cos(f*x + e)^3 + (6*c^6 - 2
9*c^5*d + 51*c^4*d^2 + 193*c^3*d^3 + 60*c^2*d^4 - 164*c*d^5 - 117*d^6)*cos(
f*x + e)^2 + (2*c^6 - 6*c^5*d + 5*c^4*d^2 + 147*c^3*d^3 + 164*c^2*d^4 - 141
*c*d^5 - 171*d^6)*cos(f*x + e))*sin(f*x + e))/((a^3*c^8 - 3*a^3*c^7*d + a^3
*c^6*d^2 + 5*a^3*c^5*d^3 - 5*a^3*c^4*d^4 - a^3*c^3*d^5 + 3*a^3*c^2*d^6 - a^
3*c*d^7)*f*cos(f*x + e)^4 + (3*a^3*c^8 - 8*a^3*c^7*d + 16*a^3*c^5*d^3 - 10*
a^3*c^4*d^4 - 8*a^3*c^3*d^5 + 8*a^3*c^2*d^6 - a^3*d^8)*f*cos(f*x + e)^3 + 3
*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2
*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^2 + (a^3*c^8 - 8*a^3*c
^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*
a^3*d^8)*f*cos(f*x + e) + (a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*
c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21708, size = 1284, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/60*(120*d^4*tan(1/2*f*x + 1/2*e)/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2
 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan
(1/2*f*x + 1/2*e)^2 - c - d)) + 120*(4*c*d^3 + 3*d^4)*(pi*floor(1/2*(f*x +
e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*
f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 +
2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*sqrt(-c^2 + d^2)) - (3*a^12*c^8*tan(
1/2*f*x + 1/2*e)^5 - 24*a^12*c^7*d*tan(1/2*f*x + 1/2*e)^5 + 84*a^12*c^6*d^2
*tan(1/2*f*x + 1/2*e)^5 - 168*a^12*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 + 210*a^1
2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 - 168*a^12*c^3*d^5*tan(1/2*f*x + 1/2*e)^5
+ 84*a^12*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 24*a^12*c*d^7*tan(1/2*f*x + 1/2*
e)^5 + 3*a^12*d^8*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^8*tan(1/2*f*x + 1/2*e)
^3 + 100*a^12*c^7*d*tan(1/2*f*x + 1/2*e)^3 - 420*a^12*c^6*d^2*tan(1/2*f*x +
 1/2*e)^3 + 980*a^12*c^5*d^3*tan(1/2*f*x + 1/2*e)^3 - 1400*a^12*c^4*d^4*tan
(1/2*f*x + 1/2*e)^3 + 1260*a^12*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 700*a^12*c
^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 220*a^12*c*d^7*tan(1/2*f*x + 1/2*e)^3 - 30*
a^12*d^8*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^8*tan(1/2*f*x + 1/2*e) - 180*a^
12*c^7*d*tan(1/2*f*x + 1/2*e) + 1020*a^12*c^6*d^2*tan(1/2*f*x + 1/2*e) - 31
80*a^12*c^5*d^3*tan(1/2*f*x + 1/2*e) + 5850*a^12*c^4*d^4*tan(1/2*f*x + 1/2*
e) - 6540*a^12*c^3*d^5*tan(1/2*f*x + 1/2*e) + 4380*a^12*c^2*d^6*tan(1/2*f*x
 + 1/2*e) - 1620*a^12*c*d^7*tan(1/2*f*x + 1/2*e) + 255*a^12*d^8*tan(1/2*f*x
 + 1/2*e))/(a^15*c^10 - 10*a^15*c^9*d + 45*a^15*c^8*d^2 - 120*a^15*c^7*d^3
+ 210*a^15*c^6*d^4 - 252*a^15*c^5*d^5 + 210*a^15*c^4*d^6 - 120*a^15*c^3*d^7
 + 45*a^15*c^2*d^8 - 10*a^15*c*d^9 + a^15*d^10))/f
```

3.233 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=368

$$-\frac{d^3(20c^2+30cd+13d^2)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^3f(c-d)^{11/2}(c+d)^{5/2}}+\frac{d(142c^2d^2-30c^3d+4c^4+525cd^3+304d^4)\tan(e+fx)}{30a^3f(c-d)^5(c+d)^2(c+d\sec(e+fx))}+\frac{d(-3}{3(}$$

```
[Out] -((d^3*(20*c^2 + 30*c*d + 13*d^2)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sq
rt[c + d]])/(a^3*(c - d)^(11/2)*(c + d)^(5/2)*f)) + (d*(4*c^3 - 30*c^2*d +
146*c*d^2 + 195*d^3)*Tan[e + f*x])/(30*a^3*(c - d)^4*(c + d)*f*(c + d*Sec[e
 + f*x])^2) + Tan[e + f*x]/(5*(c - d)*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e
 + f*x])^2) + ((2*c - 11*d)*Tan[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sec[e +
f*x])^2*(c + d*Sec[e + f*x])^2) + ((2*c^2 - 15*c*d + 76*d^2)*Tan[e + f*x])/
(15*(c - d)^3*f*(a^3 + a^3*Sec[e + f*x])*(c + d*Sec[e + f*x])^2) + (d*(4*c^
4 - 30*c^3*d + 142*c^2*d^2 + 525*c*d^3 + 304*d^4)*Tan[e + f*x])/(30*a^3*(c
- d)^5*(c + d)^2*f*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.706437, antiderivative size = 414, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3987, 103, 151, 152, 12, 93, 205}

$$\frac{d^3(20c^2+30cd+13d^2)\tan(e+fx)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2f(c-d)^{11/2}(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(142c^2d^2-30c^3d+4c^4+525cd^3+304d^4)\tan(e+fx)}{30f(c-d)^5(c+d)^2\left(a^3\sec(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c^2 + 39*c*d + 22*d^2)*Tan[e + f*x])/(10*(c - d)^3*(c + d)^2*f*(a + a*S
ec[e + f*x])^3) + ((4*c^3 - 26*c^2*d - 184*c*d^2 - 109*d^3)*Tan[e + f*x])/(
30*a*(c - d)^4*(c + d)^2*f*(a + a*Sec[e + f*x])^2) + (d^3*(20*c^2 + 30*c*d
+ 13*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a
 - a*Sec[e + f*x]])]*Tan[e + f*x])/(a^2*(c - d)^(11/2)*(c + d)^(5/2)*f*Sqrt
[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((4*c^4 - 30*c^3*d + 142*c
^2*d^2 + 525*c*d^3 + 304*d^4)*Tan[e + f*x])/(30*(c - d)^5*(c + d)^2*f*(a^3
+ a^3*Sec[e + f*x])) - (d*Tan[e + f*x])/(2*(c^2 - d^2)*f*(a + a*Sec[e + f*x
])^3*(c + d*Sec[e + f*x])^2) - (3*d*(2*c + d)*Tan[e + f*x])/(2*(c^2 - d^2)^
2*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x]))
```

Rule 3987

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[(
a^2*g*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]),
Subst[Int[((g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^n)/Sqrt[a - b*x], x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{7/2}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} - \frac{\tan(e+fx)\,\text{Sul}}{2\left(c^2-d^2\right)}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} - \frac{3}{2\left(c^2-d^2\right)^2 f(a}$$

$$= \frac{\left(2c^2+39cd+22d^2\right)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} - \frac{d\tan(e+f}{2\left(c^2-d^2\right)f(a+a\sec(e+fx)}$$

$$= \frac{\left(2c^2+39cd+22d^2\right)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{\left(4c^3-26c^2d-184cd^2-109d\right.}{30a(c-d)^4(c+d)^2f(a+a\,\text{s}}$$

$$= \frac{\left(2c^2+39cd+22d^2\right)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{\left(4c^3-26c^2d-184cd^2-109d\right.}{30a(c-d)^4(c+d)^2f(a+a\,\text{s}}$$

$$= \frac{\left(2c^2+39cd+22d^2\right)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{\left(4c^3-26c^2d-184cd^2-109d\right.}{30a(c-d)^4(c+d)^2f(a+a\,\text{s}}$$

$$= \frac{\left(2c^2+39cd+22d^2\right)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{\left(4c^3-26c^2d-184cd^2-109d\right.}{30a(c-d)^4(c+d)^2f(a+a\,\text{s}}$$

$$= \frac{\left(2c^2+39cd+22d^2\right)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{\left(4c^3-26c^2d-184cd^2-109d\right.}{30a(c-d)^4(c+d)^2f(a+a\,\text{s}}$$

Mathematica [C] time = 7.69333, size = 1096, normalized size = 2.98

$$\frac{4\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)(d+c\cos(e+fx))^3\sec\left(\frac{e}{2}\right)\left(23d\sin\left(\frac{e}{2}\right)-8c\sin\left(\frac{e}{2}\right)\right)\sec^6(e+fx)}{15(d-c)^4f(\sec(e+fx)a+a)^3(c+d\sec(e+fx))^3} + \frac{\left(20c^2+30dc+13d^2\right)\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] (4*Cos[e/2 + (f*x)/2]^4*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e + f*x]^6*(-8*
c*Sin[e/2] + 23*d*Sin[e/2]))/(15*(-c + d)^4*f*(a + a*Sec[e + f*x])^3*(c + d
*Sec[e + f*x])^3) + ((20*c^2 + 30*c*d + 13*d^2)*Cos[e/2 + (f*x)/2]^6*(d + c
*Cos[e + f*x])^3*Sec[e + f*x]^6*(((-8*I)*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e]/(S
qrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*S
qrt[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*
Cos[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (8*d^3*ArcTan[Sec
[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e]
)/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c
*Sin[e + (f*x)/2])]*Sin[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])
))/((-c + d)^5*(c + d)^2*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) - (
2*Cos[e/2 + (f*x)/2]*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e + f*x]^6*Sin[(f*
```

```
x)/2])/(5*(-c + d)^3*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) + (4*
Cos[e/2 + (f*x)/2]^3*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e + f*x]^6*(-8*c*S
in[(f*x)/2] + 23*d*Sin[(f*x)/2]))/(15*(-c + d)^4*f*(a + a*Sec[e + f*x])^3*(
c + d*Sec[e + f*x])^3) - (8*Cos[e/2 + (f*x)/2]^5*(d + c*Cos[e + f*x])^3*Sec
[e/2]*Sec[e + f*x]^6*(7*c^2*Sin[(f*x)/2] - 44*c*d*Sin[(f*x)/2] + 127*d^2*Si
n[(f*x)/2]))/(15*(-c + d)^5*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3
) + (4*Cos[e/2 + (f*x)/2]^6*(d + c*Cos[e + f*x])*Sec[e]*Sec[e + f*x]^6*(d^6
*Sin[e] - c*d^5*Sin[f*x]))/(c^2*(-c + d)^4*(c + d)*f*(a + a*Sec[e + f*x])^3
*(c + d*Sec[e + f*x])^3) - (4*Cos[e/2 + (f*x)/2]^6*(d + c*Cos[e + f*x])^2*S
ec[e]*Sec[e + f*x]^6*(-11*c^2*d^5*Sin[e] - 6*c*d^6*Sin[e] + 2*d^7*Sin[e] +
10*c^3*d^4*Sin[f*x] + 6*c^2*d^5*Sin[f*x] - c*d^6*Sin[f*x]))/(c^2*(-c + d)^5
*(c + d)^2*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) - (2*Cos[e/2 +
(f*x)/2]^2*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^6*Tan[e/2])/(5*(-c + d)^3*f*
(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3)
```

Maple [A] time = 0.116, size = 365, normalized size = 1.

$$\frac{1}{4fa^3}\left(\frac{1}{\left(c^3-3c^2d+3d^2c-d^3\right)\left(c^2-2cd+d^2\right)}\left(\frac{c^2}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5-\frac{2cd}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5+\frac{d^2}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x)
```

```
[Out] 1/4/f/a^3*(1/(c^3-3*c^2*d+3*c*d^2-d^3)/(c^2-2*c*d+d^2)*(1/5*tan(1/2*f*x+1/2
*e)^5*c^2-2/5*tan(1/2*f*x+1/2*e)^5*c*d+1/5*tan(1/2*f*x+1/2*e)^5*d^2-2/3*tan
(1/2*f*x+1/2*e)^3*c^2+10/3*tan(1/2*f*x+1/2*e)^3*c*d-8/3*tan(1/2*f*x+1/2*e)^
3*d^2+tan(1/2*f*x+1/2*e)*c^2-8*c*d*tan(1/2*f*x+1/2*e)+31*tan(1/2*f*x+1/2*e)
*d^2)+16*d^3/(c-d)^5*((-1/4*d*(10*c^2-3*c*d-7*d^2)/(c^2+2*c*d+d^2)*tan(1/2*
f*x+1/2*e)^3+5/4*d*(2*c+d)/(c+d)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*
c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2-1/4*(20*c^2+30*c*d+13*d^2)/(c^2+2*c*d+d^2)/
((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.895089, size = 5948, normalized size = 16.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/60*(15*(20*c^2*d^5 + 30*c*d^6 + 13*d^7 + (20*c^4*d^3 + 30*c^3*d^4 + 13*
c^2*d^5)*cos(f*x + e)^5 + (60*c^4*d^3 + 130*c^3*d^4 + 99*c^2*d^5 + 26*c*d^6
)*cos(f*x + e)^4 + (60*c^4*d^3 + 210*c^3*d^4 + 239*c^2*d^5 + 108*c*d^6 + 13
*d^7)*cos(f*x + e)^3 + (20*c^4*d^3 + 150*c^3*d^4 + 253*c^2*d^5 + 168*c*d^6
+ 39*d^7)*cos(f*x + e)^2 + (40*c^3*d^4 + 120*c^2*d^5 + 116*c*d^6 + 39*d^7)*
cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f
*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d
^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(4*c^6*d^2 - 30*c^
5*d^3 + 138*c^4*d^4 + 555*c^3*d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8 + (14
*c^8 - 60*c^7*d + 78*c^6*d^2 + 480*c^5*d^3 + 312*c^4*d^4 - 330*c^3*d^5 - 41
9*c^2*d^6 - 90*c*d^7 + 15*d^8)*cos(f*x + e)^4 + (12*c^8 - 62*c^7*d + 114*c^
6*d^2 + 1056*c^5*d^3 + 1626*c^4*d^4 - 81*c^3*d^5 - 1707*c^2*d^6 - 913*c*d^7
 - 45*d^8)*cos(f*x + e)^3 + (4*c^8 - 6*c^7*d - 28*c^6*d^2 + 828*c^5*d^3 + 2
400*c^4*d^4 + 1197*c^3*d^5 - 1897*c^2*d^6 - 2019*c*d^7 - 479*d^8)*cos(f*x +
 e)^2 + (8*c^7*d - 48*c^6*d^2 + 186*c^5*d^3 + 1224*c^4*d^4 + 1539*c^3*d^5 -
 459*c^2*d^6 - 1733*c*d^7 - 717*d^8)*cos(f*x + e))*sin(f*x + e))/((a^3*c^11
 - 3*a^3*c^10*d + 8*a^3*c^8*d^3 - 6*a^3*c^7*d^4 - 6*a^3*c^6*d^5 + 8*a^3*c^5
*d^6 - 3*a^3*c^3*d^8 + a^3*c^2*d^9)*f*cos(f*x + e)^5 + (3*a^3*c^11 - 7*a^3*
c^10*d - 6*a^3*c^9*d^2 + 24*a^3*c^8*d^3 - 2*a^3*c^7*d^4 - 30*a^3*c^6*d^5 +
12*a^3*c^5*d^6 + 16*a^3*c^4*d^7 - 9*a^3*c^3*d^8 - 3*a^3*c^2*d^9 + 2*a^3*c*d
^10)*f*cos(f*x + e)^4 + (3*a^3*c^11 - 3*a^3*c^10*d - 17*a^3*c^9*d^2 + 21*a^
3*c^8*d^3 + 30*a^3*c^7*d^4 - 46*a^3*c^6*d^5 - 18*a^3*c^5*d^6 + 42*a^3*c^4*d
^7 - a^3*c^3*d^8 - 15*a^3*c^2*d^9 + 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)
^3 + (a^3*c^11 + 3*a^3*c^10*d - 15*a^3*c^9*d^2 - a^3*c^8*d^3 + 42*a^3*c^7*d
^4 - 18*a^3*c^6*d^5 - 46*a^3*c^5*d^6 + 30*a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 17
*a^3*c^2*d^9 - 3*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e)^2 + (2*a^3*c^10*d
- 3*a^3*c^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7*d^4 + 12*a^3*c^6*d^5 - 30*a^3*
c^5*d^6 - 2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3
*a^3*d^11)*f*cos(f*x + e) + (a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 -
6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f)
, -1/30*(15*(20*c^2*d^5 + 30*c*d^6 + 13*d^7 + (20*c^4*d^3 + 30*c^3*d^4 + 13
*c^2*d^5)*cos(f*x + e)^5 + (60*c^4*d^3 + 130*c^3*d^4 + 99*c^2*d^5 + 26*c*d^
6)*cos(f*x + e)^4 + (60*c^4*d^3 + 210*c^3*d^4 + 239*c^2*d^5 + 108*c*d^6 + 1
3*d^7)*cos(f*x + e)^3 + (20*c^4*d^3 + 150*c^3*d^4 + 253*c^2*d^5 + 168*c*d^6
 + 39*d^7)*cos(f*x + e)^2 + (40*c^3*d^4 + 120*c^2*d^5 + 116*c*d^6 + 39*d^7)
*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) +
c)/((c^2 - d^2)*sin(f*x + e))) - (4*c^6*d^2 - 30*c^5*d^3 + 138*c^4*d^4 + 55
5*c^3*d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8 + (14*c^8 - 60*c^7*d + 78*c^6
*d^2 + 480*c^5*d^3 + 312*c^4*d^4 - 330*c^3*d^5 - 419*c^2*d^6 - 90*c*d^7 + 1
5*d^8)*cos(f*x + e)^4 + (12*c^8 - 62*c^7*d + 114*c^6*d^2 + 1056*c^5*d^3 + 1
626*c^4*d^4 - 81*c^3*d^5 - 1707*c^2*d^6 - 913*c*d^7 - 45*d^8)*cos(f*x + e)^
3 + (4*c^8 - 6*c^7*d - 28*c^6*d^2 + 828*c^5*d^3 + 2400*c^4*d^4 + 1197*c^3*d
^5 - 1897*c^2*d^6 - 2019*c*d^7 - 479*d^8)*cos(f*x + e)^2 + (8*c^7*d - 48*c^
6*d^2 + 186*c^5*d^3 + 1224*c^4*d^4 + 1539*c^3*d^5 - 459*c^2*d^6 - 1733*c*d^
7 - 717*d^8)*cos(f*x + e))*sin(f*x + e))/((a^3*c^11 - 3*a^3*c^10*d + 8*a^3*
c^8*d^3 - 6*a^3*c^7*d^4 - 6*a^3*c^6*d^5 + 8*a^3*c^5*d^6 - 3*a^3*c^3*d^8 + a
^3*c^2*d^9)*f*cos(f*x + e)^5 + (3*a^3*c^11 - 7*a^3*c^10*d - 6*a^3*c^9*d^2 +
 24*a^3*c^8*d^3 - 2*a^3*c^7*d^4 - 30*a^3*c^6*d^5 + 12*a^3*c^5*d^6 + 16*a^3*
c^4*d^7 - 9*a^3*c^3*d^8 - 3*a^3*c^2*d^9 + 2*a^3*c*d^10)*f*cos(f*x + e)^4 +
(3*a^3*c^11 - 3*a^3*c^10*d - 17*a^3*c^9*d^2 + 21*a^3*c^8*d^3 + 30*a^3*c^7*d
^4 - 46*a^3*c^6*d^5 - 18*a^3*c^5*d^6 + 42*a^3*c^4*d^7 - a^3*c^3*d^8 - 15*a^
3*c^2*d^9 + 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^3 + (a^3*c^11 + 3*a^3*c
^10*d - 15*a^3*c^9*d^2 - a^3*c^8*d^3 + 42*a^3*c^7*d^4 - 18*a^3*c^6*d^5 - 46
*a^3*c^5*d^6 + 30*a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 17*a^3*c^2*d^9 - 3*a^3*c*d
^10 + 3*a^3*d^11)*f*cos(f*x + e)^2 + (2*a^3*c^10*d - 3*a^3*c^9*d^2 - 9*a^3*
c^8*d^3 + 16*a^3*c^7*d^4 + 12*a^3*c^6*d^5 - 30*a^3*c^5*d^6 - 2*a^3*c^4*d^7
```

```
+ 24*a^3*c^3*d^8 - 6*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e
) + (a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6 - 6*a^3*c^
4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.53807, size = 1916, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(60*(20*c^2*d^3 + 30*c*d^4 + 13*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2
)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e
))/sqrt(-c^2 + d^2)))/((a^3*c^7 - 3*a^3*c^6*d + a^3*c^5*d^2 + 5*a^3*c^4*d^3
 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 + 3*a^3*c*d^6 - a^3*d^7)*sqrt(-c^2 + d^2)) -
 (3*a^12*c^12*tan(1/2*f*x + 1/2*e)^5 - 36*a^12*c^11*d*tan(1/2*f*x + 1/2*e)^
5 + 198*a^12*c^10*d^2*tan(1/2*f*x + 1/2*e)^5 - 660*a^12*c^9*d^3*tan(1/2*f*x
 + 1/2*e)^5 + 1485*a^12*c^8*d^4*tan(1/2*f*x + 1/2*e)^5 - 2376*a^12*c^7*d^5*
tan(1/2*f*x + 1/2*e)^5 + 2772*a^12*c^6*d^6*tan(1/2*f*x + 1/2*e)^5 - 2376*a^
12*c^5*d^7*tan(1/2*f*x + 1/2*e)^5 + 1485*a^12*c^4*d^8*tan(1/2*f*x + 1/2*e)^
5 - 660*a^12*c^3*d^9*tan(1/2*f*x + 1/2*e)^5 + 198*a^12*c^2*d^10*tan(1/2*f*x
 + 1/2*e)^5 - 36*a^12*c*d^11*tan(1/2*f*x + 1/2*e)^5 + 3*a^12*d^12*tan(1/2*f
*x + 1/2*e)^5 - 10*a^12*c^12*tan(1/2*f*x + 1/2*e)^3 + 150*a^12*c^11*d*tan(1
/2*f*x + 1/2*e)^3 - 990*a^12*c^10*d^2*tan(1/2*f*x + 1/2*e)^3 + 3850*a^12*c^
9*d^3*tan(1/2*f*x + 1/2*e)^3 - 9900*a^12*c^8*d^4*tan(1/2*f*x + 1/2*e)^3 + 1
7820*a^12*c^7*d^5*tan(1/2*f*x + 1/2*e)^3 - 23100*a^12*c^6*d^6*tan(1/2*f*x +
 1/2*e)^3 + 21780*a^12*c^5*d^7*tan(1/2*f*x + 1/2*e)^3 - 14850*a^12*c^4*d^8*
tan(1/2*f*x + 1/2*e)^3 + 7150*a^12*c^3*d^9*tan(1/2*f*x + 1/2*e)^3 - 2310*a^
12*c^2*d^10*tan(1/2*f*x + 1/2*e)^3 + 450*a^12*c*d^11*tan(1/2*f*x + 1/2*e)^3
 - 40*a^12*d^12*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^12*tan(1/2*f*x + 1/2*e)
- 270*a^12*c^11*d*tan(1/2*f*x + 1/2*e) + 2340*a^12*c^10*d^2*tan(1/2*f*x + 1
/2*e) - 11850*a^12*c^9*d^3*tan(1/2*f*x + 1/2*e) + 38475*a^12*c^8*d^4*tan(1/
2*f*x + 1/2*e) - 84780*a^12*c^7*d^5*tan(1/2*f*x + 1/2*e) + 131040*a^12*c^6*
d^6*tan(1/2*f*x + 1/2*e) - 144180*a^12*c^5*d^7*tan(1/2*f*x + 1/2*e) + 11272
5*a^12*c^4*d^8*tan(1/2*f*x + 1/2*e) - 61350*a^12*c^3*d^9*tan(1/2*f*x + 1/2*
e) + 22140*a^12*c^2*d^10*tan(1/2*f*x + 1/2*e) - 4770*a^12*c*d^11*tan(1/2*f*
x + 1/2*e) + 465*a^12*d^12*tan(1/2*f*x + 1/2*e))/(a^15*c^15 - 15*a^15*c^14*
d + 105*a^15*c^13*d^2 - 455*a^15*c^12*d^3 + 1365*a^15*c^11*d^4 - 3003*a^15*
c^10*d^5 + 5005*a^15*c^9*d^6 - 6435*a^15*c^8*d^7 + 6435*a^15*c^7*d^8 - 5005
*a^15*c^6*d^9 + 3003*a^15*c^5*d^10 - 1365*a^15*c^4*d^11 + 455*a^15*c^3*d^12
 - 105*a^15*c^2*d^13 + 15*a^15*c*d^14 - a^15*d^15) + 60*(10*c^2*d^4*tan(1/2
*f*x + 1/2*e)^3 - 3*c*d^5*tan(1/2*f*x + 1/2*e)^3 - 7*d^6*tan(1/2*f*x + 1/2*
```

```
e)^3 - 10*c^2*d^4*tan(1/2*f*x + 1/2*e) - 15*c*d^5*tan(1/2*f*x + 1/2*e) - 5*
d^6*tan(1/2*f*x + 1/2*e))/((a^3*c^7 - 3*a^3*c^6*d + a^3*c^5*d^2 + 5*a^3*c^4
*d^3 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 + 3*a^3*c*d^6 - a^3*d^7)*(c*tan(1/2*f*x
+ 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

3.234 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{d}f}$$

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[d]*f)
```

Rubi [A] time = 0.154608, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {3980, 206}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[d]*f)
```

Rule 3980

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[(-2*b)/f, Subst
[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
+ d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx &= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{1-adx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}\\ &= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{d}f}\end{aligned}$$

Mathematica [A] time = 0.226638, size = 102, normalized size = 1.67

$$\frac{\sqrt{2}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\sqrt{c\cos(e+fx)+d}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c\cos(e+fx)+d}}\right)}{\sqrt{d}f\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],
x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]
]]*Sqrt[d + c*Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(S
qrt[d]*f*Sqrt[c + d*Sec[e + f*x]])
```

Maple [B] time = 0.364, size = 302, normalized size = 5.

$$-\frac{\sqrt{2}\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}{f\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d+c\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\left(\ln\left(2\frac{1}{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*2^(1/2)/(-d)^(1/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*((d+c*cos(f*x
+e))/cos(f*x+e))^(1/2)*cos(f*x+e)*(ln(2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*
x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)+c*cos(f*x+
e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))-ln(-2*(2^(1/2)*(-d)^(1/2)*(
-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-c*sin(f*x+e)-d*sin(f*x
+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/(-1+cos(f*x+e)+sin(f*x+e))))*(-1+cos(f*x
+e))/sin(f*x+e)^2/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c), x
)
```

Fricas [B] time = 0.966249, size = 738, normalized size = 12.1

$$\left[\frac{\sqrt{\frac{a}{d}}\log\left(-\frac{8\,acd\cos\left(fx+e\right)+\left(ac^{2}-6\,acd+ad^{2}\right)\cos\left(fx+e\right)^{3}+4\left(2\,d^{2}\cos\left(fx+e\right)+\left(cd-d^{2}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{a}{d}}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)+d}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)+8\,ad^{2}+(a}{\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}}\right.}{2\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*sqrt(a/d)*log(-(8*a*c*d*cos(f*x + e) + (a*c^2 - 6*a*c*d + a*d^2)*cos(f
*x + e)^3 + 4*(2*d^2*cos(f*x + e) + (c*d - d^2)*cos(f*x + e)^2)*sqrt(a/d)*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x +
e))*sin(f*x + e) + 8*a*d^2 + (a*c^2 + 2*a*c*d - 7*a*d^2)*cos(f*x + e)^2)/(c
os(f*x + e)^3 + cos(f*x + e)^2))/f, sqrt(-a/d)*arctan(-2*d*sqrt(-a/d)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e)/((a*c - a*d)*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)
*cos(f*x + e)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/sqrt(c + d*sec(e + f*x)),
x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.235 $\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{2}\sqrt{c-d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}+\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

```
[Out] (Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqr
t[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*f) + (2*Sqrt[d]*
ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d
*Sec[e + f*x]])])/(Sqrt[a]*f)
```

Rubi [A] time = 0.462287, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3981, 3983, 203, 3980, 206}

$$\frac{\sqrt{2}\sqrt{c-d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}+\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqr
t[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*f) + (2*Sqrt[d]*
ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d
*Sec[e + f*x]])])/(Sqrt[a]*f)
```

Rule 3981

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> -Dist[(b*c - a*d)/d,
 Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x],
x] + Dist[b/d, Int[(Csc[e + f*x]*Sqrt[c + d*Csc[e + f*x]])/Sqrt[a + b*Csc[e
 + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 3983

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[(-2*a)/(b*f), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3980

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[(-2*b)/f, Subst
[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
+ d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} - (-c+d)\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+f}$$

$$= -\frac{(2(c-d))\,\text{Subst}\left(\int \frac{1}{2+(ac-ad)x^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} - \frac{(2d)\,\text{Sub}}{}$$

$$= \frac{\sqrt{2}\sqrt{c-d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f} + \frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(}{\sqrt{a+a\sec(e+fx)}\sqrt{c}}\right.}{\sqrt{a}f}$$

Mathematica [A] time = 16.9484, size = 187, normalized size = 1.34

$$\frac{\sqrt{c}\sin(e+fx)\sqrt{c+d\sec(e+fx)}\left(2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{c\cos(e+fx)+d}}{\sqrt{d}\sqrt{c-c\cos(e+fx)}}\right)-\sqrt{2}\sqrt{c-d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{c\cos(e+fx)+d}}{\sqrt{c-d}\sqrt{c-c\cos(e+fx)}}\right)\right)}{f\sqrt{a(\sec(e+fx)+1)}\sqrt{c-c\cos(e+fx)}\sqrt{c\cos(e+fx)+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],
x]
```

```
[Out] (Sqrt[c]*(-(Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + c*Cos[e +
f*x]])/(Sqrt[c - d]*Sqrt[c - c*Cos[e + f*x]])]) + 2*Sqrt[d]*ArcTanh[(Sqrt[c
]*Sqrt[d + c*Cos[e + f*x]])/(Sqrt[d]*Sqrt[c - c*Cos[e + f*x]])])*Sqrt[c + d
*Sec[e + f*x]]*Sin[e + f*x])/(f*Sqrt[c - c*Cos[e + f*x]]*Sqrt[d + c*Cos[e +
 f*x]]*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.366, size = 504, normalized size = 3.6

$$-\frac{\sqrt{2}\cos(fx+e)(-1+\cos(fx+e))}{af(\sin(fx+e))^2}\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\left(-\ln\left(\frac{1}{\sin(fx+e)}\left(\sqrt{-2\frac{d+c\cos}{1+\cos(}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f/a/(c-d)^(1/2)*2^(1/2)/(-d)^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(
1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*cos(f*x+e)*(-ln(1/(c-d)^(1/2)*((-2*(d+
c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(c-d)^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*c
os(f*x+e)+c-d)/sin(f*x+e))*2^(1/2)*(-d)^(1/2)*c+ln(1/(c-d)^(1/2)*((-2*(d+c*
cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(c-d)^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos
(f*x+e)+c-d)/sin(f*x+e))*2^(1/2)*(-d)^(1/2)*d+d*ln(2*(2^(1/2)*(-d)^(1/2)*(-
2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-c*sin(f*x+e)-d*sin(f*x+
e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(c-d)^(1/2)-d*
ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*
x+e)-c*sin(f*x+e)-d*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/(-1+cos(f*x+e
)+sin(f*x+e)))*(c-d)^(1/2))*(-1+cos(f*x+e))/sin(f*x+e)^2/(-2*(d+c*cos(f*x+e
))/(1+cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sec\left(fx+e\right)+c}\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sec(f*x + e)/sqrt(a*sec(f*x + e) + a), x
)
```

Fricas [A] time = 1.14313, size = 2603, normalized size = 18.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(-(c - d)/a)*log(-(2*sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) - (3*c - d)*cos(f*x + e)^2 - 2*(c + d)*cos(f*x + e) + c
 - 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + sqrt(d/a)*log(-((c^2 - 6*c
*d + d^2)*cos(f*x + e)^3 + 4*((c - d)*cos(f*x + e)^2 + 2*d*cos(f*x + e))*sq
rt(d/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/c
os(f*x + e))*sin(f*x + e) + 8*c*d*cos(f*x + e) + (c^2 + 2*c*d - 7*d^2)*cos(
f*x + e)^2 + 8*d^2)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/f, 1/2*(2*sqrt(2)*s
qrt((c - d)/a)*arctan(-sqrt(2)*sqrt((c - d)/a)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/((c - d)*s
in(f*x + e))) + sqrt(d/a)*log(-((c^2 - 6*c*d + d^2)*cos(f*x + e)^3 + 4*((c
- d)*cos(f*x + e)^2 + 2*d*cos(f*x + e))*sqrt(d/a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sin(f*x + e) + 8*c*d
*cos(f*x + e) + (c^2 + 2*c*d - 7*d^2)*cos(f*x + e)^2 + 8*d^2)/(cos(f*x + e)
^3 + cos(f*x + e)^2)))/f, 1/2*(sqrt(2)*sqrt(-(c - d)/a)*log(-(2*sqrt(2)*sqr
t(-(c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
+ d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - (3*c - d)*cos(f*x + e)^2 - 2
*(c + d)*cos(f*x + e) + c - 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2
```

```
*sqrt(-d/a)*arctan(-2*sqrt(-d/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((c - d)*co
s(f*x + e)^2 + (c + d)*cos(f*x + e) + 2*d)))/f, (sqrt(2)*sqrt((c - d)/a)*ar
ctan(-sqrt(2)*sqrt((c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(
(c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/((c - d)*sin(f*x + e))) + s
qrt(-d/a)*arctan(-2*sqrt(-d/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt
((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((c - d)*cos(
f*x + e)^2 + (c + d)*cos(f*x + e) + 2*d)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + d\sec\left(e + fx\right)}\sec\left(e + fx\right)}{\sqrt{a\left(\sec\left(e + fx\right) + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*sec(e + f*x))*sec(e + f*x)/sqrt(a*(sec(e + f*x) + 1)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sec\left(fx + e\right) + c}\sec\left(fx + e\right)}{\sqrt{a\sec\left(fx + e\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sec(f*x + e)/sqrt(a*sec(f*x + e) + a), x
)
```

3.236 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[
e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c - d]*f)
```

Rubi [A] time = 0.157262, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {3983, 203}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[
e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c - d]*f)
```

Rule 3983

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[(-2*a)/(b*f), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{2+(ac-ad)x^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}\\
&= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f}
\end{aligned}$$

Mathematica [A] time = 0.217764, size = 107, normalized size = 1.37

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c\cos(e+fx)+d}\tan^{-1}\left(\frac{\sqrt{c-d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c\cos(e+fx)+d}}\right)}{f\sqrt{c-d}\sqrt{a(\sec(e+fx)+1)}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),
x]
```

```
[Out] (2*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]]]*Cos[(e +
 f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f*x])/(Sqrt[c - d]*f*Sqrt[a*(1 +
Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]])
```

Maple [B] time = 0.331, size = 170, normalized size = 2.2

$$2\,\frac{\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}{af\sqrt{c-d}\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d+c\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\ln\left(-\frac{1}{\sin\left(fx+e\right)}\left(\sqrt{c-d}\cos\left(fx+e\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f/a/(c-d)^(1/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*cos(f*x+e)*(-1+cos(
f*x+e))*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*ln(-((c-d)^(1/2)*cos(f*x+e)-(-2
*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-(c-d)^(1/2))/sin(f*x+e))
/sin(f*x+e)^2/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)),
 x)
```

Fricas [A] time = 0.600526, size = 625, normalized size = 8.01

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{ac-ad}}\log\left(-\frac{2\sqrt{2}(c-d)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)+d}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{ac-ad}}\cos\left(fx+e\right)\sin\left(fx+e\right)-(3\,c-d)\cos\left(fx+e\right)^{2}-2\,(c+d)\cos\left(fx+e\right)+c-3\,d}{\cos\left(fx+e\right)^{2}+2\,\cos\left(fx+e\right)+1}\right)}{2\,f},-\frac{\sqrt{2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/(a*c - a*d))*log(-(2*sqrt(2)*(c - d)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/(a*c
 - a*d))*cos(f*x + e)*sin(f*x + e) - (3*c - d)*cos(f*x + e)^2 - 2*(c + d)*c
os(f*x + e) + c - 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))/f, -sqrt(2)*a
rctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
+ d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a*d)*sin(f*x + e)))/(sqrt(a*c -
 a*d)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x)))
, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.237 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{d}f}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Se
c[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c - d]*f)) + (2*ArcTa
nh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[
e + f*x]])])/(Sqrt[a]*Sqrt[d]*f)
```

Rubi [A] time = 0.528938, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3985, 3983, 203, 3980, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{d}f}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Se
c[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c - d]*f)) + (2*ArcTa
nh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[
e + f*x]])])/(Sqrt[a]*Sqrt[d]*f)
```

Rule 3985

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*S
qrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> -Dist[a/b, Int[Csc
[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] + Dis
t[1/b, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/Sqrt[c + d*Csc[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3983

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[(-2*a)/(b*f), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3980

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[(-2*b)/f, Subst
```

```
[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
+ d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} - \int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{1-adx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} + \frac{2\,\text{Subst}\left(\int \frac{1}{2+(ac-}\right.}{}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f} + \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec}}\right.}{\sqrt{a}\sqrt{d}f}$$

Mathematica [A] time = 0.27414, size = 171, normalized size = 1.21

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c\cos(e+fx)+d}\left(\sqrt{2}\sqrt{c-d}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c\cos(e+fx)+d}}\right)-\sqrt{d}\tan^{-1}\left(\frac{\sqrt{c-d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c\cos(e+fx)+d}}\right)\right)}{\sqrt{d}f\sqrt{c-d}\sqrt{a(\sec(e+fx)+1)}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]
),x]
```

```
[Out] (2*(-(Sqrt[d]*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]
]]) + Sqrt[2]*Sqrt[c - d]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/Sqrt[d
 + c*Cos[e + f*x]]])*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f*x]
)/(Sqrt[c - d]*Sqrt[d]*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]
])
```

Maple [B] time = 0.345, size = 403, normalized size = 2.9

$$-\frac{\sqrt{2}\cos(fx+e)\left(-1+\cos(fx+e)\right)}{af\left(\sin(fx+e)\right)^2}\sqrt{\frac{a\left(1+\cos(fx+e)\right)}{\cos(fx+e)}}\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\left(\ln\left(-\frac{1}{\sin(fx+e)}\left(\sqrt{c-d}\cos(fx+e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f/a/(c-d)^(1/2)*2^(1/2)/(-d)^(1/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)
*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)*(-1+cos(f*x+e))*(ln(-((c-d)
```

```
^(1/2)*cos(f*x+e)-(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-(c-
d)^(1/2))/sin(f*x+e))*2^(1/2)*(-d)^(1/2)+ln(2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*
cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)+c*co
s(f*x+e)-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(c-d)^(1/2)-ln(-2*(2^
(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-c*si
n(f*x+e)-d*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/(-1+cos(f*x+e)+sin(f*x
+e)))*(c-d)^(1/2))/sin(f*x+e)^2/(-2*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^2}{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)
), x)
```

Fricas [A] time = 1.33339, size = 2743, normalized size = 19.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*d*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/
(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c +
d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + sqrt(a*
d)*log(-(8*a*c*d*cos(f*x + e) + (a*c^2 - 6*a*c*d + a*d^2)*cos(f*x + e)^3 +
4*((c - d)*cos(f*x + e)^2 + 2*d*cos(f*x + e))*sqrt(a*d)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sin(f*x + e) +
 8*a*d^2 + (a*c^2 + 2*a*c*d - 7*a*d^2)*cos(f*x + e)^2)/(cos(f*x + e)^3 + co
s(f*x + e)^2)))/(a*d*f), 1/2*(2*sqrt(2)*a*d*arctan(sqrt(2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a*c - a*d)*sin(f*x + e)))/sqrt(a*c - a*d) + sqrt(a*d)*log(-(8*a*c*d
*cos(f*x + e) + (a*c^2 - 6*a*c*d + a*d^2)*cos(f*x + e)^3 + 4*((c - d)*cos(f
*x + e)^2 + 2*d*cos(f*x + e))*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sin(f*x + e) + 8*a*d^2 + (a*c^
2 + 2*a*c*d - 7*a*d^2)*cos(f*x + e)^2)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/
(a*d*f), 1/2*(sqrt(2)*a*d*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*
sqrt(-1/(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 +
 2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) +
 2*sqrt(-a*d)*arctan(-2*sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((a*c - a
*d)*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))))/(a*d*f), (sqrt(2)*
a*d*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
+ e) + d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a*d)*sin(f*x + e)))/sqrt(a
```

```
*c - a*d) + sqrt(-a*d)*arctan(-2*sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/
((a*c - a*d)*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))))/(a*d*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x
))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.238 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{d}f\sqrt{c+d}}$$

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Se
c[e + f*x]])])/(Sqrt[d]*Sqrt[c + d]*f)
```

Rubi [A] time = 0.136476, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {3967, 205}

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{d}f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Se
c[e + f*x]])])/(Sqrt[d]*Sqrt[c + d]*f)
```

Rule 3967

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b)/f, Subst[Int
[1/(b*c + a*d + d*x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx &= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{ac+ad+dx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\ &= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{d}\sqrt{c+d}f}\end{aligned}$$

Mathematica [A] time = 0.226029, size = 94, normalized size = 1.54

$$\frac{\sqrt{2}\sqrt{\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\sqrt{\cos(e+fx)}}\right)}{\sqrt{d}f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e
+ f*x]])]*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(
Sqrt[d]*Sqrt[c + d]*f)
```

Maple [B] time = 0.255, size = 431, normalized size = 7.1

$$\frac{\sqrt{2}}{2f}\left(\ln\left(-2\frac{1}{\sqrt{(c+d)(c-d)}\sin\left(fx+e\right)+c\cos\left(fx+e\right)-d\cos\left(fx+e\right)-c+d}\left(\sqrt{-2\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{d}{c-d}}\sqrt{2}c\sin\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] 1/2/f*2^(1/2)/(d/(c-d))^(1/2)/((c+d)*(c-d))^(1/2)*(ln(-2*((-2*cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^
(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)-((c+d)*(c-d))^(1/2)
*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(
1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))-ln(2*((-2*cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2
)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos
(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)
*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)))*(-2*cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Fricas [B] time = 1.04114, size = 801, normalized size = 13.13

$$\left[\frac{\sqrt{-\frac{a}{cd+d^2}}\log\left(-\frac{\left(ac^2+8\,acd+8\,ad^2\right)\cos\left(fx+e\right)^3+ad^2+\left(ac^2+2\,acd\right)\cos\left(fx+e\right)^2-4\left(\left(c^2d+3\,cd^2+2\,d^3\right)\cos\left(fx+e\right)^2-\left(cd^2+d^3\right)\cos\left(fx+e\right)\right)\sqrt{-\frac{a}{cd+d^2}}\sqrt{\frac{a\cos(fx+}{\cos(fx}}}{c^2\cos\left(fx+e\right)^3+\left(c^2+2\,cd\right)\cos\left(fx+e\right)^2+d^2+\left(2\,cd+d^2\right)\cos\left(fx+e\right)}\right.}{2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [1/2*sqrt(-a/(c*d + d^2))*log(-((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3
+ a*d^2 + (a*c^2 + 2*a*c*d)*cos(f*x + e)^2 - 4*((c^2*d + 3*c*d^2 + 2*d^3)*c
os(f*x + e)^2 - (c*d^2 + d^3)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - (6*a*c*d + 7*a*d^2)*cos(f*x +
e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2
)*cos(f*x + e)))/f, sqrt(a/(c*d + d^2))*arctan(2*(c*d + d^2)*sqrt(a/(c*d +
d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((a
*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.239 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=149

$$\frac{2\sqrt{a}g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{df}-\frac{2\sqrt{a}\sqrt{c}g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{df\sqrt{c+d}}$$

```
[Out] (2*Sqrt[a]*g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[g*Sec[e + f
*x]]*Sqrt[a + a*Sec[e + f*x]])])/(d*f) - (2*Sqrt[a]*Sqrt[c]*g^(3/2)*ArcTanh
[(Sqrt[a]*Sqrt[c]*Sqrt[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*Sec[e + f*x]]*S
qrt[a + a*Sec[e + f*x]])])/(d*Sqrt[c + d]*f)
```

Rubi [A] time = 0.56998, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3970, 3802, 208, 3965}

$$\frac{2\sqrt{a}g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{df}-\frac{2\sqrt{a}\sqrt{c}g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{df\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e + f*x]),
x]
```


```
[Out] (2*Sqrt[a]*g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[g*Sec[e + f
*x]]*Sqrt[a + a*Sec[e + f*x]])])/(d*f) - (2*Sqrt[a]*Sqrt[c]*g^(3/2)*ArcTanh
[(Sqrt[a]*Sqrt[c]*Sqrt[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*Sec[e + f*x]]*S
qrt[a + a*Sec[e + f*x]])])/(d*Sqrt[c + d]*f)
```


Rule 3970

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[g/d,
Int[Sqrt[g*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(c*g)/d, I
nt[(Sqrt[g*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0]
```

Rule 3802

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/f, Subst[Int[1/(b - d*x^2), x], x, (b*C
ot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] &&  !GtQ[(a*d)/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3965

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b*g
```

```
)/f, Subst[Int[1/(b*c + a*d - c*g*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[g*Csc
[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f, g}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{g\int\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx}{d} - \frac{(cg)\int\frac{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+f}}{c+d\sec(e+fx)}}{d}$$

$$= -\frac{(2ag^2)\,\text{Subst}\left(\int\frac{1}{a-gx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{df} + \frac{(2acg^2)\,\text{Subs}}{}$$

$$= \frac{2\sqrt{a}g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{df} - \frac{2\sqrt{a}\sqrt{c}g^{3/2}\tanh^{-1}\left(\frac{}{\sqrt{c+d}\sqrt{g}}\right.}{d\sqrt{c+c}}$$

Mathematica [C] time = 1.32184, size = 427, normalized size = 2.87

$$\frac{\left(\sqrt{2}-2i\right)g^2\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\left(i\left(2\sqrt{c+d}\log\left(2\sin\left(\frac{1}{2}(e+fx)\right)+\sqrt{2}\right)+2\sqrt{c}\log\left(\sqrt{2}\sqrt{c+d}-2\sqrt{}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e +
f*x]),x]
```

```
[Out] ((-2*I + Sqrt[2])*g^2*(2*Sqrt[c + d]*ArcTan[(Cos[(e + f*x)/4] - (-1 + Sqrt[
2])*Sin[(e + f*x)/4])/((1 + Sqrt[2])*Cos[(e + f*x)/4] - Sin[(e + f*x)/4])]
+ 2*Sqrt[c + d]*ArcTan[(Cos[(e + f*x)/4] - (1 + Sqrt[2])*Sin[(e + f*x)/4])/
((-1 + Sqrt[2])*Cos[(e + f*x)/4] - Sin[(e + f*x)/4])] + I*(2*Sqrt[c + d]*Lo
g[Sqrt[2] + 2*Sin[(e + f*x)/2]] - Sqrt[c + d]*Log[2 - Sqrt[2]*Cos[(e + f*x)
/2] - Sqrt[2]*Sin[(e + f*x)/2]] - Sqrt[c + d]*Log[2 + Sqrt[2]*Cos[(e + f*x)
/2] - Sqrt[2]*Sin[(e + f*x)/2]] + 2*Sqrt[c]*Log[Sqrt[2]*Sqrt[c + d] - 2*Sqr
t[c]*Sin[(e + f*x)/2]] - 2*Sqrt[c]*Log[Sqrt[2]*Sqrt[c + d] + 2*Sqrt[c]*Sin[
(e + f*x)/2]]))*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(4*(I + Sqrt[2
])*d*Sqrt[c + d]*f*Sqrt[g*Sec[e + f*x]])
```

Maple [B] time = 0.379, size = 568, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] -2/f*(c-d)/((c+d)*(c-d))^(1/2)/(c-d+((c+d)*(c-d))^(1/2))/(-c+d+((c+d)*(c-d)
)^(1/2))/(c/(c-d))^(1/2)*(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2*(-1+cos(f*x+e))^
2*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(((c+d)*(c-d))^(1/2)*arctanh(1/2*(1
/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1-sin(f*x+e)))*(c/(c-d))^(1/2)-((c+d)*(c
-d))^(1/2)*arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1+sin(f*x+e)))*
(c/(c-d))^(1/2)+c*ln(2*(-2*(1/(1+cos(f*x+e)))^(1/2)*(c/(c-d))^(1/2)*c*sin(f
```

```
*x+e)+2*(1/(1+cos(f*x+e)))^(1/2)*(c/(c-d))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))
^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c
-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))-c*ln(2*(2*(1/(1+cos(f
*x+e)))^(1/2)*(c/(c-d))^(1/2)*c*sin(f*x+e)-2*(1/(1+cos(f*x+e)))^(1/2)*(c/(c
-d))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)+c*sin(f*x+e)-d*sin(f
*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*c
os(f*x+e)+c-d)))/sin(f*x+e)^4/(1/(1+cos(f*x+e)))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="maxima")

[Out] Exception raised: RuntimeError
```

Fricas [A] time = 21.0628, size = 2753, normalized size = 18.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="fricas")

[Out] [1/2*(sqrt(a*c*g/(c + d))*g*log((a*c^2*g*cos(f*x + e)^3 - (7*a*c^2 + 6*a*c*
d)*g*cos(f*x + e)^2 + 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)
*cos(f*x + e))*sqrt(a*c*g/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sqrt(g/cos(f*x + e))*sin(f*x + e) + (2*a*c*d + a*d^2)*g*cos(f*x + e) + (8*a
*c^2 + 8*a*c*d + a*d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)
^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) + sqrt(a*g)*g*log((a*g*cos(f*x + e)
^3 - 7*a*g*cos(f*x + e)^2 - 4*sqrt(a*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) +
8*a*g)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/(d*f), -1/2*(2*sqrt(-a*c*g/(c +
d))*g*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt(-a*c*g/(c
 + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))/(a*c*g*
sin(f*x + e))) - sqrt(a*g)*g*log((a*g*cos(f*x + e)^3 - 7*a*g*cos(f*x + e)^2
 - 4*sqrt(a*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) + 8*a*g)/(cos(f*x + e)^3 +
cos(f*x + e)^2)))/(d*f), 1/2*(2*sqrt(-a*g)*g*arctan(2*sqrt(-a*g)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
)/(a*g*cos(f*x + e)^2 - a*g*cos(f*x + e) - 2*a*g)) + sqrt(a*c*g/(c + d))*g*
log((a*c^2*g*cos(f*x + e)^3 - (7*a*c^2 + 6*a*c*d)*g*cos(f*x + e)^2 + 4*((c^
2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*cos(f*x + e))*sqrt(a*c*g/(c
 + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x
 + e) + (2*a*c*d + a*d^2)*g*cos(f*x + e) + (8*a*c^2 + 8*a*c*d + a*d^2)*g)/(
c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos
(f*x + e))))/(d*f), (sqrt(-a*g)*g*arctan(2*sqrt(-a*g)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*g*cos(
f*x + e)^2 - a*g*cos(f*x + e) - 2*a*g)) - sqrt(-a*c*g/(c + d))*g*arctan(1/2
*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt(-a*c*g/(c + d))*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))/(a*c*g*sin(f*x + e))))/
```

```
(d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="giac")

[Out] Timed out
```

3.240 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=122

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}-\frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)\sqrt{c+d}}$$

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])
/(Sqrt[a]*(c - d)*f) - (2*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sq
rt[c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```

Rubi [A] time = 0.301194, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3972, 3795, 203, 3967, 205}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}-\frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])
/(Sqrt[a]*(c - d)*f) - (2*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sq
rt[c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```

Rule 3972

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[
Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[d/(b*c - a*d), Int[(Cs
c[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^
2 - d^2, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3967

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b)/f, Subst[Int
[1/(b*c + a*d + d*x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

]

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} - \frac{d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{a(c-d)}$$
$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{(c-d)f} + \frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{ac+ad+dx^2}\,dx,\right.}{(c-d)f}$$
$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} - \frac{2\sqrt{d}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{c+d}f}$$

Mathematica [C] time = 33.619, size = 228531, normalized size = 1873.2

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.238, size = 518, normalized size = 4.3

$$\frac{1}{2f(c-d)a}\left(2\sqrt{(c+d)(c-d)}\ln\left(-\frac{1}{\sin(fx+e)}\left(-\sqrt{-2\frac{\cos(fx+e)}{1+\cos(fx+e)}}\sin(fx+e)+\cos(fx+e)-1\right)\right)\sqrt{\frac{d}{c-d}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f/(d/(c-d))^(1/2)/(c-d)/((c+d)*(c-d))^(1/2)/a*(2*((c+d)*(c-d))^(1/2)*ln
(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/sin(f*x+e
))*(d/(c-d))^(1/2)-d*2^(1/2)*ln(-2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d
/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+
e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(
f*x+e)-d*cos(f*x+e)-c+d))+d*2^(1/2)*ln(2*((-2*cos(f*x+e)/(1+cos(f*x+e)))^(1
/2)*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*si
n(f*x+e)+d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-
c*cos(f*x+e)+d*cos(f*x+e)+c-d))*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/co
s(f*x+e)*a*(1+cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Fricas [A] time = 1.21029, size = 2403, normalized size = 19.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] [-1/2*(sqrt(2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x +
 e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + sqrt(-d/(a*c + a*d))*log(
-((c^2 + 8*c*d + 8*d^2)*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 - 4*(
(c^2 + 3*c*d + 2*d^2)*cos(f*x + e)^2 - (c*d + d^2)*cos(f*x + e))*sqrt(-d/(a
*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + d^2 - (6*
c*d + 7*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)
^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))/((c - d)*f), -1/2*(sqrt(2)*sqrt(-1
/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f
*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)
^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(d/(a*c + a*d))*arctan(2*(c + d)*sqrt(d/(
a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e)/((c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)))/((c - d)*f), -1/
2*(sqrt(-d/(a*c + a*d))*log(-((c^2 + 8*c*d + 8*d^2)*cos(f*x + e)^3 + (c^2 +
 2*c*d)*cos(f*x + e)^2 - 4*((c^2 + 3*c*d + 2*d^2)*cos(f*x + e)^2 - (c*d + d
^2)*cos(f*x + e))*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sin(f*x + e) + d^2 - (6*c*d + 7*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3
+ (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) + 2*sqr
t(2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e)))/sqrt(a))/((c - d)*f), -(sqrt(d/(a*c + a*d))*arctan(2*
(c + d)*sqrt(d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e)/((c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)) +
 sqrt(2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/((c - d)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.241 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=124

$$\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\sqrt{d}f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])
])/(Sqrt[a]*(c - d)*f)) + (2*c*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[
c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[d]*Sqrt[c + d]*f)
```

Rubi [A] time = 0.367964, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3976, 3795, 203, 3967, 205}

$$\frac{2c\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\sqrt{d}f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```


```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])
])/(Sqrt[a]*(c - d)*f)) + (2*c*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[
c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[d]*Sqrt[c + d]*f)
```


Rule 3976

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(
csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> -Dist[a/(b*c - a*d), I
nt[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[c/(b*c - a*d), Int[
(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ
[c^2 - d^2, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3967

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b)/f, Subst[Int
[1/(b*c + a*d + d*x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

]

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = -\frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} + \frac{c\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{a(c-d)}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{(c-d)f} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{ac+ad+dx^2}\,dx, x\right.}{(c-d)f}$$
$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} + \frac{2c\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{d}\sqrt{c+d}f}$$

Mathematica [A] time = 0.41069, size = 141, normalized size = 1.14

$$-\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(\sqrt{d}\sqrt{c+d}\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\cos(e+fx)}}\right)-\sqrt{2}c\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\sqrt{\cos(e+fx)}}\right)\right)}{\sqrt{d}f(c-d)\sqrt{c+d}\sqrt{\cos(e+fx)}\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-2*(Sqrt[d]*Sqrt[c + d]*ArcTan[Sin[(e + f*x)/2]/Sqrt[Cos[e + f*x]]] - Sqrt
[2]*c*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f
*x]])])*Cos[(e + f*x)/2])/((c - d)*Sqrt[d]*Sqrt[c + d]*f*Sqrt[Cos[e + f*x]]
*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.249, size = 518, normalized size = 4.2

$$\frac{1}{2\,af\,(c-d)}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{-2\,\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\left(c\sqrt{2}\ln\left(-2\,\frac{1}{\sqrt{(c+d)\,(c-d)}\sin\left(fx+e\right)+c\cos\left(fx+e\right)-d\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f/a/((c+d)*(c-d))^(1/2)/(c-d)/(d/(c-d))^(1/2)*(1/cos(f*x+e)*a*(1+cos(f*
x+e)))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(c*2^(1/2)*ln(-2*((-2*cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^(1/2)*
(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)-((c+d)*(c
-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2))/(((c+d
)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))-c*2^(1/2)*ln(2*((
-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(d/(c-d))^(1/2)*2^(1/2)*c*sin(f*x+e)-2^
(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*d*sin(f*x+e)+((c
```

```
+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/
(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d))-2*((c+d)*(c
-d))^(1/2)*ln(-(-(-2*cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)
-1)/sin(f*x+e))*(d/(c-d))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^{2}}{\sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x
)
```

Fricas [A] time = 1.50967, size = 2543, normalized size = 20.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt(2)*(a*c*d + a*d^2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e
)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - sqrt(-a*
c*d - a*d^2)*c*log(-((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 + a*d^2 + (
a*c^2 + 2*a*c*d)*cos(f*x + e)^2 - 4*sqrt(-a*c*d - a*d^2)*((c + 2*d)*cos(f*x
 + e)^2 - d*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x +
 e) - (6*a*c*d + 7*a*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)
*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))/((a*c^2*d - a*d^3)*f)
, -1/2*(sqrt(2)*(a*c*d + a*d^2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x +
e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 2*sqrt(
a*c*d + a*d^2)*c*arctan(2*sqrt(a*c*d + a*d^2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*cos(f*x + e)*sin(f*x + e)/((a*c + 2*a*d)*cos(f*x + e)^2 - a*d +
(a*c + a*d)*cos(f*x + e))))/((a*c^2*d - a*d^3)*f), 1/2*(sqrt(-a*c*d - a*d^2
)*c*log(-((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 + a*d^2 + (a*c^2 + 2*a
*c*d)*cos(f*x + e)^2 - 4*sqrt(-a*c*d - a*d^2)*((c + 2*d)*cos(f*x + e)^2 - d
*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - (6*a*
c*d + 7*a*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x +
e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) + 2*sqrt(2)*(a*c*d + a*d^2)*arcta
n(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin
(f*x + e)))/sqrt(a))/((a*c^2*d - a*d^3)*f), (sqrt(a*c*d + a*d^2)*c*arctan(2
*sqrt(a*c*d + a*d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e)/((a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))
) + sqrt(2)*(a*c*d + a*d^2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/((a*c^2*d - a*d^3)*f)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a(\sec(e+fx)+1)}(c+d\sec(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))),
 x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

3.242 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=167

$$\frac{2\sqrt{c}g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}f(c-d)}$$

```
[Out] -((Sqrt[2]*g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[2]*Sqrt[g*S
ec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*f)) + (2*Sqrt[c]*
g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[c]*Sqrt[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*
Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```

Rubi [A] time = 0.582828, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3974, 3808, 208, 3965}

$$\frac{2\sqrt{c}g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}f(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] -((Sqrt[2]*g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[2]*Sqrt[g*S
ec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*f)) + (2*Sqrt[c]*
g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[c]*Sqrt[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*
Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```

Rule 3974

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> -Dist[(a*g
)/(b*c - a*d), Int[Sqrt[g*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] +
Dist[(c*g)/(b*c - a*d), Int[(Sqrt[g*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])
/(c + d*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3965

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b*g
```

```
)/f, Subst[Int[1/(b*c + a*d - c*g*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[g*Csc
[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f, g}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = -\frac{g\int \frac{\sqrt{g\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} + \frac{(cg)\int \frac{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{a(c-d)}$$
$$= \frac{(2g^2)\,\text{Subst}\left(\int \frac{1}{2a-gx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{(c-d)f} - \frac{(2cg^2)\,\text{Subst}}{}$$
$$= -\frac{\sqrt{2}g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} + \frac{2\sqrt{c}g^{3/2}\tanh^{-1}\left(\frac{}{\sqrt{c+d}\sqrt{}}\right)}{\sqrt{a}(c-d)}$$

Mathematica [A] time = 0.394346, size = 198, normalized size = 1.19

$$\frac{g\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{g\sec(e+fx)}\left(\sqrt{2}\sqrt{c}\left(\log\left(\sqrt{2}\sqrt{c+d}+2\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sqrt{2}\sqrt{c+d}-2\sqrt{c}\sin\left(\frac{1}{2}(e+f\right.\right.\right.}{f(c-d)\sqrt{c+d}\sqrt{a(\sec}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] (g*Cos[(e + f*x)/2]*(2*Sqrt[c + d]*Log[Cos[(e + f*x)/4] - Sin[(e + f*x)/4]]
 - 2*Sqrt[c + d]*Log[Cos[(e + f*x)/4] + Sin[(e + f*x)/4]] + Sqrt[2]*Sqrt[c]
*(-Log[Sqrt[2]*Sqrt[c + d] - 2*Sqrt[c]*Sin[(e + f*x)/2]] + Log[Sqrt[2]*Sqrt
[c + d] + 2*Sqrt[c]*Sin[(e + f*x)/2]]))*Sqrt[g*Sec[e + f*x]])/((c - d)*Sqrt
[c + d]*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.338, size = 473, normalized size = 2.8

$$\frac{(\cos(fx+e))^2(-1+\cos(fx+e))^2}{af(c-d)(\sin(fx+e))^4}\left(\frac{g}{\cos(fx+e)}\right)^{\frac{3}{2}}\sqrt{\frac{a(1+\cos(fx+e))}{\cos(fx+e)}}\left(\text{Arcsinh}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\sqrt{2}\sqrt{\frac{c}{c-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f/a/(c/(c-d))^(1/2)/(c-d)/((c+d)*(c-d))^(1/2)*(g/cos(f*x+e))^(3/2)*(1/cos
(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^2*(-1+cos(f*x+e))^2*(arcsinh((-1
+cos(f*x+e))/sin(f*x+e))*2^(1/2)*(c/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)+c*ln(2
*(-2*(1/(1+cos(f*x+e)))^(1/2)*(c/(c-d))^(1/2)*c*sin(f*x+e)+2*(1/(1+cos(f*x+
e)))^(1/2)*(c/(c-d))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*si
n(f*x+e)+d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+
c*cos(f*x+e)-d*cos(f*x+e)-c+d))-c*ln(2*(2*(1/(1+cos(f*x+e)))^(1/2)*(c/(c-d)
)^(1/2)*c*sin(f*x+e)-2*(1/(1+cos(f*x+e)))^(1/2)*(c/(c-d))^(1/2)*d*sin(f*x+e
)+((c+d)*(c-d))^(1/2)*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1
/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d))/sin(f
```

```
*x+e)^4/(1/(1+cos(f*x+e)))^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.56623, size = 2654, normalized size = 15.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*g*sqrt(g/a)*log((2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - g*cos(f*x
 + e)^2 + 2*g*cos(f*x + e) + 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) +
sqrt(c*g/(a*c + a*d))*g*log((c^2*g*cos(f*x + e)^3 - (7*c^2 + 6*c*d)*g*cos(f
*x + e)^2 + 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*cos(f*x +
 e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/c
os(f*x + e))*sin(f*x + e) + (2*c*d + d^2)*g*cos(f*x + e) + (8*c^2 + 8*c*d +
 d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d
+ d^2)*cos(f*x + e))))/((c - d)*f), 1/2*(2*sqrt(2)*g*sqrt(-g/a)*arctan(sqrt
(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))
*cos(f*x + e)/(g*sin(f*x + e))) - sqrt(c*g/(a*c + a*d))*g*log((c^2*g*cos(f*
x + e)^3 - (7*c^2 + 6*c*d)*g*cos(f*x + e)^2 + 4*((c^2 + c*d)*cos(f*x + e)^2
 - (2*c^2 + 3*c*d + d^2)*cos(f*x + e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) + (2*c*d + d^2)
*g*cos(f*x + e) + (8*c^2 + 8*c*d + d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c
*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))/((c - d)*f), -1/2*
(sqrt(2)*g*sqrt(g/a)*log((2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - g*cos(f*x + e)^
2 + 2*g*cos(f*x + e) + 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 2*sqrt
(-c*g/(a*c + a*d))*g*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))
*sqrt(-c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(
f*x + e))/(c*g*sin(f*x + e))))/((c - d)*f), (sqrt(2)*g*sqrt(-g/a)*arctan(sq
rt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e
))*cos(f*x + e)/(g*sin(f*x + e))) + sqrt(-c*g/(a*c + a*d))*g*arctan(1/2*(c*
cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt(-c*g/(a*c + a*d))*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))/(c*g*sin(f*x + e))))/((c
- d)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g \sec \left(e+fx\right)\right)^{\frac{3}{2}}}{\sqrt{a \left(\sec \left(e+fx\right)+1\right)} \left(c+d \sec \left(e+fx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e +
 f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g \sec \left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{a \sec \left(fx+e\right)+a} \left(d \sec \left(fx+e\right)+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e)
+ c)), x)
```

3.243 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=231

$$-\frac{2c^{3/2}g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}df(c-d)\sqrt{c+d}}+\frac{\sqrt{2}g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}f(c-d)}+\frac{2g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}}{\sqrt{a\sec(e+fx)}}\right.}{\sqrt{a}df}$$

```
[Out] (2*g^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[g*Sec[e + f*x]]*Sqr
t[a + a*Sec[e + f*x]])])/(Sqrt[a]*d*f) + (Sqrt[2]*g^(5/2)*ArcTanh[(Sqrt[a]*
Sqrt[g]*Tan[e + f*x])/(Sqrt[2]*Sqrt[g*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
])])/(Sqrt[a]*(c - d)*f) - (2*c^(3/2)*g^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[c]*Sqrt
[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
])])/(Sqrt[a]*(c - d)*d*Sqrt[c + d]*f)
```

Rubi [A] time = 0.815862, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3978, 3965, 208, 4023, 3808, 3802}

$$-\frac{2c^{3/2}g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}df(c-d)\sqrt{c+d}}+\frac{\sqrt{2}g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}f(c-d)}+\frac{2g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}}{\sqrt{a\sec(e+fx)}}\right.}{\sqrt{a}df}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] (2*g^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[g*Sec[e + f*x]]*Sqr
t[a + a*Sec[e + f*x]])])/(Sqrt[a]*d*f) + (Sqrt[2]*g^(5/2)*ArcTanh[(Sqrt[a]*
Sqrt[g]*Tan[e + f*x])/(Sqrt[2]*Sqrt[g*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
])])/(Sqrt[a]*(c - d)*f) - (2*c^(3/2)*g^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[c]*Sqrt
[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
])])/(Sqrt[a]*(c - d)*d*Sqrt[c + d]*f)
```

Rule 3978

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(5/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> -Dist[(c^2
*g^2)/(d*(b*c - a*d)), Int[(Sqrt[g*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])/
(c + d*Csc[e + f*x]), x], x] + Dist[g^2/(d*(b*c - a*d)), Int[(Sqrt[g*Csc[e
+ f*x]]*(a*c + (b*c - a*d)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x]
/; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rule 3965

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[(-2*b*g
)/f, Subst[Int[1/(b*c + a*d - c*g*x^2), x], x, (b*Cot[e + f*x])/(Sqrt[g*Csc
[e + f*x]]*Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f, g}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3802

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/f, Subst[Int[1/(b - d*x^2), x], x, (b*C
ot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] &&  !GtQ[(a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx &= \frac{g^2\int \frac{\sqrt{g\sec(e+fx)}(ac+(ac-ad)\sec(e+fx))}{\sqrt{a+a\sec(e+fx)}}\,dx}{a(c-d)d} - \frac{\left(c^2g^2\right)\int \frac{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx}}{c+d\sec(e+fx)}}{a(c-d)d}\\ &= \frac{g^2\int \frac{\sqrt{g\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} + \frac{g^2\int \sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx}{ad} + \frac{(2}{}\\ &= -\frac{2c^{3/2}g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)d\sqrt{c+d}f} - \frac{\left(2g^3\right)\text{Subst}\left(\int \frac{1}{2a-g}\right.}{}\\ &= \frac{2g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}df} + \frac{\sqrt{2}g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan}{\sqrt{2}\sqrt{g\sec(e+fx)}}\right.}{\sqrt{a}(c-d)f}\end{aligned}$$

Mathematica [A] time = 0.391366, size = 155, normalized size = 0.67

$$\frac{2g^2\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{g\sec(e+fx)}\left(\sqrt{2}\left((c-d)\sqrt{c+d}\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)-c^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)\right)\right.}{df(c-d)\sqrt{c+d}\sqrt{a(\sec(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] (2*g^2*(d*Sqrt[c + d]*ArcTanh[Sin[(e + f*x)/2]] + Sqrt[2]*((c - d)*Sqrt[c +
 d]*ArcTanh[Sqrt[2]*Sin[(e + f*x)/2]] - c^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[c]*Si
n[(e + f*x)/2])/Sqrt[c + d]]))*Cos[(e + f*x)/2]*Sqrt[g*Sec[e + f*x]])/((c -
```

```
d)*d*Sqrt[c + d]*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] time = 0.346, size = 726, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f/a/(c/(c-d))^(1/2)/(-c+d+((c+d)*(c-d))^(1/2))/(c-d+((c+d)*(c-d))^(1/2))/
((c+d)*(c-d))^(1/2)*(g/cos(f*x+e))^(5/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1
/2)*cos(f*x+e)^3*(-1+cos(f*x+e))^3*(arcsinh((-1+cos(f*x+e))/sin(f*x+e))*2^(
1/2)*(c/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*d+arctanh(1/2*(1/(1+cos(f*x+e)))^(
1/2)*(cos(f*x+e)+1-sin(f*x+e)))*(c/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c-arcta
nh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1-sin(f*x+e)))*(c/(c-d))^(1/2)*
((c+d)*(c-d))^(1/2)*d-arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1+si
n(f*x+e)))*(c/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c+arctanh(1/2*(1/(1+cos(f*x+
e)))^(1/2)*(cos(f*x+e)+1+sin(f*x+e)))*(c/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*d
+ln(2*(-2*(1/(1+cos(f*x+e)))^(1/2)*(c/(c-d))^(1/2)*c*sin(f*x+e)+2*(1/(1+cos
(f*x+e)))^(1/2)*(c/(c-d))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)
-c*sin(f*x+e)+d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*
x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))*c^2-ln(2*(2*(1/(1+cos(f*x+e)))^(1/2)*(
c/(c-d))^(1/2)*c*sin(f*x+e)-2*(1/(1+cos(f*x+e)))^(1/2)*(c/(c-d))^(1/2)*d*si
n(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c
-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d))
*c^2)/sin(f*x+e)^6/(1/(1+cos(f*x+e)))^(5/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.244 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx$

Optimal. Leaf size=250

$$\frac{\left(95ac^3d+80acd^3+112bc^2d^2+12bc^4+16bd^4\right)\tan(e+fx)}{30f}+\frac{\left(24ac^2d^2+8ac^4+3ad^4+16bc^3d+12bcd^3\right)\tanh^{-1}(\sin(e}{8f}$$

```
[Out] ((8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*ArcTanh[Sin[e
 + f*x]])/(8*f) + ((12*b*c^4 + 95*a*c^3*d + 112*b*c^2*d^2 + 80*a*c*d^3 + 16
*b*d^4)*Tan[e + f*x])/(30*f) + (d*(24*b*c^3 + 130*a*c^2*d + 116*b*c*d^2 + 4
5*a*d^3)*Sec[e + f*x]*Tan[e + f*x])/(120*f) + ((12*b*c^2 + 35*a*c*d + 16*b*
d^2)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(60*f) + ((4*b*c + 5*a*d)*(c + d*
Sec[e + f*x])^3*Tan[e + f*x])/(20*f) + (b*(c + d*Sec[e + f*x])^4*Tan[e + f*
x])/(5*f)
```

Rubi [A] time = 0.500991, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{\left(95ac^3d+80acd^3+112bc^2d^2+12bc^4+16bd^4\right)\tan(e+fx)}{30f}+\frac{\left(24ac^2d^2+8ac^4+3ad^4+16bc^3d+12bcd^3\right)\tanh^{-1}(\sin(e}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```


```
[Out] ((8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*ArcTanh[Sin[e
 + f*x]])/(8*f) + ((12*b*c^4 + 95*a*c^3*d + 112*b*c^2*d^2 + 80*a*c*d^3 + 16
*b*d^4)*Tan[e + f*x])/(30*f) + (d*(24*b*c^3 + 130*a*c^2*d + 116*b*c*d^2 + 4
5*a*d^3)*Sec[e + f*x]*Tan[e + f*x])/(120*f) + ((12*b*c^2 + 35*a*c*d + 16*b*
d^2)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(60*f) + ((4*b*c + 5*a*d)*(c + d*
Sec[e + f*x])^3*Tan[e + f*x])/(20*f) + (b*(c + d*Sec[e + f*x])^4*Tan[e + f*
x])/(5*f)
```


Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx = \frac{b(c+d\sec(e+fx))^4\tan(e+fx)}{5f}+\frac{1}{5}\int \sec(e+fx)(c+d$$
$$= \frac{(4bc+5ad)(c+d\sec(e+fx))^3\tan(e+fx)}{20f}+\frac{b(c+d\sec(e$$
$$= \frac{\left(12bc^2+35acd+16bd^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{60f}+$$
$$= \frac{d\left(24bc^3+130ac^2d+116bcd^2+45ad^3\right)\sec(e+fx)\tan(e+}{120f}$$
$$= \frac{d\left(24bc^3+130ac^2d+116bcd^2+45ad^3\right)\sec(e+fx)\tan(e+}{120f}$$
$$= \frac{\left(8ac^4+16bc^3d+24ac^2d^2+12bcd^3+3ad^4\right)\tanh^{-1}(\sin(e+}{8f}$$
$$= \frac{\left(8ac^4+16bc^3d+24ac^2d^2+12bcd^3+3ad^4\right)\tanh^{-1}(\sin(e+}{8f}$$

Mathematica [A] time = 4.42882, size = 201, normalized size = 0.8

$$15\left(a\left(24c^2d^2+8c^4+3d^4\right)+4bcd\left(4c^2+3d^2\right)\right)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(8\left(10d^2\left(2acd+b\left(3c^2+d^2\right)\right)\tan^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (15*(4*b*c*d*(4*c^2 + 3*d^2) + a*(8*c^4 + 24*c^2*d^2 + 3*d^4))*ArcTanh[Sin[
e + f*x]] + Tan[e + f*x]*(15*d*(3*a*d*(8*c^2 + d^2) + 4*b*(4*c^3 + 3*c*d^2)
)*Sec[e + f*x] + 30*d^3*(4*b*c + a*d)*Sec[e + f*x]^3 + 8*(15*(4*a*c*d*(c^2
+ d^2) + b*(c^4 + 6*c^2*d^2 + d^4)) + 10*d^2*(2*a*c*d + b*(3*c^2 + d^2))*Ta
n[e + f*x]^2 + 3*b*d^4*Tan[e + f*x]^4)))/(120*f)
```

Maple [A] time = 0.048, size = 431, normalized size = 1.7

$$\frac{ac^4\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+4\,\frac{ac^3d\tan\left(fx+e\right)}{f}+3\,\frac{ac^2d^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{f}+3\,\frac{ac^2d^2\ln\left(\sec\left(fx+e\right)\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x)
```

```
[Out] 1/f*a*c^4*ln(sec(f*x+e)+tan(f*x+e))+4/f*a*c^3*d*tan(f*x+e)+3/f*a*c^2*d^2*se
c(f*x+e)*tan(f*x+e)+3/f*a*c^2*d^2*ln(sec(f*x+e)+tan(f*x+e))+8/3/f*a*c*d^3*t
an(f*x+e)+4/3/f*a*c*d^3*tan(f*x+e)*sec(f*x+e)^2+1/4/f*a*d^4*tan(f*x+e)*sec(
f*x+e)^3+3/8/f*a*d^4*sec(f*x+e)*tan(f*x+e)+3/8/f*a*d^4*ln(sec(f*x+e)+tan(f*
x+e))+1/f*b*c^4*tan(f*x+e)+2/f*b*c^3*d*sec(f*x+e)*tan(f*x+e)+2/f*b*c^3*d*ln
(sec(f*x+e)+tan(f*x+e))+4/f*b*c^2*d^2*tan(f*x+e)+2/f*b*c^2*d^2*tan(f*x+e)*s
ec(f*x+e)^2+1/f*b*c*d^3*tan(f*x+e)*sec(f*x+e)^3+3/2/f*b*c*d^3*sec(f*x+e)*ta
n(f*x+e)+3/2/f*b*c*d^3*ln(sec(f*x+e)+tan(f*x+e))+8/15/f*b*d^4*tan(f*x+e)+1/
5/f*b*d^4*tan(f*x+e)*sec(f*x+e)^4+4/15/f*b*d^4*tan(f*x+e)*sec(f*x+e)^2
```

Maxima [A] time = 1.15576, size = 512, normalized size = 2.05

$$480\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)bc^2d^2+320\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)acd^3+16\left(3\,\tan\left(fx+e\right)^5+10\,\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/240*(480*(tan(f*x + e)^3 + 3*tan(f*x + e))*b*c^2*d^2 + 320*(tan(f*x + e)^
3 + 3*tan(f*x + e))*a*c*d^3 + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15
*tan(f*x + e))*b*d^4 - 60*b*c*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(s
in(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin
(f*x + e) - 1)) - 15*a*d^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x
+ e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x +
e) - 1)) - 240*b*c^3*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
 e) + 1) + log(sin(f*x + e) - 1)) - 360*a*c^2*d^2*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a*c^4*lo
g(sec(f*x + e) + tan(f*x + e)) + 240*b*c^4*tan(f*x + e) + 960*a*c^3*d*tan(f
*x + e))/f
```

Fricas [A] time = 0.616146, size = 687, normalized size = 2.75

$$15\left(8\,ac^4+16\,bc^3d+24\,ac^2d^2+12\,bcd^3+3\,ad^4\right)\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\left(8\,ac^4+16\,bc^3d+24\,ac^2d^2+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/240*(15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*cos(
f*x + e)^5*log(sin(f*x + e) + 1) - 15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2
+ 12*b*c*d^3 + 3*a*d^4)*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(24*b*d^4
 + 8*(15*b*c^4 + 60*a*c^3*d + 60*b*c^2*d^2 + 40*a*c*d^3 + 8*b*d^4)*cos(f*x
+ e)^4 + 15*(16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*cos(f*x + e)
^3 + 16*(15*b*c^2*d^2 + 10*a*c*d^3 + 2*b*d^4)*cos(f*x + e)^2 + 30*(4*b*c*d^
3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec\left(e + fx\right)\right)\left(c + d\sec\left(e + fx\right)\right)^4 \sec\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**4*sec(e + f*x), x)
```

Giac [B] time = 1.45158, size = 1207, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] 1/120*(15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*log(
abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 +
12*b*c*d^3 + 3*a*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(120*b*c^4*tan
(1/2*f*x + 1/2*e)^9 + 480*a*c^3*d*tan(1/2*f*x + 1/2*e)^9 - 240*b*c^3*d*tan(
1/2*f*x + 1/2*e)^9 - 360*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^9 + 720*b*c^2*d^2*t
an(1/2*f*x + 1/2*e)^9 + 480*a*c*d^3*tan(1/2*f*x + 1/2*e)^9 - 300*b*c*d^3*ta
n(1/2*f*x + 1/2*e)^9 - 75*a*d^4*tan(1/2*f*x + 1/2*e)^9 + 120*b*d^4*tan(1/2*
f*x + 1/2*e)^9 - 480*b*c^4*tan(1/2*f*x + 1/2*e)^7 - 1920*a*c^3*d*tan(1/2*f*
x + 1/2*e)^7 + 480*b*c^3*d*tan(1/2*f*x + 1/2*e)^7 + 720*a*c^2*d^2*tan(1/2*f
*x + 1/2*e)^7 - 1920*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 1280*a*c*d^3*tan(1/
2*f*x + 1/2*e)^7 + 120*b*c*d^3*tan(1/2*f*x + 1/2*e)^7 + 30*a*d^4*tan(1/2*f*
x + 1/2*e)^7 - 160*b*d^4*tan(1/2*f*x + 1/2*e)^7 + 720*b*c^4*tan(1/2*f*x + 1
/2*e)^5 + 2880*a*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 2400*b*c^2*d^2*tan(1/2*f*x
+ 1/2*e)^5 + 1600*a*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 464*b*d^4*tan(1/2*f*x +
1/2*e)^5 - 480*b*c^4*tan(1/2*f*x + 1/2*e)^3 - 1920*a*c^3*d*tan(1/2*f*x + 1/
2*e)^3 - 480*b*c^3*d*tan(1/2*f*x + 1/2*e)^3 - 720*a*c^2*d^2*tan(1/2*f*x + 1
/2*e)^3 - 1920*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 1280*a*c*d^3*tan(1/2*f*x
+ 1/2*e)^3 - 120*b*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 30*a*d^4*tan(1/2*f*x + 1/
2*e)^3 - 160*b*d^4*tan(1/2*f*x + 1/2*e)^3 + 120*b*c^4*tan(1/2*f*x + 1/2*e)
+ 480*a*c^3*d*tan(1/2*f*x + 1/2*e) + 240*b*c^3*d*tan(1/2*f*x + 1/2*e) + 360
*a*c^2*d^2*tan(1/2*f*x + 1/2*e) + 720*b*c^2*d^2*tan(1/2*f*x + 1/2*e) + 480*
a*c*d^3*tan(1/2*f*x + 1/2*e) + 300*b*c*d^3*tan(1/2*f*x + 1/2*e) + 75*a*d^4*
tan(1/2*f*x + 1/2*e) + 120*b*d^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e
)^2 - 1)^5)/f
```

3.245 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=180

$$\frac{\left(4ad\left(4c^2+d^2\right)+3b\left(c^3+4cd^2\right)\right)\tan(e+fx)}{6f}+\frac{\left(8ac^3+12acd^2+12bc^2d+3bd^3\right)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{d\left(20acd+6bc\right.}{}$$

```
[Out] ((8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*ArcTanh[Sin[e + f*x]])/(8*f)
 + ((4*a*d*(4*c^2 + d^2) + 3*b*(c^3 + 4*c*d^2))*Tan[e + f*x])/(6*f) + (d*(6
*b*c^2 + 20*a*c*d + 9*b*d^2)*Sec[e + f*x]*Tan[e + f*x])/(24*f) + ((3*b*c +
4*a*d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(12*f) + (b*(c + d*Sec[e + f*x]
)^3*Tan[e + f*x])/(4*f)
```

Rubi [A] time = 0.356296, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{\left(4ad\left(4c^2+d^2\right)+3b\left(c^3+4cd^2\right)\right)\tan(e+fx)}{6f}+\frac{\left(8ac^3+12acd^2+12bc^2d+3bd^3\right)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{d\left(20acd+6bc\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*ArcTanh[Sin[e + f*x]])/(8*f)
 + ((4*a*d*(4*c^2 + d^2) + 3*b*(c^3 + 4*c*d^2))*Tan[e + f*x])/(6*f) + (d*(6
*b*c^2 + 20*a*c*d + 9*b*d^2)*Sec[e + f*x]*Tan[e + f*x])/(24*f) + ((3*b*c +
4*a*d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(12*f) + (b*(c + d*Sec[e + f*x]
)^3*Tan[e + f*x])/(4*f)
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx = \frac{b(c+d\sec(e+fx))^3\tan(e+fx)}{4f} + \frac{1}{4}\int \sec(e+fx)(c+d$$

$$= \frac{(3bc+4ad)(c+d\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{b(c+d\sec(e$$

$$= \frac{d\left(6bc^2+20acd+9bd^2\right)\sec(e+fx)\tan(e+fx)}{24f} + \frac{(3bc+$$

$$= \frac{d\left(6bc^2+20acd+9bd^2\right)\sec(e+fx)\tan(e+fx)}{24f} + \frac{(3bc+$$

$$= \frac{\left(8ac^3+12bc^2d+12acd^2+3bd^3\right)\tanh^{-1}(\sin(e+fx))}{8f} + \frac{d}{}$$

$$= \frac{\left(8ac^3+12bc^2d+12acd^2+3bd^3\right)\tanh^{-1}(\sin(e+fx))}{8f} + \frac{(4$$

Mathematica [A] time = 1.08466, size = 143, normalized size = 0.79

$$\frac{3\left(4a\left(2c^3+3cd^2\right)+3bd\left(4c^2+d^2\right)\right)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(8\left(d^2(ad+3bc)\tan^2(e+fx)+3ad\left(3c^2+d^2\right.\right.\right.}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (3*(3*b*d*(4*c^2 + d^2) + 4*a*(2*c^3 + 3*c*d^2))*ArcTanh[Sin[e + f*x]] + Ta
n[e + f*x]*(9*d*(4*a*c*d + b*(4*c^2 + d^2))*Sec[e + f*x] + 6*b*d^3*Sec[e +
f*x]^3 + 8*(3*a*d*(3*c^2 + d^2) + 3*b*(c^3 + 3*c*d^2) + d^2*(3*b*c + a*d)*T
an[e + f*x]^2)))/(24*f)
```

Maple [A] time = 0.04, size = 290, normalized size = 1.6

$$\frac{ac^3\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+3\,\frac{c^2da\tan\left(fx+e\right)}{f}+\frac{3\,acd^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}+\frac{3\,acd^2\ln\left(\sec\left(fx+e\right.\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x)
```

```
[Out] 1/f*a*c^3*ln(sec(f*x+e)+tan(f*x+e))+3/f*a*c^2*d*tan(f*x+e)+3/2/f*a*d^2*c*se
c(f*x+e)*tan(f*x+e)+3/2/f*a*d^2*c*ln(sec(f*x+e)+tan(f*x+e))+2/3/f*a*d^3*tan
(f*x+e)+1/3/f*a*d^3*tan(f*x+e)*sec(f*x+e)^2+1/f*c^3*b*tan(f*x+e)+3/2/f*b*c^
2*d*sec(f*x+e)*tan(f*x+e)+3/2/f*b*c^2*d*ln(sec(f*x+e)+tan(f*x+e))+2/f*d^2*c
*b*tan(f*x+e)+1/f*d^2*c*b*tan(f*x+e)*sec(f*x+e)^2+1/4/f*b*d^3*tan(f*x+e)*se
c(f*x+e)^3+3/8/f*b*d^3*sec(f*x+e)*tan(f*x+e)+3/8/f*b*d^3*ln(sec(f*x+e)+tan(
f*x+e))
```

Maxima [A] time = 1.16779, size = 359, normalized size = 1.99

$$\frac{48\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)bcd^{2}+16\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)ad^{3}-3\,bd^{3}\left(\frac{2\left(3\,\sin\left(fx+e\right)^{3}-5\,\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^{4}-2\,\sin\left(fx+e\right)^{2}+1}-3\,\mathrm{l}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(48*(tan(f*x + e)^3 + 3*tan(f*x + e))*b*c*d^2 + 16*(tan(f*x + e)^3 + 3
*tan(f*x + e))*a*d^3 - 3*b*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(
f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*
x + e) - 1)) - 36*b*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*
x + e) + 1) + log(sin(f*x + e) - 1)) - 36*a*c*d^2*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 48*a*c^3*log
(sec(f*x + e) + tan(f*x + e)) + 48*b*c^3*tan(f*x + e) + 144*a*c^2*d*tan(f*x
 + e))/f
```

Fricas [A] time = 0.578471, size = 510, normalized size = 2.83

$$\frac{3\left(8\,ac^{3}+12\,bc^{2}d+12\,acd^{2}+3\,bd^{3}\right)\cos\left(fx+e\right)^{4}\log\left(\sin\left(fx+e\right)+1\right)-3\left(8\,ac^{3}+12\,bc^{2}d+12\,acd^{2}+3\,bd^{3}\right)\cos\left(fx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*cos(f*x + e)^4*log(si
n(f*x + e) + 1) - 3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*cos(f*x +
 e)^4*log(-sin(f*x + e) + 1) + 2*(6*b*d^3 + 8*(3*b*c^3 + 9*a*c^2*d + 6*b*c*
d^2 + 2*a*d^3)*cos(f*x + e)^3 + 9*(4*b*c^2*d + 4*a*c*d^2 + b*d^3)*cos(f*x +
 e)^2 + 8*(3*b*c*d^2 + a*d^3)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)^{3}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**3*sec(e + f*x), x)
```

Giac [B] time = 1.25672, size = 833, normalized size = 4.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/24*(3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*log(abs(tan(1/2*f*x +
 1/2*e) + 1)) - 3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*log(abs(tan
(1/2*f*x + 1/2*e) - 1)) - 2*(24*b*c^3*tan(1/2*f*x + 1/2*e)^7 + 72*a*c^2*d*t
an(1/2*f*x + 1/2*e)^7 - 36*b*c^2*d*tan(1/2*f*x + 1/2*e)^7 - 36*a*c*d^2*tan(
1/2*f*x + 1/2*e)^7 + 72*b*c*d^2*tan(1/2*f*x + 1/2*e)^7 + 24*a*d^3*tan(1/2*f
*x + 1/2*e)^7 - 15*b*d^3*tan(1/2*f*x + 1/2*e)^7 - 72*b*c^3*tan(1/2*f*x + 1/
2*e)^5 - 216*a*c^2*d*tan(1/2*f*x + 1/2*e)^5 + 36*b*c^2*d*tan(1/2*f*x + 1/2*
e)^5 + 36*a*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 120*b*c*d^2*tan(1/2*f*x + 1/2*e)
^5 - 40*a*d^3*tan(1/2*f*x + 1/2*e)^5 - 9*b*d^3*tan(1/2*f*x + 1/2*e)^5 + 72*
b*c^3*tan(1/2*f*x + 1/2*e)^3 + 216*a*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 36*b*c^
2*d*tan(1/2*f*x + 1/2*e)^3 + 36*a*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 120*b*c*d^
2*tan(1/2*f*x + 1/2*e)^3 + 40*a*d^3*tan(1/2*f*x + 1/2*e)^3 - 9*b*d^3*tan(1/
2*f*x + 1/2*e)^3 - 24*b*c^3*tan(1/2*f*x + 1/2*e) - 72*a*c^2*d*tan(1/2*f*x +
 1/2*e) - 36*b*c^2*d*tan(1/2*f*x + 1/2*e) - 36*a*c*d^2*tan(1/2*f*x + 1/2*e)
 - 72*b*c*d^2*tan(1/2*f*x + 1/2*e) - 24*a*d^3*tan(1/2*f*x + 1/2*e) - 15*b*d
^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

3.246 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=115

$$\frac{2\left(3acd+b\left(c^2+d^2\right)\right)\tan(e+fx)}{3f}+\frac{\left(a\left(2c^2+d^2\right)+2bcd\right)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{d(3ad+2bc)\tan(e+fx)\sec(e+fx)}{6f}$$

```
[Out] ((2*b*c*d + a*(2*c^2 + d^2))*ArcTanh[Sin[e + f*x]])/(2*f) + (2*(3*a*c*d + b
*(c^2 + d^2))*Tan[e + f*x])/(3*f) + (d*(2*b*c + 3*a*d)*Sec[e + f*x]*Tan[e +
 f*x])/(6*f) + (b*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f)
```

Rubi [A] time = 0.185081, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4002, 3997, 3787, 3770, 3767, 8}

$$\frac{2\left(3acd+b\left(c^2+d^2\right)\right)\tan(e+fx)}{3f}+\frac{\left(a\left(2c^2+d^2\right)+2bcd\right)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{d(3ad+2bc)\tan(e+fx)\sec(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((2*b*c*d + a*(2*c^2 + d^2))*ArcTanh[Sin[e + f*x]])/(2*f) + (2*(3*a*c*d + b
*(c^2 + d^2))*Tan[e + f*x])/(3*f) + (d*(2*b*c + 3*a*d)*Sec[e + f*x]*Tan[e +
 f*x])/(6*f) + (b*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f)
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx = \frac{b(c+d\sec(e+fx))^2\tan(e+fx)}{3f}+\frac{1}{3}\int \sec(e+fx)(c+d$$

$$= \frac{d(2bc+3ad)\sec(e+fx)\tan(e+fx)}{6f}+\frac{b(c+d\sec(e+fx)}{3f}$$

$$= \frac{d(2bc+3ad)\sec(e+fx)\tan(e+fx)}{6f}+\frac{b(c+d\sec(e+fx)}{3f}$$

$$= \frac{\left(2bcd+a\left(2c^2+d^2\right)\right)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{d(2bc+3ad)}{}$$

$$= \frac{\left(2bcd+a\left(2c^2+d^2\right)\right)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{2\left(3acd+b\left(c\right.\right.}{}$$

Mathematica [A] time = 0.593982, size = 88, normalized size = 0.77

$$\frac{3\left(a\left(2c^2+d^2\right)+2bcd\right)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(3d(ad+2bc)\sec(e+fx)+12acd+6b\left(c^2+d^2\right)+2bd^2\tan\right.}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (3*(2*b*c*d + a*(2*c^2 + d^2))*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(12*a*c
*d + 6*b*(c^2 + d^2) + 3*d*(2*b*c + a*d)*Sec[e + f*x] + 2*b*d^2*Tan[e + f*x
]^2))/(6*f)
```

Maple [A] time = 0.035, size = 174, normalized size = 1.5

$$\frac{c^2a\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f}+2\frac{acd\tan\left(fx+e\right)}{f}+\frac{ad^2\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f}+\frac{ad^2\ln\left(\sec\left(fx+e\right)+\tan\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x)
```

```
[Out] 1/f*c^2*a*ln(sec(f*x+e)+tan(f*x+e))+2/f*a*c*d*tan(f*x+e)+1/2/f*a*d^2*sec(f*
x+e)*tan(f*x+e)+1/2/f*a*d^2*ln(sec(f*x+e)+tan(f*x+e))+1/f*c^2*b*tan(f*x+e)+
1/f*b*c*d*sec(f*x+e)*tan(f*x+e)+1/f*b*c*d*ln(sec(f*x+e)+tan(f*x+e))+2/3/f*d
^2*b*tan(f*x+e)+1/3/f*d^2*b*tan(f*x+e)*sec(f*x+e)^2
```

Maxima [A] time = 1.02573, size = 223, normalized size = 1.94

$$\frac{4\left(\tan\left(fx+e\right)^{3}+3\tan\left(fx+e\right)\right)bd^{2}-6\,bcd\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-3\,ad^{2}\left(\frac{2\sin\left(\cdots\right.}{\sin\left(\cdots\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*b*d^2 - 6*b*c*d*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 3*a*
d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(
f*x + e) - 1)) + 12*a*c^2*log(sec(f*x + e) + tan(f*x + e)) + 12*b*c^2*tan(f
*x + e) + 24*a*c*d*tan(f*x + e))/f
```

Fricas [A] time = 0.560209, size = 371, normalized size = 3.23

$$\frac{3\left(2\,ac^{2}+2\,bcd+ad^{2}\right)\cos\left(fx+e\right)^{3}\log\left(\sin\left(fx+e\right)+1\right)-3\left(2\,ac^{2}+2\,bcd+ad^{2}\right)\cos\left(fx+e\right)^{3}\log\left(-\sin\left(fx+e\right)+1\cdots\right.}{12\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(2*a*c^2 + 2*b*c*d + a*d^2)*cos(f*x + e)^3*log(sin(f*x + e) + 1) -
3*(2*a*c^2 + 2*b*c*d + a*d^2)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*
b*d^2 + 2*(3*b*c^2 + 6*a*c*d + 2*b*d^2)*cos(f*x + e)^2 + 3*(2*b*c*d + a*d^2
)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)^{2}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**2*sec(e + f*x), x)
```

Giac [B] time = 1.34212, size = 419, normalized size = 3.64

$$\frac{3\left(2\,ac^{2}+2\,bcd+ad^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-3\left(2\,ac^{2}+2\,bcd+ad^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{2\left(6\,bc^{2}\tan\left(\frac{1}{2}fx+\cdots\right.\right.}{\cdots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] 1/6*(3*(2*a*c^2 + 2*b*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*(
2*a*c^2 + 2*b*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(6*b*c^2*
tan(1/2*f*x + 1/2*e)^5 + 12*a*c*d*tan(1/2*f*x + 1/2*e)^5 - 6*b*c*d*tan(1/2*
f*x + 1/2*e)^5 - 3*a*d^2*tan(1/2*f*x + 1/2*e)^5 + 6*b*d^2*tan(1/2*f*x + 1/2
*e)^5 - 12*b*c^2*tan(1/2*f*x + 1/2*e)^3 - 24*a*c*d*tan(1/2*f*x + 1/2*e)^3 -
 4*b*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*b*c^2*tan(1/2*f*x + 1/2*e) + 12*a*c*d*t
an(1/2*f*x + 1/2*e) + 6*b*c*d*tan(1/2*f*x + 1/2*e) + 3*a*d^2*tan(1/2*f*x +
1/2*e) + 6*b*d^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)/f
```

3.247 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=61

$$\frac{(ad+bc)\tan(e+fx)}{f}+\frac{(2ac+bd)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{bd\tan(e+fx)\sec(e+fx)}{2f}$$

```
[Out] ((2*a*c + b*d)*ArcTanh[Sin[e + f*x]])/(2*f) + ((b*c + a*d)*Tan[e + f*x])/f
+ (b*d*Sec[e + f*x]*Tan[e + f*x])/(2*f)
```

Rubi [A] time = 0.0750499, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3997, 3787, 3770, 3767, 8}

$$\frac{(ad+bc)\tan(e+fx)}{f}+\frac{(2ac+bd)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{bd\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

```
[Out] ((2*a*c + b*d)*ArcTanh[Sin[e + f*x]])/(2*f) + ((b*c + a*d)*Tan[e + f*x])/f
+ (b*d*Sec[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx = \frac{bd\sec(e+fx)\tan(e+fx)}{2f} + \frac{1}{2}\int \sec(e+fx)(2ac+bd+2(\ldots$$
$$= \frac{bd\sec(e+fx)\tan(e+fx)}{2f} + (bc+ad)\int \sec^2(e+fx)\,dx + \ldots$$
$$= \frac{(2ac+bd)\tanh^{-1}(\sin(e+fx))}{2f} + \frac{bd\sec(e+fx)\tan(e+fx\ldots}{2f}$$
$$= \frac{(2ac+bd)\tanh^{-1}(\sin(e+fx))}{2f} + \frac{(bc+ad)\tan(e+fx)}{f} + \ldots$$

Mathematica [A] time = 0.0269827, size = 75, normalized size = 1.23

$$\frac{ac\tanh^{-1}(\sin(e+fx))}{f} + \frac{ad\tan(e+fx)}{f} + \frac{bc\tan(e+fx)}{f} + \frac{bd\tanh^{-1}(\sin(e+fx))}{2f} + \frac{bd\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*c*ArcTanh[Sin[e + f*x]])/f + (b*d*ArcTanh[Sin[e + f*x]])/(2*f) + (b*c*Ta
n[e + f*x])/f + (a*d*Tan[e + f*x])/f + (b*d*Sec[e + f*x]*Tan[e + f*x])/(2*f
)
```

Maple [A] time = 0.026, size = 86, normalized size = 1.4

$$\frac{ac\ln\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{f} + \frac{ad\tan\left(fx+e\right)}{f} + \frac{bc\tan\left(fx+e\right)}{f} + \frac{db\sec\left(fx+e\right)\tan\left(fx+e\right)}{2f} + \frac{db\ln\left(\sec\left(\ldots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

```
[Out] 1/f*a*c*ln(sec(f*x+e)+tan(f*x+e))+1/f*a*d*tan(f*x+e)+1/f*b*c*tan(f*x+e)+1/2
*b*d*sec(f*x+e)*tan(f*x+e)/f+1/2/f*d*b*ln(sec(f*x+e)+tan(f*x+e))
```

Maxima [A] time = 1.01459, size = 119, normalized size = 1.95

$$-\frac{bd\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1} - \log\left(\sin\left(fx+e\right)+1\right) + \log\left(\sin\left(fx+e\right)-1\right)\right) - 4ac\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right) - 4bc\tan\left(\ldots\right.}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] -1/4*(b*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) - 4*a*c*log(sec(f*x + e) + tan(f*x + e)) - 4*b*c*tan(f
```

```
*x + e) - 4*a*d*tan(f*x + e))/f
```

Fricas [A] time = 0.535161, size = 247, normalized size = 4.05

$$\frac{(2\,ac+bd)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)-(2\,ac+bd)\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)+2\left(bd+2\,(bc+ad)\,\mathrm{c}\cdots\right.}{4\,f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/4*((2*a*c + b*d)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a*c + b*d)*cos
(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(b*d + 2*(b*c + a*d)*cos(f*x + e))*s
in(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))*sec(e + f*x), x)
```

Giac [B] time = 1.34489, size = 219, normalized size = 3.59

$$\frac{(2\,ac+bd)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-(2\,ac+bd)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(2\,bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\,ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-bd\,\mathrm{t}\cdots\right.}{\cdots}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((2*a*c + b*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - (2*a*c + b*d)*log(a
bs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(2*b*c*tan(1/2*f*x + 1/2*e)^3 + 2*a*d*tan
(1/2*f*x + 1/2*e)^3 - b*d*tan(1/2*f*x + 1/2*e)^3 - 2*b*c*tan(1/2*f*x + 1/2*
e) - 2*a*d*tan(1/2*f*x + 1/2*e) - b*d*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x +
1/2*e)^2 - 1)^2)/f
```

3.248 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=76

$$\frac{b\tanh^{-1}(\sin(e+fx))}{df}-\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{df\sqrt{c-d}\sqrt{c+d}}$$

```
[Out] (b*ArcTanh[Sin[e + f*x]])/(d*f) - (2*(b*c - a*d)*ArcTanh[(Sqrt[c - d]*Tan[(
e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*d*Sqrt[c + d]*f)
```

Rubi [A] time = 0.12879, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3998, 3770, 3831, 2659, 208}

$$\frac{b\tanh^{-1}(\sin(e+fx))}{df}-\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{df\sqrt{c-d}\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

```
[Out] (b*ArcTanh[Sin[e + f*x]])/(d*f) - (2*(b*c - a*d)*ArcTanh[(Sqrt[c - d]*Tan[(
e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*d*Sqrt[c + d]*f)
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{b\int \sec(e+fx)\,dx}{d} + \frac{(-bc+ad)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{d}$$
$$= \frac{b\tanh^{-1}(\sin(e+fx))}{df} - \frac{(bc-ad)\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d^2}$$
$$= \frac{b\tanh^{-1}(\sin(e+fx))}{df} - \frac{(2(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^2 f}$$
$$= \frac{b\tanh^{-1}(\sin(e+fx))}{df} - \frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,d\sqrt{c+d}\,f}$$

Mathematica [A] time = 0.208298, size = 112, normalized size = 1.47

$$\frac{\frac{2(bc-ad)\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + b\left(\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

```
[Out] ((2*(b*c - a*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[
c^2 - d^2] + b*(-Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]]))/(d*f)
```

Maple [A] time = 0.058, size = 135, normalized size = 1.8

$$\frac{b}{fd}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right) - \frac{b}{fd}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right) + 2\frac{a}{f\sqrt{(c+d)(c-d)}}\mathrm{Artanh}\left(\frac{\tan(1/2\,fx+e/2)(c-d)}{\sqrt{(c+d)(c-d)}}\right) - 2\frac{}{fd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] 1/f*b/d*ln(tan(1/2*f*x+1/2*e)+1)-1/f*b/d*ln(tan(1/2*f*x+1/2*e)-1)+2/f/((c+d
)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))*a-2/f/
d/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2))
*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95888, size = 707, normalized size = 9.3

$$\left[-\frac{(bc-ad)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2+2\,\sqrt{c^2-d^2}\left(d\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)+2\,c^2-d^2}{c^2\cos\left(fx+e\right)^2+2\,cd\cos\left(fx+e\right)+d^2}\right)-\left(bc^2-bd^2\right)\log\left(\sin\left(fx\right.\right.}{2\left(c^2d-d^3\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [-1/2*((b*c - a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*
cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^
2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - (b*c^2 - b*d^2)
*log(sin(f*x + e) + 1) + (b*c^2 - b*d^2)*log(-sin(f*x + e) + 1))/((c^2*d -
d^3)*f), -1/2*(2*(b*c - a*d)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*c
os(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (b*c^2 - b*d^2)*log(sin(f*x
+ e) + 1) + (b*c^2 - b*d^2)*log(-sin(f*x + e) + 1))/((c^2*d - d^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sec\left(e+fx\right)\right)\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Giac [A] time = 1.29107, size = 178, normalized size = 2.34

$$\frac{\frac{b\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{d}-\frac{b\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{d}+\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)(bc-ad)}{\sqrt{-c^2+d^2}\,d}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (b*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d - b*log(abs(tan(1/2*f*x + 1/2*e) -
1))/d + 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(
1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(b*c - a*d)/(
sqrt(-c^2 + d^2)*d))/f
```

3.249 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=99

$$\frac{(bc-ad)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}+\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{3/2}}$$

```
[Out] (2*(a*c - b*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c - d
)^(3/2)*(c + d)^(3/2)*f) + ((b*c - a*d)*Tan[e + f*x])/((c^2 - d^2)*f*(c + d
*Sec[e + f*x]))
```

Rubi [A] time = 0.143887, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{(bc-ad)\tan(e+fx)}{f\left(c^2-d^2\right)(c+d\sec(e+fx))}+\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*(a*c - b*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c - d
)^(3/2)*(c + d)^(3/2)*f) + ((b*c - a*d)*Tan[e + f*x])/((c^2 - d^2)*f*(c + d
*Sec[e + f*x]))
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{(bc-ad)\tan(e+fx)}{\left(c^2-d^2\right)f(c+d\sec(e+fx))}+\frac{\int \frac{(-ac+bd)\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{-c^2+d^2}$$
$$= \frac{(bc-ad)\tan(e+fx)}{\left(c^2-d^2\right)f(c+d\sec(e+fx))}+\frac{(ac-bd)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c^2-d^2}$$
$$= \frac{(bc-ad)\tan(e+fx)}{\left(c^2-d^2\right)f(c+d\sec(e+fx))}+\frac{(ac-bd)\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d\left(c^2-d^2\right)}$$
$$= \frac{(bc-ad)\tan(e+fx)}{\left(c^2-d^2\right)f(c+d\sec(e+fx))}+\frac{(2(ac-bd))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx,x,\tan\left(\right.\right.}{d\left(c^2-d^2\right)f}$$
$$= \frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{3/2}f}+\frac{(bc-ad)\tan(e+fx)}{\left(c^2-d^2\right)f(c+d\sec(e+fx))}$$

Mathematica [A] time = 0.360421, size = 97, normalized size = 0.98

$$\frac{\frac{(bc-ad)\sin(e+fx)}{(c-d)(c+d)(c\cos(e+fx)+d)}-\frac{2(ac-bd)\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((-2*(a*c - b*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2
 - d^2)^(3/2) + ((b*c - a*d)*Sin[e + f*x])/((c - d)*(c + d)*(d + c*Cos[e +
f*x])))/f
```

Maple [A] time = 0.067, size = 132, normalized size = 1.3

$$\frac{1}{f}\left(2\,\frac{(ad-bc)\tan\left(1/2\,fx+e/2\right)}{\left(c^2-d^2\right)\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2c-\left(\tan\left(1/2\,fx+e/2\right)\right)^2d-c-d\right)}+2\,\frac{ac-db}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}\mathrm{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 1/f*(2*(a*d-b*c)/(c^2-d^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1
/2*f*x+1/2*e)^2*d-c-d)+2*(a*c-b*d)/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(
tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.529548, size = 861, normalized size = 8.7

$$\left[\frac{\left(acd - bd^2 + \left(ac^2 - bcd\right)\cos\left(fx+e\right)\right)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos\left(fx+e\right) - \left(c^2-2\,d^2\right)\cos\left(fx+e\right)^2 + 2\,\sqrt{c^2-d^2}\left(d\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right) + 2\,c^2 - d^2}{c^2\cos\left(fx+e\right)^2 + 2\,cd\cos\left(fx+e\right) + d^2}\right) + \cdots}{2\left(\left(c^5 - 2\,c^3d^2 + cd^4\right)f\cos\left(fx+e\right) + \left(c^4d - 2\,c^2d^3 + d^5\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((a*c*d - b*d^2 + (a*c^2 - b*c*d)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2
*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos
(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(
f*x + e) + d^2)) + 2*(b*c^3 - a*c^2*d - b*c*d^2 + a*d^3)*sin(f*x + e))/((c^
5 - 2*c^3*d^2 + c*d^4)*f*cos(f*x + e) + (c^4*d - 2*c^2*d^3 + d^5)*f), ((a*c
*d - b*d^2 + (a*c^2 - b*c*d)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c
^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b*c^3 - a*c^2
*d - b*c*d^2 + a*d^3)*sin(f*x + e))/((c^5 - 2*c^3*d^2 + c*d^4)*f*cos(f*x +
e) + (c^4*d - 2*c^2*d^3 + d^5)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\sec\left(e + fx\right)\right)\sec\left(e + fx\right)}{\left(c + d\sec\left(e + fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x))**2, x)
```

Giac [A] time = 1.23511, size = 242, normalized size = 2.44

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)(ac-bd)}{(c^2-d^2)\sqrt{-c^2+d^2}}+\frac{bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)(c^2-d^2)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*
x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(a*c - b*d)/((c^2 -
 d^2)*sqrt(-c^2 + d^2)) + (b*c*tan(1/2*f*x + 1/2*e) - a*d*tan(1/2*f*x + 1/2
*e))/((c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)*(c^2 -
d^2)))/f
```

3.250 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=166

$$-\frac{\left(3bcd-a\left(2c^2+d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{5/2}(c+d)^{5/2}}-\frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{2f\left(c^2-d^2\right)^2(c+d\sec(e+fx))}+\frac{(bc-ad)\tan(e+fx)}{2f\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

```
[Out] -(((3*b*c*d - a*(2*c^2 + d^2))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[
c + d]])/((c - d)^(5/2)*(c + d)^(5/2)*f)) + ((b*c - a*d)*Tan[e + f*x])/(2*(
c^2 - d^2)*f*(c + d*Sec[e + f*x])^2) - ((3*a*c*d - b*(c^2 + 2*d^2))*Tan[e +
 f*x])/(2*(c^2 - d^2)^2*f*(c + d*Sec[e + f*x]))
```

Rubi [A] time = 0.300656, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$-\frac{\left(3bcd-a\left(2c^2+d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{5/2}(c+d)^{5/2}}-\frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{2f\left(c^2-d^2\right)^2(c+d\sec(e+fx))}+\frac{(bc-ad)\tan(e+fx)}{2f\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] -(((3*b*c*d - a*(2*c^2 + d^2))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[
c + d]])/((c - d)^(5/2)*(c + d)^(5/2)*f)) + ((b*c - a*d)*Tan[e + f*x])/(2*(
c^2 - d^2)*f*(c + d*Sec[e + f*x])^2) - ((3*a*c*d - b*(c^2 + 2*d^2))*Tan[e +
 f*x])/(2*(c^2 - d^2)^2*f*(c + d*Sec[e + f*x]))
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{\int \frac{\sec(e+fx)(-2(ac-bd)-(bc-ad)\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx}{2\left(c^2-d^2\right)}$$

$$= \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))} + \frac{\int \frac{(-3b}{}}{}$$

$$= \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))} - \frac{(3bcd}{}$$

$$= \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))} - \frac{(3bcd}{}$$

$$= \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{\left(3acd-b\left(c^2+2d^2\right)\right)\tan(e+fx)}{2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))} - \frac{(3bcd}{}$$

$$= \frac{\left(2ac^2-3bcd+ad^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{5/2}(c+d)^{5/2}f} + \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+f}$$

Mathematica [A] time = 0.822732, size = 172, normalized size = 1.04

$$\frac{\frac{\left(ad\left(d^2-4c^2\right)+bc\left(2c^2+d^2\right)\right)\sin(e+fx)}{c(c-d)^2(c+d)^2(c\cos(e+fx)+d)} - \frac{2\left(a\left(2c^2+d^2\right)-3bcd\right)\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{5/2}} + \frac{d(ad-bc)\sin(e+fx)}{c(c-d)(c+d)(c\cos(e+fx)+d)^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((-2*(-3*b*c*d + a*(2*c^2 + d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]])/(c^2 - d^2)^(5/2) + (d*(-(b*c) + a*d)*Sin[e + f*x])/(c*(c - d)
*(c + d)*(d + c*Cos[e + f*x])^2) + ((a*d*(-4*c^2 + d^2) + b*c*(2*c^2 + d^2)
)*Sin[e + f*x])/(c*(c - d)^2*(c + d)^2*(d + c*Cos[e + f*x])))/(2*f)
```

Maple [A] time = 0.079, size = 236, normalized size = 1.4

$$\frac{1}{f}\left(-2\frac{1}{\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2c-\left(\tan\left(1/2\,fx+e/2\right)\right)^2d-c-d\right)^2}\left(-1/2\frac{\left(4\,acd+ad^2-2\,bc^2-bcd-2\,d^2b\right)\left(\tan\left(1/2\,f\right.\right.}{(c-d)\left(c^2+2\,cd+d^2\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x)
```

```
[Out] 1/f*(-2*(-1/2*(4*a*c*d+a*d^2-2*b*c^2-b*c*d-2*b*d^2)/(c-d)/(c^2+2*c*d+d^2)*t
an(1/2*f*x+1/2*e)^3+1/2*(4*a*c*d-a*d^2-2*b*c^2+b*c*d-2*b*d^2)/(c+d)/(c^2-2*
c*d+d^2)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d
-c-d)^2+(2*a*c^2+a*d^2-3*b*c*d)/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arc
tanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.628727, size = 1631, normalized size = 9.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] [1/4*((2*a*c^2*d^2 - 3*b*c*d^3 + a*d^4 + (2*a*c^4 - 3*b*c^3*d + a*c^2*d^2)*
cos(f*x + e)^2 + 2*(2*a*c^3*d - 3*b*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sqrt(c
^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c
^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e
)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b*c^4*d - 3*a*c^3*d^2 + b*c^2*d^3 + 3
*a*c*d^4 - 2*b*d^5 + (2*b*c^5 - 4*a*c^4*d - b*c^3*d^2 + 5*a*c^2*d^3 - b*c*d
^4 - a*d^5)*cos(f*x + e))*sin(f*x + e))/((c^8 - 3*c^6*d^2 + 3*c^4*d^4 - c^2
*d^6)*f*cos(f*x + e)^2 + 2*(c^7*d - 3*c^5*d^3 + 3*c^3*d^5 - c*d^7)*f*cos(f*
x + e) + (c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)*f), 1/2*((2*a*c^2*d^2 - 3*
b*c*d^3 + a*d^4 + (2*a*c^4 - 3*b*c^3*d + a*c^2*d^2)*cos(f*x + e)^2 + 2*(2*a
*c^3*d - 3*b*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt
(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b*c^4*d -
3*a*c^3*d^2 + b*c^2*d^3 + 3*a*c*d^4 - 2*b*d^5 + (2*b*c^5 - 4*a*c^4*d - b*c^
3*d^2 + 5*a*c^2*d^3 - b*c*d^4 - a*d^5)*cos(f*x + e))*sin(f*x + e))/((c^8 -
3*c^6*d^2 + 3*c^4*d^4 - c^2*d^6)*f*cos(f*x + e)^2 + 2*(c^7*d - 3*c^5*d^3 +
3*c^3*d^5 - c*d^7)*f*cos(f*x + e) + (c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)
*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\sec(e + fx))\sec(e + fx)}{(c + d\sec(e + fx))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x))**3, x)
```

Giac [B] time = 1.48116, size = 564, normalized size = 3.4

$$\frac{\left(2\,ac^2-3\,bcd+ad^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,c+2\,d\right)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(c^4-2\,c^2d^2+d^4\right)\sqrt{-c^2+d^2}}-\frac{2\,bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-4\,ac^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-bc^2d\tan\left(\frac{1}{2}\,fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] ((2*a*c^2 - 3*b*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2
*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 +
 d^2)))/((c^4 - 2*c^2*d^2 + d^4)*sqrt(-c^2 + d^2)) - (2*b*c^3*tan(1/2*f*x +
 1/2*e)^3 - 4*a*c^2*d*tan(1/2*f*x + 1/2*e)^3 - b*c^2*d*tan(1/2*f*x + 1/2*e)
^3 + 3*a*c*d^2*tan(1/2*f*x + 1/2*e)^3 + b*c*d^2*tan(1/2*f*x + 1/2*e)^3 + a*
d^3*tan(1/2*f*x + 1/2*e)^3 - 2*b*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*b*c^3*tan(1
/2*f*x + 1/2*e) + 4*a*c^2*d*tan(1/2*f*x + 1/2*e) - b*c^2*d*tan(1/2*f*x + 1/
2*e) + 3*a*c*d^2*tan(1/2*f*x + 1/2*e) - b*c*d^2*tan(1/2*f*x + 1/2*e) - a*d^
3*tan(1/2*f*x + 1/2*e) - 2*b*d^3*tan(1/2*f*x + 1/2*e))/((c^4 - 2*c^2*d^2 +
d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

3.251 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=237

$$\frac{\left(2ac^3+3acd^2-4bc^2d-bd^3\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{7/2}(c+d)^{7/2}}+\frac{\left(-11ac^2d-4ad^3+2bc^3+13bcd^2\right)\tan(e+fx)}{6f\left(c^2-d^2\right)^3(c+d\sec(e+fx))}+\frac{\left(-5acd+2b\right.}{6f\left(c^2-d^2\right.}$$

```
[Out] ((2*a*c^3 - 4*b*c^2*d + 3*a*c*d^2 - b*d^3)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*
x)/2])/Sqrt[c + d]])/((c - d)^(7/2)*(c + d)^(7/2)*f) + ((b*c - a*d)*Tan[e +
 f*x])/(3*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^3) + ((2*b*c^2 - 5*a*c*d + 3*b
*d^2)*Tan[e + f*x])/(6*(c^2 - d^2)^2*f*(c + d*Sec[e + f*x])^2) + ((2*b*c^3
- 11*a*c^2*d + 13*b*c*d^2 - 4*a*d^3)*Tan[e + f*x])/(6*(c^2 - d^2)^3*f*(c +
d*Sec[e + f*x]))
```

Rubi [A] time = 0.511888, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{\left(2ac^3+3acd^2-4bc^2d-bd^3\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{7/2}(c+d)^{7/2}}+\frac{\left(-11ac^2d-4ad^3+2bc^3+13bcd^2\right)\tan(e+fx)}{6f\left(c^2-d^2\right)^3(c+d\sec(e+fx))}+\frac{\left(-5acd+2b\right.}{6f\left(c^2-d^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```


```
[Out] ((2*a*c^3 - 4*b*c^2*d + 3*a*c*d^2 - b*d^3)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*
x)/2])/Sqrt[c + d]])/((c - d)^(7/2)*(c + d)^(7/2)*f) + ((b*c - a*d)*Tan[e +
 f*x])/(3*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^3) + ((2*b*c^2 - 5*a*c*d + 3*b
*d^2)*Tan[e + f*x])/(6*(c^2 - d^2)^2*f*(c + d*Sec[e + f*x])^2) + ((2*b*c^3
- 11*a*c^2*d + 13*b*c*d^2 - 4*a*d^3)*Tan[e + f*x])/(6*(c^2 - d^2)^3*f*(c +
d*Sec[e + f*x]))
```


Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \frac{(bc-ad)\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} - \frac{\int \frac{\sec(e+fx)(-3(ac-bd)-2(bc-ad)\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx}{3\left(c^2-d^2\right)}$$

$$= \frac{(bc-ad)\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{\left(2bc^2-5acd+3bd^2\right)\tan(e+fx)}{6\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))^2} + \frac{\int \frac{\sec(}{}}{}$$

$$= \frac{(bc-ad)\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{\left(2bc^2-5acd+3bd^2\right)\tan(e+fx)}{6\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))^2} + \frac{\left(2bc^3\right.}{}$$

$$= \frac{(bc-ad)\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{\left(2bc^2-5acd+3bd^2\right)\tan(e+fx)}{6\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))^2} + \frac{\left(2bc^3\right.}{}$$

$$= \frac{(bc-ad)\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{\left(2bc^2-5acd+3bd^2\right)\tan(e+fx)}{6\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))^2} + \frac{\left(2bc^3\right.}{}$$

$$= \frac{(bc-ad)\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{\left(2bc^2-5acd+3bd^2\right)\tan(e+fx)}{6\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))^2} + \frac{\left(2bc^3\right.}{}$$

$$= \frac{\left(2ac^3-4bc^2d+3acd^2-bd^3\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{7/2}(c+d)^{7/2}f} + \frac{(bc-ad)\tan(}{3\left(c^2-d^2\right)f(c+d\,\mathrm{s}}$$

Mathematica [A] time = 1.03578, size = 405, normalized size = 1.71

$$\frac{\sec^3(e+fx)(a+b\sec(e+fx))(c\cos(e+fx)+d)\left(\frac{24\left(a\left(2c^3+3cd^2\right)-bd\left(4c^2+d^2\right)\right)(c\cos(e+fx)+d)^3\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + 54ac^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]^3*(a + b*Sec[e + f*x])*((24*(-(b*d*(4*c^
2 + d^2)) + a*(2*c^3 + 3*c*d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c
```

```
^2 - d^2]]*(d + c*Cos[e + f*x])^3)/Sqrt[c^2 - d^2] - 6*b*c^5*Sin[e + f*x] +
 18*a*c^4*d*Sin[e + f*x] - 18*b*c^3*d^2*Sin[e + f*x] + 39*a*c^2*d^3*Sin[e +
 f*x] - 51*b*c*d^4*Sin[e + f*x] + 18*a*d^5*Sin[e + f*x] - 12*b*c^4*d*Sin[2*
(e + f*x)] + 54*a*c^3*d^2*Sin[2*(e + f*x)] - 54*b*c^2*d^3*Sin[2*(e + f*x)]
+ 6*a*c*d^4*Sin[2*(e + f*x)] + 6*b*d^5*Sin[2*(e + f*x)] - 6*b*c^5*Sin[3*(e
+ f*x)] + 18*a*c^4*d*Sin[3*(e + f*x)] - 10*b*c^3*d^2*Sin[3*(e + f*x)] - 5*a
*c^2*d^3*Sin[3*(e + f*x)] + b*c*d^4*Sin[3*(e + f*x)] + 2*a*d^5*Sin[3*(e + f
*x)]))/(24*(-c^2 + d^2)^3*f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^4)
```

Maple [A] time = 0.092, size = 376, normalized size = 1.6

$$\frac{1}{f}\left(-2\frac{1}{\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2c-\left(\tan\left(1/2\,fx+e/2\right)\right)^2d-c-d\right)^3}\left(-1/2\,\frac{\left(6\,ac^2d+3\,acd^2+2\,ad^3-2\,bc^3-2\,bc^2d-6\,bcd^2\right.}{(c-d)\left(c^3+3\,c^2d+3\,d^2c\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x)
```

```
[Out] 1/f*(-2*(-1/2*(6*a*c^2*d+3*a*c*d^2+2*a*d^3-2*b*c^3-2*b*c^2*d-6*b*c*d^2-b*d^
3)/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5+2/3*(9*a*c^2*d+a*d^
3-3*b*c^3-7*b*c*d^2)/(c^2+2*c*d+d^2)/(c^2-2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1
/2*(6*a*c^2*d-3*a*c*d^2+2*a*d^3-2*b*c^3+2*b*c^2*d-6*b*c*d^2+b*d^3)/(c+d)/(c
^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2
*f*x+1/2*e)^2*d-c-d)^3+(2*a*c^3+3*a*c*d^2-4*b*c^2*d-b*d^3)/(c^6-3*c^4*d^2+3
*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d)*(
c-d))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.751108, size = 2707, normalized size = 11.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] [1/12*(3*(2*a*c^3*d^3 - 4*b*c^2*d^4 + 3*a*c*d^5 - b*d^6 + (2*a*c^6 - 4*b*c^
5*d + 3*a*c^4*d^2 - b*c^3*d^3)*cos(f*x + e)^3 + 3*(2*a*c^5*d - 4*b*c^4*d^2
+ 3*a*c^3*d^3 - b*c^2*d^4)*cos(f*x + e)^2 + 3*(2*a*c^4*d^2 - 4*b*c^3*d^3 +
```

```
3*a*c^2*d^4 - b*c*d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e
) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*s
in(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2))
 + 2*(2*b*c^5*d^2 - 11*a*c^4*d^3 + 11*b*c^3*d^4 + 7*a*c^2*d^5 - 13*b*c*d^6
+ 4*a*d^7 + (6*b*c^7 - 18*a*c^6*d + 4*b*c^5*d^2 + 23*a*c^4*d^3 - 11*b*c^3*d
^4 - 7*a*c^2*d^5 + b*c*d^6 + 2*a*d^7)*cos(f*x + e)^2 + 3*(2*b*c^6*d - 9*a*c
^5*d^2 + 7*b*c^4*d^3 + 8*a*c^3*d^4 - 10*b*c^2*d^5 + a*c*d^6 + b*d^7)*cos(f*
x + e))*sin(f*x + e))/((c^11 - 4*c^9*d^2 + 6*c^7*d^4 - 4*c^5*d^6 + c^3*d^8)
*f*cos(f*x + e)^3 + 3*(c^10*d - 4*c^8*d^3 + 6*c^6*d^5 - 4*c^4*d^7 + c^2*d^9
)*f*cos(f*x + e)^2 + 3*(c^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^1
0)*f*cos(f*x + e) + (c^8*d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f)
, 1/6*(3*(2*a*c^3*d^3 - 4*b*c^2*d^4 + 3*a*c*d^5 - b*d^6 + (2*a*c^6 - 4*b*c^
5*d + 3*a*c^4*d^2 - b*c^3*d^3)*cos(f*x + e)^3 + 3*(2*a*c^5*d - 4*b*c^4*d^2
+ 3*a*c^3*d^3 - b*c^2*d^4)*cos(f*x + e)^2 + 3*(2*a*c^4*d^2 - 4*b*c^3*d^3 +
3*a*c^2*d^4 - b*c*d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d
^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (2*b*c^5*d^2 - 11*a*
c^4*d^3 + 11*b*c^3*d^4 + 7*a*c^2*d^5 - 13*b*c*d^6 + 4*a*d^7 + (6*b*c^7 - 18
*a*c^6*d + 4*b*c^5*d^2 + 23*a*c^4*d^3 - 11*b*c^3*d^4 - 7*a*c^2*d^5 + b*c*d^
6 + 2*a*d^7)*cos(f*x + e)^2 + 3*(2*b*c^6*d - 9*a*c^5*d^2 + 7*b*c^4*d^3 + 8*
a*c^3*d^4 - 10*b*c^2*d^5 + a*c*d^6 + b*d^7)*cos(f*x + e))*sin(f*x + e))/((c
^11 - 4*c^9*d^2 + 6*c^7*d^4 - 4*c^5*d^6 + c^3*d^8)*f*cos(f*x + e)^3 + 3*(c^
10*d - 4*c^8*d^3 + 6*c^6*d^5 - 4*c^4*d^7 + c^2*d^9)*f*cos(f*x + e)^2 + 3*(c
^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^10)*f*cos(f*x + e) + (c^8*
d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\sec\left(e+fx\right)\right)\sec\left(e+fx\right)}{\left(c+d\sec\left(e+fx\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x))**4, x)
```

Giac [B] time = 1.4312, size = 980, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] -1/3*(3*(2*a*c^3 - 4*b*c^2*d + 3*a*c*d^2 - b*d^3)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x +
1/2*e))/sqrt(-c^2 + d^2)))/((c^6 - 3*c^4*d^2 + 3*c^2*d^4 - d^6)*sqrt(-c^2 +
 d^2)) + (6*b*c^5*tan(1/2*f*x + 1/2*e)^5 - 18*a*c^4*d*tan(1/2*f*x + 1/2*e)^
5 - 6*b*c^4*d*tan(1/2*f*x + 1/2*e)^5 + 27*a*c^3*d^2*tan(1/2*f*x + 1/2*e)^5
+ 12*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^5 - 6*a*c^2*d^3*tan(1/2*f*x + 1/2*e)^5
- 27*b*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 + 3*a*c*d^4*tan(1/2*f*x + 1/2*e)^5 +
12*b*c*d^4*tan(1/2*f*x + 1/2*e)^5 - 6*a*d^5*tan(1/2*f*x + 1/2*e)^5 + 3*b*d^
5*tan(1/2*f*x + 1/2*e)^5 - 12*b*c^5*tan(1/2*f*x + 1/2*e)^3 + 36*a*c^4*d*tan
```

```
(1/2*f*x + 1/2*e)^3 - 16*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 32*a*c^2*d^3*ta
n(1/2*f*x + 1/2*e)^3 + 28*b*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 4*a*d^5*tan(1/2*
f*x + 1/2*e)^3 + 6*b*c^5*tan(1/2*f*x + 1/2*e) - 18*a*c^4*d*tan(1/2*f*x + 1/
2*e) + 6*b*c^4*d*tan(1/2*f*x + 1/2*e) - 27*a*c^3*d^2*tan(1/2*f*x + 1/2*e) +
 12*b*c^3*d^2*tan(1/2*f*x + 1/2*e) - 6*a*c^2*d^3*tan(1/2*f*x + 1/2*e) + 27*
b*c^2*d^3*tan(1/2*f*x + 1/2*e) - 3*a*c*d^4*tan(1/2*f*x + 1/2*e) + 12*b*c*d^
4*tan(1/2*f*x + 1/2*e) - 6*a*d^5*tan(1/2*f*x + 1/2*e) - 3*b*d^5*tan(1/2*f*x
 + 1/2*e))/((c^6 - 3*c^4*d^2 + 3*c^2*d^4 - d^6)*(c*tan(1/2*f*x + 1/2*e)^2 -
 d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))/f
```

3.252 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=247

$$\frac{d^2\left(a^2d^2-4abcd+6b^2c^2\right)\tan(e+fx)}{b^3f}+\frac{d(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f}+\frac{d^3(4bc-ad)\tan}{2b^2}$$

```
[Out] (d^3*(4*b*c - a*d)*ArcTanh[Sin[e + f*x]])/(2*b^2*f) + (d*(2*b*c - a*d)*(2*b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*ArcTanh[Sin[e + f*x]])/(b^4*f) + (2*(b*c - a*
d)^4*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*
Sqrt[a + b]*f) + (d^4*Tan[e + f*x])/(b*f) + (d^2*(6*b^2*c^2 - 4*a*b*c*d + a
^2*d^2)*Tan[e + f*x])/(b^3*f) + (d^3*(4*b*c - a*d)*Sec[e + f*x]*Tan[e + f*x
])/(2*b^2*f) + (d^4*Tan[e + f*x]^3)/(3*b*f)
```

Rubi [A] time = 0.444554, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3988, 2952, 2659, 208, 3770, 3767, 8, 3768}

$$\frac{d^2\left(a^2d^2-4abcd+6b^2c^2\right)\tan(e+fx)}{b^3f}+\frac{d(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f}+\frac{d^3(4bc-ad)\tan}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x]),x]
```


```
[Out] (d^3*(4*b*c - a*d)*ArcTanh[Sin[e + f*x]])/(2*b^2*f) + (d*(2*b*c - a*d)*(2*b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*ArcTanh[Sin[e + f*x]])/(b^4*f) + (2*(b*c - a*
d)^4*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*
Sqrt[a + b]*f) + (d^4*Tan[e + f*x])/(b*f) + (d^2*(6*b^2*c^2 - 4*a*b*c*d + a
^2*d^2)*Tan[e + f*x])/(b^3*f) + (d^3*(4*b*c - a*d)*Sec[e + f*x]*Tan[e + f*x
])/(2*b^2*f) + (d^4*Tan[e + f*x]^3)/(3*b*f)
```


Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rule 2952

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx = \int \frac{(d+c\cos(e+fx))^4\sec^4(e+fx)}{b+a\cos(e+fx)}\,dx$$

$$= \int \left(\frac{(bc-ad)^4}{b^4(b+a\cos(e+fx))} + \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)\sec(e+fx)}{b^4} + \frac{d^2}{}\right.$$

$$= \frac{d^4\int\sec^4(e+fx)\,dx}{b} + \frac{(bc-ad)^4\int\frac{1}{b+a\cos(e+fx)}\,dx}{b^4} + \frac{\left(d^3(4bc-ad)\right)\int\sec^3(e+}{b^2}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f} + \frac{d^3(4bc-ad)\sec(e+}{2b^2f}$$

$$= \frac{d^3(4bc-ad)\tanh^{-1}(\sin(e+fx))}{2b^2f} + \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)\tanh^{-1}(}{b^4f}$$

Mathematica [B] time = 4.34563, size = 580, normalized size = 2.35

$$\frac{\cos^3(e+fx)(a\cos(e+fx)+b)(c+d\sec(e+fx))^4\left(\frac{4bd^2\left(3a^2d^2-12abcd+2b^2\left(9c^2+d^2\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)} + \frac{4bd^2\left(3a^2d^2-12abcd+2b^2\left(9c^2+d^2\right)\right)\sin}{\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+f\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]^3*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^4*((-24*(b*c - a*
d)^4*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
- 6*d*(8*a^2*b*c*d^2 - 2*a^3*d^3 + 4*b^3*c*(2*c^2 + d^2) - a*b^2*d*(12*c^2
+ d^2))*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 6*d*(-8*a^2*b*c*d^2 + 2*
a^3*d^3 - 4*b^3*c*(2*c^2 + d^2) + a*b^2*d*(12*c^2 + d^2))*Log[Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]] + (b^2*d^3*(-3*a*d + b*(12*c + d)))/(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^2 + (2*b^3*d^4*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^3 + (4*b*d^2*(-12*a*b*c*d + 3*a^2*d^2 + 2*b^2*(9*c^2 + d
^2))*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (2*b^3*d^4*S
in[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - (b^2*d^3*(-3*a*d
 + b*(12*c + d)))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (4*b*d^2*(-12*a
*b*c*d + 3*a^2*d^2 + 2*b^2*(9*c^2 + d^2))*Sin[(e + f*x)/2])/(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])))/(12*b^4*f*(d + c*Cos[e + f*x])^4*(a + b*Sec[e + f*
x]))
```

Maple [B] time = 0.098, size = 1066, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x)
```

```
[Out] 2/f/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2
))*c^4-1/3/f*d^4/b/(tan(1/2*f*x+1/2*e)+1)^3-1/f*d^4/b/(tan(1/2*f*x+1/2*e)+1
)+1/2/f*d^4/b/(tan(1/2*f*x+1/2*e)+1)^2-1/3/f*d^4/b/(tan(1/2*f*x+1/2*e)-1)^3
-1/f*d^4/b/(tan(1/2*f*x+1/2*e)-1)-1/2/f*d^4/b/(tan(1/2*f*x+1/2*e)-1)^2+12/f
/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/
2))*a^2*c^2*d^2-8/f/b^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e
)/((a+b)*(a-b))^(1/2))*a^3*c*d^3-8/f/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*ta
n(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*a*c^3*d+2/f*d^3/b/(tan(1/2*f*x+1/2*e)
-1)^2*c-1/2/f*d^4/b^2/(tan(1/2*f*x+1/2*e)+1)*a-6/f*d^2/b/(tan(1/2*f*x+1/2*e
)+1)*c^2+2/f*d^3/b/(tan(1/2*f*x+1/2*e)+1)*c+1/2/f*d^4/b^2/(tan(1/2*f*x+1/2*
e)+1)^2*a-2/f*d^3/b/(tan(1/2*f*x+1/2*e)+1)^2*c-1/f*d^4/b^4*ln(tan(1/2*f*x+1
/2*e)+1)*a^3-1/2/f*d^4/b^2*ln(tan(1/2*f*x+1/2*e)+1)*a+4/f*d/b*ln(tan(1/2*f*
x+1/2*e)+1)*c^3+2/f*d^3/b*ln(tan(1/2*f*x+1/2*e)+1)*c-1/f*d^4/b^3/(tan(1/2*f
*x+1/2*e)+1)*a^2+1/f*d^4/b^4*ln(tan(1/2*f*x+1/2*e)-1)*a^3+1/2/f*d^4/b^2*ln(
tan(1/2*f*x+1/2*e)-1)*a-4/f*d/b*ln(tan(1/2*f*x+1/2*e)-1)*c^3-2/f*d^3/b*ln(t
an(1/2*f*x+1/2*e)-1)*c-1/f*d^4/b^3/(tan(1/2*f*x+1/2*e)-1)*a^2-1/2/f*d^4/b^2
/(tan(1/2*f*x+1/2*e)-1)*a-6/f*d^2/b/(tan(1/2*f*x+1/2*e)-1)*c^2+2/f*d^3/b/(t
an(1/2*f*x+1/2*e)-1)*c-1/2/f*d^4/b^2/(tan(1/2*f*x+1/2*e)-1)^2*a+6/f*d^2/b^2
*ln(tan(1/2*f*x+1/2*e)-1)*a*c^2+4/f*d^3/b^2/(tan(1/2*f*x+1/2*e)-1)*a*c+2/f/
b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2
))*a^4*d^4+4/f*d^3/b^3*ln(tan(1/2*f*x+1/2*e)+1)*a^2*c-6/f*d^2/b^2*ln(tan(1/
2*f*x+1/2*e)+1)*a*c^2+4/f*d^3/b^2/(tan(1/2*f*x+1/2*e)+1)*a*c-4/f*d^3/b^3*ln
(tan(1/2*f*x+1/2*e)-1)*a^2*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + d \sec\left(e + fx\right)\right)^4 \sec\left(e + fx\right)}{a + b \sec\left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**4*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Giac [B] time = 1.46134, size = 848, normalized size = 3.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(8*b^3*c^3*d - 12*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + 4*b^3*c*d^3 - 2*a^
3*d^4 - a*b^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^4 - 3*(8*b^3*c^3*d
- 12*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + 4*b^3*c*d^3 - 2*a^3*d^4 - a*b^2*d^4)*l
og(abs(tan(1/2*f*x + 1/2*e) - 1))/b^4 - 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2
*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*s
gn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sq
rt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^4) - 2*(36*b^2*c^2*d^2*tan(1/2*f*x + 1
/2*e)^5 - 24*a*b*c*d^3*tan(1/2*f*x + 1/2*e)^5 - 12*b^2*c*d^3*tan(1/2*f*x +
1/2*e)^5 + 6*a^2*d^4*tan(1/2*f*x + 1/2*e)^5 + 3*a*b*d^4*tan(1/2*f*x + 1/2*e
)^5 + 6*b^2*d^4*tan(1/2*f*x + 1/2*e)^5 - 72*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e
)^3 + 48*a*b*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 12*a^2*d^4*tan(1/2*f*x + 1/2*e)
^3 - 4*b^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 36*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e)
 - 24*a*b*c*d^3*tan(1/2*f*x + 1/2*e) + 12*b^2*c*d^3*tan(1/2*f*x + 1/2*e) +
6*a^2*d^4*tan(1/2*f*x + 1/2*e) - 3*a*b*d^4*tan(1/2*f*x + 1/2*e) + 6*b^2*d^4
*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*b^3))/f
```

3.253 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=170

$$\frac{d\left(a^2d^2-3abcd+3b^2c^2\right)\tanh^{-1}(\sin(e+fx))}{b^3f}+\frac{d^2(3bc-ad)\tan(e+fx)}{b^2f}+\frac{2(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^3f\sqrt{a-b}\sqrt{a+b}}+$$

```
[Out] (d^3*ArcTanh[Sin[e + f*x]])/(2*b*f) + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*
ArcTanh[Sin[e + f*x]])/(b^3*f) + (2*(b*c - a*d)^3*ArcTanh[(Sqrt[a - b]*Tan[
(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*f) + (d^2*(3*b*c -
 a*d)*Tan[e + f*x])/(b^2*f) + (d^3*Sec[e + f*x]*Tan[e + f*x])/(2*b*f)
```

Rubi [A] time = 0.350474, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3988, 2952, 2659, 208, 3770, 3767, 8, 3768}

$$\frac{d\left(a^2d^2-3abcd+3b^2c^2\right)\tanh^{-1}(\sin(e+fx))}{b^3f}+\frac{d^2(3bc-ad)\tan(e+fx)}{b^2f}+\frac{2(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^3f\sqrt{a-b}\sqrt{a+b}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (d^3*ArcTanh[Sin[e + f*x]])/(2*b*f) + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*
ArcTanh[Sin[e + f*x]])/(b^3*f) + (2*(b*c - a*d)^3*ArcTanh[(Sqrt[a - b]*Tan[
(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*f) + (d^2*(3*b*c -
 a*d)*Tan[e + f*x])/(b^2*f) + (d^3*Sec[e + f*x]*Tan[e + f*x])/(2*b*f)
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rule 2952

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx = \int \frac{(d+c\cos(e+fx))^3\sec^3(e+fx)}{b+a\cos(e+fx)}\,dx$$

$$= \int \left(\frac{(bc-ad)^3}{b^3(b+a\cos(e+fx))} + \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)\sec(e+fx)}{b^3} + \frac{d^2(3bc-ad)}{b}\right.$$

$$= \frac{d^3\int \sec^3(e+fx)\,dx}{b} + \frac{(bc-ad)^3\int \frac{1}{b+a\cos(e+fx)}\,dx}{b^3} + \frac{\left(d^2(3bc-ad)\right)\int \sec^2(e+}{b^2}$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)\tanh^{-1}(\sin(e+fx))}{b^3f} + \frac{d^3\sec(e+fx)\tan(e+fx)}{2bf} + \frac{d^3}{}$$

$$= \frac{d^3\tanh^{-1}(\sin(e+fx))}{2bf} + \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)\tanh^{-1}(\sin(e+fx))}{b^3f} + \frac{2(bc}{}$$

Mathematica [B] time = 1.40604, size = 389, normalized size = 2.29

$$\frac{\cos^2(e+fx)(a\cos(e+fx)+b)(c+d\sec(e+fx))^3\left(-2d\left(2a^2d^2-6abcd+b^2\left(6c^2+d^2\right)\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]^2*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^3*((8*(-(b*c) + a
*d)^3*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

```
 - 2*d*(-6*a*b*c*d + 2*a^2*d^2 + b^2*(6*c^2 + d^2))*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]] + 2*d*(-6*a*b*c*d + 2*a^2*d^2 + b^2*(6*c^2 + d^2))*Log[Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b^2*d^3)/(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^2 + (4*b*d^2*(3*b*c - a*d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - (b^2*d^3)/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (4*
b*d^2*(3*b*c - a*d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
))/(4*b^3*f*(d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x]))
```

Maple [B] time = 0.081, size = 593, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x)
```

```
[Out] -2/f/b^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))
^(1/2))*a^3*d^3+6/f/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e
)/((a+b)*(a-b))^(1/2))*a^2*c*d^2-6/f/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*ta
n(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*a*c^2*d+2/f/((a+b)*(a-b))^(1/2)*arcta
nh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*c^3-1/2/f*d^3/b/(tan(1/2*f
*x+1/2*e)+1)^2+1/f*d^3/b^3*ln(tan(1/2*f*x+1/2*e)+1)*a^2-3/f*d^2/b^2*ln(tan(
1/2*f*x+1/2*e)+1)*a*c+3/f*d/b*ln(tan(1/2*f*x+1/2*e)+1)*c^2+1/2/f*d^3/b*ln(t
an(1/2*f*x+1/2*e)+1)+1/f*d^3/b^2/(tan(1/2*f*x+1/2*e)+1)*a-3/f*d^2/b/(tan(1/
2*f*x+1/2*e)+1)*c+1/2/f*d^3/b/(tan(1/2*f*x+1/2*e)+1)+1/2/f*d^3/b/(tan(1/2*f
*x+1/2*e)-1)^2-1/f*d^3/b^3*ln(tan(1/2*f*x+1/2*e)-1)*a^2+3/f*d^2/b^2*ln(tan(
1/2*f*x+1/2*e)-1)*a*c-3/f*d/b*ln(tan(1/2*f*x+1/2*e)-1)*c^2-1/2/f*d^3/b*ln(t
an(1/2*f*x+1/2*e)-1)+1/f*d^3/b^2/(tan(1/2*f*x+1/2*e)-1)*a-3/f*d^2/b/(tan(1/
2*f*x+1/2*e)-1)*c+1/2/f*d^3/b/(tan(1/2*f*x+1/2*e)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 85.8349, size = 1621, normalized size = 9.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [-1/4*(2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(a^2 - b^2
)*cos(f*x + e)^2*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 - 2
```

```
*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(
f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) - (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3
*b - a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*cos(f*x + e)^2*log(sin(f*x
 + e) + 1) + (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2*a^4 -
a^2*b^2 - b^4)*d^3)*cos(f*x + e)^2*log(-sin(f*x + e) + 1) - 2*((a^2*b^2 - b
^4)*d^3 + 2*(3*(a^2*b^2 - b^4)*c*d^2 - (a^3*b - a*b^3)*d^3)*cos(f*x + e))*s
in(f*x + e))/((a^2*b^3 - b^5)*f*cos(f*x + e)^2), 1/4*(4*(b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*
(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e)))*cos(f*x + e)^2 + (6*(a^2*b
^2 - b^4)*c^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*co
s(f*x + e)^2*log(sin(f*x + e) + 1) - (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3*b -
a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*cos(f*x + e)^2*log(-sin(f*x + e
) + 1) + 2*((a^2*b^2 - b^4)*d^3 + 2*(3*(a^2*b^2 - b^4)*c*d^2 - (a^3*b - a*b
^3)*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*b^3 - b^5)*f*cos(f*x + e)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + d\sec\left(e + fx\right)\right)^3 \sec\left(e + fx\right)}{a + b\sec\left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**3*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Giac [B] time = 1.34235, size = 474, normalized size = 2.79

$$\frac{\left(6\,b^2c^2d-6\,abcd^2+2\,a^2d^3+b^2d^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{b^3} - \frac{\left(6\,b^2c^2d-6\,abcd^2+2\,a^2d^3+b^2d^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{b^3} - \frac{4\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\left(\pi\left\lfloor \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/2*((6*b^2*c^2*d - 6*a*b*c*d^2 + 2*a^2*d^3 + b^2*d^3)*log(abs(tan(1/2*f*x
+ 1/2*e) + 1))/b^3 - (6*b^2*c^2*d - 6*a*b*c*d^2 + 2*a^2*d^3 + b^2*d^3)*log(
abs(tan(1/2*f*x + 1/2*e) - 1))/b^3 - 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((
a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/(sqrt(-
a^2 + b^2)*b^3) - 2*(6*b*c*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a*d^3*tan(1/2*f*x
 + 1/2*e)^3 - b*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*b*c*d^2*tan(1/2*f*x + 1/2*e)
 + 2*a*d^3*tan(1/2*f*x + 1/2*e) - b*d^3*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x
 + 1/2*e)^2 - 1)^2*b^2))/f
```

3.254 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=103

$$\frac{d(2bc-ad)\tanh^{-1}(\sin(e+fx))}{b^2f}+\frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^2f\sqrt{a-b}\sqrt{a+b}}+\frac{d^2\tan(e+fx)}{bf}$$

```
[Out] (d*(2*b*c - a*d)*ArcTanh[Sin[e + f*x]])/(b^2*f) + (2*(b*c - a*d)^2*ArcTanh[
(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*f
) + (d^2*Tan[e + f*x])/(b*f)
```

Rubi [A] time = 0.297448, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3988, 2952, 2659, 208, 3770, 3767, 8}

$$\frac{d(2bc-ad)\tanh^{-1}(\sin(e+fx))}{b^2f}+\frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^2f\sqrt{a-b}\sqrt{a+b}}+\frac{d^2\tan(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (d*(2*b*c - a*d)*ArcTanh[Sin[e + f*x]])/(b^2*f) + (2*(b*c - a*d)^2*ArcTanh[
(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*f
) + (d^2*Tan[e + f*x])/(b*f)
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rule 2952

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx = \int \frac{(d+c\cos(e+fx))^2\sec^2(e+fx)}{b+a\cos(e+fx)}\,dx$$

$$= \int \left(\frac{(bc-ad)^2}{b^2(b+a\cos(e+fx))} + \frac{d(2bc-ad)\sec(e+fx)}{b^2} + \frac{d^2\sec^2(e+fx)}{b}\right) dx$$

$$= \frac{d^2\int \sec^2(e+fx)\,dx}{b} + \frac{(bc-ad)^2\int \frac{1}{b+a\cos(e+fx)}\,dx}{b^2} + \frac{(d(2bc-ad))\int \sec(e+fx)$$

$$= \frac{d(2bc-ad)\tanh^{-1}(\sin(e+fx))}{b^2f} - \frac{d^2\,\mathrm{Subst}(\int 1\,dx, x, -\tan(e+fx))}{bf} + \frac{(2(bc-$$

$$= \frac{d(2bc-ad)\tanh^{-1}(\sin(e+fx))}{b^2f} + \frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}f} + \frac{d^2\,\mathrm{t}$$

Mathematica [A] time = 0.804488, size = 135, normalized size = 1.31

$$\frac{d\left(bd\tan(e+fx)-(2bc-ad)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)-\frac{2(b}{}}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x]),x]
```

```
[Out] ((-2*(b*c - a*d)^2*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + d*(-((2*b*c - a*d)*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) + b*d*Tan[e + f*x]))/(b^2*f)
```

Maple [B] time = 0.065, size = 288, normalized size = 2.8

$$2\frac{a^2d^2}{fb^2\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,fx+e/2)}{\sqrt{(a+b)(a-b)}}\right)-4\frac{acd}{fb\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,fx+e/2)}{\sqrt{(a+b)(a-b)}}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x)
```

```
[Out] 2/f/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^
(1/2))*a^2*d^2-4/f/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/(
(a+b)*(a-b))^(1/2))*a*c*d+2/f/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x
+1/2*e)/((a+b)*(a-b))^(1/2))*c^2-1/f*d^2/b/(tan(1/2*f*x+1/2*e)+1)-1/f*d^2/b
^2*ln(tan(1/2*f*x+1/2*e)+1)*a+2/f*d/b*ln(tan(1/2*f*x+1/2*e)+1)*c-1/f*d^2/b/
(tan(1/2*f*x+1/2*e)-1)+1/f*d^2/b^2*ln(tan(1/2*f*x+1/2*e)-1)*a-2/f*d/b*ln(ta
n(1/2*f*x+1/2*e)-1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 12.7316, size = 1139, normalized size = 11.06

$$\Bigg[\frac{2\left(a^2b-b^3\right)d^2\sin\left(fx+e\right)+\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{a^2-b^2}\cos\left(fx+e\right)\log\left(\frac{2\,ab\cos\left(fx+e\right)-\left(a^2-2\,b^2\right)\cos\left(fx+e\right)^2+2\,\sqrt{a^2-b^2}(}{a^2\cos\left(fx+e\right)^2+2\,ab\cos(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/2*(2*(a^2*b - b^3)*d^2*sin(f*x + e) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sq
rt(a^2 - b^2)*cos(f*x + e)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x
+ e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)
/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) + (2*(a^2*b - b^3)*c*d -
(a^3 - a*b^2)*d^2)*cos(f*x + e)*log(sin(f*x + e) + 1) - (2*(a^2*b - b^3)*c*
d - (a^3 - a*b^2)*d^2)*cos(f*x + e)*log(-sin(f*x + e) + 1))/((a^2*b^2 - b^4
)*f*cos(f*x + e)), 1/2*(2*(a^2*b - b^3)*d^2*sin(f*x + e) + 2*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e)
 + a)/((a^2 - b^2)*sin(f*x + e)))*cos(f*x + e) + (2*(a^2*b - b^3)*c*d - (a^
3 - a*b^2)*d^2)*cos(f*x + e)*log(sin(f*x + e) + 1) - (2*(a^2*b - b^3)*c*d -
 (a^3 - a*b^2)*d^2)*cos(f*x + e)*log(-sin(f*x + e) + 1))/((a^2*b^2 - b^4)*f
*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c+d\sec\left(e+fx\right)\right)^2\sec\left(e+fx\right)}{a+b\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**2*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Giac [B] time = 1.41525, size = 273, normalized size = 2.65

$$-\frac{\frac{2\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)b}-\frac{\left(2\,bcd-ad^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{b^2}+\frac{\left(2\,bcd-ad^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{b^2}+\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\cdots\right.}{\sqrt{-a^2+b^2}\cdots}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -(2*d^2*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*b) - (2*b*c*d -
a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^2 + (2*b*c*d - a*d^2)*log(abs(t
an(1/2*f*x + 1/2*e) - 1))/b^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) -
b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^2))/f
```

3.255 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=76

$$\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{bf\sqrt{a-b}\sqrt{a+b}}+\frac{d\tanh^{-1}(\sin(e+fx))}{bf}$$

```
[Out] (d*ArcTanh[Sin[e + f*x]])/(b*f) + (2*(b*c - a*d)*ArcTanh[(Sqrt[a - b]*Tan[(
e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*f)
```

Rubi [A] time = 0.126988, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3998, 3770, 3831, 2659, 208}

$$\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{bf\sqrt{a-b}\sqrt{a+b}}+\frac{d\tanh^{-1}(\sin(e+fx))}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x]),x]
```

```
[Out] (d*ArcTanh[Sin[e + f*x]])/(b*f) + (2*(b*c - a*d)*ArcTanh[(Sqrt[a - b]*Tan[(
e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*f)
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx = \frac{d\int \sec(e+fx)\,dx}{b} + \frac{(bc-ad)\int \frac{\sec(e+fx)}{a+b\sec(e+fx)}\,dx}{b}$$
$$= \frac{d\tanh^{-1}(\sin(e+fx))}{bf} + \frac{(bc-ad)\int \frac{1}{1+\frac{a\cos(e+fx)}{b}}\,dx}{b^2}$$
$$= \frac{d\tanh^{-1}(\sin(e+fx))}{bf} + \frac{(2(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{b^2 f}$$
$$= \frac{d\tanh^{-1}(\sin(e+fx))}{bf} + \frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,f}$$

Mathematica [A] time = 0.183553, size = 112, normalized size = 1.47

$$\frac{\frac{2(ad-bc)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + d\left(\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x]),x]
```

```
[Out] ((2*(-(b*c) + a*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + d*(-Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]]))/(b*f)
```

Maple [A] time = 0.063, size = 135, normalized size = 1.8

$$-2\,\frac{ad}{fb\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan\left(1/2\,fx+e/2\right)}{\sqrt{(a+b)(a-b)}}\right)+2\,\frac{c}{f\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan\left(1/2\,fx+e/2\right)}{\sqrt{(a+b)(a-b)}}\right)+\frac{}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x)
```

```
[Out] -2/f/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(
1/2))*a*d+2/f/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(
a-b))^(1/2))*c+1/f*d/b*ln(tan(1/2*f*x+1/2*e)+1)-1/f*d/b*ln(tan(1/2*f*x+1/2*
e)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53203, size = 694, normalized size = 9.13

$$\left[\frac{\left(a^2-b^2\right)d\log\left(\sin\left(fx+e\right)+1\right)-\left(a^2-b^2\right)d\log\left(-\sin\left(fx+e\right)+1\right)-\sqrt{a^2-b^2}(bc-ad)\log\left(\frac{2\,ab\cos\left(fx+e\right)-\left(a^2-2\,b^2\right)\cos\cdots}{a^2\cdots}\right.}{2\left(a^2b-b^3\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [1/2*((a^2 - b^2)*d*log(sin(f*x + e) + 1) - (a^2 - b^2)*d*log(-sin(f*x + e)
 + 1) - sqrt(a^2 - b^2)*(b*c - a*d)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)
*cos(f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a
^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)))/((a^2*b - b^3)*
f), 1/2*((a^2 - b^2)*d*log(sin(f*x + e) + 1) - (a^2 - b^2)*d*log(-sin(f*x +
 e) + 1) + 2*sqrt(-a^2 + b^2)*(b*c - a*d)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f
*x + e) + a)/((a^2 - b^2)*sin(f*x + e))))/((a^2*b - b^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+d\sec\left(e+fx\right)\right)\sec\left(e+fx\right)}{a+b\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c + d*sec(e + f*x))*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Giac [A] time = 1.28417, size = 178, normalized size = 2.34

$$\frac{\frac{d\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{b}-\frac{d\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{b}-\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-a^2+b^2}}\right)\right)(bc-ad)}{\sqrt{-a^2+b^2}b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (d*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b - d*log(abs(tan(1/2*f*x + 1/2*e) -
1))/b - 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(
1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))*(b*c - a*d)/(
sqrt(-a^2 + b^2)*b))/f
```

3.256 $\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=121

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\sqrt{a+b}(bc-ad)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\sqrt{c+d}(bc-ad)}$$

```
[Out] (2*b*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt
[a + b]*(b*c - a*d)*f) - (2*d*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c
 + d]])/(Sqrt[c - d]*Sqrt[c + d]*(b*c - a*d)*f)
```

Rubi [A] time = 0.274576, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3988, 3001, 2659, 208}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\sqrt{a+b}(bc-ad)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*b*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt
[a + b]*(b*c - a*d)*f) - (2*d*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c
 + d]])/(Sqrt[c - d]*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx &= \int \frac{\cos(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))}\,dx \\ &= \frac{b\int \frac{1}{b+a\cos(e+fx)}\,dx}{bc-ad} - \frac{d\int \frac{1}{d+c\cos(e+fx)}\,dx}{bc-ad} \\ &= \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)f} - \frac{(2d)\,\mathrm{Subst}\left(\int \frac{}{c+d-}\right.}{} \\ &= \frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(bc-ad)f} - \frac{2d\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}(bc-ad)f}\end{aligned}$$

Mathematica [A] time = 0.231845, size = 119, normalized size = 0.98

$$-\frac{2b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}(bc-ad)} - \frac{2d\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-2*b*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2
]*(b*c - a*d)*f) - (2*d*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]
])/((-(b*c) + a*d)*Sqrt[c^2 - d^2]*f)
```

Maple [A] time = 0.083, size = 108, normalized size = 0.9

$$\frac{1}{f}\left(-2\frac{b}{(ad-bc)\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan\left(1/2\,fx+e/2\right)}{\sqrt{(a+b)(a-b)}}\right)+2\frac{d}{(ad-bc)\sqrt{(c+d)(c-d)}}\mathrm{Artanh}\left(\frac{\tan\left(1/2\,f\right.}{\sqrt{(c\cdot}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] 1/f*(-2*b/(a*d-b*c)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((
a+b)*(a-b))^(1/2))+2*d/(a*d-b*c)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/
2*e)*(c-d)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.49626, size = 2214, normalized size = 18.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [-1/2*((a^2 - b^2)*sqrt(c^2 - d^2)*d*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2
)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*
c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + (b*c^2 - b*d^
2)*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 -
 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*co
s(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)))/(((a^2*b - b^3)*c^3 - (a^3 - a*b
^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f), -1/2*((a^2 - b^2)*
sqrt(c^2 - d^2)*d*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 +
2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos
(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(b*c^2 - b*d^2)*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))
))/(((a^2*b - b^3)*c^3 - (a^3 - a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 -
 a*b^2)*d^3)*f), -1/2*(2*(a^2 - b^2)*sqrt(-c^2 + d^2)*d*arctan(-sqrt(-c^2 +
 d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b*c^2 - b*d^2)*sq
rt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 - 2*sq
rt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x
 + e)^2 + 2*a*b*cos(f*x + e) + b^2)))/(((a^2*b - b^3)*c^3 - (a^3 - a*b^2)*c
^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f), -((a^2 - b^2)*sqrt(-c^2
 + d^2)*d*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*
x + e))) - (b*c^2 - b*d^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos
(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))))/(((a^2*b - b^3)*c^3 - (a^3 - a*
b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(e + fx\right)}{\left(a + b\sec\left(e + fx\right)\right)\left(c + d\sec\left(e + fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/((a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)
```

Giac [B] time = 1.34189, size = 710, normalized size = 5.87

$$\frac{\frac{\left(\sqrt{-c^2+d^2}b(c-2d)|c-d|+\sqrt{-c^2+d^2}ad|c-d|+\sqrt{-c^2+d^2}|-bc+ad||c-d|\right)\left(\pi\left\lfloor \frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-\frac{2ac-2bd+\sqrt{-4(ac+bc+ad+bd)(ac-bc-ad+bd)+4(ac-bd)^2}}{ac-bc-ad+bd}}}\right)\right)}{(bc-ad)^2\left(c^2-2cd+d^2\right)+\left(c^3-2c^2d+cd^2\right)a|-bc+ad|-\left(c^2d-2cd^2+d^3\right)b|-bc+ad|} + \frac{\left(\sqrt{-a^2+b^2}b\cdots\right.}{\cdots}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] ((sqrt(-c^2 + d^2)*b*(c - 2*d)*abs(c - d) + sqrt(-c^2 + d^2)*a*d*abs(c - d)
 + sqrt(-c^2 + d^2)*abs(-b*c + a*d)*abs(c - d))*(pi*floor(1/2*(f*x + e)/pi
+ 1/2) + arctan(2*sqrt(1/2)*tan(1/2*f*x + 1/2*e)/sqrt(-(2*a*c - 2*b*d + sqr
t(-4*(a*c + b*c + a*d + b*d)*(a*c - b*c - a*d + b*d) + 4*(a*c - b*d)^2))/(a
*c - b*c - a*d + b*d))))/((b*c - a*d)^2*(c^2 - 2*c*d + d^2) + (c^3 - 2*c^2*
d + c*d^2)*a*abs(-b*c + a*d) - (c^2*d - 2*c*d^2 + d^3)*b*abs(-b*c + a*d)) +
 (sqrt(-a^2 + b^2)*b*c*abs(a - b) + sqrt(-a^2 + b^2)*(a - 2*b)*d*abs(a - b)
 - sqrt(-a^2 + b^2)*abs(-b*c + a*d)*abs(a - b))*(pi*floor(1/2*(f*x + e)/pi
+ 1/2) + arctan(2*sqrt(1/2)*tan(1/2*f*x + 1/2*e)/sqrt(-(2*a*c - 2*b*d - sqr
t(-4*(a*c + b*c + a*d + b*d)*(a*c - b*c - a*d + b*d) + 4*(a*c - b*d)^2))/(a
*c - b*c - a*d + b*d))))/((a^2 - 2*a*b + b^2)*(b*c - a*d)^2 - (a^3 - 2*a^2*
b + a*b^2)*c*abs(-b*c + a*d) + (a^2*b - 2*a*b^2 + b^3)*d*abs(-b*c + a*d)))/
f
```

3.257 $\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=187

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\sqrt{a+b}(bc-ad)^2}+\frac{d^2\sin(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c\cos(e+fx)+d)}-\frac{2d\left(-acd+2bc^2-bd^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+f\ldots\right.\right.}{\sqrt{c+d}}\right.}{f(c-d)^{3/2}(c+d)^{3/2}(bc-ad)^2}$$

```
[Out] (2*b^2*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sq
rt[a + b]*(b*c - a*d)^2*f) - (2*d*(2*b*c^2 - a*c*d - b*d^2)*ArcTanh[(Sqrt[c
 - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c - d)^(3/2)*(c + d)^(3/2)*(b*c - a
*d)^2*f) + (d^2*Sin[e + f*x])/((b*c - a*d)*(c^2 - d^2)*f*(d + c*Cos[e + f*x
]))
```

Rubi [A] time = 0.612432, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3988, 3056, 3001, 2659, 208}

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\sqrt{a+b}(bc-ad)^2}+\frac{d^2\sin(e+fx)}{f\left(c^2-d^2\right)(bc-ad)(c\cos(e+fx)+d)}-\frac{2d\left(-acd+2bc^2-bd^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+f\ldots\right.\right.}{\sqrt{c+d}}\right.}{f(c-d)^{3/2}(c+d)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (2*b^2*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sq
rt[a + b]*(b*c - a*d)^2*f) - (2*d*(2*b*c^2 - a*c*d - b*d^2)*ArcTanh[(Sqrt[c
 - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c - d)^(3/2)*(c + d)^(3/2)*(b*c - a
*d)^2*f) + (d^2*Sin[e + f*x])/((b*c - a*d)*(c^2 - d^2)*f*(d + c*Cos[e + f*x
]))
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \int \frac{\cos^2(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))^2}\,dx$$

$$= \frac{d^2\sin(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(d+c\cos(e+fx))} + \frac{\int \frac{-bcd-\left(acd-b\left(c^2-d^2\right)\right)\cos(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))}}{(bc-ad)\left(c^2-d^2\right)}$$

$$= \frac{d^2\sin(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(d+c\cos(e+fx))} + \frac{b^2\int \frac{1}{b+a\cos(e+fx)}\,dx}{(bc-ad)^2} + \frac{\left(d\left(a\right.\right.}{}$$

$$= \frac{d^2\sin(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(d+c\cos(e+fx))} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\right.}{(bc-ad}$$

$$= \frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(bc-ad)^2f} - \frac{2d\left(2bc^2-acd-bd^2\right)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\text{ta}}{}\right.}{(c-d)^{3/2}(c+d)^{3/2}(bc-ad)^2}$$

Mathematica [A] time = 0.713981, size = 229, normalized size = 1.22

$$\frac{-2b^2\left(c^2-d^2\right)^{3/2}(c\cos(e+fx)+d)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right) - d\sqrt{a^2-b^2}\left(d\sqrt{c^2-d^2}(ad-bc)\sin(e+fx) - 2\left(-acd\right.\right.}{f\sqrt{a^2-b^2}(c-d)(c+d)\sqrt{c^2-d^2}(bc-ad)^2(c\cos(e+fx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (-2*b^2*(c^2 - d^2)^(3/2)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^
2]]*(d + c*Cos[e + f*x]) - Sqrt[a^2 - b^2]*d*(-2*(2*b*c^2 - a*c*d - b*d^2)*
ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x]) +
 d*(-(b*c) + a*d)*Sqrt[c^2 - d^2]*Sin[e + f*x]))/(Sqrt[a^2 - b^2]*(c - d)*(
c + d)*(b*c - a*d)^2*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x]))
```

Maple [A] time = 0.1, size = 208, normalized size = 1.1

$$\frac{1}{f}\left(2\,\frac{b^2}{(ad-bc)^2\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan\left(1/2\,fx+e/2\right)}{\sqrt{(a+b)(a-b)}}\right)-2\,\frac{d}{(ad-bc)^2}\left(-\frac{d\,(ad-bc)\tan\left(\right.}{\left(c^2-d^2\right)\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2c-\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x)
```

```
[Out] 1/f*(2*b^2/(a*d-b*c)^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)
/((a+b)*(a-b))^(1/2))-2*d/(a*d-b*c)^2*(-d*(a*d-b*c)/(c^2-d^2)*tan(1/2*f*x+1
/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)-(a*c*d-2*b*c^2+b*
d^2)/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+d
)*(c-d))^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.47005, size = 459, normalized size = 2.45

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-a^2+b^2}}\right)\right)b^2}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-a^2+b^2}}+\frac{d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(bc^3-ac^2d-bcd^2+ad^3\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)}-\frac{(2b\cdots}{\cdots}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*
x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))*b^2/((b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*sqrt(-a^2 + b^2)) + d^2*tan(1/2*f*x + 1/2*e)/((b*c^3 - a*
c^2*d - b*c*d^2 + a*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)
^2 - c - d)) - (2*b*c^2*d - a*c*d^2 - b*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1
/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e
))/sqrt(-c^2 + d^2)))/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 - b^2*c^2*d^2 +
 2*a*b*c*d^3 - a^2*d^4)*sqrt(-c^2 + d^2)))/f
```

3.258 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=379

$$\frac{d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\tan(e+fx)}{b^4f}+\frac{d^2\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)\tanh^{-1}(\sin(e+fx))}{b^5f}-\frac{(bc}{b^4f\left(a^2-\right.}$$

```
[Out] (d^4*(5*b*c - 2*a*d)*ArcTanh[Sin[e + f*x]])/(2*b^3*f) + (d^2*(10*b^3*c^3 -
20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*ArcTanh[Sin[e + f*x]])/(b^5*f)
 + (2*(b*c - a*d)^5*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a
*(a - b)^(3/2)*b^3*(a + b)^(3/2)*f) + (2*(b*c - a*d)^4*(b*c + 4*a*d)*ArcTan
h[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b^5*Sqrt[a +
b]*f) - ((b*c - a*d)^5*Sin[e + f*x])/(b^4*(a^2 - b^2)*f*(b + a*Cos[e + f*x]
)) + (d^5*Tan[e + f*x])/(b^2*f) + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2
)*Tan[e + f*x])/(b^4*f) + (d^4*(5*b*c - 2*a*d)*Sec[e + f*x]*Tan[e + f*x])/(
2*b^3*f) + (d^5*Tan[e + f*x]^3)/(3*b^2*f)
```

Rubi [A] time = 0.667993, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {3988, 2952, 2664, 12, 2659, 208, 3770, 3767, 8, 3768}

$$\frac{d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\tan(e+fx)}{b^4f}+\frac{d^2\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)\tanh^{-1}(\sin(e+fx))}{b^5f}-\frac{(bc}{b^4f\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + b*Sec[e + f*x])^2,x]
```


```
[Out] (d^4*(5*b*c - 2*a*d)*ArcTanh[Sin[e + f*x]])/(2*b^3*f) + (d^2*(10*b^3*c^3 -
20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*ArcTanh[Sin[e + f*x]])/(b^5*f)
 + (2*(b*c - a*d)^5*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a
*(a - b)^(3/2)*b^3*(a + b)^(3/2)*f) + (2*(b*c - a*d)^4*(b*c + 4*a*d)*ArcTan
h[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b^5*Sqrt[a +
b]*f) - ((b*c - a*d)^5*Sin[e + f*x])/(b^4*(a^2 - b^2)*f*(b + a*Cos[e + f*x]
)) + (d^5*Tan[e + f*x])/(b^2*f) + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2
)*Tan[e + f*x])/(b^4*f) + (d^4*(5*b*c - 2*a*d)*Sec[e + f*x]*Tan[e + f*x])/(
2*b^3*f) + (d^5*Tan[e + f*x]^3)/(3*b^2*f)
```


Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rule 2952

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
```

```
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d+c\cos(e+fx))^5\sec^4(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \int \left(\frac{(-bc+ad)^5}{ab^4(b+a\cos(e+fx))^2} + \frac{(-bc+ad)^4(bc+4ad)}{ab^5(b+a\cos(e+fx))} + \frac{d^2\left(10b^3c^3-20ab^2c^2d+\right.}{}\right.$$

$$= \frac{d^5\int\sec^4(e+fx)\,dx}{b^2} + \frac{\left(d^4(5bc-2ad)\right)\int\sec^3(e+fx)\,dx}{b^3} - \frac{(bc-ad)^5\int\frac{1}{(b+a\cos(}}{ab^4}$$

$$= \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)\tanh^{-1}(\sin(e+fx))}{b^5f} - \frac{(bc-ad}{b^4\left(a^2-b^2\right)f}$$

$$= \frac{d^4(5bc-2ad)\tanh^{-1}(\sin(e+fx))}{2b^3f} + \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)}{b^5f}$$

$$= \frac{d^4(5bc-2ad)\tanh^{-1}(\sin(e+fx))}{2b^3f} + \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)}{b^5f}$$

$$= \frac{d^4(5bc-2ad)\tanh^{-1}(\sin(e+fx))}{2b^3f} + \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)}{b^5f}$$

Mathematica [B] time = 6.54493, size = 1137, normalized size = 3.

$$\frac{(b+a\cos(e+fx))\left(12d^5\sin(e+fx)b^5+60c^2d^3\sin(e+fx)b^5+6c^5\sin(2(e+fx))b^5+30cd^4\sin(2(e+fx))b^5+4d^5\sin(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (-2*(b*c - a*d)^4*(-(a*b*c) - 4*a^2*d + 5*b^2*d)*ArcTanh[((-a + b)*Tan[(e +
 f*x)/2])/Sqrt[a^2 - b^2]]*Cos[e + f*x]^3*(b + a*Cos[e + f*x])^2*(c + d*Sec
[e + f*x])^5)/(b^5*Sqrt[a^2 - b^2]*(-a^2 + b^2)*f*(d + c*Cos[e + f*x])^5*(a
 + b*Sec[e + f*x])^2) + ((-20*b^3*c^3*d^2 + 40*a*b^2*c^2*d^3 - 30*a^2*b*c*d
^4 - 5*b^3*c*d^4 + 8*a^3*d^5 + 2*a*b^2*d^5)*Cos[e + f*x]^3*(b + a*Cos[e + f
*x])^2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(c + d*Sec[e + f*x])^5)/(2*
b^5*f*(d + c*Cos[e + f*x])^5*(a + b*Sec[e + f*x])^2) + ((20*b^3*c^3*d^2 - 4
0*a*b^2*c^2*d^3 + 30*a^2*b*c*d^4 + 5*b^3*c*d^4 - 8*a^3*d^5 - 2*a*b^2*d^5)*C
os[e + f*x]^3*(b + a*Cos[e + f*x])^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2
]]*(c + d*Sec[e + f*x])^5)/(2*b^5*f*(d + c*Cos[e + f*x])^5*(a + b*Sec[e + f
*x])^2) + ((b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^5*(-60*a^2*b^3*c^2*d^3
*Sin[e + f*x] + 60*b^5*c^2*d^3*Sin[e + f*x] + 45*a^3*b^2*c*d^4*Sin[e + f*x]
 - 45*a*b^4*c*d^4*Sin[e + f*x] - 12*a^4*b*d^5*Sin[e + f*x] + 12*b^5*d^5*Sin
[e + f*x] + 6*b^5*c^5*Sin[2*(e + f*x)] - 30*a*b^4*c^4*d*Sin[2*(e + f*x)] +
60*a^2*b^3*c^3*d^2*Sin[2*(e + f*x)] - 120*a^3*b^2*c^2*d^3*Sin[2*(e + f*x)]
+ 60*a*b^4*c^2*d^3*Sin[2*(e + f*x)] + 90*a^4*b*c*d^4*Sin[2*(e + f*x)] - 90*
a^2*b^3*c*d^4*Sin[2*(e + f*x)] + 30*b^5*c*d^4*Sin[2*(e + f*x)] - 24*a^5*d^5
*Sin[2*(e + f*x)] + 22*a^3*b^2*d^5*Sin[2*(e + f*x)] - 4*a*b^4*d^5*Sin[2*(e
+ f*x)] - 60*a^2*b^3*c^2*d^3*Sin[3*(e + f*x)] + 60*b^5*c^2*d^3*Sin[3*(e + f
*x)] + 45*a^3*b^2*c*d^4*Sin[3*(e + f*x)] - 45*a*b^4*c*d^4*Sin[3*(e + f*x)]
- 12*a^4*b*d^5*Sin[3*(e + f*x)] + 8*a^2*b^3*d^5*Sin[3*(e + f*x)] + 4*b^5*d^
5*Sin[3*(e + f*x)] + 3*b^5*c^5*Sin[4*(e + f*x)] - 15*a*b^4*c^4*d*Sin[4*(e +
```

```
f*x)] + 30*a^2*b^3*c^3*d^2*Sin[4*(e + f*x)] - 60*a^3*b^2*c^2*d^3*Sin[4*(e
+ f*x)] + 30*a*b^4*c^2*d^3*Sin[4*(e + f*x)] + 45*a^4*b*c*d^4*Sin[4*(e + f*x
)] - 30*a^2*b^3*c*d^4*Sin[4*(e + f*x)] - 12*a^5*d^5*Sin[4*(e + f*x)] + 7*a^
3*b^2*d^5*Sin[4*(e + f*x)] + 2*a*b^4*d^5*Sin[4*(e + f*x)]))/(24*b^4*(-a^2 +
 b^2)*f*(d + c*Cos[e + f*x])^5*(a + b*Sec[e + f*x])^2)
```

Maple [B] time = 0.145, size = 1870, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x)
```

```
[Out] 40/f/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b
)*(a-b))^(1/2))*a*c^3*d^2-10/f*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a
-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*c^4*d+10/f/b^3/(a^2-b^2)*tan(1/
2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)*a^4*c*d^4-
20/f/b^2/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-tan(1/2*f*x+1
/2*e)^2*b-a-b)*a^3*c^2*d^3+20/f/b/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x
+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)*a^2*c^3*d^2+8/f/b^5/(a+b)/(a-b)/((a
+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*a^6*
d^5-10/f/b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*
e)/((a+b)*(a-b))^(1/2))*a^4*d^5-1/2/f*d^5/b^2/(tan(1/2*f*x+1/2*e)-1)^2-1/3/
f*d^5/b^2/(tan(1/2*f*x+1/2*e)+1)^3-1/f*d^5/b^2/(tan(1/2*f*x+1/2*e)+1)+1/2/f
*d^5/b^2/(tan(1/2*f*x+1/2*e)+1)^2-1/3/f*d^5/b^2/(tan(1/2*f*x+1/2*e)-1)^3-1/
f*d^5/b^2/(tan(1/2*f*x+1/2*e)-1)+40/f/b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*a^4*c^2*d^3-20/f/b^2/(
a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b
))^(1/2))*a^3*c^3*d^2+40/f/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*a^3*c*d^4-60/f/b/(a+b)/(a-b)/((a+
b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*a^2*c
^2*d^3-30/f/b^4/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1
/2*e)/((a+b)*(a-b))^(1/2))*a^5*c*d^4-15/f*d^4/b^4*ln(tan(1/2*f*x+1/2*e)-1)*
a^2*c+20/f*d^3/b^3*ln(tan(1/2*f*x+1/2*e)-1)*a*c^2+10/f*d^4/b^3/(tan(1/2*f*x
+1/2*e)-1)*a*c+2/f*b/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-t
an(1/2*f*x+1/2*e)^2*b-a-b)*c^5+15/f*d^4/b^4*ln(tan(1/2*f*x+1/2*e)+1)*a^2*c-
20/f*d^3/b^3*ln(tan(1/2*f*x+1/2*e)+1)*a*c^2+10/f*d^4/b^3/(tan(1/2*f*x+1/2*e
)+1)*a*c+1/f*d^5/b^3/(tan(1/2*f*x+1/2*e)+1)^2*a-10/f/(a^2-b^2)*tan(1/2*f*x+
1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)*a*c^4*d-2/f/b^4/
(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b
-a-b)*a^5*d^5+2/f/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x
+1/2*e)/((a+b)*(a-b))^(1/2))*c^5*a-5/2/f*d^4/b^2/(tan(1/2*f*x+1/2*e)+1)^2*c
+4/f*d^5/b^5*ln(tan(1/2*f*x+1/2*e)-1)*a^3+1/f*d^5/b^3*ln(tan(1/2*f*x+1/2*e)
-1)*a-10/f*d^2/b^2*ln(tan(1/2*f*x+1/2*e)-1)*c^3-5/2/f*d^4/b^2*ln(tan(1/2*f*
x+1/2*e)-1)*c-3/f*d^5/b^4/(tan(1/2*f*x+1/2*e)-1)*a^2-1/f*d^5/b^3/(tan(1/2*f
*x+1/2*e)-1)*a-10/f*d^3/b^2/(tan(1/2*f*x+1/2*e)-1)*c^2+5/2/f*d^4/b^2/(tan(1
/2*f*x+1/2*e)-1)*c-1/f*d^5/b^3/(tan(1/2*f*x+1/2*e)-1)^2*a+5/2/f*d^4/b^2/(ta
n(1/2*f*x+1/2*e)-1)^2*c-4/f*d^5/b^5*ln(tan(1/2*f*x+1/2*e)+1)*a^3-1/f*d^5/b^
3*ln(tan(1/2*f*x+1/2*e)+1)*a+10/f*d^2/b^2*ln(tan(1/2*f*x+1/2*e)+1)*c^3+5/2/
f*d^4/b^2*ln(tan(1/2*f*x+1/2*e)+1)*c-3/f*d^5/b^4/(tan(1/2*f*x+1/2*e)+1)*a^2
-1/f*d^5/b^3/(tan(1/2*f*x+1/2*e)+1)*a-10/f*d^3/b^2/(tan(1/2*f*x+1/2*e)+1)*c
^2+5/2/f*d^4/b^2/(tan(1/2*f*x+1/2*e)+1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + d\sec\left(e + fx\right)\right)^5 \sec\left(e + fx\right)}{\left(a + b\sec\left(e + fx\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**5/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))**5*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Giac [B] time = 1.44164, size = 1197, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*(a*b^5*c^5 - 5*b^6*c^4*d - 10*a^3*b^3*c^3*d^2 + 20*a*b^5*c^3*d^2 +
 20*a^4*b^2*c^2*d^3 - 30*a^2*b^4*c^2*d^3 - 15*a^5*b*c*d^4 + 20*a^3*b^3*c*d^
4 + 4*a^6*d^5 - 5*a^4*b^2*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a -
2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 +
 b^2)))/((a^2*b^5 - b^7)*sqrt(-a^2 + b^2)) - 12*(b^5*c^5*tan(1/2*f*x + 1/2*
e) - 5*a*b^4*c^4*d*tan(1/2*f*x + 1/2*e) + 10*a^2*b^3*c^3*d^2*tan(1/2*f*x +
1/2*e) - 10*a^3*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e) + 5*a^4*b*c*d^4*tan(1/2*f*
x + 1/2*e) - a^5*d^5*tan(1/2*f*x + 1/2*e))/((a^2*b^4 - b^6)*(a*tan(1/2*f*x
```

```
+ 1/2*e)^2 - b*tan(1/2*f*x + 1/2*e)^2 - a - b)) - 3*(20*b^3*c^3*d^2 - 40*a*
b^2*c^2*d^3 + 30*a^2*b*c*d^4 + 5*b^3*c*d^4 - 8*a^3*d^5 - 2*a*b^2*d^5)*log(a
bs(tan(1/2*f*x + 1/2*e) + 1))/b^5 + 3*(20*b^3*c^3*d^2 - 40*a*b^2*c^2*d^3 +
30*a^2*b*c*d^4 + 5*b^3*c*d^4 - 8*a^3*d^5 - 2*a*b^2*d^5)*log(abs(tan(1/2*f*x
 + 1/2*e) - 1))/b^5 + 2*(60*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 60*a*b*c*d
^4*tan(1/2*f*x + 1/2*e)^5 - 15*b^2*c*d^4*tan(1/2*f*x + 1/2*e)^5 + 18*a^2*d^
5*tan(1/2*f*x + 1/2*e)^5 + 6*a*b*d^5*tan(1/2*f*x + 1/2*e)^5 + 6*b^2*d^5*tan
(1/2*f*x + 1/2*e)^5 - 120*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 120*a*b*c*d^
4*tan(1/2*f*x + 1/2*e)^3 - 36*a^2*d^5*tan(1/2*f*x + 1/2*e)^3 - 4*b^2*d^5*ta
n(1/2*f*x + 1/2*e)^3 + 60*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 60*a*b*c*d^4*t
an(1/2*f*x + 1/2*e) + 15*b^2*c*d^4*tan(1/2*f*x + 1/2*e) + 18*a^2*d^5*tan(1/
2*f*x + 1/2*e) - 6*a*b*d^5*tan(1/2*f*x + 1/2*e) + 6*b^2*d^5*tan(1/2*f*x + 1
/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*b^4))/f
```

3.259 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=297

$$\frac{d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f}-\frac{(bc-ad)^4\sin(e+fx)}{b^3f\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{2d^3(2bc-ad)\tan(e+fx)}{b^3f}+\frac{2(bc\cdots}{\cdots}$$

```
[Out] (d^4*ArcTanh[Sin[e + f*x]])/(2*b^2*f) + (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2
*d^2)*ArcTanh[Sin[e + f*x]])/(b^4*f) + (2*(b*c - a*d)^4*ArcTanh[(Sqrt[a - b
]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*b^2*(a + b)^(3/2)*f) + (
2*(b*c - a*d)^3*(b*c + 3*a*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a
 + b]])/(a*Sqrt[a - b]*b^4*Sqrt[a + b]*f) - ((b*c - a*d)^4*Sin[e + f*x])/(b
^3*(a^2 - b^2)*f*(b + a*Cos[e + f*x])) + (2*d^3*(2*b*c - a*d)*Tan[e + f*x])
/(b^3*f) + (d^4*Sec[e + f*x]*Tan[e + f*x])/(2*b^2*f)
```

Rubi [A] time = 0.526849, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {3988, 2952, 2664, 12, 2659, 208, 3770, 3767, 8, 3768}

$$\frac{d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f}-\frac{(bc-ad)^4\sin(e+fx)}{b^3f\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{2d^3(2bc-ad)\tan(e+fx)}{b^3f}+\frac{2(bc\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (d^4*ArcTanh[Sin[e + f*x]])/(2*b^2*f) + (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2
*d^2)*ArcTanh[Sin[e + f*x]])/(b^4*f) + (2*(b*c - a*d)^4*ArcTanh[(Sqrt[a - b
]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*b^2*(a + b)^(3/2)*f) + (
2*(b*c - a*d)^3*(b*c + 3*a*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a
 + b]])/(a*Sqrt[a - b]*b^4*Sqrt[a + b]*f) - ((b*c - a*d)^4*Sin[e + f*x])/(b
^3*(a^2 - b^2)*f*(b + a*Cos[e + f*x])) + (2*d^3*(2*b*c - a*d)*Tan[e + f*x])
/(b^3*f) + (d^4*Sec[e + f*x]*Tan[e + f*x])/(2*b^2*f)
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rule 2952

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d+c\cos(e+fx))^4\sec^3(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \int \left(-\frac{(-bc+ad)^4}{ab^3(b+a\cos(e+fx))^2} - \frac{(-bc+ad)^3(bc+3ad)}{ab^4(b+a\cos(e+fx))} + \frac{d^2\left(6b^2c^2-8abcd+3a\right.}{b^4}\right.$$

$$= \frac{d^4\int\sec^3(e+fx)\,dx}{b^2} - \frac{(bc-ad)^4\int\frac{1}{(b+a\cos(e+fx))^2}\,dx}{ab^3} + \frac{\left(2d^3(2bc-ad)\right)\int\sec^2(e}{b^3}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f} - \frac{(bc-ad)^4\sin(e+fx)}{b^3\left(a^2-b^2\right)f(b+a\cos(e+f}$$

$$= \frac{d^4\tanh^{-1}(\sin(e+fx))}{2b^2f} + \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f} + \frac{2(b}{}$$

$$= \frac{d^4\tanh^{-1}(\sin(e+fx))}{2b^2f} + \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f} + \frac{2(b}{}$$

$$= \frac{d^4\tanh^{-1}(\sin(e+fx))}{2b^2f} + \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f} + \frac{2(b}{}$$

Mathematica [A] time = 3.46889, size = 511, normalized size = 1.72

$$\frac{\cos^2(e+fx)(a\cos(e+fx)+b)(c+d\sec(e+fx))^4\left(-2d^2\left(6a^2d^2-16abcd+b^2\left(12c^2+d^2\right)\right)(a\cos(e+fx)+b)\log\left(\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]^2*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^4*((8*(-(b*c) + a
*d)^3*(a*b*c + 3*a^2*d - 4*b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[
a^2 - b^2]]*(b + a*Cos[e + f*x]))/(a^2 - b^2)^(3/2) - 2*d^2*(-16*a*b*c*d +
6*a^2*d^2 + b^2*(12*c^2 + d^2))*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]] + 2*d^2*(-16*a*b*c*d + 6*a^2*d^2 + b^2*(12*c^2 + d^2))*(
b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b^2*d^4*(b
+ a*Cos[e + f*x]))/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (8*b*d^3*(2*b*
c - a*d)*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]) - (b^2*d^4*(b + a*Cos[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^2 + (8*b*d^3*(2*b*c - a*d)*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(C
os[(e + f*x)/2] + Sin[(e + f*x)/2]) + (4*b*(b*c - a*d)^4*Sin[e + f*x])/((-a
 + b)*(a + b))))/(4*b^4*f*(d + c*Cos[e + f*x])^4*(a + b*Sec[e + f*x])^2)
```

Maple [B] time = 0.118, size = 1249, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x)
```

```
[Out] 8/f/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((
a+b)*(a-b))^(1/2))*a^3*d^4-8/f*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a
-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*c^3*d+24/f/(a+b)/(a-b)/((a+b)*(
a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*a*c^2*d^2
-6/f/b^4/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/(
(a+b)*(a-b))^(1/2))*a^5*d^4-8/f/b^2/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f
*x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)*a^3*c*d^3+12/f/b/(a^2-b^2)*tan(1/
2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)*a^2*c^2*d^
2+6/f*d^2/b^2*ln(tan(1/2*f*x+1/2*e)+1)*c^2-1/2/f*d^4/b^2/(tan(1/2*f*x+1/2*e
)+1)^2+1/2/f*d^4/b^2*ln(tan(1/2*f*x+1/2*e)+1)+1/2/f*d^4/b^2/(tan(1/2*f*x+1/
2*e)+1)+1/2/f*d^4/b^2/(tan(1/2*f*x+1/2*e)-1)^2-1/2/f*d^4/b^2*ln(tan(1/2*f*x
+1/2*e)-1)+1/2/f*d^4/b^2/(tan(1/2*f*x+1/2*e)-1)-24/f/b/(a+b)/(a-b)/((a+b)*(
a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*a^2*c*d^3
+16/f/b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/
((a+b)*(a-b))^(1/2))*a^4*c*d^3-12/f/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arc
tanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*a^3*c^2*d^2+2/f/(a+b)/(a
-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2
))*c^4*a+2/f/b^3/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-tan(1
/2*f*x+1/2*e)^2*b-a-b)*a^4*d^4-8/f/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*
x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)*a*c^3*d+2/f*d^4/b^3/(tan(1/2*f*x+1
/2*e)+1)*a-4/f*d^3/b^2/(tan(1/2*f*x+1/2*e)+1)*c-3/f*d^4/b^4*ln(tan(1/2*f*x+
1/2*e)-1)*a^2-6/f*d^2/b^2*ln(tan(1/2*f*x+1/2*e)-1)*c^2+2/f*d^4/b^3/(tan(1/2
*f*x+1/2*e)-1)*a-4/f*d^3/b^2/(tan(1/2*f*x+1/2*e)-1)*c+3/f*d^4/b^4*ln(tan(1/
2*f*x+1/2*e)+1)*a^2+2/f*b/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^
2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)*c^4-8/f*d^3/b^3*ln(tan(1/2*f*x+1/2*e)+1)*a*
c+8/f*d^3/b^3*ln(tan(1/2*f*x+1/2*e)-1)*a*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+d\sec \left(e+fx\right)\right)^4\sec \left(e+fx\right)}{\left(a+b\sec \left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))**4*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Giac [B] time = 1.32983, size = 770, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*(a*b^4*c^4 - 4*b^5*c^3*d - 6*a^3*b^2*c^2*d^2 + 12*a*b^4*c^2*d^2 + 8
*a^4*b*c*d^3 - 12*a^2*b^3*c*d^3 - 3*a^5*d^4 + 4*a^3*b^2*d^4)*(pi*floor(1/2*
(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan
(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/((a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) -
4*(b^4*c^4*tan(1/2*f*x + 1/2*e) - 4*a*b^3*c^3*d*tan(1/2*f*x + 1/2*e) + 6*a^
2*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e) - 4*a^3*b*c*d^3*tan(1/2*f*x + 1/2*e) + a
^4*d^4*tan(1/2*f*x + 1/2*e))/((a^2*b^3 - b^5)*(a*tan(1/2*f*x + 1/2*e)^2 - b
*tan(1/2*f*x + 1/2*e)^2 - a - b)) - (12*b^2*c^2*d^2 - 16*a*b*c*d^3 + 6*a^2*
d^4 + b^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^4 + (12*b^2*c^2*d^2 - 1
6*a*b*c*d^3 + 6*a^2*d^4 + b^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/b^4 +
 2*(8*b*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 4*a*d^4*tan(1/2*f*x + 1/2*e)^3 - b*d
^4*tan(1/2*f*x + 1/2*e)^3 - 8*b*c*d^3*tan(1/2*f*x + 1/2*e) + 4*a*d^4*tan(1/
2*f*x + 1/2*e) - b*d^4*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^
2*b^3))/f
```

3.260 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=228

$$-\frac{(bc-ad)^3\sin(e+fx)}{b^2f\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3f}+\frac{2(bc-ad)^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+\right.}{\sqrt{a+b}}\right.}{ab^3f\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] (d^2*(3*b*c - 2*a*d)*ArcTanh[Sin[e + f*x]])/(b^3*f) + (2*(b*c - a*d)^3*ArcT
anh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*b*(a + b)
^(3/2)*f) + (2*(b*c - a*d)^2*(b*c + 2*a*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*
x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b^3*Sqrt[a + b]*f) - ((b*c - a*d)^3*Sin
[e + f*x])/(b^2*(a^2 - b^2)*f*(b + a*Cos[e + f*x])) + (d^3*Tan[e + f*x])/(b
^2*f)
```

Rubi [A] time = 0.467264, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {3988, 2952, 2664, 12, 2659, 208, 3770, 3767, 8}

$$-\frac{(bc-ad)^3\sin(e+fx)}{b^2f\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3f}+\frac{2(bc-ad)^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+\right.}{\sqrt{a+b}}\right.}{ab^3f\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*ArcTanh[Sin[e + f*x]])/(b^3*f) + (2*(b*c - a*d)^3*ArcT
anh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*b*(a + b)
^(3/2)*f) + (2*(b*c - a*d)^2*(b*c + 2*a*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*
x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b^3*Sqrt[a + b]*f) - ((b*c - a*d)^3*Sin
[e + f*x])/(b^2*(a^2 - b^2)*f*(b + a*Cos[e + f*x])) + (d^3*Tan[e + f*x])/(b
^2*f)
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rule 2952

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
```

```
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d+c\cos(e+fx))^3\sec^2(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \int \left(\frac{(-bc+ad)^3}{ab^2(b+a\cos(e+fx))^2} + \frac{(-bc+ad)^2(bc+2ad)}{ab^3(b+a\cos(e+fx))} + \frac{d^2(3bc-2ad)\sec(e+ \ldots}{b^3}\right.$$

$$= \frac{d^3\int\sec^2(e+fx)\,dx}{b^2} + \frac{\left(d^2(3bc-2ad)\right)\int\sec(e+fx)\,dx}{b^3} - \frac{(bc-ad)^3\int\frac{\ldots}{(b+a\cos\ldots}}{ab^2}$$

$$= \frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3f} - \frac{(bc-ad)^3\sin(e+fx)}{b^2\left(a^2-b^2\right)f(b+a\cos(e+fx))} + \frac{(bc \ldots}{\ldots}$$

$$= \frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3f} + \frac{2(bc-ad)^2(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\ldots}{\sqrt{a+\ldots}}\right.}{a\sqrt{a-b}b^3\sqrt{a+b}f}$$

$$= \frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3f} + \frac{2(bc-ad)^2(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\ldots}{\sqrt{a+\ldots}}\right.}{a\sqrt{a-b}b^3\sqrt{a+b}f}$$

$$= \frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3f} + \frac{2(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}b(a+b)^{3/2}f} + \ldots$$

Mathematica [A] time = 1.57867, size = 362, normalized size = 1.59

$$\frac{\cos(e+fx)(a\cos(e+fx)+b)(c+d\sec(e+fx))^3\left(-\frac{2(bc-ad)^2\left(2a^2d+abc-3b^2d\right)(a\cos(e+fx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + d^2(2ad \ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^3*((-2*(b*c - a*d)^
2*(a*b*c + 2*a^2*d - 3*b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2
- b^2]]*(b + a*Cos[e + f*x]))/(a^2 - b^2)^(3/2) + d^2*(-3*b*c + 2*a*d)*(b +
 a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + d^2*(3*b*c - 2*
a*d)*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b*d^3
*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
]) + (b*d^3*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2]) + (b*(b*c - a*d)^3*Sin[e + f*x])/((-a + b)*(a + b))))/(b^3*f*
(d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x])^2)
```

Maple [B] time = 0.092, size = 790, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x)
```

```
[Out] -2/f/b^2/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-tan(1/2*f*x+1
/2*e)^2*b-a-b)*a^3*d^3+6/f/b/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*
e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)*a^2*c*d^2-6/f/(a^2-b^2)*tan(1/2*f*x+1/2*
e)/(tan(1/2*f*x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)*a*c^2*d+2/f*b/(a^2-b
^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)*
c^3+4/f/b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e
)/((a+b)*(a-b))^(1/2))*a^4*d^3-6/f/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*a^3*c*d^2-6/f/b/(a+b)/(a-
b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2)
)*a^2*d^3+2/f/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2
*e)/((a+b)*(a-b))^(1/2))*c^3*a+12/f/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*a*c*d^2-6/f*b/(a+b)/(a-b)/((
a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*c^2
*d-1/f*d^3/b^2/(tan(1/2*f*x+1/2*e)+1)-2/f*d^3/b^3*ln(tan(1/2*f*x+1/2*e)+1)*
a+3/f*d^2/b^2*ln(tan(1/2*f*x+1/2*e)+1)*c-1/f*d^3/b^2/(tan(1/2*f*x+1/2*e)-1)
+2/f*d^3/b^3*ln(tan(1/2*f*x+1/2*e)-1)*a-3/f*d^2/b^2*ln(tan(1/2*f*x+1/2*e)-1
)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 154.86, size = 2800, normalized size = 12.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*(((a^2*b^3*c^3 - 3*a*b^4*c^2*d - 3*(a^4*b - 2*a^2*b^3)*c*d^2 + (2*a^5
- 3*a^3*b^2)*d^3)*cos(f*x + e)^2 + (a*b^4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2 -
2*a*b^4)*c*d^2 + (2*a^4*b - 3*a^2*b^3)*d^3)*cos(f*x + e))*sqrt(a^2 - b^2)*l
og((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(
b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b
*cos(f*x + e) + b^2)) + ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*
a^4*b^2 + a^2*b^4)*d^3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d
^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(sin(f*x + e) + 1)
 - ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*d^
3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d^2 - 2*(a^5*b - 2*a^3
*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*((a^4*b^2 - 2*a
^2*b^4 + b^6)*d^3 - ((a^2*b^4 - b^6)*c^3 - 3*(a^3*b^3 - a*b^5)*c^2*d + 3*(a
^4*b^2 - a^2*b^4)*c*d^2 - (2*a^5*b - 3*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*
sin(f*x + e))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*f*cos(f*x + e)^2 + (a^4*b^4 -
2*a^2*b^6 + b^8)*f*cos(f*x + e)), 1/2*(2*((a^2*b^3*c^3 - 3*a*b^4*c^2*d - 3*
(a^4*b - 2*a^2*b^3)*c*d^2 + (2*a^5 - 3*a^3*b^2)*d^3)*cos(f*x + e)^2 + (a*b^
4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2 - 2*a*b^4)*c*d^2 + (2*a^4*b - 3*a^2*b^3)*d
```

```
^3)*cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e)
 + a)/((a^2 - b^2)*sin(f*x + e))) + ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 -
 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*d^3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^
4 + b^6)*c*d^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(sin(f
*x + e) + 1) - ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 +
 a^2*b^4)*d^3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d^2 - 2*(a
^5*b - 2*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*((a
^4*b^2 - 2*a^2*b^4 + b^6)*d^3 - ((a^2*b^4 - b^6)*c^3 - 3*(a^3*b^3 - a*b^5)*
c^2*d + 3*(a^4*b^2 - a^2*b^4)*c*d^2 - (2*a^5*b - 3*a^3*b^3 + a*b^5)*d^3)*co
s(f*x + e))*sin(f*x + e))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*f*cos(f*x + e)^2 +
 (a^4*b^4 - 2*a^2*b^6 + b^8)*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+d\sec\left(e+fx\right)\right)^3\sec\left(e+fx\right)}{\left(a+b\sec\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))**3*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Giac [B] time = 1.43682, size = 757, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -(2*(a*b^3*c^3 - 3*b^4*c^2*d - 3*a^3*b*c*d^2 + 6*a*b^3*c*d^2 + 2*a^4*d^3 -
3*a^2*b^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a
*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/((a^2*b^
3 - b^5)*sqrt(-a^2 + b^2)) - 2*(b^3*c^3*tan(1/2*f*x + 1/2*e)^3 - 3*a*b^2*c^
2*d*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*b*c*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a^3*d
^3*tan(1/2*f*x + 1/2*e)^3 + a^2*b*d^3*tan(1/2*f*x + 1/2*e)^3 + a*b^2*d^3*ta
n(1/2*f*x + 1/2*e)^3 - b^3*d^3*tan(1/2*f*x + 1/2*e)^3 - b^3*c^3*tan(1/2*f*x
 + 1/2*e) + 3*a*b^2*c^2*d*tan(1/2*f*x + 1/2*e) - 3*a^2*b*c*d^2*tan(1/2*f*x
+ 1/2*e) + 2*a^3*d^3*tan(1/2*f*x + 1/2*e) + a^2*b*d^3*tan(1/2*f*x + 1/2*e)
- a*b^2*d^3*tan(1/2*f*x + 1/2*e) - b^3*d^3*tan(1/2*f*x + 1/2*e))/((a*tan(1/
2*f*x + 1/2*e)^4 - b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
a + b)*(a^2*b^2 - b^4)) - (3*b*c*d^2 - 2*a*d^3)*log(abs(tan(1/2*f*x + 1/2*e
) + 1))/b^3 + (3*b*c*d^2 - 2*a*d^3)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/b^3)
/f
```

3.261 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=198

$$\frac{2\left(b^2c^2-a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^2f\sqrt{a-b}\sqrt{a+b}}-\frac{(bc-ad)^2\sin(e+fx)}{bf\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{af(a-b)^{3/2}(a+b)^{3/2}}+d$$

```
[Out] (d^2*ArcTanh[Sin[e + f*x]])/(b^2*f) + (2*(b*c - a*d)^2*ArcTanh[(Sqrt[a - b]
*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*f) + (2*(b^
2*c^2 - a^2*d^2)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*Sq
rt[a - b]*b^2*Sqrt[a + b]*f) - ((b*c - a*d)^2*Sin[e + f*x])/(b*(a^2 - b^2)*
f*(b + a*Cos[e + f*x]))
```

Rubi [A] time = 0.366955, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3988, 2952, 2664, 12, 2659, 208, 3770}

$$\frac{2\left(b^2c^2-a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^2f\sqrt{a-b}\sqrt{a+b}}-\frac{(bc-ad)^2\sin(e+fx)}{bf\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{af(a-b)^{3/2}(a+b)^{3/2}}+d$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x])^2,x]
```


```
[Out] (d^2*ArcTanh[Sin[e + f*x]])/(b^2*f) + (2*(b*c - a*d)^2*ArcTanh[(Sqrt[a - b]
*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*f) + (2*(b^
2*c^2 - a^2*d^2)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*Sq
rt[a - b]*b^2*Sqrt[a + b]*f) - ((b*c - a*d)^2*Sin[e + f*x])/(b*(a^2 - b^2)*
f*(b + a*Cos[e + f*x]))
```


Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rule 2952

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
```

```
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d+c\cos(e+fx))^2\sec(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \int \left(-\frac{(-bc+ad)^2}{ab(b+a\cos(e+fx))^2} + \frac{b^2c^2-a^2d^2}{ab^2(b+a\cos(e+fx))} + \frac{d^2\sec(e+fx)}{b^2}\right)dx$$

$$= \frac{d^2\int \sec(e+fx)\,dx}{b^2} - \frac{(bc-ad)^2\int \frac{1}{(b+a\cos(e+fx))^2}\,dx}{ab} + \frac{\left(b^2c^2-a^2d^2\right)\int \frac{1}{b+a\cos(}}{ab^2}$$

$$= \frac{d^2\tanh^{-1}(\sin(e+fx))}{b^2f} - \frac{(bc-ad)^2\sin(e+fx)}{b\left(a^2-b^2\right)f(b+a\cos(e+fx))} + \frac{(bc-ad)^2\int \frac{}{b+a\,c}}{ab\left(a^2-\right.}$$

$$= \frac{d^2\tanh^{-1}(\sin(e+fx))}{b^2f} + \frac{2\left(b^2c^2-a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^2\sqrt{a+b}f} - \frac{(b}{b\left(a^2-\right.}$$

$$= \frac{d^2\tanh^{-1}(\sin(e+fx))}{b^2f} + \frac{2\left(b^2c^2-a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^2\sqrt{a+b}f} - \frac{(b}{b\left(a^2-\right.}$$

$$= \frac{d^2\tanh^{-1}(\sin(e+fx))}{b^2f} + \frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}f} + \frac{2\left(b^2c^2-a\right.}{}$$

Mathematica [A] time = 0.669054, size = 180, normalized size = 0.91

$$\frac{\frac{2\left(a^3d^2-ab^2\left(c^2+2d^2\right)+2b^3cd\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{b(bc-ad)^2\sin(e+fx)}{(b-a)(a+b)(a\cos(e+fx)+b)} + d^2\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right) +}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] ((2*(2*b^3*c*d + a^3*d^2 - a*b^2*(c^2 + 2*d^2))*ArcTanh[((-a + b)*Tan[(e +
f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - d^2*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]] + d^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b*(b*c -
 a*d)^2*Sin[e + f*x])/((-a + b)*(a + b)*(b + a*Cos[e + f*x])))/(b^2*f)
```

Maple [B] time = 0.088, size = 486, normalized size = 2.5

$$2\,{\frac {\tan \left(1/2\,fx+e/2 \right) {a}^{2}{d}^{2}}{fb \left({a}^{2}-{b}^{2} \right) \left(\left(\tan \left(1/2\,fx+e/2 \right) \right) ^{2}a- \left(\tan \left(1/2\,fx+e/2 \right) \right) ^{2}b-a-b \right) }}-4\,{\frac {\tan \left(1/2\,fx+e/2 \right) ac}{f \left({a}^{2}-{b}^{2} \right) \left(\left(\tan \left(1/2\,fx+e/2 \right) \right) ^{2}a- \left(\tan \left(\right.\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x)
```

```
[Out] 2/f/b/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-tan(1/2*f*x+1/2*
e)^2*b-a-b)*a^2*d^2-4/f/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*
a-tan(1/2*f*x+1/2*e)^2*b-a-b)*a*c*d+2/f*b/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan
(1/2*f*x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)*c^2-2/f/b^2/(a+b)/(a-b)/((a
+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*a^3*
d^2+2/f/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((
a+b)*(a-b))^(1/2))*c^2*a+4/f/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*a*d^2-4/f*b/(a+b)/(a-b)/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))*c*d+1/f*d^2/
b^2*ln(tan(1/2*f*x+1/2*e)+1)-1/f*d^2/b^2*ln(tan(1/2*f*x+1/2*e)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 21.6519, size = 1719, normalized size = 8.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*((a*b^3*c^2 - 2*b^4*c*d - (a^3*b - 2*a*b^3)*d^2 + (a^2*b^2*c^2 - 2*a*
b^3*c*d - (a^4 - 2*a^2*b^2)*d^2)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*c
os(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x +
```

```
e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x +
e) + b^2)) - ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b
^3 + b^5)*d^2)*log(sin(f*x + e) + 1) + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f
*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*log(-sin(f*x + e) + 1) + 2*((a^2*b
^3 - b^5)*c^2 - 2*(a^3*b^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*sin(f*x +
e))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*f*cos(f*x + e) + (a^4*b^3 - 2*a^2*b^5 +
b^7)*f), 1/2*(2*(a*b^3*c^2 - 2*b^4*c*d - (a^3*b - 2*a*b^3)*d^2 + (a^2*b^2*c
^2 - 2*a*b^3*c*d - (a^4 - 2*a^2*b^2)*d^2)*cos(f*x + e))*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) + (
(a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)
*log(sin(f*x + e) + 1) - ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4
*b - 2*a^2*b^3 + b^5)*d^2)*log(-sin(f*x + e) + 1) - 2*((a^2*b^3 - b^5)*c^2
- 2*(a^3*b^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*sin(f*x + e))/((a^5*b^2
- 2*a^3*b^4 + a*b^6)*f*cos(f*x + e) + (a^4*b^3 - 2*a^2*b^5 + b^7)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + d \sec \left(e + fx\right)\right)^2 \sec \left(e + fx\right)}{\left(a + b \sec \left(e + fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))**2*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Giac [A] time = 1.44071, size = 374, normalized size = 1.89

$$\frac{\frac{d^2 \log\left(\left|\tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) + 1\right|\right)}{b^2} - \frac{d^2 \log\left(\left|\tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - 1\right|\right)}{b^2} - \frac{2\left(ab^2c^2 - 2b^3cd - a^3d^2 + 2ab^2d^2\right)\left(\pi\left\lfloor \frac{fx+e}{2\pi} + \frac{1}{2}\right\rfloor \operatorname{sgn}(2a - 2b) + \arctan\left(\frac{a\tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - b\tan\left(\frac{1}{2} f\right.}{\sqrt{-a^2 + b^2}}\right.\right.}{\left(a^2b^2 - b^4\right)\sqrt{-a^2 + b^2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] (d^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^2 - d^2*log(abs(tan(1/2*f*x + 1/2
*e) - 1))/b^2 - 2*(a*b^2*c^2 - 2*b^3*c*d - a^3*d^2 + 2*a*b^2*d^2)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) -
b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/((a^2*b^2 - b^4)*sqrt(-a^2 + b^2
)) + 2*(b^2*c^2*tan(1/2*f*x + 1/2*e) - 2*a*b*c*d*tan(1/2*f*x + 1/2*e) + a^2
*d^2*tan(1/2*f*x + 1/2*e))/((a^2*b - b^3)*(a*tan(1/2*f*x + 1/2*e)^2 - b*tan
(1/2*f*x + 1/2*e)^2 - a - b)))/f
```

3.262 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f(a-b)^{3/2}(a+b)^{3/2}}-\frac{(bc-ad)\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))}$$

```
[Out] (2*(a*c - b*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/((a - b
)^(3/2)*(a + b)^(3/2)*f) - ((b*c - a*d)*Tan[e + f*x])/((a^2 - b^2)*f*(a + b
*Sec[e + f*x]))
```

Rubi [A] time = 0.136388, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4003, 12, 3831, 2659, 208}

$$\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f(a-b)^{3/2}(a+b)^{3/2}}-\frac{(bc-ad)\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (2*(a*c - b*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/((a - b
)^(3/2)*(a + b)^(3/2)*f) - ((b*c - a*d)*Tan[e + f*x])/((a^2 - b^2)*f*(a + b
*Sec[e + f*x]))
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx &= -\frac{(bc-ad)\tan(e+fx)}{\left(a^2-b^2\right)f(a+b\sec(e+fx))}+\frac{\int \frac{(-ac+bd)\sec(e+fx)}{a+b\sec(e+fx)}\,dx}{-a^2+b^2}\\
&= -\frac{(bc-ad)\tan(e+fx)}{\left(a^2-b^2\right)f(a+b\sec(e+fx))}+\frac{(ac-bd)\int \frac{\sec(e+fx)}{a+b\sec(e+fx)}\,dx}{a^2-b^2}\\
&= -\frac{(bc-ad)\tan(e+fx)}{\left(a^2-b^2\right)f(a+b\sec(e+fx))}+\frac{(ac-bd)\int \frac{1}{1+\frac{a\cos(e+fx)}{b}}\,dx}{b\left(a^2-b^2\right)}\\
&= -\frac{(bc-ad)\tan(e+fx)}{\left(a^2-b^2\right)f(a+b\sec(e+fx))}+\frac{(2(ac-bd))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\right.}{b\left(a^2-b^2\right)f}\\
&= \frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}f}-\frac{(bc-ad)\tan(e+fx)}{\left(a^2-b^2\right)f(a+b\sec(e+fx))}
\end{aligned}$$

Mathematica [A] time = 0.36182, size = 97, normalized size = 0.97

$$\frac{\frac{(ad-bc)\sin(e+fx)}{(a-b)(a+b)(a\cos(e+fx)+b)}-\frac{2(ac-bd)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] ((-2*(a*c - b*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2
 - b^2)^(3/2) + ((-(b*c) + a*d)*Sin[e + f*x])/((a - b)*(a + b)*(b + a*Cos[e
 + f*x])))/f
```

Maple [A] time = 0.068, size = 132, normalized size = 1.3

$$\frac{1}{f}\left(-2\,\frac{(ad-bc)\tan\left(1/2\,fx+e/2\right)}{\left(a^2-b^2\right)\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^2a-\left(\tan\left(1/2\,fx+e/2\right)\right)^2b-a-b\right)}+2\,\frac{ac-db}{(a+b)\,(a-b)\,\sqrt{(a+b)\,(a-b)}}\mathrm{Artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x)
```

```
[Out] 1/f*(-2*(a*d-b*c)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-tan(
1/2*f*x+1/2*e)^2*b-a-b)+2*(a*c-b*d)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.527234, size = 861, normalized size = 8.61

$$\left[\frac{\left(abc - b^2d + \left(a^2c - abd\right)\cos\left(fx+e\right)\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos\left(fx+e\right) - \left(a^2 - 2\,b^2\right)\cos\left(fx+e\right)^2 + 2\,\sqrt{a^2-b^2}\left(b\cos\left(fx+e\right)+a\right)\sin\left(fx+e\right) + 2\,a^2 - b^2}{a^2\cos\left(fx+e\right)^2 + 2\,ab\cos\left(fx+e\right) + b^2}\right)}{2\left(\left(a^5 - 2\,a^3b^2 + ab^4\right)f\cos\left(fx+e\right) + \left(a^4b - 2\,a^2b^3 + b^5\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((a*b*c - b^2*d + (a^2*c - a*b*d)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2
*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos
(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(
f*x + e) + b^2)) - 2*((a^2*b - b^3)*c - (a^3 - a*b^2)*d)*sin(f*x + e))/((a^
5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*f), ((a*b
*c - b^2*d + (a^2*c - a*b*d)*cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) - ((a^2*b - b^3)
*c - (a^3 - a*b^2)*d)*sin(f*x + e))/((a^5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x +
e) + (a^4*b - 2*a^2*b^3 + b^5)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + d\sec\left(e + fx\right)\right)\sec\left(e + fx\right)}{\left(a + b\sec\left(e + fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Giac [A] time = 1.29692, size = 243, normalized size = 2.43

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-a^2+b^2}}\right)\right)(ac-bd)}{(a^2-b^2)\sqrt{-a^2+b^2}}-\frac{bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a-b\right)(a^2-b^2)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*
x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))*(a*c - b*d)/((a^2 -
 b^2)*sqrt(-a^2 + b^2)) - (b*c*tan(1/2*f*x + 1/2*e) - a*d*tan(1/2*f*x + 1/2
*e))/((a*tan(1/2*f*x + 1/2*e)^2 - b*tan(1/2*f*x + 1/2*e)^2 - a - b)*(a^2 -
b^2)))/f
```

3.263 $\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=186

$$-\frac{b^2\sin(e+fx)}{f\left(a^2-b^2\right)(bc-ad)(a\cos(e+fx)+b)}+\frac{2b\left(-2a^2d+abc+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f(a-b)^{3/2}(a+b)^{3/2}(bc-ad)^2}+\frac{2d^2\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e\cdots\right.\right.}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\sqrt{c+d}(bc-a\cdots}$$

```
[Out] (2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[
a + b]])/((a - b)^(3/2)*(a + b)^(3/2)*(b*c - a*d)^2*f) + (2*d^2*ArcTanh[(Sq
rt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*Sqrt[c + d]*(b*c - a
*d)^2*f) - (b^2*Sin[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*(b + a*Cos[e + f*x
]))
```

Rubi [A] time = 0.604509, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3988, 3056, 3001, 2659, 208}

$$-\frac{b^2\sin(e+fx)}{f\left(a^2-b^2\right)(bc-ad)(a\cos(e+fx)+b)}+\frac{2b\left(-2a^2d+abc+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f(a-b)^{3/2}(a+b)^{3/2}(bc-ad)^2}+\frac{2d^2\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e\cdots\right.\right.}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\sqrt{c+d}(bc-a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + b*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[
a + b]])/((a - b)^(3/2)*(a + b)^(3/2)*(b*c - a*d)^2*f) + (2*d^2*ArcTanh[(Sq
rt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*Sqrt[c + d]*(b*c - a
*d)^2*f) - (b^2*Sin[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*(b + a*Cos[e + f*x
]))
```

Rule 3988

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \int \frac{\cos^2(e+fx)}{(b+a\cos(e+fx))^2(d+c\cos(e+fx))}\,dx$$

$$= -\frac{b^2\sin(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(b+a\cos(e+fx))} - \frac{\int \frac{-abd-\left(abc-a^2d+b^2d\right)\cos(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))}}{\left(a^2-b^2\right)(bc-ad)}$$

$$= -\frac{b^2\sin(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(b+a\cos(e+fx))} + \frac{d^2\int \frac{1}{d+c\cos(e+fx)}\,dx}{(bc-ad)^2} + \frac{(b\,(\ldots}{\ldots}$$

$$= -\frac{b^2\sin(e+fx)}{\left(a^2-b^2\right)(bc-ad)f(b+a\cos(e+fx))} + \frac{\left(2d^2\right)\mathrm{Subst}\left(\int \frac{1}{c+d+(-c+d)x\ldots}\right)}{(bc-a\ldots}$$

$$= \frac{2b\left(abc-2a^2d+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}(bc-ad)^2f} + \frac{2d^2\tanh^{-1}\left(\frac{\sqrt{c-d}\,\mathrm{ta}\ldots}{\ldots}\right)}{\sqrt{c-d}\sqrt{c+d}(bc\ldots}$$

Mathematica [A] time = 1.01619, size = 176, normalized size = 0.95

$$\frac{\frac{2d^2\left(a^2-b^2\right)\tanh^{-1}\left(\frac{(d-c)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{2b\left(-2a^2d+abc+b^2d\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{b^2(bc-ad)\sin(e+fx)}{a\cos(e+fx)+b}}{f(b-a)(a+b)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + b*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

```
[Out] ((2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^
2 - b^2]])/Sqrt[a^2 - b^2] + (2*(a^2 - b^2)*d^2*ArcTanh[((-c + d)*Tan[(e +
f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + (b^2*(b*c - a*d)*Sin[e + f*x])
/(b + a*Cos[e + f*x]))/((-a + b)*(a + b)*(b*c - a*d)^2*f)
```

Maple [A] time = 0.105, size = 210, normalized size = 1.1

$$\frac{1}{f}\left(2\frac{b}{(ad-bc)^{2}}\left(-\frac{b(ad-bc)\tan\left(1/2\,fx+e/2\right)}{\left(a^{2}-b^{2}\right)\left(\left(\tan\left(1/2\,fx+e/2\right)\right)^{2}a-\left(\tan\left(1/2\,fx+e/2\right)\right)^{2}b-a-b\right)}-\frac{2\,a^{2}d-abc-b^{2}d}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\mathrm{A}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x)
```

```
[Out] 1/f*(2*b/(a*d-b*c)^2*(-b*(a*d-b*c)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*
x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)-(2*a^2*d-a*b*c-b^2*d)/(a+b)/(a-b)/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2)))+
2*d^2/(a*d-b*c)^2/((c+d)*(c-d))^(1/2)*arctanh(tan(1/2*f*x+1/2*e)*(c-d)/((c+
d)*(c-d))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))**2/(c+d*sec(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32131, size = 458, normalized size = 2.46

$$2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d^2}{(b^2c^2-2abcd+a^2d^2)\sqrt{-c^2+d^2}}+\frac{b^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{(a^2bc-b^3c-a^3d+ab^2d)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a-b\right)}-\frac{(ab\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f
*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d^2/((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*sqrt(-c^2 + d^2)) + b^2*tan(1/2*f*x + 1/2*e)/((a^2*b*c -
 b^3*c - a^3*d + a*b^2*d)*(a*tan(1/2*f*x + 1/2*e)^2 - b*tan(1/2*f*x + 1/2*e
)^2 - a - b)) - (a*b^2*c - 2*a^2*b*d + b^3*d)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*
e))/sqrt(-a^2 + b^2)))/((a^2*b^2*c^2 - b^4*c^2 - 2*a^3*b*c*d + 2*a*b^3*c*d
+ a^4*d^2 - a^2*b^2*d^2)*sqrt(-a^2 + b^2)))/f
```

3.264 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=213

$$\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{df}-\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\,\text{s}\ldots}{}}}{df(c+d)\sqrt{-\tan\ldots}}$$

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*(b*c - a*d)*EllipticPi[(2*d)/(c + d
), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e
 + f*x])/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[
-Tan[e + f*x]^2])
```

Rubi [A] time = 0.293255, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3969, 3832, 3973}

$$\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{df}-\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\Pi\ldots}{df(c+d)\sqrt{-\tan^2(e+fx)\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*(b*c - a*d)*EllipticPi[(2*d)/(c + d
), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e
 + f*x])/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[
-Tan[e + f*x]^2])
```

Rule 3969

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d, Int[Csc[e + f*
x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/d, Int[Csc[e + f*x]/
(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3973

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[(-2*Cot[e + f*x]*Sq
rt[(a + b*Csc[e + f*x])/(a + b)]*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 -
```

```
Csc[e + f*x]]/Sqrt[2]], (2*b)/(a + b)])/(f*(c + d)*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[-Cot[e + f*x]^2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx}{d}$$
$$= \frac{2\sqrt{a+b}\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+f}{a-b}}}{df}$$

Mathematica [A] time = 3.65297, size = 185, normalized size = 0.87

$$\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\sqrt{a+b\sec(e+fx)}\left((a-b)(c+d)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right.\right.\right.}{f(c-d)(c+d)(a\cos(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)*(c + d)*EllipticF[ArcSin[
Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*(-(b*c) + a*d)*EllipticPi[(c - d)/(
c + d), -ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b*Sec[e + f*x
]])/((c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Maple [A] time = 0.33, size = 355, normalized size = 1.7

$$2\,\frac{\left(1+\cos\left(fx+e\right)\right)^{2}\left(-1+\cos\left(fx+e\right)\right)}{f\left(c+d\right)\left(c-d\right)\left(a\cos\left(fx+e\right)+b\right)\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{(a+b)\left(1+\cos\left(fx+\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] 2/f/(c+d)/(c-d)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))
^2*(EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*c+EllipticF
((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*d-EllipticF((-1+cos(f*x+
e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*c-EllipticF((-1+cos(f*x+e))/sin(f*x+e
),((a-b)/(a+b))^(1/2))*b*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c
+d),((a-b)/(a+b))^(1/2))*a*d+2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/
(c+d),((a-b)/(a+b))^(1/2))*b*c)*(-1+cos(f*x+e))/(a*cos(f*x+e)+b)/sin(f*x+e)
^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sec\left(e+fx\right)}\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

3.265 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=196

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}}{df\sqrt{\frac{a+b}{c+d}}}$$

```
[Out] (2*Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(c
 + d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c + d
))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a +
 b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*S
ec[e + f*x]))]*(a + b*Sec[e + f*x]))/(d*Sqrt[(a + b)/(c + d)]*f)
```

Rubi [A] time = 0.192992, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3982}

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}}{df\sqrt{\frac{a+b}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (2*Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(c
 + d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c + d
))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a +
 b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*S
ec[e + f*x]))]*(a + b*Sec[e + f*x]))/(d*Sqrt[(a + b)/(c + d)]*f)
```

Rule 3982

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(-2*(a + b*Csc[
e + f*x])*Sqrt[-(((b*c - a*d)*(1 - Csc[e + f*x]))/((c + d)*(a + b*Csc[e + f
*x])))]*Sqrt[((b*c - a*d)*(1 + Csc[e + f*x]))/((c - d)*(a + b*Csc[e + f*x])
)]*EllipticPi[(b*(c + d))/(d*(a + b)), ArcSin[(Sqrt[(a + b)/(c + d)]*Sqrt[c
 + d*Csc[e + f*x]])/Sqrt[a + b*Csc[e + f*x]]], ((a - b)*(c + d))/((a + b)*(
c - d))])/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\cot(e+fx)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+}{(c+d)(a+b\sec(e+}}}{d\sqrt{\frac{a+b}{c+d}}f}$$

Mathematica [C] time = 32.4728, size = 44216, normalized size = 225.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],
x]
```

```
[Out] Result too large to show
```

Maple [A] time = 0.404, size = 351, normalized size = 1.8

$$2\frac{\cos\left(fx+e\right)\left(\sin\left(fx+e\right)\right)^{2}}{f\left(-1+\cos\left(fx+e\right)\right)\left(d+c\cos\left(fx+e\right)\right)\left(a\cos\left(fx+e\right)+b\right)}\left(\mathrm{EllipticF}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\sqrt{\frac{a-b}{a+b}},\sqrt{\frac{(c-d)(a+b)}{(a-b)(c+d)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f/((a-b)/(a+b))^(1/2)*(EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(
f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a-EllipticF((-1+cos(f*x+e))*((a-b)/
(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b+2*EllipticPi((-1
+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),(a+b)/(a-b),((c-d)/(c+d))^(1/2)
/((a-b)/(a+b))^(1/2))*b)*cos(f*x+e)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(
(d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^2*(1/(c+d)*(d+c*cos(f*x+e))/(
1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)/(-1+co
s(f*x+e))/(d+c*cos(f*x+e))/(a*cos(f*x+e)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sec\left(e+fx\right)}\sec\left(e+fx\right)}{\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/sqrt(c + d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c), x
)
```

3.266 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=192

$$\frac{2\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f\sqrt{c+d}(bc-ad)}$$

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e
+ f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a - b
)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f
*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x]
)))]*(c + d*Sec[e + f*x]))/(Sqrt[c + d]*(b*c - a*d)*f)
```

Rubi [A] time = 0.168126, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3984}

$$\frac{2\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e
+ f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a - b
)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f
*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x]
)))]*(c + d*Sec[e + f*x]))/(Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 3984

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[(-2*(c + d*Csc[
e + f*x])*Sqrt[((b*c - a*d)*(1 - Csc[e + f*x]))/((a + b)*(c + d*Csc[e + f*x
]))]*Sqrt[-(((b*c - a*d)*(1 + Csc[e + f*x]))/((a - b)*(c + d*Csc[e + f*x]))
)]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c
 + d*Csc[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*
Rt[(c + d)/(a + b), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\sqrt{a+b}\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}}{\sqrt{c+d}(bc-ad)f}$$

Mathematica [A] time = 3.48568, size = 233, normalized size = 1.21

$$\frac{4\sin^2\left(\frac{1}{2}(e+fx)\right)\csc(e+fx)\sqrt{a+b\sec(e+fx)}\sqrt{\frac{(c+d)\cot^2\left(\frac{1}{2}(e+fx)\right)}{c-d}}\sqrt{\frac{(a+b)\csc^2\left(\frac{1}{2}(e+fx)\right)(c\cos(e+fx)+d)}{ad-bc}}\text{EllipticF}\left(\sin^{-1}\left(\ldots\right.\right.}{f(a+b)\sqrt{c+d\sec(e+fx)}\sqrt{\frac{(c+d)\csc^2\left(\frac{1}{2}(e+fx)\right)(a\cos(e+fx)+b)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),
x]
```

```
[Out] (4*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((a + b)*(d + c*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqr
t[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]/Sqrt[2]
], (2*(b*c - a*d))/((a + b)*(c - d))]*Sqrt[a + b*Sec[e + f*x]]*Sin[(e + f*x
)/2]^2)/((a + b)*f*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(
b*c - a*d)]*Sqrt[c + d*Sec[e + f*x]])
```

Maple [A] time = 0.389, size = 219, normalized size = 1.1

$$2\,\frac{\left(\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)}{f\left(-1+\cos\left(fx+e\right)\right)\left(d+c\cos\left(fx+e\right)\right)\left(a\cos\left(fx+e\right)+b\right)}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\cos\left(fx+e\right)}}\sqrt{\frac{d+c\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d+c\ldots}{(c+d)\left(1\ldots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f/((a-b)/(a+b))^(1/2)*sin(f*x+e)^2*cos(f*x+e)*(1/cos(f*x+e)*(a*cos(f*x+e)
+b))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1
+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*Ellipti
cF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))
^(1/2))/(-1+cos(f*x+e))/(d+c*cos(f*x+e))/(a*cos(f*x+e)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}\sec\left(fx+e\right)}{bd\sec\left(fx+e\right)^{2}+ac+(bc+ad)\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)*sec(f*x + e)/(b*
d*sec(f*x + e)^2 + a*c + (b*c + a*d)*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x))),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)),
 x)
```

3.267 $\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx$

Optimal. Leaf size=110

$$\frac{2\cot(e+fx)(4-5\sec(e+fx))\sqrt{\frac{1-\sec(e+fx)}{4-5\sec(e+fx)}}\sqrt{\frac{\sec(e+fx)+1}{4-5\sec(e+fx)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{3\sec(e+fx)+2}}{\sqrt{5}\sqrt{5\sec(e+fx)-4}}\right),45\right)}{f}$$

```
[Out] (2*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[2 + 3*Sec[e + f*x]]/(Sqrt[5]*Sqrt[-4
+ 5*Sec[e + f*x]])], 45]*(4 - 5*Sec[e + f*x])*Sqrt[(1 - Sec[e + f*x])/(4 -
5*Sec[e + f*x])]*Sqrt[(1 + Sec[e + f*x])/(4 - 5*Sec[e + f*x])])/f
```

Rubi [A] time = 0.12409, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3984}

$$\frac{2\cot(e+fx)(4-5\sec(e+fx))\sqrt{\frac{1-\sec(e+fx)}{4-5\sec(e+fx)}}\sqrt{\frac{\sec(e+fx)+1}{4-5\sec(e+fx)}}F\left(\sin^{-1}\left(\frac{\sqrt{3\sec(e+fx)+2}}{\sqrt{5}\sqrt{5\sec(e+fx)-4}}\right)\middle|45\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[2 + 3*Sec[e + f*x]]*Sqrt[-4 + 5*Sec[e + f*x]]),x]
```

```
[Out] (2*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[2 + 3*Sec[e + f*x]]/(Sqrt[5]*Sqrt[-4
+ 5*Sec[e + f*x]])], 45]*(4 - 5*Sec[e + f*x])*Sqrt[(1 - Sec[e + f*x])/(4 -
5*Sec[e + f*x])]*Sqrt[(1 + Sec[e + f*x])/(4 - 5*Sec[e + f*x])])/f
```

Rule 3984

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[(-2*(c + d*Csc[
e + f*x])*Sqrt[((b*c - a*d)*(1 - Csc[e + f*x]))/((a + b)*(c + d*Csc[e + f*x
]))]*Sqrt[-(((b*c - a*d)*(1 + Csc[e + f*x]))/((a - b)*(c + d*Csc[e + f*x]))
)]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c
 + d*Csc[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*
Rt[(c + d)/(a + b), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \frac{2\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{2+3\sec(e+fx)}}{\sqrt{5}\sqrt{-4+5\sec(e+fx)}}\right)\middle|45\right)(4-5\sec(e+fx))\sqrt{\frac{1}{4}}}{f}$$

Mathematica [A] time = 1.75091, size = 176, normalized size = 1.6

$$-\frac{4\sin^4\left(\frac{1}{2}(e+fx)\right)\sqrt{-\cot^2\left(\frac{1}{2}(e+fx)\right)}\csc(e+fx)\sec(e+fx)\sqrt{-(2\cos(e+fx)+3)\csc^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{-(4\cos(e}}{3\sqrt{5}f\sqrt{3\sec(e+fx)+2}\sqrt{5\sec(e+fx)-4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[2 + 3*Sec[e + f*x]]*Sqrt[-4 + 5*Sec[e + f*x]])
,x]
```

```
[Out] (-4*Sqrt[-Cot[(e + f*x)/2]^2]*Sqrt[-((3 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]^
2)]*Sqrt[-((-5 + 4*Cos[e + f*x])*Csc[(e + f*x)/2]^2)]*Csc[e + f*x]*Elliptic
F[ArcSin[Sqrt[5/22]*Sqrt[(-5 + 4*Cos[e + f*x])/(-1 + Cos[e + f*x])]], 44/45
]*Sec[e + f*x]*Sin[(e + f*x)/2]^4)/(3*Sqrt[5]*f*Sqrt[2 + 3*Sec[e + f*x]]*Sq
rt[-4 + 5*Sec[e + f*x]])
```

Maple [C] time = 0.339, size = 177, normalized size = 1.6

$$\frac{\frac{i}{5}\sqrt{5}\sqrt{10}\cos\left(fx+e\right)\left(\sin\left(fx+e\right)\right)^{2}}{f\left(8\left(\cos\left(fx+e\right)\right)^{3}-6\left(\cos\left(fx+e\right)\right)^{2}-17\cos\left(fx+e\right)+15\right)}\sqrt{\frac{2\cos\left(fx+e\right)+3}{1+\cos\left(fx+e\right)}}\sqrt{-2\frac{4\cos\left(fx+e\right)-5}{1+\cos\left(fx+e\right)}}\sqrt{\frac{2\cos(}{\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/5*I/f*5^(1/2)*10^(1/2)*((2*cos(f*x+e)+3)/(1+cos(f*x+e)))^(1/2)*(-2*(4*cos
(f*x+e)-5)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)*sin(f*x+e)^2*((2*cos(f*x+e)+3)/
cos(f*x+e))^(1/2)*(-(4*cos(f*x+e)-5)/cos(f*x+e))^(1/2)*EllipticF(1/5*I*(-1+
cos(f*x+e))*5^(1/2)/sin(f*x+e),3*5^(1/2))/(8*cos(f*x+e)^3-6*cos(f*x+e)^2-17
*cos(f*x+e)+15)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{5\sec\left(fx+e\right)-4}\sqrt{3\sec\left(fx+e\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(5*sec(f*x + e) - 4)*sqrt(3*sec(f*x + e) + 2)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{5\sec\left(fx+e\right)-4}\sqrt{3\sec\left(fx+e\right)+2}\sec\left(fx+e\right)}{15\sec\left(fx+e\right)^{2}-2\sec\left(fx+e\right)-8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(5*sec(f*x + e) - 4)*sqrt(3*sec(f*x + e) + 2)*sec(f*x + e)/(15
*sec(f*x + e)^2 - 2*sec(f*x + e) - 8), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{3\sec\left(e+fx\right)+2}\sqrt{5\sec\left(e+fx\right)-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))**(1/2)/(-4+5*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(3*sec(e + f*x) + 2)*sqrt(5*sec(e + f*x) - 4)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{5\sec\left(fx+e\right)-4}\sqrt{3\sec\left(fx+e\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(5*sec(f*x + e) - 4)*sqrt(3*sec(f*x + e) + 2)),
 x)
```

3.268 $\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx$

Optimal. Leaf size=125

$$\frac{2i\cot(e+fx)\sqrt{\frac{1-\sec(e+fx)}{3\sec(e+fx)+2}}\sqrt{\frac{\sec(e+fx)+1}{3\sec(e+fx)+2}}(3\sec(e+fx)+2)\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{5}\sqrt{4-5\sec(e+fx)}}{\sqrt{3\sec(e+fx)+2}}\right),\frac{1}{45}\right)}{3\sqrt{5}f}$$

```
[Out] (((2*I)/3)*Cot[e + f*x]*EllipticF[I*ArcSinh[(Sqrt[5]*Sqrt[4 - 5*Sec[e + f*x
]])/Sqrt[2 + 3*Sec[e + f*x]]], 1/45]*Sqrt[(1 - Sec[e + f*x])/(2 + 3*Sec[e +
 f*x])]*Sqrt[(1 + Sec[e + f*x])/(2 + 3*Sec[e + f*x])]*(2 + 3*Sec[e + f*x]))
/(Sqrt[5]*f)
```

Rubi [A] time = 0.119524, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {3984}

$$\frac{2i\cot(e+fx)\sqrt{\frac{1-\sec(e+fx)}{3\sec(e+fx)+2}}\sqrt{\frac{\sec(e+fx)+1}{3\sec(e+fx)+2}}(3\sec(e+fx)+2)F\left(i\sinh^{-1}\left(\frac{\sqrt{5}\sqrt{4-5\sec(e+fx)}}{\sqrt{3\sec(e+fx)+2}}\right)|\frac{1}{45}\right)}{3\sqrt{5}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[4 - 5*Sec[e + f*x]]*Sqrt[2 + 3*Sec[e + f*x]]),x]
```

```
[Out] (((2*I)/3)*Cot[e + f*x]*EllipticF[I*ArcSinh[(Sqrt[5]*Sqrt[4 - 5*Sec[e + f*x
]])/Sqrt[2 + 3*Sec[e + f*x]]], 1/45]*Sqrt[(1 - Sec[e + f*x])/(2 + 3*Sec[e +
 f*x])]*Sqrt[(1 + Sec[e + f*x])/(2 + 3*Sec[e + f*x])]*(2 + 3*Sec[e + f*x]))
/(Sqrt[5]*f)
```

Rule 3984

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[(-2*(c + d*Csc[
e + f*x])*Sqrt[((b*c - a*d)*(1 - Csc[e + f*x]))/((a + b)*(c + d*Csc[e + f*x
]))]*Sqrt[-(((b*c - a*d)*(1 + Csc[e + f*x]))/((a - b)*(c + d*Csc[e + f*x]))
)]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c
 + d*Csc[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*
Rt[(c + d)/(a + b), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = \frac{2i\cot(e+fx)F\left(i\sinh^{-1}\left(\frac{\sqrt{5}\sqrt{4-5\sec(e+fx)}}{\sqrt{2+3\sec(e+fx)}}\right)|\frac{1}{45}\right)\sqrt{\frac{1-\sec(e+fx)}{2+3\sec(e+fx)}}\sqrt{\frac{1+\sec(e+f}{2+3\sec(e+f}}}{3\sqrt{5}f}$$

Mathematica [A] time = 0.444602, size = 176, normalized size = 1.41

$$-\frac{4\sin^4\left(\frac{1}{2}(e+fx)\right)\sqrt{-\cot^2\left(\frac{1}{2}(e+fx)\right)}\csc(e+fx)\sec(e+fx)\sqrt{-(2\cos(e+fx)+3)\csc^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{-(4\cos(e+f}}{3\sqrt{5}f\sqrt{4-5\sec(e+fx)}\sqrt{3\sec(e+fx)+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[4 - 5*Sec[e + f*x]]*Sqrt[2 + 3*Sec[e + f*x]]),
x]
```

```
[Out] (-4*Sqrt[-Cot[(e + f*x)/2]^2]*Sqrt[-((3 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]^
2)]*Sqrt[-((-5 + 4*Cos[e + f*x])*Csc[(e + f*x)/2]^2)]*Csc[e + f*x]*Elliptic
F[ArcSin[Sqrt[5/22]*Sqrt[(-5 + 4*Cos[e + f*x])/(-1 + Cos[e + f*x])]], 44/45
]*Sec[e + f*x]*Sin[(e + f*x)/2]^4)/(3*Sqrt[5]*f*Sqrt[4 - 5*Sec[e + f*x]]*Sq
rt[2 + 3*Sec[e + f*x]])
```

Maple [A] time = 0.309, size = 170, normalized size = 1.4

$$\frac{-\frac{i}{15}\left(\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)\sqrt{10}}{f\left(8\left(\cos\left(fx+e\right)\right)^{3}-6\left(\cos\left(fx+e\right)\right)^{2}-17\cos\left(fx+e\right)+15\right)}\sqrt{\frac{4\cos\left(fx+e\right)-5}{\cos\left(fx+e\right)}}\sqrt{\frac{2\cos\left(fx+e\right)+3}{\cos\left(fx+e\right)}}\sqrt{-2\frac{4\cos\left(fx+e\right)-5}{1+\cos\left(fx+e\right)}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/15*I/f*sin(f*x+e)^2*cos(f*x+e)*((4*cos(f*x+e)-5)/cos(f*x+e))^(1/2)*((2*c
os(f*x+e)+3)/cos(f*x+e))^(1/2)*(-2*(4*cos(f*x+e)-5)/(1+cos(f*x+e)))^(1/2)*1
0^(1/2)*((2*cos(f*x+e)+3)/(1+cos(f*x+e)))^(1/2)*EllipticF(3*I*(-1+cos(f*x+e
))/sin(f*x+e),1/15*5^(1/2))/(8*cos(f*x+e)^3-6*cos(f*x+e)^2-17*cos(f*x+e)+15
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{3\sec\left(fx+e\right)+2}\sqrt{-5\sec\left(fx+e\right)+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(3*sec(f*x + e) + 2)*sqrt(-5*sec(f*x + e) + 4))
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{3\sec\left(fx+e\right)+2}\sqrt{-5\sec\left(fx+e\right)+4}\sec\left(fx+e\right)}{15\sec\left(fx+e\right)^{2}-2\sec\left(fx+e\right)-8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(3*sec(f*x + e) + 2)*sqrt(-5*sec(f*x + e) + 4)*sec(f*x + e)/(
15*sec(f*x + e)^2 - 2*sec(f*x + e) - 8), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{3\sec(e+fx)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))**(1/2)/(2+3*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(4 - 5*sec(e + f*x))*sqrt(3*sec(e + f*x) + 2)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(fx+e)}{\sqrt{3\sec(fx+e)+2}\sqrt{-5\sec(fx+e)+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(3*sec(f*x + e) + 2)*sqrt(-5*sec(f*x + e) + 4))
, x)
```

3.269 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=396

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}}{bdf\sqrt{\frac{a+b}{c+d}}}$$

```
[Out] (2*Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(c
 + d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c + d
))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a +
 b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*S
ec[e + f*x]))]*(a + b*Sec[e + f*x]))/(b*d*Sqrt[(a + b)/(c + d)]*f) - (2*a*S
qrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*
x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
 + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f*x])
)]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]
*(c + d*Sec[e + f*x]))/(b*Sqrt[c + d]*(b*c - a*d)*f)
```

Rubi [A] time = 0.61053, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {3985, 3984, 3982}

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}}{bdf\sqrt{\frac{a+b}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(c
 + d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c + d
))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a +
 b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*S
ec[e + f*x]))]*(a + b*Sec[e + f*x]))/(b*d*Sqrt[(a + b)/(c + d)]*f) - (2*a*S
qrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*
x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
 + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f*x])
)]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]
*(c + d*Sec[e + f*x]))/(b*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 3985

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*S
qrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> -Dist[a/b, Int[Csc
[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] + Dis
t[1/b, Int[(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/Sqrt[c + d*Csc[e + f*x]]
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3984

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[(-2*(c + d*Csc[
```

```
e + f*x])*Sqrt[((b*c - a*d)*(1 - Csc[e + f*x]))/((a + b)*(c + d*Csc[e + f*x
]))]*Sqrt[-(((b*c - a*d)*(1 + Csc[e + f*x]))/((a - b)*(c + d*Csc[e + f*x]))
)]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c
 + d*Csc[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*
Rt[(c + d)/(a + b), 2]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3982

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[(-2*(a + b*Csc[
e + f*x])*Sqrt[-(((b*c - a*d)*(1 - Csc[e + f*x]))/((c + d)*(a + b*Csc[e + f
*x])))]*Sqrt[((b*c - a*d)*(1 + Csc[e + f*x]))/((c - d)*(a + b*Csc[e + f*x])
)]*EllipticPi[(b*(c + d))/(d*(a + b)), ArcSin[(Sqrt[(a + b)/(c + d)]*Sqrt[c
 + d*Csc[e + f*x]])/Sqrt[a + b*Csc[e + f*x]]], ((a - b)*(c + d))/((a + b)*(
c - d))])/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{b} - \frac{a\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx}{b}$$

$$= \frac{2\cot(e+fx)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\mathrm{se}}{(c+d)(a+b\,\mathrm{se}}}}{bd\sqrt{\frac{a+b}{c+d}}f}$$

Mathematica [C] time = 32.3777, size = 39039, normalized size = 98.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]
),x]
```

```
[Out] Result too large to show
```

Maple [A] time = 0.411, size = 291, normalized size = 0.7

$$-2\frac{\cos(fx+e)(\sin(fx+e))^2}{f(-1+\cos(fx+e))(d+c\cos(fx+e))(a\cos(fx+e)+b)}\left(\mathrm{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\sqrt{\frac{a-b}{a+b}},\sqrt{\frac{(c-d)(a+b}{(a-b)(c+d}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f/((a-b)/(a+b))^(1/2)*(EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin
(f*x+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))-2*EllipticPi((-1+cos(f*x+e))*((a-b
)/(a+b))^(1/2)/sin(f*x+e),(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/
2)))*cos(f*x+e)*sin(f*x+e)^2*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*((d+c*co
```

```
s(f*x+e))/cos(f*x+e))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)
*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)/(-1+cos(f*x+e))/(d+c*cos(f
*x+e))/(a*cos(f*x+e)+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x))
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)
), x)
```

3.270 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=170

$$\frac{2g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{bf(a+b)\sqrt{c+d\sec(e+fx)}}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{bf\sqrt{c+d\sec(e+fx)}}$$

```
[Out] (2*d*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[2, (e + f*x)/2, (2*c)/
(c + d)]*Sqrt[g*Sec[e + f*x]])/(b*f*Sqrt[c + d*Sec[e + f*x]]) + (2*(b*c - a
*d)*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x
)/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/(b*(a + b)*f*Sqrt[c + d*Sec[e + f
*x]])
```

Rubi [A] time = 0.848721, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3971, 3859, 2807, 2805, 3975}

$$\frac{2g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{bf(a+b)\sqrt{c+d\sec(e+fx)}}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{bf\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[c + d*Sec[e + f*x]])/(a + b*Sec[e + f*x]),
x]
```

```
[Out] (2*d*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[2, (e + f*x)/2, (2*c)/
(c + d)]*Sqrt[g*Sec[e + f*x]])/(b*f*Sqrt[c + d*Sec[e + f*x]]) + (2*(b*c - a
*d)*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x
)/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/(b*(a + b)*f*Sqrt[c + d*Sec[e + f
*x]])
```

Rule 3971

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx = \frac{d\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx}{b} - \frac{(-bc+ad)\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx}{b}$$

$$= \frac{\left(dg\sqrt{d+c\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{d+c\cos(e+fx)}}\,dx}{b\sqrt{c+d\sec(e+fx)}} - \frac{\left((-bc+ad)g\sqrt{d\right.}{}$$

$$= \frac{\left(dg\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}\,dx}{b\sqrt{c+d\sec(e+fx)}} - \frac{\left((-bc+ad)g\sqrt{\frac{d+c\,c}{}}\right.}{}$$

$$= \frac{2dg\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{bf\sqrt{c+d\sec(e+fx)}} + \frac{2(bc-ad)g\sqrt{\frac{d+c\cos}{c}}}{b}$$

Mathematica [C] time = 4.02838, size = 223, normalized size = 1.31

$$-\frac{2ig\cot(e+fx)\sqrt{g\sec(e+fx)}\sqrt{-\frac{c(\cos(e+fx)-1)}{c+d}}\sqrt{\frac{c(\cos(e+fx)+1)}{c-d}}\sqrt{c+d\sec(e+fx)}\left(\Pi\left(1-\frac{c}{d};i\sinh^{-1}\left(\sqrt{\frac{1}{c-d}}\sqrt{d+c\cos(e\right.\right.\right.}{bf\sqrt{\frac{1}{c-d}}\sqrt{c\cos(e+fx)+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[c + d*Sec[e + f*x]])/(a + b*Sec[e +
f*x]),x]
```

```
[Out] ((-2*I)*g*Sqrt[-((c*(-1 + Cos[e + f*x]))/(c + d))]*Sqrt[(c*(1 + Cos[e + f*x
]))/(c - d)]*Cot[e + f*x]*(EllipticPi[1 - c/d, I*ArcSinh[Sqrt[(c - d)^(-1)]
*Sqrt[d + c*Cos[e + f*x]]], (-c + d)/(c + d)] - EllipticPi[(a*(-c + d))/(-(
b*c) + a*d), I*ArcSinh[Sqrt[(c - d)^(-1)]*Sqrt[d + c*Cos[e + f*x]]], (-c +
d)/(c + d)])*Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(b*Sqrt[(c - d)
^(-1)]*f*Sqrt[d + c*Cos[e + f*x]])
```

Maple [C] time = 0.393, size = 465, normalized size = 2.7

$$\frac{-2i\left(\cos\left(fx+e\right)\right)^{2}}{fb\left(a+b\right)\left(a-b\right)\left(d+c\cos\left(fx+e\right)\right)}\sqrt{\frac{d+c\cos\left(fx+e\right)}{\left(c+d\right)\left(1+\cos\left(fx+e\right)\right)}}\left(\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{c-d}{c+d}}\right)abc-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x)
```

```
[Out] -2*I/f/b/(a+b)/(a-b)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(Ellip
ticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*a*b*c-EllipticF(I*(
-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*a*b*d+EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*b^2*c-EllipticF(I*(-1+cos(f*x+e))/s
in(f*x+e),(-(c-d)/(c+d))^(1/2))*b^2*d+2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*
x+e),-(a-b)/(a+b),I*((c-d)/(c+d))^(1/2))*a^2*d-2*EllipticPi(I*(-1+cos(f*x+e
))/sin(f*x+e),-(a-b)/(a+b),I*((c-d)/(c+d))^(1/2))*a*b*c-2*EllipticPi(I*(-1+
cos(f*x+e))/sin(f*x+e),-1,I*((c-d)/(c+d))^(1/2))*a^2*d+2*EllipticPi(I*(-1+c
os(f*x+e))/sin(f*x+e),-1,I*((c-d)/(c+d))^(1/2))*b^2*d)*((d+c*cos(f*x+e))/co
s(f*x+e))^(1/2)*(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2/(d+c*cos(f*x+e))/(1/(1+co
s(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sec\left(fx+e\right)+c}\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*(g*sec(f*x + e))^(3/2)/(b*sec(f*x + e) +
 a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**(1/2)/(a+b*sec(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sec\left(fx+e\right)+c}\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*(g*sec(f*x + e))^(3/2)/(b*sec(f*x + e) +
 a), x)
```

3.271 $\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=83

$$\frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{f(a+b)\sqrt{c+d\sec(e+fx)}}$$

```
[Out] (2*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x)
/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/((a + b)*f*Sqrt[c + d*Sec[e + f*x]
])
```

Rubi [A] time = 0.433414, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3975, 2807, 2805}

$$\frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{f(a+b)\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(3/2)/((a + b*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),
x]
```

```
[Out] (2*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x)
/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/((a + b)*f*Sqrt[c + d*Sec[e + f*x]
])
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\left(g\sqrt{d+c\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{(b+a\cos(e+fx))\sqrt{d+c\cos(e+fx)}}\,dx}{\sqrt{c+d\sec(e+fx)}}$$
$$= \frac{\left(g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{(b+a\cos(e+fx))\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}\,dx}{\sqrt{c+d\sec(e+fx)}}$$
$$= \frac{2g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{(a+b)f\sqrt{c+d\sec(e+fx)}}$$

Mathematica [A] time = 0.242946, size = 83, normalized size = 1.

$$\frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{f(a+b)\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/((a + b*Sec[e + f*x])*Sqrt[c + d*Sec[e + f
*x]]),x]
```

```
[Out] (2*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x)
/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/((a + b)*f*Sqrt[c + d*Sec[e + f*x]
])
```

Maple [C] time = 0.355, size = 236, normalized size = 2.8

$$\frac{-2i\left(\cos\left(fx+e\right)\right)^2}{f\left(a+b\right)\left(a-b\right)\left(d+c\cos\left(fx+e\right)\right)}\left(a\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{c-d}{c+d}}\right)+b\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x)
```

```
[Out] -2*I/f/(a+b)/(a-b)*(a*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))
^(1/2))+b*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))-2*a*
EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-(a-b)/(a+b),I*((c-d)/(c+d))^(1/2))
)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(g/cos(f*x+e))^(3/2)*cos(
f*x+e)^2*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)/(d+c*cos(f*x+e))/(1/(1+cos(f*x
+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sec\left(fx+e\right)+a\right)\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/((b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e)
+ c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\sec\left(e+fx\right)\right)^{\frac{3}{2}}}{\left(a+b\sec\left(e+fx\right)\right)\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)/((a + b*sec(e + f*x))*sqrt(c + d*sec(e + f
*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sec\left(fx+e\right)+a\right)\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/((b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e)
+ c)), x)
```

3.272 $\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx$

Optimal. Leaf size=168

$$\frac{2(ac-bd)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{af(a+b)\sqrt{c+d\sec(e+fx)}}+\frac{2d\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{af\sqrt{c+d\sec(e+fx)}}$$

```
[Out] (2*d*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[2, (e + f*x)/2, (2*c)/(c
 + d)]*Sqrt[g*Sec[e + f*x]])/(a*f*Sqrt[c + d*Sec[e + f*x]]) + (2*(a*c - b*d
)*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*b)/(a + b), (e + f*x)/2,
 (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/(a*(a + b)*f*Sqrt[c + d*Sec[e + f*x]]
)
```

Rubi [A] time = 1.06961, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2962, 3971, 3859, 2807, 2805, 3975}

$$\frac{2(ac-bd)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{af(a+b)\sqrt{c+d\sec(e+fx)}}+\frac{2d\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{af\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a + b*Cos[e + f*x]),x]
```


```
[Out] (2*d*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[2, (e + f*x)/2, (2*c)/(c
 + d)]*Sqrt[g*Sec[e + f*x]])/(a*f*Sqrt[c + d*Sec[e + f*x]]) + (2*(a*c - b*d
)*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*b)/(a + b), (e + f*x)/2,
 (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/(a*(a + b)*f*Sqrt[c + d*Sec[e + f*x]]
)
```


Rule 2962

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
(c_))^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist
[g^m, Int[(g*Csc[e + f*x])^(p - m)*(b + a*Csc[e + f*x])^m*(c + d*Csc[e + f*
x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[m]
```

Rule 3971

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = \frac{\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{b+a\sec(e+fx)}\,dx}{g}$$

$$= \frac{d\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx}{ag} + \frac{(ac-bd)\int \frac{(g\sec(e+fx))^{3/2}}{(b+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx}{ag}$$

$$= \frac{\left(d\sqrt{d+c\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{d+c\cos(e+fx)}}\,dx}{a\sqrt{c+d\sec(e+fx)}} + \frac{\left((ac-bd)\sqrt{d+\cdots\right.}{\cdots}$$

$$= \frac{\left(d\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}\,dx}{a\sqrt{c+d\sec(e+fx)}} + \frac{\left((ac-bd)\sqrt{\frac{d+c\cos(\cdots}{c+d}}\cdots\right.}{\cdots}$$

$$= \frac{2d\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{af\sqrt{c+d\sec(e+fx)}} + \frac{2(ac-bd)\sqrt{\frac{d+c\cos(\cdots}{c+\cdots}}\cdots}{a(\cdots}$$

Mathematica [C] time = 3.94151, size = 222, normalized size = 1.32

$$-\frac{2i\cot(e+fx)\sqrt{g\sec(e+fx)}\sqrt{-\frac{c(\cos(e+fx)-1)}{c+d}}\sqrt{\frac{c(\cos(e+fx)+1)}{c-d}}\sqrt{c+d\sec(e+fx)}\left(\Pi\left(1-\frac{c}{d};i\sinh^{-1}\left(\sqrt{\frac{1}{c-d}}\sqrt{d+c\cos(\cdots\right.\right.\right.}{af\sqrt{\frac{1}{c-d}}\sqrt{c\cos(e+fx)+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a + b*Cos[e + f*
x]),x]
```

```
[Out] ((-2*I)*Sqrt[-((c*(-1 + Cos[e + f*x]))/(c + d))]*Sqrt[(c*(1 + Cos[e + f*x])
)/(c - d)]*Cot[e + f*x]*(EllipticPi[1 - c/d, I*ArcSinh[Sqrt[(c - d)^(-1)]*S
qrt[d + c*Cos[e + f*x]]], (-c + d)/(c + d)] - EllipticPi[(b*(-c + d))/(-(a*
c) + b*d), I*ArcSinh[Sqrt[(c - d)^(-1)]*Sqrt[d + c*Cos[e + f*x]]], (-c + d)
```

```
/(c + d)])*Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a*Sqrt[(c - d)^(
-1)]*f*Sqrt[d + c*Cos[e + f*x]])
```

Maple [C] time = 0.397, size = 479, normalized size = 2.9

$$\frac{-2i\cos\left(fx+e\right)\left(\sin\left(fx+e\right)\right)^{2}}{af\left(a+b\right)\left(a-b\right)\left(-1+\cos\left(fx+e\right)\right)\left(d+c\cos\left(fx+e\right)\right)}\left(\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{c-d}{c+d}}\right)a^{2}c-\mathrm{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x)
```

```
[Out] -2*I/f/a/(a+b)/(a-b)*(EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))
^(1/2))*a^2*c-EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*
a^2*d+EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*a*b*c-El
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*a*b*d+2*Elliptic
Pi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((c-d)/(c+d))^(1/2))*a^2*d-2*EllipticP
i(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((c-d)/(c+d))^(1/2))*b^2*d-2*EllipticPi
(I*(-1+cos(f*x+e))/sin(f*x+e),(a-b)/(a+b),I*((c-d)/(c+d))^(1/2))*a*b*c+2*El
lipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),(a-b)/(a+b),I*((c-d)/(c+d))^(1/2))*b^
2*d)*cos(f*x+e)*sin(f*x+e)^2*(g/cos(f*x+e))^(1/2)*((d+c*cos(f*x+e))/cos(f*x
+e))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)
))^(1/2)/(-1+cos(f*x+e))/(d+c*cos(f*x+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sec\left(fx+e\right)+c}\sqrt{g\sec\left(fx+e\right)}}{b\cos\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sqrt(g*sec(f*x + e))/(b*cos(f*x + e) + a
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{g\sec\left(e+fx\right)}\sqrt{c+d\sec\left(e+fx\right)}}{a+b\cos\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e))**(1/2)/(a+b*cos(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*sec(e + f*x))*sqrt(c + d*sec(e + f*x))/(a + b*cos(e + f*x))
, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sec\left(fx+e\right)+c}\sqrt{g\sec\left(fx+e\right)}}{b\cos\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sqrt(g*sec(f*x + e))/(b*cos(f*x + e) + a
), x)
```

3.273 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

Optimal. Leaf size=95

$$\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}E\left(\sin^{-1}\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}$$

```
[Out] (EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(
1 + Sec[e + f*x])^(-1)]*Sqrt[a + b*Sec[e + f*x]])/(c*f*Sqrt[(a + b*Sec[e +
f*x])/((a + b)*(1 + Sec[e + f*x]))])
```

Rubi [A] time = 0.119371, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {3968}

$$\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}E\left(\sin^{-1}\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e + f*x]),x]
```

```
[Out] (EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(
1 + Sec[e + f*x])^(-1)]*Sqrt[a + b*Sec[e + f*x]])/(c*f*Sqrt[(a + b*Sec[e +
f*x])/((a + b)*(1 + Sec[e + f*x]))])
```

Rule 3968

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> -Simp[(Sqrt[a + b*Csc[e
+ f*x]]*Sqrt[c/(c + d*Csc[e + f*x])]*EllipticE[ArcSin[(c*Cot[e + f*x])/(c +
 d*Csc[e + f*x])], -((b*c - a*d)/(b*c + a*d))])/(d*f*Sqrt[(c*d*(a + b*Csc[e
 + f*x]))/((b*c + a*d)*(c + d*Csc[e + f*x]))]), x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{E\left(\sin^{-1}\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{a+b\sec(e+fx)}}{cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(1+\sec(e+fx))}}}$$

Mathematica [B] time = 6.32919, size = 264, normalized size = 2.78

$$\frac{\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sqrt{\sec(e+fx)+1}\sec^4\left(\frac{1}{2}(e+fx)\right)E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)}{\left(\frac{1}{\cos(e+fx)+1}\right)^{3/2}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}} + \frac{\left(\sin\left(\frac{3}{2}(e+f\right.\right.}{}\right.}{4cf(\sec(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e + f*x]),x]
```

```
[Out] (Cos[(e + f*x)/2]^2*Sqrt[Sec[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*((2*Sqrt[Co
s[e + f*x]/(1 + Cos[e + f*x])]*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/
(a + b)]*Sec[(e + f*x)/2]^4*Sqrt[1 + Sec[e + f*x]])/(((1 + Cos[e + f*x])^(-
1))^(3/2)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]) + (Sec[(
e + f*x)/2]^5*Sqrt[1 + Sec[e + f*x]]*(-Sin[(e + f*x)/2] + Sin[(3*(e + f*x))
/2]))/((1 + Cos[e + f*x])^(-1))^(3/2) - 8*Sqrt[Sec[e + f*x]]*(Sin[e + f*x]
- Tan[(e + f*x)/2])))/(4*c*f*(1 + Sec[e + f*x]))
```

Maple [A] time = 0.306, size = 153, normalized size = 1.6

$$-\frac{(-a-b)\left(-1+\cos\left(fx+e\right)\right)\left(1+\cos\left(fx+e\right)\right)^{2}}{fc\left(a\cos\left(fx+e\right)+b\right)\left(\sin\left(fx+e\right)\right)^{2}}\mathrm{EllipticE}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)},\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{a\cos\left(fx+e\right)+b}{(a+b)\left(1+\cos\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x)
```

```
[Out] -1/c/f*(-a-b)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*(1/
(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1
/2)*(-1+cos(f*x+e))*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(1+cos(f*x+e))^2/
(a*cos(f*x+e)+b)/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{c\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(c*sec(f*x + e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{c\sec\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(c*sec(f*x + e) + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a+b\sec\left(e+fx\right)}\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/(sec(e + f*x) + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{c\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(c*sec(f*x + e) + c), x)
```

3.274 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

Optimal. Leaf size=295

$$\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g\sin(e+fx)\sqrt{g\sec(e+fx)}(a\cos(e+fx)+b)}{f(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}}+\frac{g}{}$$

```
[Out] (g*(b + a*Cos[e + f*x])*EllipticE[(e + f*x)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e
+ f*x]])/(c*f*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[a + b*Sec[e + f*x]])
+ ((a - b)*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e + f*x)/2, (2*a
)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + (2*b*g*Sq
rt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/(a + b)]*
Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) - (g*(b + a*Cos[e + f*
x])*Sqrt[g*Sec[e + f*x]]*Sin[e + f*x])/(f*(c + c*Cos[e + f*x])*Sqrt[a + b*S
ec[e + f*x]])
```

Rubi [A] time = 0.9223, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.282, Rules used = {3971, 3859, 2807, 2805, 3975, 2768, 2752, 2663, 2661, 2655, 2653}

$$-\frac{g\sin(e+fx)\sqrt{g\sec(e+fx)}(a\cos(e+fx)+b)}{f(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}}+\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}F\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}+\frac{g\sqrt{g\sec}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e + f*x]),
x]
```


```
[Out] (g*(b + a*Cos[e + f*x])*EllipticE[(e + f*x)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e
+ f*x]])/(c*f*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[a + b*Sec[e + f*x]])
+ ((a - b)*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e + f*x)/2, (2*a
)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + (2*b*g*Sq
rt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/(a + b)]*
Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) - (g*(b + a*Cos[e + f*
x])*Sqrt[g*Sec[e + f*x]]*Sin[e + f*x])/(f*(c + c*Cos[e + f*x])*Sqrt[a + b*S
ec[e + f*x]])
```


Rule 3971

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = -\left((-a+b)\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx\right) + \frac{b\int \frac{(g\sec(e+fx}{\sqrt{a+b\sec(e}}}{c}$$

$$= -\frac{\left((-a+b)g\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{\sqrt{b+a\cos(e+fx)}(c+c\cos(e+f}}{\sqrt{a+b\sec(e+fx)}}$$

$$= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} + \frac{\left(a(-a+b)g\sqrt{b+a\,c\right.}{(}$$

$$= \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} - \frac{g(b+a\cos(e+}{f(c+c\cos(}$$

$$= \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} - \frac{g(b+a\cos(e+}{f(c+c\cos(}$$

$$= \frac{g(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}} + \frac{(a-b)g\sqrt{\frac{b+a\,co}{a}}}{}$$

Mathematica [F] time = 18.9168, size = 0, normalized size = 0.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e +
f*x]),x]
```

```
[Out] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e +
f*x]), x]
```

Maple [C] time = 0.355, size = 292, normalized size = 1.

$$\frac{i\left(\cos\left(fx+e\right)\right)^2}{fc\left(a\cos\left(fx+e\right)+b\right)}\sqrt{\frac{a\cos\left(fx+e\right)+b}{(a+b)\left(1+\cos\left(fx+e\right)\right)}}\left(2\,a\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{a-b}{a+b}}\right)-2\,b\mathrm{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x)
```

```
[Out] I/c/f*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(2*a*EllipticF(I*(-1+
cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-2*b*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-a*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e)
,(-(a-b)/(a+b))^(1/2))-b*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+
b))^(1/2))+4*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((a-b)/(a+b))^(1/
2))*b)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(g/cos(f*x+e))^(3/2)*cos(f*x+e
)^2/(a*cos(f*x+e)+b)/(1/(1+cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{c\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(c*sec(f*x + e) +
 c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)*(a+b*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{c\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(c*sec(f*x + e) +
 c), x)
```

3.275 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$

Optimal. Leaf size=209

$$\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}E\left(\sin^{-1}\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf(a-b)\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{Elliptic}}{cf(a-b)}$$

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[e + f*x]))/(a - b))])/((a - b)*c*f) + (EllipticE[ArcSin[Tan[e + f*x
]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[
a + b*Sec[e + f*x]])/((a - b)*c*f*Sqrt[(a + b*Sec[e + f*x])/((a + b)*(1 + S
ec[e + f*x]))])
```

Rubi [A] time = 0.28782, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3972, 3832, 3968}

$$\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}E\left(\sin^{-1}\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf(a-b)\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\right.}{cf(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```


```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[e + f*x]))/(a - b))])/((a - b)*c*f) + (EllipticE[ArcSin[Tan[e + f*x
]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[
a + b*Sec[e + f*x]])/((a - b)*c*f*Sqrt[(a + b*Sec[e + f*x])/((a + b)*(1 + S
ec[e + f*x]))])
```


Rule 3972

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[
Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[d/(b*c - a*d), Int[(Cs
c[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^
2 - d^2, 0])
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3968

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> -Simp[(Sqrt[a + b*Csc[e
+ f*x]]*Sqrt[c/(c + d*Csc[e + f*x])]*EllipticE[ArcSin[(c*Cot[e + f*x])/(c +
 d*Csc[e + f*x])], -((b*c - a*d)/(b*c + a*d))])/(d*f*Sqrt[(c*d*(a + b*Csc[e
 + f*x]))/((b*c + a*d)*(c + d*Csc[e + f*x]))]), x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = -\frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{(a-b)c} - \frac{c\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx}{-ac+bc}$$

$$= -\frac{2\sqrt{a+b}\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(}{}}}{(a-b)cf}$$

Mathematica [B] time = 17.7217, size = 2173, normalized size = 10.4

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

```
[Out] (Cos[e/2 + (f*x)/2]^2*(b + a*Cos[e + f*x])*Sec[e + f*x]^2*((2*Sin[e + f*x])
/(-a + b) - (2*Tan[(e + f*x)/2])/(-a + b)))/(f*Sqrt[a + b*Sec[e + f*x]]*(c
+ c*Sec[e + f*x])) - (2*Cos[e/2 + (f*x)/2]^2*(-(b/((-a + b)*Sqrt[b + a*Cos[
e + f*x]]*Sqrt[Sec[e + f*x]])) - (a*Sqrt[Sec[e + f*x]])/((-a + b)*Sqrt[b +
a*Cos[e + f*x]]) + (b*Sqrt[Sec[e + f*x]])/((-a + b)*Sqrt[b + a*Cos[e + f*x]
]) - (a*Cos[2*(e + f*x)]*Sqrt[Sec[e + f*x]])/((-a + b)*Sqrt[b + a*Cos[e + f
*x]]))*Sec[e + f*x]^(3/2)*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*((a - b)*El
lipticE[ArcSin[Sqrt[(a - b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sq
rt[((b + a*Cos[e + f*x])*Sec[(e + f*x)/2]^2)/(a + b)] + Sqrt[2]*Sqrt[(a - b
)/(a + b)]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*(b + a*Cos[e + f*x])*Tan[(
e + f*x)/2])*(-1 + Tan[(e + f*x)/2]^2))/(((a - b)/(a + b))^(3/2)*(a + b)*f*
Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e
 + f*x])*((-2*Sec[(e + f*x)/2]^2*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Tan[
(e + f*x)/2]*((a - b)*EllipticE[ArcSin[Sqrt[(a - b)/(a + b)]*Tan[(e + f*x)/
2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[e + f*x])*Sec[(e + f*x)/2]^2)/(a + b
)] + Sqrt[2]*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*(b
 + a*Cos[e + f*x])*Tan[(e + f*x)/2]))/(((a - b)/(a + b))^(3/2)*(a + b)*Sqrt
[b + a*Cos[e + f*x]]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]) - (a*Sqrt[Cos[(
e + f*x)/2]^2*Sec[e + f*x]]*Sin[e + f*x]*((a - b)*EllipticE[ArcSin[Sqrt[(a
- b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[e + f*x]
)*Sec[(e + f*x)/2]^2)/(a + b)] + Sqrt[2]*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[e +
 f*x]/(1 + Cos[e + f*x])]*(b + a*Cos[e + f*x])*Tan[(e + f*x)/2])*(-1 + Tan[
(e + f*x)/2]^2))/(((a - b)/(a + b))^(3/2)*(a + b)*(b + a*Cos[e + f*x])^(3/2
)*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]) + (Sqrt[Cos[(e + f*x)/2]^2*Sec[e +
 f*x]]*((a - b)*EllipticE[ArcSin[Sqrt[(a - b)/(a + b)]*Tan[(e + f*x)/2]], (
a + b)/(a - b)]*Sqrt[((b + a*Cos[e + f*x])*Sec[(e + f*x)/2]^2)/(a + b)] + S
qrt[2]*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*(b + a*C
os[e + f*x])*Tan[(e + f*x)/2])*(-(Sec[(e + f*x)/2]^4*Sin[e + f*x]) + 2*Cos[
e + f*x]*Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2])*(-1 + Tan[(e + f*x)/2]^2))/((
(a - b)/(a + b))^(3/2)*(a + b)*Sqrt[b + a*Cos[e + f*x]]*(Cos[e + f*x]*Sec[(
e + f*x)/2]^4)^(3/2)) - (2*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*(-1 + Tan[
```

```
(e + f*x)/2]^2)*((Sqrt[(a - b)/(a + b)]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x]
)]*(b + a*Cos[e + f*x])*Sec[(e + f*x)/2]^2)/Sqrt[2] - Sqrt[2]*a*Sqrt[(a - b
)/(a + b)]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sin[e + f*x]*Tan[(e + f*x)
/2] + (Sqrt[(a - b)/(a + b)]*(b + a*Cos[e + f*x])*((Cos[e + f*x]*Sin[e + f*
x])/(1 + Cos[e + f*x])^2 - Sin[e + f*x]/(1 + Cos[e + f*x]))*Tan[(e + f*x)/2
])/(Sqrt[2]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]) + ((a - b)*EllipticE[Arc
Sin[Sqrt[(a - b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*(-((a*Sec[(e
+ f*x)/2]^2*Sin[e + f*x])/(a + b)) + ((b + a*Cos[e + f*x])*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2])/(a + b)))/(2*Sqrt[((b + a*Cos[e + f*x])*Sec[(e + f*x)/
2]^2)/(a + b)]) + ((a - b)*Sqrt[(a - b)/(a + b)]*Sec[(e + f*x)/2]^2*Sqrt[((
b + a*Cos[e + f*x])*Sec[(e + f*x)/2]^2)/(a + b)]*Sqrt[1 - Tan[(e + f*x)/2]^
2])/(2*Sqrt[1 - ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])))/(((a - b)/(a + b))
^(3/2)*(a + b)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^
4]) - (((a - b)*EllipticE[ArcSin[Sqrt[(a - b)/(a + b)]*Tan[(e + f*x)/2]], (
a + b)/(a - b)]*Sqrt[((b + a*Cos[e + f*x])*Sec[(e + f*x)/2]^2)/(a + b)] + S
qrt[2]*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*(b + a*C
os[e + f*x])*Tan[(e + f*x)/2])*(-1 + Tan[(e + f*x)/2]^2)*(-(Cos[(e + f*x)/2
]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e +
f*x]))/(((a - b)/(a + b))^(3/2)*(a + b)*Sqrt[b + a*Cos[e + f*x]]*Sqrt[Cos[e
 + f*x]*Sec[(e + f*x)/2]^4]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]))
```

Maple [A] time = 0.293, size = 225, normalized size = 1.1

$$-\frac{\left(1+\cos\left(fx+e\right)\right)^{2}\left(-1+\cos\left(fx+e\right)\right)}{fc\left(a-b\right)\left(a\cos\left(fx+e\right)+b\right)\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\left(a+b\right)\left(1+\cos\left(fx+e\right)\right)}}\left(2\,\mathrm{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/c/f/(a-b)*(1+cos(f*x+e))^2*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
(-1+cos(f*x+e))*(2*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2)
)*b-a*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))-b*EllipticE
((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2)))/(a*cos(f*x+e)+b)/sin(f*x+
e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{bc\sec\left(fx+e\right)^{2}+(a+b)c\sec\left(fx+e\right)+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(b*c*sec(f*x + e)^2 + (a + b
)*c*sec(f*x + e) + a*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}\sec\left(e+fx\right)+\sqrt{a+b\sec\left(e+fx\right)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a + b*sec(e + f*x))*sec(e + f*x) + sqrt(a + b*s
ec(e + f*x))), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x)
```

3.276 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$

Optimal. Leaf size=214

$$\frac{2a\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bcf(a-b)}-\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+ \dots}}{cf(a-b)\sqrt{\dots}}$$

```
[Out] (2*a*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/((a - b)*b*c*f) - (EllipticE[ArcSin[Tan[e +
f*x]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sq
rt[a + b*Sec[e + f*x]])/((a - b)*c*f*Sqrt[(a + b*Sec[e + f*x])/((a + b)*(1
+ Sec[e + f*x]))])
```

Rubi [A] time = 0.360072, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {3976, 3832, 3968}

$$\frac{2a\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bcf(a-b)}-\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}E\left(\text{si}\dots\right.}{cf(a-b)\sqrt{\frac{a+b\sec(e\dots}{(a+b)(\sec(e\dots}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

```
[Out] (2*a*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/((a - b)*b*c*f) - (EllipticE[ArcSin[Tan[e +
f*x]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sq
rt[a + b*Sec[e + f*x]])/((a - b)*c*f*Sqrt[(a + b*Sec[e + f*x])/((a + b)*(1
+ Sec[e + f*x]))])
```

Rule 3976

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(
csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> -Dist[a/(b*c - a*d), I
nt[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[c/(b*c - a*d), Int[
(Csc[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(c + d*Csc[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ
[c^2 - d^2, 0])
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3968

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> -Simp[(Sqrt[a + b*Csc[e
+ f*x]]*Sqrt[c/(c + d*Csc[e + f*x])]*EllipticE[ArcSin[(c*Cot[e + f*x])/(c +
 d*Csc[e + f*x])], -((b*c - a*d)/(b*c + a*d))])/(d*f*Sqrt[(c*d*(a + b*Csc[e
 + f*x]))/((b*c + a*d)*(c + d*Csc[e + f*x]))]), x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \frac{a\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{(a-b)c} + \frac{c\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx}{-ac+bc}$$

$$= \frac{2a\sqrt{a+b}\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(}{}}}{(a-b)bcf}$$

Mathematica [A] time = 5.11181, size = 156, normalized size = 0.73

$$\frac{4\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\left((a+b)E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)-2a\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\right.\right.}{cf(b-a)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}(\cos(e+fx)+1)^2\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^4*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x])
)]*((a + b)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*a*Elli
pticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))/((-a + b)*c*f*Sqrt[Cos[e
 + f*x]/(1 + Cos[e + f*x])]*(1 + Cos[e + f*x])^2*Sqrt[a + b*Sec[e + f*x]])
```

Maple [A] time = 0.312, size = 224, normalized size = 1.1

$$\frac{\left(1+\cos\left(fx+e\right)\right)^2\left(-1+\cos\left(fx+e\right)\right)}{fc\left(a-b\right)\left(a\cos\left(fx+e\right)+b\right)\left(\sin\left(fx+e\right)\right)^2}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\left(a+b\right)\left(1+\cos\left(fx+e\right)\right)}}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/c/f/(a-b)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))^2*(
-1+cos(f*x+e))*(2*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))
*a-a*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))-b*EllipticE(
(-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2)))/(a*cos(f*x+e)+b)/sin(f*x+e
)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)^2}{bc\sec\left(fx+e\right)^2+(a+b)c\sec\left(fx+e\right)+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)^2/(b*c*sec(f*x + e)^2 + (a +
 b)*c*sec(f*x + e) + a*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^2\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}\sec\left(e+fx\right)+\sqrt{a+b\sec\left(e+fx\right)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a + b*sec(e + f*x))*sec(e + f*x) + sqrt(a +
b*sec(e + f*x))), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x
)
```

3.277 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$

Optimal. Leaf size=229

$$\frac{g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g\sin(e+fx)\sqrt{g\sec(e+fx)}(a\cos(e+fx)+b)}{f(a-b)(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}}+\frac{g\sqrt{g\sec(e\cdots}}{cf(a\cdots}$$

```
[Out] (g*(b + a*Cos[e + f*x])*EllipticE[(e + f*x)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e
+ f*x]])/((a - b)*c*f*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[a + b*Sec[e +
 f*x]]) + (g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e + f*x)/2, (2*a
)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) - (g*(b + a
*Cos[e + f*x])*Sqrt[g*Sec[e + f*x]]*Sin[e + f*x])/((a - b)*f*(c + c*Cos[e +
 f*x])*Sqrt[a + b*Sec[e + f*x]])
```

Rubi [A] time = 0.439518, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18, Rules used = {3975, 2768, 2752, 2663, 2661, 2655, 2653}

$$-\frac{g\sin(e+fx)\sqrt{g\sec(e+fx)}(a\cos(e+fx)+b)}{f(a-b)(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}}+\frac{g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}F\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}+\frac{g\sqrt{g\sec(e+fx)}(a\cdots}{cf(a-b)\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),
x]
```

```
[Out] (g*(b + a*Cos[e + f*x])*EllipticE[(e + f*x)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e
+ f*x]])/((a - b)*c*f*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[a + b*Sec[e +
 f*x]]) + (g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e + f*x)/2, (2*a
)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) - (g*(b + a
*Cos[e + f*x])*Sqrt[g*Sec[e + f*x]]*Sin[e + f*x])/((a - b)*f*(c + c*Cos[e +
 f*x])*Sqrt[a + b*Sec[e + f*x]])
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
```

```
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \frac{\left(g\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{\sqrt{b+a\cos(e+fx)}(c+c\cos(e+fx))}\,dx}{\sqrt{a+b\sec(e+fx)}}$$

$$= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} - \frac{\left(ag\sqrt{b+a\cos(e+\right.}{(a-}$$

$$= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} + \frac{\left(g\sqrt{b+a\cos(e+f\right.}{2c}$$

$$= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} + \frac{\left(g(b+a\cos(e+fx\right.}{2(a-b)c}$$

$$= \frac{g(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(a-b)cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}} + \frac{g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}}{c}$$

Mathematica [C] time = 8.38203, size = 1019, normalized size = 4.45

$$\frac{(b+a\cos(e+fx))(g\sec(e+fx))^{3/2}\left(\frac{2\csc(e)}{(b-a)f}+\frac{2\sec\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}+\frac{fx}{2}\right)\sin\left(\frac{fx}{2}\right)}{(b-a)f}\right)\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{\sqrt{a+b\sec(e+fx)}(\sec(e+fx)c+c)}+\frac{a\sqrt{b+a\cos(e+fx)}\csc\left(\frac{e}{2}\right)\sec\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f
*x])),x]
```

```
[Out] (Cos[e/2 + (f*x)/2]^2*(b + a*Cos[e + f*x])*(g*Sec[e + f*x])^(3/2)*((2*Csc[e
])/((-a + b)*f) + (2*Sec[e/2]*Sec[e/2 + (f*x)/2]*Sin[(f*x)/2])/((-a + b)*f)
))/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])) + (AppellF1[1/2, 1/2, 1/
2, 3/2, (Csc[e]*(b - a*Sqrt[1 + Cot[e]^2]*Sin[e]*Sin[f*x - ArcTan[Cot[e]]])
)/(a*Sqrt[1 + Cot[e]^2]*(1 + (b*Csc[e])/(a*Sqrt[1 + Cot[e]^2]))), (Csc[e]*(
b - a*Sqrt[1 + Cot[e]^2]*Sin[e]*Sin[f*x - ArcTan[Cot[e]]]))/(a*Sqrt[1 + Cot
[e]^2]*(-1 + (b*Csc[e])/(a*Sqrt[1 + Cot[e]^2])))]*Cos[e/2 + (f*x)/2]^2*Sqrt
[b + a*Cos[e + f*x]]*Csc[e/2]*Sec[e/2]*(g*Sec[e + f*x])^(3/2)*Sec[f*x - Arc
Tan[Cot[e]]]*Sqrt[(a*Sqrt[1 + Cot[e]^2] - a*Sqrt[1 + Cot[e]^2]*Sin[f*x - Ar
cTan[Cot[e]]])/(a*Sqrt[1 + Cot[e]^2] - b*Csc[e])]*Sqrt[(a*Sqrt[1 + Cot[e]^2
] + a*Sqrt[1 + Cot[e]^2]*Sin[f*x - ArcTan[Cot[e]]])/(a*Sqrt[1 + Cot[e]^2] +
 b*Csc[e])]*Sqrt[b - a*Sqrt[1 + Cot[e]^2]*Sin[e]*Sin[f*x - ArcTan[Cot[e]]]]
)/((-a + b)*f*Sqrt[1 + Cot[e]^2]*Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*
x])) + (a*Cos[e/2 + (f*x)/2]^2*Sqrt[b + a*Cos[e + f*x]]*Csc[e/2]*Sec[e/2]*(
g*Sec[e + f*x])^(3/2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[e]*(b + a*Co
s[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(a*Sqrt[1 + Tan[e]^2]*(
1 - (b*Sec[e])/(a*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(b + a*Cos[e]*Cos[f*x +
 ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(a*Sqrt[1 + Tan[e]^2]*(-1 - (b*Sec[e]
)/(a*Sqrt[1 + Tan[e]^2]))))]*Sin[f*x + ArcTan[Tan[e]]]*Tan[e])/(Sqrt[1 + Ta
n[e]^2]*Sqrt[(a*Sqrt[1 + Tan[e]^2] - a*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + T
an[e]^2])/(b*Sec[e] + a*Sqrt[1 + Tan[e]^2])]*Sqrt[(a*Sqrt[1 + Tan[e]^2] + a
*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(b*Sec[e]) + a*Sqrt[1 + Ta
n[e]^2])]*Sqrt[b + a*Cos[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])
- ((Sin[f*x + ArcTan[Tan[e]]]*Tan[e])/Sqrt[1 + Tan[e]^2] + (2*a*Cos[e]*(b +
 a*Cos[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(a^2*Cos[e]^2 + a^
2*Sin[e]^2))/Sqrt[b + a*Cos[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]
]))/(2*(-a + b)*f*Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x]))
```

Maple [C] time = 0.332, size = 222, normalized size = 1.

$$\frac{i\left(\cos\left(fx+e\right)\right)^{2}}{fc\left(a-b\right)\left(a\cos\left(fx+e\right)+b\right)}\left(2\,a\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{a-b}{a+b}}\right)-a\mathrm{EllipticE}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] I/c/f/(a-b)*(2*a*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2
))-a*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-b*Ellipti
```

```
cE(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2)))*(1/(a+b)*(a*cos(f*x+
e)+b)/(1+cos(f*x+e)))^(1/2)*(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2*(1/cos(f*x+e)
*(a*cos(f*x+e)+b))^(1/2)/(a*cos(f*x+e)+b)/(1/(1+cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
+ c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\sqrt{g\sec\left(fx+e\right)}g\sec\left(fx+e\right)}{bc\sec\left(fx+e\right)^{2}+(a+b)c\sec\left(fx+e\right)+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sqrt(g*sec(f*x + e))*g*sec(f*x + e)/(b*c*
sec(f*x + e)^2 + (a + b)*c*sec(f*x + e) + a*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\left(g\sec\left(e+fx\right)\right)^{\frac{3}{2}}}{\sqrt{a+b\sec\left(e+fx\right)}\sec\left(e+fx\right)+\sqrt{a+b\sec\left(e+fx\right)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)/(sqrt(a + b*sec(e + f*x))*sec(e + f*x) + s
qrt(a + b*sec(e + f*x))), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)+a}\left(c\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
+ c)), x)
```

3.278 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$

Optimal. Leaf size=312

$$-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}+\frac{g^2\sin(e+fx)\sqrt{g\sec(e+fx)}(a\cos(e+fx)+b)}{f(a-b)(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{\cdots}}{\cdots}$$

```
[Out] -((g^2*(b + a*Cos[e + f*x])*EllipticE[(e + f*x)/2, (2*a)/(a + b)]*Sqrt[g*Se
c[e + f*x]])/((a - b)*c*f*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[a + b*Sec
[e + f*x]])) - (g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e + f*x)/
2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + (2
*g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/(a
 + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + (g^2*(b + a*C
os[e + f*x])*Sqrt[g*Sec[e + f*x]]*Sin[e + f*x])/((a - b)*f*(c + c*Cos[e + f
*x])*Sqrt[a + b*Sec[e + f*x]])
```

Rubi [A] time = 0.88935, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.282, Rules used = {3979, 3859, 2807, 2805, 3975, 2768, 2752, 2663, 2661, 2655, 2653}

$$\frac{g^2\sin(e+fx)\sqrt{g\sec(e+fx)}(a\cos(e+fx)+b)}{f(a-b)(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}F\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+f\cdots}}{cf(a-b)\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),
x]
```

```
[Out] -((g^2*(b + a*Cos[e + f*x])*EllipticE[(e + f*x)/2, (2*a)/(a + b)]*Sqrt[g*Se
c[e + f*x]])/((a - b)*c*f*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[a + b*Sec
[e + f*x]])) - (g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e + f*x)/
2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + (2
*g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/(a
 + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + (g^2*(b + a*C
os[e + f*x])*Sqrt[g*Sec[e + f*x]]*Sin[e + f*x])/((a - b)*f*(c + c*Cos[e + f
*x])*Sqrt[a + b*Sec[e + f*x]])
```

Rule 3979

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(5/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(c*g)/d,
 Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = -\left(g\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx\right)+\frac{g\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,d}{c}$$

$$= -\frac{\left(g^2\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{\sqrt{b+a\cos(e+fx)}(c+c\cos(e+fx))}\,dx}{\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{g^2(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}}+\frac{\left(ag^2\sqrt{b+a\cos(e+\right.}{(a-}$$

$$= \frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}}+\frac{g^2(b+a\cos(e}{(a-b)f(c+c}$$

$$= \frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}}+\frac{g^2(b+a\cos(e}{(a-b)f(c+c}$$

$$= -\frac{g^2(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(a-b)cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{\frac{b+a\cos(e}{a+b}}}{}$$

Mathematica [F] time = 14.6012, size = 0, normalized size = 0.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f
*x])),x]
```

```
[Out] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f
*x])), x]
```

Maple [C] time = 0.336, size = 355, normalized size = 1.1

$$\frac{i\left(-1+\cos\left(fx+e\right)\right)\left(\cos\left(fx+e\right)\right)^3}{fc\,(a-b)\left(a\cos\left(fx+e\right)+b\right)\left(\sin\left(fx+e\right)\right)^2}\left(4\,a\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{a-b}{a+b}}\right)-2\,b\mathrm{EllipticF}\left(\frac{i\left(-1\right.}{\mathrm{si}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] I/c/f/(a-b)*(4*a*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2
))-2*b*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-a*Ellip
ticE(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-b*EllipticE(I*(-1+c
os(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-4*EllipticPi(I*(-1+cos(f*x+e))/
sin(f*x+e),-1,I*((a-b)/(a+b))^(1/2))*a+4*EllipticPi(I*(-1+cos(f*x+e))/sin(f
*x+e),-1,I*((a-b)/(a+b))^(1/2))*b)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e))
)^(1/2)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(-1+cos(f*x+e))*(g/cos(f*x+e)
)^(5/2)*cos(f*x+e)^3/(a*cos(f*x+e)+b)/(1/(1+cos(f*x+e)))^(3/2)/sin(f*x+e)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g \sec\left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{b \sec\left(fx+e\right)+a}\left(c \sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
+ c)), x)
```

3.279 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=213

$$\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{df}-\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+bs}{}}}{df(c+d)\sqrt{-\tan}}$$

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*(b*c - a*d)*EllipticPi[(2*d)/(c + d
), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e
 + f*x])/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[
-Tan[e + f*x]^2])
```

Rubi [A] time = 0.277011, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3969, 3832, 3973}

$$\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{df}-\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\Pi}{df(c+d)\sqrt{-\tan^2(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*(b*c - a*d)*EllipticPi[(2*d)/(c + d
), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e
 + f*x])/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[
-Tan[e + f*x]^2])
```

Rule 3969

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d, Int[Csc[e + f*
x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/d, Int[Csc[e + f*x]/
(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3973

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[(-2*Cot[e + f*x]*Sq
rt[(a + b*Csc[e + f*x])/(a + b)]*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 -
```

```
Csc[e + f*x]]/Sqrt[2]], (2*b)/(a + b)])/(f*(c + d)*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[-Cot[e + f*x]^2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx}{d}$$
$$= \frac{2\sqrt{a+b}\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+f}{a-b}}}{df}$$

Mathematica [A] time = 0.391206, size = 185, normalized size = 0.87

$$\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\sqrt{a+b\sec(e+fx)}\left((a-b)(c+d)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right.\right.\right.}{f(c-d)(c+d)(a\cos(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)*(c + d)*EllipticF[ArcSin[
Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*(-(b*c) + a*d)*EllipticPi[(c - d)/(
c + d), -ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b*Sec[e + f*x
]])/((c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Maple [A] time = 0.343, size = 355, normalized size = 1.7

$$2\,\frac{\left(1+\cos\left(fx+e\right)\right)^{2}\left(-1+\cos\left(fx+e\right)\right)}{f\left(c+d\right)\left(c-d\right)\left(a\cos\left(fx+e\right)+b\right)\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{(a+b)\left(1+\cos\left(fx+\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] 2/f/(c+d)/(c-d)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))
^2*(EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*c+EllipticF
((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*d-EllipticF((-1+cos(f*x+
e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b*c-EllipticF((-1+cos(f*x+e))/sin(f*x+e
),((a-b)/(a+b))^(1/2))*b*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c
+d),((a-b)/(a+b))^(1/2))*a*d+2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/
(c+d),((a-b)/(a+b))^(1/2))*b*c)*(-1+cos(f*x+e))/(a*cos(f*x+e)+b)/sin(f*x+e)
^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec\left(fx+e\right)+a} \sec\left(fx+e\right)}{d \sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b \sec\left(e+fx\right)} \sec\left(e+fx\right)}{c+d \sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec\left(fx+e\right)+a} \sec\left(fx+e\right)}{d \sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

3.280 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=170

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{2g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df(c+d)\sqrt{a+b\sec(e+fx)}}$$

```
[Out] (2*b*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/
(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*f*Sqrt[a + b*Sec[e + f*x]]) - (2*(b*c - a
*d)*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x
)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f
*x]])
```

Rubi [A] time = 0.842811, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3971, 3859, 2807, 2805, 3975}

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{2g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df(c+d)\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),
x]
```

```
[Out] (2*b*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/
(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*f*Sqrt[a + b*Sec[e + f*x]]) - (2*(b*c - a
*d)*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x
)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f
*x]])
```

Rule 3971

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{b\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{(bc-ad)\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx}{d}$$

$$= \frac{\left(bg\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{b+a\cos(e+fx)}}\,dx}{d\sqrt{a+b\sec(e+fx)}} - \frac{\left((bc-ad)g\sqrt{b\cdots\right.}{}$$

$$= \frac{\left(bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}\,dx}{d\sqrt{a+b\sec(e+fx)}} - \frac{\left((bc-ad)g\sqrt{\frac{b+a\cos\cdots}{a\cdots}}\right.}{}$$

$$= \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{df\sqrt{a+b\sec(e+fx)}} - \frac{2(bc-ad)g\sqrt{\frac{b+a\cos\cdots}{a\cdots}}}{d\cdots}$$

Mathematica [C] time = 3.82805, size = 223, normalized size = 1.31

$$-\frac{2ig\cot(e+fx)\sqrt{g\sec(e+fx)}\sqrt{-\frac{a(\cos(e+fx)-1)}{a+b}}\sqrt{\frac{a(\cos(e+fx)+1)}{a-b}}\sqrt{a+b\sec(e+fx)}\left(\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(e\cdots}\right.\right.\right.}{df\sqrt{\frac{1}{a-b}}\sqrt{a\cos(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e +
f*x]),x]
```

```
[Out] ((-2*I)*g*Sqrt[-((a*(-1 + Cos[e + f*x]))/(a + b))]*Sqrt[(a*(1 + Cos[e + f*x
]))/(a - b)]*Cot[e + f*x]*(EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]
*Sqrt[b + a*Cos[e + f*x]]], (-a + b)/(a + b)] - EllipticPi[((a - b)*c)/(-(b
*c) + a*d), I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[e + f*x]]], (-a + b
)/(a + b)])*Sqrt[g*Sec[e + f*x]]*Sqrt[a + b*Sec[e + f*x]])/(Sqrt[(a - b)^(-
1)]*d*f*Sqrt[b + a*Cos[e + f*x]])
```

Maple [C] time = 0.391, size = 465, normalized size = 2.7

$$\frac{-2i\left(\cos\left(fx+e\right)\right)^{2}}{fd\left(c+d\right)\left(c-d\right)\left(a\cos\left(fx+e\right)+b\right)}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\left(a+b\right)\left(1+\cos\left(fx+e\right)\right)}}\left(\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{a-b}{a+b}}\right)acd+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] -2*I/f/d/(c+d)/(c-d)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(Ellip
ticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))*a*c*d+EllipticF(I*(
-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))*a*d^2-EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))*b*c*d-EllipticF(I*(-1+cos(f*x+e))/s
in(f*x+e),(-(a-b)/(a+b))^(1/2))*b*d^2-2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*
x+e),-1,I*((a-b)/(a+b))^(1/2))*b*c^2+2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x
+e),-1,I*((a-b)/(a+b))^(1/2))*b*d^2-2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+
e),-(c-d)/(c+d),I*((a-b)/(a+b))^(1/2))*a*c*d+2*EllipticPi(I*(-1+cos(f*x+e))
/sin(f*x+e),-(c-d)/(c+d),I*((a-b)/(a+b))^(1/2))*b*c^2)*(1/cos(f*x+e)*(a*cos
(f*x+e)+b))^(1/2)*(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2/(a*cos(f*x+e)+b)/(1/(1+
cos(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(d*sec(f*x + e) +
 c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(fx+e\right)+a}\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{d\sec\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(d*sec(f*x + e) +
 c), x)
```

3.281 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=102

$$\frac{2\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\Pi\left(\frac{2d}{c+d};\sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{f(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$

```
[Out] (2*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/
(a + b)]*Sqrt[(a + b*Sec[e + f*x])/(a + b)]*Tan[e + f*x])/((c + d)*f*Sqrt[a
 + b*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])
```

Rubi [A] time = 0.121654, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {3973}

$$\frac{2\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\Pi\left(\frac{2d}{c+d};\sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{f(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/
(a + b)]*Sqrt[(a + b*Sec[e + f*x])/(a + b)]*Tan[e + f*x])/((c + d)*f*Sqrt[a
 + b*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])
```

Rule 3973

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[(-2*Cot[e + f*x]*Sq
rt[(a + b*Csc[e + f*x])/(a + b)]*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 -
Csc[e + f*x]]/Sqrt[2]], (2*b)/(a + b)])/(f*(c + d)*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[-Cot[e + f*x]^2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2\Pi\left(\frac{2d}{c+d};\sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

Mathematica [A] time = 4.31785, size = 189, normalized size = 1.85

$$\frac{2\sqrt{\sec(e+fx)}\sqrt{\sec(e+fx)+1}\sqrt{\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\left((c+d)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+\right.\right.\right.\right.}{f(c-d)(c+d)\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((c + d)*Ellipti
cF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*d*EllipticPi[(c - d)/(c +
 d), -ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[Cos[e + f*x]*Sec[(e
+ f*x)/2]^2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]])/((c - d)*(c + d)*f*
Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[a + b*Sec[e + f*x]])
```

Maple [B] time = 0.342, size = 236, normalized size = 2.3

$$2\,\frac{\left(1+\cos\left(fx+e\right)\right)^{2}\left(-1+\cos\left(fx+e\right)\right)}{f\left(c-d\right)\left(c+d\right)\left(a\cos\left(fx+e\right)+b\right)\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\left(a+b\right)\left(1+\cos\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f/(c-d)/(c+d)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))
^2*(-1+cos(f*x+e))*(EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2
))*c+EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*d-2*Elliptic
Pi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/2))*d)/(a*cos(f*
x+e)+b)/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

3.282 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=209

$$\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bdf}-\frac{2c\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\text{I}}{df(c+d)\sqrt{-\tan^2(e}}$$

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(b*d*f) - (2*c*EllipticPi[(2*d)/(c + d), ArcSi
n[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x])
/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[-Tan[e +
 f*x]^2])
```

Rubi [A] time = 0.361243, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {3977, 3832, 3973}

$$\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bdf}-\frac{2c\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\Pi\left(\frac{2d}{c+d};\text{si}\right.}{df(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(b*d*f) - (2*c*EllipticPi[(2*d)/(c + d), ArcSi
n[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x])
/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[-Tan[e +
 f*x]^2])
```

Rule 3977

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(
csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[1/d, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[c/d, Int[Csc[e + f*x]/(Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3973

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[(-2*Cot[e + f*x]*Sq
rt[(a + b*Csc[e + f*x])/(a + b)]*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 -
```

```
Csc[e + f*x]]/Sqrt[2]], (2*b)/(a + b)])/(f*(c + d)*Sqrt[a + b*Csc[e + f*x]]
*Sqrt[-Cot[e + f*x]^2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{c\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx}{d}$$

$$= \frac{2\sqrt{a+b}\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1}{}}}{bdf}$$

Mathematica [A] time = 3.00876, size = 167, normalized size = 0.8

$$-\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sec(e+fx)\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\left((c+d)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)+2c\right.}{f(c-d)(c+d)\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((c + d)*EllipticF[ArcSin[Tan[(e
+ f*x)/2]], (a - b)/(a + b)] + 2*c*EllipticPi[(c - d)/(c + d), -ArcSin[Tan[
(e + f*x)/2]], (a - b)/(a + b)])*Sec[e + f*x])/((c - d)*(c + d)*f*Sqrt[a +
b*Sec[e + f*x]])
```

Maple [A] time = 0.339, size = 236, normalized size = 1.1

$$-2\frac{\left(1+\cos\left(fx+e\right)\right)^2\left(-1+\cos\left(fx+e\right)\right)}{f\left(c+d\right)\left(c-d\right)\left(a\cos\left(fx+e\right)+b\right)\left(\sin\left(fx+e\right)\right)^2}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{a\cos\left(fx+e\right)+}{\left(a+b\right)\left(1+\cos\left(fx\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f/(c+d)/(c-d)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e)
)^2*(EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*c+EllipticF(
(-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*d-2*c*EllipticPi((-1+cos(f*
x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/2)))*(-1+cos(f*x+e))/(a*cos(f
*x+e)+b)/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x
)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}\left(c+d\sec\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x
)
```

3.283 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=83

$$\frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{f(c+d)\sqrt{a+b\sec(e+fx)}}$$

```
[Out] (2*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x)
/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/((c + d)*f*Sqrt[a + b*Sec[e + f*x]
])
```

Rubi [A] time = 0.424288, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3975, 2807, 2805}

$$\frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{f(c+d)\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] (2*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x)
/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/((c + d)*f*Sqrt[a + b*Sec[e + f*x]
])
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx = \frac{\left(g\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))}dx}{\sqrt{a+b\sec(e+fx)}}$$
$$= \frac{\left(g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}(d+c\cos(e+fx))}dx}{\sqrt{a+b\sec(e+fx)}}$$
$$= \frac{2g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(c+d)f\sqrt{a+b\sec(e+fx)}}$$

Mathematica [A] time = 0.240321, size = 83, normalized size = 1.

$$\frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{f(c+d)\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] (2*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x)
/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/((c + d)*f*Sqrt[a + b*Sec[e + f*x]
])
```

Maple [C] time = 0.352, size = 236, normalized size = 2.8

$$\frac{-2i\left(\cos\left(fx+e\right)\right)^2}{f\left(c+d\right)\left(c-d\right)\left(a\cos\left(fx+e\right)+b\right)}\left(c\text{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\sqrt{-\frac{a-b}{a+b}}\right)+d\text{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2*I/f/(c+d)/(c-d)*(c*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))
^(1/2))+d*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-2*c*
EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-(c-d)/(c+d),I*((a-b)/(a+b))^(1/2))
)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(g/cos(f*x+e))^(3/2)*cos(
f*x+e)^2*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)/(a*cos(f*x+e)+b)/(1/(1+cos(f
*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
+ c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
+ c)), x)
```

3.284 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=166

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{2cg^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df(c+d)\sqrt{a+b\sec(e+fx)}}$$

```
[Out] (2*g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/
(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*f*Sqrt[a + b*Sec[e + f*x]]) - (2*c*g^2*Sq
rt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x)/2, (2*
a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]])
```

Rubi [A] time = 0.855632, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3979, 3859, 2807, 2805, 3975}

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{2cg^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df(c+d)\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] (2*g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/
(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*f*Sqrt[a + b*Sec[e + f*x]]) - (2*c*g^2*Sq
rt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x)/2, (2*
a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]])
```

Rule 3979

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(5/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(c*g)/d,
 Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3975

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(g*Sq
rt[g*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx = \frac{g\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}dx}{d} - \frac{(cg)\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}dx}{d}$$

$$= \frac{\left(g^2\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{b+a\cos(e+fx)}}dx}{d\sqrt{a+b\sec(e+fx)}} - \frac{\left(cg^2\sqrt{b+a \cdots\right.}{\cdots}$$

$$= \frac{\left(g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}dx}{d\sqrt{a+b\sec(e+fx)}} - \frac{\left(cg^2\sqrt{\frac{b+a\cos(e\cdots}{a+b}}\right.}{\cdots}$$

$$= \frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{df\sqrt{a+b\sec(e+fx)}} - \frac{2cg^2\sqrt{\frac{b+a\cos(e\cdots}{a+b}}}{\cdots}$$

Mathematica [C] time = 4.04738, size = 246, normalized size = 1.48

$$-\frac{2ig\cot(e+fx)(g\sec(e+fx))^{3/2}\sqrt{-\frac{a(\cos(e+fx)-1)}{a+b}}\sqrt{\frac{a(\cos(e+fx)+1)}{a-b}}\sqrt{a\cos(e+fx)+b}\left((ad-bc)\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-\cdots}}\right.\right.\right.}{bdf\sqrt{\frac{1}{a-b}}(ad-bc)\sqrt{a+b\sec(e+f\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] ((-2*I)*g*Sqrt[-((a*(-1 + Cos[e + f*x]))/(a + b))]*Sqrt[(a*(1 + Cos[e + f*x
]))/(a - b)]*Sqrt[b + a*Cos[e + f*x]]*Cot[e + f*x]*((-(b*c) + a*d)*Elliptic
Pi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[e + f*x]]], (-a + b
)/(a + b)] + b*c*EllipticPi[((a - b)*c)/(-(b*c) + a*d), I*ArcSinh[Sqrt[(a -
 b)^(-1)]*Sqrt[b + a*Cos[e + f*x]]], (-a + b)/(a + b)])*(g*Sec[e + f*x])^(3
/2))/(Sqrt[(a - b)^(-1)]*b*d*(-(b*c) + a*d)*f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [C] time = 0.354, size = 344, normalized size = 2.1

$$\frac{-2i\left(-1+\cos(fx+e)\right)\left(\cos(fx+e)\right)^3}{fd(c+d)(c-d)\left(a\cos(fx+e)+b\right)\left(\sin(fx+e)\right)^2}\left(\mathrm{EllipticF}\left(\frac{i\left(-1+\cos(fx+e)\right)}{\sin(fx+e)},\sqrt{-\frac{a-b}{a+b}}\right)dc+\mathrm{EllipticF}\left(\frac{i\cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2*I/f/d/(c+d)/(c-d)*(EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))
^(1/2))*d*c+EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))*d^
2+2*c^2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((a-b)/(a+b))^(1/2))-2
*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((a-b)/(a+b))^(1/2))*d^2-2*c^
2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-(c-d)/(c+d),I*((a-b)/(a+b))^(1/2
)))*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1/cos(f*x+e)*(a*cos(f*
x+e)+b))^(1/2)*(-1+cos(f*x+e))*(g/cos(f*x+e))^(5/2)*cos(f*x+e)^3/(a*cos(f*x
+e)+b)/(1/(1+cos(f*x+e)))^(3/2)/sin(f*x+e)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g \sec\left(fx+e\right)\right)^{\frac{5}{2}}}{\sqrt{b \sec\left(fx+e\right)+a}\left(d \sec\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
+ c)), x)
```

3.285 $\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx$

Optimal. Leaf size=67

$$\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{36c^7f}-\frac{\cot^7\left(\frac{1}{2}(e+fx)\right)}{14c^7f}+\frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{20c^7f}$$

```
[Out] Cot[(e + f*x)/2]^5/(20*c^7*f) - Cot[(e + f*x)/2]^7/(14*c^7*f) + Cot[(e + f*
x)/2]^9/(36*c^7*f)
```

Rubi [A] time = 0.298053, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {12, 270}

$$\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{36c^7f}-\frac{\cot^7\left(\frac{1}{2}(e+fx)\right)}{14c^7f}+\frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{20c^7f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^7,x]
```


```
[Out] Cot[(e + f*x)/2]^5/(20*c^7*f) - Cot[(e + f*x)/2]^7/(14*c^7*f) + Cot[(e + f*
x)/2]^9/(36*c^7*f)
```


Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx &= \frac{2\,\mathrm{Subst}\left(\int -\frac{\left(1-x^2\right)^2}{8c^7x^{10}}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}\\
&= -\frac{\mathrm{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^{10}}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{4c^7f}\\
&= -\frac{\mathrm{Subst}\left(\int \left(\frac{1}{x^{10}}-\frac{2}{x^8}+\frac{1}{x^6}\right)dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{4c^7f}\\
&= \frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{20c^7f}-\frac{\cot^7\left(\frac{1}{2}(e+fx)\right)}{14c^7f}+\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{36c^7f}
\end{aligned}$$

Mathematica [B] time = 0.914968, size = 151, normalized size = 2.25

$$\csc\left(\frac{e}{2}\right)\left(-718830\sin\left(e+\frac{fx}{2}\right)+467208\sin\left(e+\frac{3fx}{2}\right)+659400\sin\left(2e+\frac{3fx}{2}\right)-303192\sin\left(2e+\frac{5fx}{2}\right)-179640\sin\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^7,x]
```

```
[Out] (Csc[e/2]*Csc[(e + f*x)/2]^9*(-971082*Sin[(f*x)/2] - 718830*Sin[e + (f*x)/2
] + 467208*Sin[e + (3*f*x)/2] + 659400*Sin[2*e + (3*f*x)/2] - 303192*Sin[2*
e + (5*f*x)/2] - 179640*Sin[3*e + (5*f*x)/2] + 30753*Sin[3*e + (7*f*x)/2] +
 89955*Sin[4*e + (7*f*x)/2] - 13427*Sin[4*e + (9*f*x)/2] + 15*Sin[5*e + (9*
f*x)/2]))/(23063040*c^7*f)
```

Maple [A] time = 0.12, size = 49, normalized size = 0.7

$$\frac{1}{4fc^7}\left(\frac{1}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}-\frac{2}{7}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}+\frac{1}{9}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x)
```

```
[Out] 1/4/f/c^7*(1/5/tan(1/2*f*x+1/2*e)^5-2/7/tan(1/2*f*x+1/2*e)^7+1/9/tan(1/2*f*
x+1/2*e)^9)
```

Maxima [A] time = 1.02752, size = 92, normalized size = 1.37

$$-\frac{\left(\frac{90\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{63\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-35\right)\left(\cos\left(fx+e\right)+1\right)^9}{1260c^7f\sin\left(fx+e\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x, algorithm="maxima")
```

```
[Out] -1/1260*(90*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 63*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 - 35)*(cos(f*x + e) + 1)^9/(c^7*f*sin(f*x + e)^9)
```

Fricas [B] time = 0.664516, size = 309, normalized size = 4.61

$$\frac{47\cos\left(fx+e\right)^5+127\cos\left(fx+e\right)^4+101\cos\left(fx+e\right)^3+11\cos\left(fx+e\right)^2-8\cos\left(fx+e\right)+2}{315\left(c^7f\cos\left(fx+e\right)^4-4c^7f\cos\left(fx+e\right)^3+6c^7f\cos\left(fx+e\right)^2-4c^7f\cos\left(fx+e\right)+c^7f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x, algorithm="fricas")
```

```
[Out] 1/315*(47*cos(f*x + e)^5 + 127*cos(f*x + e)^4 + 101*cos(f*x + e)^3 + 11*cos
(f*x + e)^2 - 8*cos(f*x + e) + 2)/((c^7*f*cos(f*x + e)^4 - 4*c^7*f*cos(f*x
+ e)^3 + 6*c^7*f*cos(f*x + e)^2 - 4*c^7*f*cos(f*x + e) + c^7*f)*sin(f*x + e
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\tan^4(e+fx)\sec(e+fx)}{\sec^7(e+fx)-7\sec^6(e+fx)+21\sec^5(e+fx)-35\sec^4(e+fx)+35\sec^3(e+fx)-21\sec^2(e+fx)+7\sec(e+fx)-1}\,dx}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)**4/(c-c*sec(f*x+e))**7,x)
```

```
[Out] -Integral(tan(e + f*x)**4*sec(e + f*x)/(sec(e + f*x)**7 - 7*sec(e + f*x)**6
 + 21*sec(e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e
+ f*x)**2 + 7*sec(e + f*x) - 1), x)/c**7
```

Giac [A] time = 2.18048, size = 68, normalized size = 1.01

$$\frac{63\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-90\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+35}{1260\,c^7 f\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x, algorithm="giac")
```

```
[Out] 1/1260*(63*tan(1/2*f*x + 1/2*e)^4 - 90*tan(1/2*f*x + 1/2*e)^2 + 35)/(c^7*f*
tan(1/2*f*x + 1/2*e)^9)
```

3.286 $\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx$

Optimal. Leaf size=89

$$-\frac{\cot^{11}\left(\frac{1}{2}(e+fx)\right)}{88c^8f}+\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{24c^8f}-\frac{3\cot^7\left(\frac{1}{2}(e+fx)\right)}{56c^8f}+\frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{40c^8f}$$

```
[Out] Cot[(e + f*x)/2]^5/(40*c^8*f) - (3*Cot[(e + f*x)/2]^7)/(56*c^8*f) + Cot[(e
+ f*x)/2]^9/(24*c^8*f) - Cot[(e + f*x)/2]^11/(88*c^8*f)
```

Rubi [A] time = 0.335691, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {12, 270}

$$-\frac{\cot^{11}\left(\frac{1}{2}(e+fx)\right)}{88c^8f}+\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{24c^8f}-\frac{3\cot^7\left(\frac{1}{2}(e+fx)\right)}{56c^8f}+\frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{40c^8f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^8,x]
```

```
[Out] Cot[(e + f*x)/2]^5/(40*c^8*f) - (3*Cot[(e + f*x)/2]^7)/(56*c^8*f) + Cot[(e
+ f*x)/2]^9/(24*c^8*f) - Cot[(e + f*x)/2]^11/(88*c^8*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{16c^8x^{12}}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^{12}}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{8c^8f}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x^{12}}-\frac{3}{x^{10}}+\frac{3}{x^8}-\frac{1}{x^6}\right)dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{8c^8f}\\
&= \frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{40c^8f}-\frac{3\cot^7\left(\frac{1}{2}(e+fx)\right)}{56c^8f}+\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{24c^8f}-\frac{\cot^{11}\left(\frac{1}{2}(e+fx)\right)}{88c^8f}
\end{aligned}$$

Mathematica [A] time = 1.07155, size = 175, normalized size = 1.97

$$-\frac{\csc\left(\frac{e}{2}\right)\left(486024\sin\left(e+\frac{fx}{2}\right)-351450\sin\left(e+\frac{3fx}{2}\right)-299970\sin\left(2e+\frac{3fx}{2}\right)+145695\sin\left(2e+\frac{5fx}{2}\right)+180015\sin\left(3e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^8,x]
```

```
[Out] -(Csc[e/2]*Csc[(e + f*x)/2]^11*(425964*Sin[(f*x)/2] + 486024*Sin[e + (f*x)/
2] - 351450*Sin[e + (3*f*x)/2] - 299970*Sin[2*e + (3*f*x)/2] + 145695*Sin[2
*e + (5*f*x)/2] + 180015*Sin[3*e + (5*f*x)/2] - 63580*Sin[3*e + (7*f*x)/2]
- 44990*Sin[4*e + (7*f*x)/2] + 6710*Sin[4*e + (9*f*x)/2] + 15004*Sin[5*e +
(9*f*x)/2] - 1975*Sin[5*e + (11*f*x)/2] + Sin[6*e + (11*f*x)/2]))/(15375360
*c^8*f)
```

Maple [A] time = 0.135, size = 62, normalized size = 0.7

$$\frac{1}{8fc^8}\left(-\frac{1}{11}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-11}+\frac{1}{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-5}-\frac{3}{7}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-7}+\frac{1}{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^{-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x)
```

```
[Out] 1/8/f/c^8*(-1/11/tan(1/2*f*x+1/2*e)^11+1/5/tan(1/2*f*x+1/2*e)^5-3/7/tan(1/2
*f*x+1/2*e)^7+1/3/tan(1/2*f*x+1/2*e)^9)
```

Maxima [A] time = 0.985534, size = 119, normalized size = 1.34

$$\frac{\left(\frac{385\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{495\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{231\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}-105\right)\left(\cos\left(fx+e\right)+1\right)^{11}}{9240\,c^8f\sin\left(fx+e\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x, algorithm="maxima")
```

```
[Out] 1/9240*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 495*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 231*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 105)*(cos(f*x +
e) + 1)^11/(c^8*f*sin(f*x + e)^11)
```

Fricas [A] time = 0.523058, size = 377, normalized size = 4.24

$$\frac{152\cos\left(fx+e\right)^6+395\cos\left(fx+e\right)^5+289\cos\left(fx+e\right)^4+15\cos\left(fx+e\right)^3-19\cos\left(fx+e\right)^2+10\cos\left(fx+e\right.}{1155\left(c^8f\cos\left(fx+e\right)^5-5c^8f\cos\left(fx+e\right)^4+10c^8f\cos\left(fx+e\right)^3-10c^8f\cos\left(fx+e\right)^2+5c^8f\cos\left(fx+e\right)-c^8f\right)\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x, algorithm="fricas")
```

```
[Out] 1/1155*(152*cos(f*x + e)^6 + 395*cos(f*x + e)^5 + 289*cos(f*x + e)^4 + 15*c
os(f*x + e)^3 - 19*cos(f*x + e)^2 + 10*cos(f*x + e) - 2)/((c^8*f*cos(f*x +
e)^5 - 5*c^8*f*cos(f*x + e)^4 + 10*c^8*f*cos(f*x + e)^3 - 10*c^8*f*cos(f*x
+ e)^2 + 5*c^8*f*cos(f*x + e) - c^8*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^4(e+fx)\sec(e+fx)}{\sec^8(e+fx)-8\sec^7(e+fx)+28\sec^6(e+fx)-56\sec^5(e+fx)+70\sec^4(e+fx)-56\sec^3(e+fx)+28\sec^2(e+fx)-8\sec(e+fx)+1}\,dx}{c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)**4/(c-c*sec(f*x+e))**8,x)
```

```
[Out] Integral(tan(e + f*x)**4*sec(e + f*x)/(sec(e + f*x)**8 - 8*sec(e + f*x)**7
+ 28*sec(e + f*x)**6 - 56*sec(e + f*x)**5 + 70*sec(e + f*x)**4 - 56*sec(e +
 f*x)**3 + 28*sec(e + f*x)**2 - 8*sec(e + f*x) + 1), x)/c**8
```

Giac [A] time = 2.26592, size = 86, normalized size = 0.97

$$\frac{231\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-495\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+385\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-105}{9240\,c^{8}f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x, algorithm="giac")
```

```
[Out] 1/9240*(231*tan(1/2*f*x + 1/2*e)^6 - 495*tan(1/2*f*x + 1/2*e)^4 + 385*tan(1
/2*f*x + 1/2*e)^2 - 105)/(c^8*f*tan(1/2*f*x + 1/2*e)^11)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```